POWER SYSTEM TRACKING STATE ESTIMATION

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by

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To Luzia and Clarice

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ABSTRACT

This thesis considers the problem of on-line updating of a power system control centre data base using telemetered information. The approach adopted to solve the problem is State Estimation, in which statistical techniques are used to reduce the level of uncertainty existent in the original data.

The thesis contains two contributions to the subject: the first one is related to the development of an improved version of a static state estimator which uses as criterion the sum of the moduli of the estimate residuals; the second is the development of a class of tracking state estimators based on the combination of non-quadratic static algorithms, including the one referred to above, with a procedure for the prediction of the system state based on time-series techniques. The main objective of both developments was to obtain estimators with an acceptable performance in the presence of gross measurement and topological errors.

The results of a comprehensive comparative study of the performance of the developed algorithms against previously available methods is also presented.

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ABOUT THE AUTHOR

The author of this thesis was born in Aquidauana, Mato Grosso, Brazil, on December 13, 1947. He received his B.Sc. in Electrical Engineering from the Federal University of Paraná, Brazil, in 1971 and the M.Sc. in Electrical Engineering from the Federal University of Rio de Janeiro, Brazil, in 1973. From January 1974 to August 1977 he has been working as a lecturer at the Federal University of Rio de Janeiro. In this post he was involved in lecturing undergraduate and postgraduate courses and research in power system engineering. From October 1977 to the present time he was granted a leave of absence from his job and a scholarship from the National Council for Scientific and Technological Development - CNPq, Brazil, to attend a Ph.D. course at UMIST. As a result of the research work involved in this course, three papers were published which are listed below. After the completion of the course, he will return to his lecturing post in Brazil.

Published papers

1. "Power System Tracking State Estimation and Bad Data Processing", presented at the 12th IEEE-PICA Conference, Philadelphia, USA, May 1981. Co-authors: P.A. Cooke and A. Brameller.
2. "A Fast Decoupled State Estimator Using Linear Programming", presented at the 2nd International Symposium SPSO-81, Wroclaw, Poland, June 1981. Co-author: A. Brameller.
3. "Nonquadratic State Estimation: A Comparison of Methods", presented at the 7th PSCC Conference, Lausanne, Switzerland, July 1981. Co-authors: S.H. Karaki and A. Brameller.

DECLARATION

The work referred to in this thesis has not been submitted in support of an application for another degree of this or any other university or other institution of learning.

D.M. FALCÃO

LIST OF PRINCIPAL SYMBOLS* AND ABBREVIATIONS

WLC	weighted least squares
BDS	bad data suppression
QSR	quadratic-square root
PLC	piece-wise linear criterion
LP	linear programming
FDSE	fast decoupled state estimator (WLS)
b.f.s.	basic feasible solution
r.h.s.	right hand side
$\underline{x}$	state vector
$\underline{Z}$	measurement vector
$\underline{v}$	error vector
$h(\cdot)$	nonlinear vector valued function relating state and measured variables
$\underline{H}$	Jacobian matrix
$\underline{R}$	matrix of weights
$\underline{A}$	gain matrix (WLS estimator)
$\underline{\Lambda}$	gain matrix (BDS estimator)
$\underline{V}$, $\underline{\theta}$	voltage magnitude, phase angle vectors
P, Q	active reactive injections or line flows
r,x,y	network element resistance, reactance and shunt susceptance
σ	measurement error standard deviation
E { }	expected value
N (μ,σ)	normally distributed random variable with mean μ and standard deviation σ

(*) underlined symbols represent vectors or matrices

M	number of measurements
N	number of states

sub/superscripts

k	time interval
i	iteration counter
p, q	active, reactive variables
m	measurement number

CHAPTER I

INTRODUCTION

Modern society depends largely on electricity for industrial production, transport, home comfort, leisure, medical care etc. Interruptions in the power supply, even of a small duration, can cause serious economic losses, discomfort and even put life at risk. Also the quality of the electrical energy delivered, measured by deviations of voltage and frequency from nominal values, should attain certain standards in order to make possible the safe use of sophisticated electrical and electronic devices available nowadays in industry, business in general and at home. These requirements have to be met in the most economic way and with a minimum of environmental deterioration.

The achievement of a reliable supply of electricity is obtained by measures taken both at the planning and operating stages. In the system planning, some spare capacity should be allocated in order to allow the system to cope with future eventualities. This reserve margin represents a large investment in extra equipment and has to be limited. Therefore, regardless of the strength planned into a power system, part of the task of maintaining a high level of reliability is performed in its day-to-day operation.

A basic function for ensuring a secure operation of the power system is the close monitoring of the current state of operation. As a result of the system's growth in size and complexity and interconnection with neighbouring systems, a large amount of telemetered data should be gathered in order to assess the system operation condition at some instant of time. Process control digital computers have been used in the last two decades to help the operators in handling these data. This function requires some sort of data processing algorithm which can also be used to "clean up" the inherent error introduced by the telemetering system, before it can be displayed to the operator and used by other application programs. One possible approach in the development of such algorithms is the use of the technique called State Estimation.

The first works in power system state estimation were published in 1970 and contained the results of three independent groups: The MIT group[79,80,81], the BPA group[22,55,56] and the AEP group[27]. From then on the subject has called the attention of many researchers from universities, research centres and industry. Reference (78), published in 1974 contains a review of most of the work produced in the early years of these studies. In Britain, CERL has performed some investigation on the subject with views of application to the CEGB system[36,49,73]. Some of those methods have been implemented in practice with reported acceptable performance[33].

At UMIST an extensive research effort in power system state estimation has been carried out[10,11,51,58,60,68]. This work has covered the subject from an initial assessment of its applicability to the development and improvement of new algorithms. The work was oriented towards the development of general methods, which could be used in any power system, rather than particular versions only suitable to specific applications. Another preoccupation in these studies was the development of efficient algorithms in terms of storage and computing time requirements, which involved the use of advanced programming techniques, for instance sparsity. As major results of this work, efficient algorithms for static state estimation, including facilities for detection and identification of gross measurement errors and network topology determination, were produced[11,51].

The work reported in this thesis is a natural extension of the one referred to above. From co-operation between the research group at UMIST and industry, it was understood that to have the previously developed algorithms implemented efficiently in a process control computer for real-time operation, it would be necessary to make them operate in a dynamic mode, i. e. to make the algorithms able to continuously update a data base with information obtained from consecutive scans, using as much information from the past as possible, in order to reduce time requirement and improve the redundancy of the available information.

In the first step of the project, a comparative study between two available approaches to the problem, namely dynamic[22] and tracking[62] estimation, was carried out in order to assess which one would be the most adequate. The dynamic approach, which has already been analysed in reference (58), was found to have inadequate theoretical background and serious computational difficulties. The second one, which extends the static state estimation technique to the time varying case, has the advantage of simplicity and previously accumulated experience and, therefore, was the one chosen. A fast decoupled algorithm reported in references (12) and (51) and a co-ordination technique suggested in (51) together with a static incremental model developed using time-series techniques, were used to develop a first tracking state estimator.

In parallel with this work, an investigation on a newly proposed estimation method using linear programming[48] was carried out. The method presented potential advantages in detection and identification of gross measurement and topological errors in the tracking mode of operation, but large storage and computing time requirements. As a result of this work a new algorithm was developed which has a much better computational performance. This algorithm was later extended to operate in the tracking mode.

The presentation of the material studied in the research project is organised as follows: Chapter II analyses the benefits of using state estimation in a modern energy control centre; Chapter III defines the power system problem in the framework of a general estimation theory and compares the different possible approaches to the problem; Chapter IV presents a review of the existent static estimation algorithms which are used as basis for the algorithms developed in this work; in Chapter V the improved version of the estimator using linear programming is presented; Chapter VI describes the two developed tracking state estimators and their interface with related programs; Chapter VII presents the result of simulation studies performed to compare the developed algorithms with the ones previously available; in Chapter VIII a summary of the main results obtained in the research project is presented together with some suggestions for further research.

CHAPTER II

THE ROLE OF STATE ESTIMATION IN POWER SYSTEM COMPUTER CONTROL

In this chapter the advantages in using state estimation as a data processing scheme in a modern power system control centre are discussed. The discussion is mainly based on the effectiveness of state estimation in helping security monitoring and enhancement. A general description of the basic functions performed by the state estimator as well as its interface with other components of the on-line software are also presented.

2.1 INTRODUCTION

In the daily operation of a power system a series of control actions are required to achieve prespecified standards of quality and continuity of supply and economy. These actions may be taken at local level, i.e. power plants, substations, etc., or at a central site. Local control usually only needs information obtained locally and has a specific task, such as regulation, switching, protective relaying, etc. Centralised control is usually associated with functions in which an overall view of the system is required. Two separated centralised control systems have been in use for a relatively long time: supervisory control and generation control. The first one is primarily concerned with simple alarm functions (overloads, overvoltages, etc.), equipment status and facilities to remotely actuate station equipment.

Generation control is used for the automatic control of the generator's output in order to meet the continuous changes in demand.

In the small and isolated power systems of the past, supervisory and generation control were performed by the operators, both at the control centre and remote stations, communicating among themselves by telephone. As a result of the power systems' growth in size and complexity and interconnection with other systems, the amount of information needed at the dispatch centre increased considerably. Speed, accuracy and reliability in transmitting the information also became major requirements. This new context forced the introduction of remote telemetering and hardware in the control centres to process, store and display information and send back control actions automatically or with help from the operators.

Traditionally, the supervisory and generation control functions each had its own central and remote hardware. Generation control was initially performed using an analogue system which eventually has evolved to a full digital system. Somewhere in this evolution, an optimising control level (economic dispatch) was introduced in the initially regulatory type only generation control. Similarly supervisory control evolved from a hardwired and mimic board type of equipment to digital computer driven CRT's.

The type of equipment and philosophy pictured above were the ones used in most electricity companies around the world by the mid-sixties. By that time, motivated by serious operational problems, which in some cases have produced blackouts lasting for several hours, power system engineers in the United States started the study of a more comprehensive and effective type of centralised control based on system engineering techniques. The key feature of this new approach is the introduction of the concept of system security which lead to the so-called security control[33].

The implementation of security control requires an integration of the conventional supervisory and generation control, a large and more sophisticated data acquisition and processing system and the inclusion of new facilities directly related to the assessment and enhancement of system security. A more powerful and complex computer system both in hardware and software terms will also be required. The investment needed to implement this type of control may be justified, apart from its ability in reducing the probability of occurrence of blackouts, by savings in operation costs and in delaying, or even eliminating, the addition of new equipment, as it allows the power system to be operated more closely to its full capacity[63,77]. This new philosophy in power system control spread rapidly around the world and today a large number of companies have, or are going to have in the near future, control centres designed according to these new concepts[33,34].

2.2 BASIC CONCEPTS ON SECURITY CONTROL[26,32,33,34,39]

The power system operation may be characterised by three sets of constraints: load, operating and security constraints. The load constraints are a set of equations describing the behaviour of system components. The operating constraints are a set of inequalities representing operating limits on system variables. The security constraints are associated with minimum levels of reserve that the system should maintain in order to cope with eventualities. They reflect all load and operating constraints associated with emergencies.

Depending on the fact that the above constraints are satisfied or not, the power system operation may be categorised in four operating states: normal, alert, emergency and restorative. A system is in the normal state if all the constraints are satisfied. If any of the security constraints are violated, the system goes into the alert state in which a disturbance may cause the violation of an operating constraint, i.e. an emergency condition. The restorative state is associated with the period in which actions are taken to bring the system from the emergency to the normal state.

System security is the ability of the power system to undergo a disturbance without getting into an emergency condition. The objective of the security control is to keep the system in the secure region, i.e. in the normal state, by shifting generation and other actions whenever security constraints are violated.

2.3 BASIC CONFIGURATION OF A CONTROL CENTRE SOFTWARE[4,26,32]

In Figure 2.1, a basic software configuration for an energy control centre designed using the concepts of system security is presented. The main components of this configuration are:

i. Data Acquisition System Its main function is to interface the communication channels with the control computer. It handles logical information about switch status and digitalised values of analogue variables (voltages, power flows, etc.). It also performs some limit and error checking, conversion to engineering units, etc.

ii. Raw Data Processing This is the function responsible for the production of an updated model of the power system and the present operating state. The first task is performed by the network configurator[51,76,91], which processes the status information and the second one by some processing scheme (e.g. state estimation) for the analogue measurements.

iii. Security Monitoring and Analysis The security monitoring function checks whether the present operating point satisfies load and operating constraints. If so, the ability of the system to undergo a specified set of disturbances is tested by the contingency evaluation function. If at least one contingency may bring the system to the emergency state, the security constrained optimisation is activated to find a secure operating point.

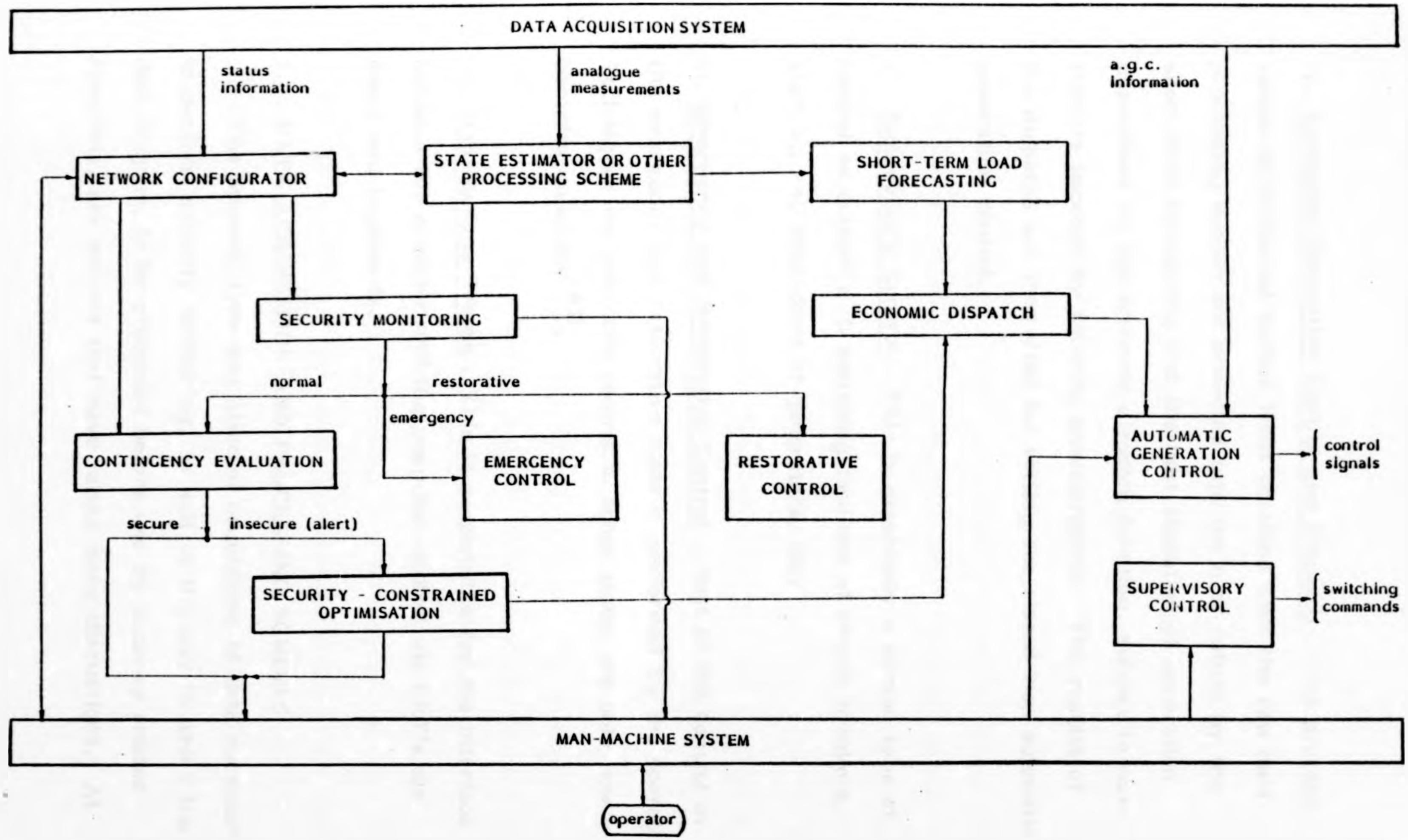


Figure 2.1 Basic configuration of a security-control oriented software

iv. Automatic Generation Control and Dispatch The present values of individual busbar loads obtained from the raw data processing module are projected into the near future by the short-term forecasting and the best allocation of generation determined by the economic dispatch function, subject to constraints imposed by security considerations. The results of the dispatch are then used for setting the closed loop automatic generation control.

v. Supervisory Control This is essentially a manual type of control associated with switching facilities of circuit breakers, start-up and shut-down of generators, etc.

vi. Emergency and Restorative Control Most of the control in the emergency and restorative state is performed by the operator. Techniques for automatic control in those states are only now becoming available[83].

vii. Man-Machine System This is responsible for the interface between the operator and the computer control via CRT's displays and keyboards.

2.4 DATA ACQUISITION AND PROCESSING SCHEMES

The amount, type and place of acquisition of data necessary to perform security monitoring, as well as the way in which the data is going to be processed before use by security related functions, are matters that have raised many discussions. At least three basic schemes have been put forward by power system engineers[26]:

i. Measurement of selected quantities which allow the direct monitoring of some key facilities.

ii. Measurement of a minimum set of quantities enough to perform an on-line load flow which would allow the monitoring of all system components.

iii. Measurement of a larger number of quantities (redundant set) followed by a systematic data processing method in which uncertainties inherent to the measurement system are taken into account.

A disadvantage of the first scheme is that the monitoring is incomplete. A facility which is not considered important under one operating condition may become important in another. It also does not produce a complete and consistent set of initial conditions for functions like security analysis, for instance.

The second scheme corrects to some extent the shortcomings of the first one. However, it also has some inconveniences. First of all, it restricts the choice of measurement type and location to the set required by the load flow. Secondly, it cannot be performed if some data is missing. A third and major problem is that if one data is incorrect, the result of the calculation will certainly be seriously affected and probably will be useless.

The third scheme overcomes all the above problems associated with the first two by the use of redundant information and statistical techniques to compensate for the noisy, erroneous and missing data. It also has the advantage of allowing a cross-checking of the network configurator results. The processing module of this scheme is called the State Estimator.

Although apparently requiring a more expensive data acquisition system (due to the need of redundancy), state estimation, in fact, may become more economical than the two other methods for the same level of reliability on the results. This is so because the other two schemes not making use of the correlated information existent among measurements, have to rely on multiplicity and high accuracy in each individual data point.

2.5 THE STATE ESTIMATION APPROACH

The state estimator is a computer program designed with the objective of systematically processing raw telemetered data received through the data acquisition system in order to produce a consistent and reliable estimate of the current power system operating point. It works in close association with the network configurator from which it receives the model of the network and measurement system and returns information regarding inconsistency in that model for further analysis. The final results of the state estimator and network configurator are then used by the other functions implemented in the control centre (see Figure 2.1).

The information received by the state estimator contains a certain amount of error. Such error may be classified as:

i. Measurement noise: the 'normal' error associated with the data acquisition process. It results from the limited accuracy of transducers, TP's and TC's, conversion and communication equipment.

ii. Gross measurement error: caused by a partial or total meter-communication failure or by observations taken during transients.

iii. Topological errors: the result of an unreported change of circuit breaker or switch status or by a failure in the network configurator which produce wrong information about the connectivity of the network.

iv. Parameter errors: represents the uncertainty in the values of the system parameters.

The measurement noise usually has a magnitude comparable to the uncertainties in the operating constraints against which the results of the estimation will be checked[49]. Therefore, its influence on the accuracy of the estimation is not so important. However, it introduces a certain degree of inconsistency in the data which should be removed before it is transferred to other application programs.

Parameter errors have a significant influence only in the early stages of operation of a real-time system, when data previously used for off-line studies is checked against on-line information. After a period of 'debugging' this kind of error will become very small and eventually disappear.

Gross measurement error and topological error, usually called bad data [*], are the main source of troubles in power system state estimation. Firstly, because if these errors remain undetected they will certainly affect substantially the accuracy of the estimation. Secondly, because the unpredictable nature of these errors make detection and identification a very laborious task.

The nature of the errors above suggests a multiple stage approach to the state estimation problem. Filtering techniques developed in the fields of control engineering and statistics[25,82] are well suited for the treatment of measurement noise. However, they usually fail in the presence of bad data. These types of error are better dealt with by a combination of pre-filtering logical checks on the income raw measurements and post-filtering statistical tests on the results of the filtering stage[49,50,73].

In Figure 2.2 a basic state estimation scheme, and its relationship with the network configurator, is shown. The raw measurements obtained from the data acquisition system goes into a pre-filtering stage in which simple logical checks (e.g. sum of power flows entering a node) are made.

(*) In this thesis both gross measurement and topological errors will be often referred to indiscriminately as bad data. A specific reference to one or another type of error will be made whenever a clear distinction becomes necessary.

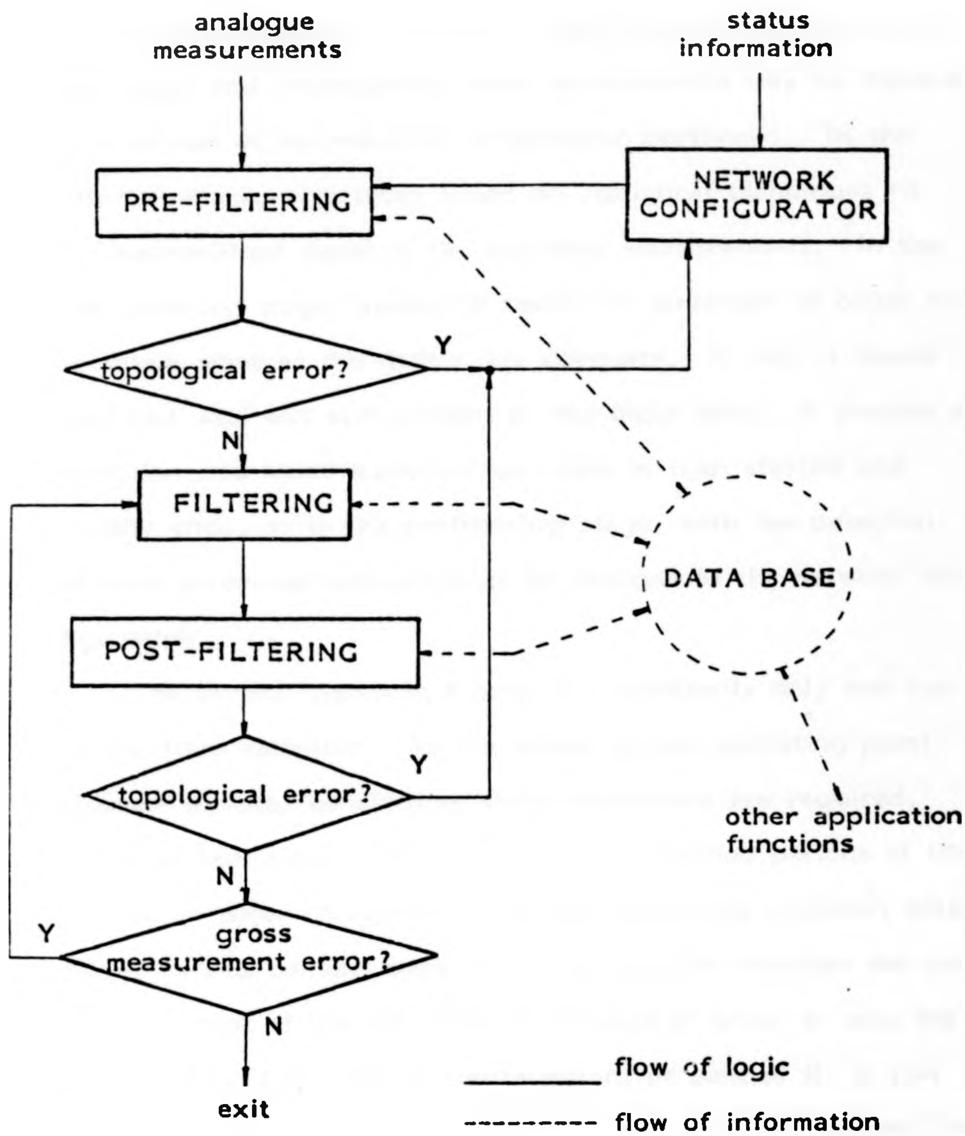


Figure 2.2: Basic state estimation scheme

If large discrepancies are found, bad data may be spotted at this stage and consequently some measurements may be discarded or a re-run of the network configurator performed. In the filtering stage, algorithms based on statistical techniques fit the mathematical model in the real-time measurements. In the post-filtering stage, statistical tests are performed in order to ascertain whether the fitting was adequate. If not, it means that bad data was still present in the input data. A process of detection and identification of bad data is then started and usually ends, as in the prefiltering stage, with the detection of some erroneous measurements or changes in the network configuration.

The scheme shown in Figure 2.2 represents only one run of the state estimator. As the power system operating point changes in time, consecutive state estimations are required. These estimations are performed at pre-specified periods of time or when a major change in the system operating condition takes place. In this context some form of interaction between the consecutive runs of the estimation is desired in order to ease the overall task. The network configurator, or part of it, is run only when a change in configuration takes place or an error in the present configuration is detected.

2.6 APPLICATIONS AND ADVANTAGES OF STATE ESTIMATION

The implementation of the security control, as described before, requires the use of advanced functions like contingency evaluation and security constrained optimisation. These functions use complex numerical techniques like fast load flow and optimisation methods to analyse and correct on real-time the power system operating point. A reliable, consistent and complete data base containing a description of the present state of the system, as well as the correct network configuration, is a fundamental requirement for the success of these functions. State estimation is the most efficient way of updating this data base in real-time. Its ability to cope with measurement noise, detection and identification of gross measurement and topological errors and the flexibility of producing good results, even if some data is missing, represents a great advantage over other approaches.

In the on-line environment the state estimator will also help the task of producing equivalents of the neighbouring system, which requires for its calculation the values of the voltages at border busbars. The economic dispatch also benefits from the results of the state estimation using the estimated values of the individual load busbars projected to the near future.

State estimation also has off-line applications. One of them, which was indicated earlier in this chapter, is related to the 'debugging' of system parameter errors, bias type errors due to meter miscalibration, etc., using a modified state estimation program and data obtained on-line[21]. Another off-line application is the determination of the optimal meter location in order to facilitate the on-line task of the state estimator[60,61]. This last application is very important as it can produce considerable savings in meter and communication hardware.

2.7 CONCLUSIONS

Security control is at present an established design philosophy for energy control centres. This type of control requires a reliable real-time data base. Among the many ways of updating this data base, state estimation has been gaining more acceptance due to its advantages in terms of reliability, flexibility and even economy. An initial reluctance to the use of the new technique has been steadily disappearing, due mainly to the research effort put into the development of state estimation algorithms with more acceptable computer requirements. The increased use of state estimation in the power industry can be observed in a recent published survey of control centres around the world[34].

CHAPTER III

STATIC, DYNAMIC AND TRACKING STATE ESTIMATION: DEFINITIONS AND COMPARISON OF FORMULATIONS

In this chapter the power system state estimation problem is formulated within the framework of the general estimation theory. The difficulties in obtaining suitable models, which would enable the use of standard techniques, is outlined and simplified models are indicated. This formulation, together with some practical operational requirements, is used to define and compare the three approaches available for the problem, namely static, dynamic and tracking state estimation.

3.1 INTRODUCTION

The first studies related to estimation problems go back to the sixteenth century. Gauss first used the least-squares method for the study of planet and comet motion using telescopic observations in 1795[84]. More recent important contributions to the subject were introduced by Fisher, Kolmogorov and Wiener. The general framework of estimation theory was established mainly in the work of Kalman in which the recently introduced state space formulation was used. The basic ideas of the so-called "Kalman filter" are contained in reference[50]. These techniques have been largely tested with success in practical applications mainly in the aerospace industry.

A very basic formulation of the estimation problem can be stated as: "infer the properties of a system from a set of observations". In a more particular and useful formulation the "properties" of interest are summarised in a set of variable components of a state vector. The observations are also arranged in a vector of information or, more adequately in the case of physical systems, measurements. The estimation problem is then restated as: "obtain the best estimation of the state vector from the available measurements". The word "best" appears in the above definition to account for the general impossibility of finding an exact solution for the problem due to the presence of errors in the measurements. The problem thus defined is called the state estimation problem.

A solution for the state estimation problem is obtained by minimisation of some function of the error in the estimates. The function mostly used since the early days of estimation is the sum of the squares of the errors. However, other functions can be used depending on the particular objective of the estimation. If statistics of the measurement errors are known, estimators can be derived by optimising corresponding statistics of the estimates. In most cases the estimators obtained by the two approaches are identical, the only difference between them being on the properties attached to the solution.

In the power system industry, state estimation theory has found several applications. The particular interest in this thesis is focused on the estimation of the so-called "static state" which defines a power system operating point in the "quasi-static" mode of operation[35]. In order to obtain efficient and reliable estimation methods, the general estimation techniques should be adapted to the power system problem due to facts like absence of adequate models, high dimensionality, eventual large errors in the measurements, etc.

3.2 STATE-TRANSITION MODELS FOR DYNAMIC SYSTEMS[25,82,92]

A basic step in the development of state estimation algorithms is the establishment of an adequate model for the process being observed and for the observations themselves. These models must contain a description of the time evolution of the state vector (if time varying) from initial conditions, including a definition of the statistics associated with eventual random disturbances in the state. The model of the observations must show a relationship between the measured variables and the state at the instant in which the measurements are taken as well as the statistics of the measurement error. State space models are nowadays almost universally used in system engineering applications and therefore they will be adopted in this thesis. As all the algorithms developed in this work are directed for use in digital computers, the discrete time formulation is the most adequate for the development of the models.

An adequate model for the purposes of this chapter, in a relatively general form, is given by

$$\underline{x}_{k+1} = \underline{\phi}(\underline{x}_k, \underline{u}_k, k) + \underline{w}_k \tag{3.1}$$

$$\underline{Z}_k = \underline{h}(\underline{x}_k, k) + \underline{v}_k \tag{3.2}$$

where

k = time interval

$\underline{x}_k$ = state vector (nx1)

$\underline{Z}_k$ = measurement vector (mx1)

$\underline{w}_k$, $\underline{v}_k$ = random vectors representing uncertain processes for which statistics are usually known

$\underline{u}_k$ = known (deterministic) input

$\underline{\phi}$, $\underline{h}$ = non-linear vector valued functions

$\underline{x}_o$ = uncertain initial state vector

The model given by (3.1) and (3.2), although corresponding to many important practical situations, introduces certain problems in the derivation of the estimation algorithms. The usual approach is to initially use a simpler model and, after the derivation of the estimator equations, to extend them to more general cases if necessary.

A very commonly used model for estimation purposes is obtained from the previous one by the introduction of two simplifications:

a. linearisation of $\underline{\phi}$ and $\underline{h}$
b. assumption that $\underline{w}_k$ and $\underline{v}_k$ are "white processes"[82], i.e. processes in which no correlation in time exists.

The new model, in which the explicit dependence on the input $\underline{u}_k$ is dropped for simplicity, is then given by:

$$\underline{x}_{k+1} = \underline{\Phi}_k \, \underline{x}_k + \underline{w}_k \tag{3.3}$$

$$\underline{Z}_k = \underline{H}_k \, \underline{x}_k + \underline{v}_k \tag{3.4}$$

where $\underline{\Phi}_k$ and $\underline{H}_k$ are (nxn) and (mxn) matrices respectively.

The first and second statistics of $\underline{w}_k$, $\underline{v}_k$ and $\underline{x}_o$ are assumed to be known and given by

$$E\{\underline{w}_k\} = 0 \qquad (3.5)$$

$$E\{\underline{w}_k \underline{w}_j^T\} = \begin{cases} \underline{Q} & \text{if } k=j \\ \underline{0} & \text{if } k \neq j \end{cases} \qquad (3.6)$$

$$E\{\underline{v}_k\} = \underline{0} \qquad (3.7)$$

$$E\{\underline{v}_k \; \underline{v}_j^T\} = \begin{cases} \underline{R} & \text{if } k=j \\ \underline{0} & \text{if } k \neq j \end{cases} \qquad (3.8)$$

$$E\{\underline{x}_o\} = \hat{\underline{x}}_o \qquad (3.9)$$

$$E\{(\hat{\underline{x}}_o - \underline{x}_o)(\hat{\underline{x}}_o - \underline{x}_o)^T\} = \underline{P}_o \qquad (3.10)$$

where x_o represent the true but unknown value of the initial state and $\underline{Q}$, $\underline{R}$ and $\underline{P}_o$ are (mxm), (nxn) and (nxn) covariance matrices, respectively.

3.3 THE ESTIMATION PROBLEM[25,82,92]

As already stated in the introduction to this chapter, the state estimation problem is concerned with obtaining the "best" estimation of the vectors $\underline{x}_k$ (k = 1,2,...) from measured values of $\underline{Z}_k$ (k = 1,2,...). According to the specific time in which the estimation is required and the amount of information available, three different problems can arise:

a. Filtering: estimate $\underline{x}_k$ from $\underline{Z}_k$, $0 \leq k \leq K$

b. Prediction: estimate $\underline{x}_{k+\Delta}$ from $\underline{Z}_k$, $0 \leq k \leq K$, $\Delta \geq 0$

c. Smoothing: estimate $\underline{x}_\Delta$, $0 \leq \Delta \leq K$, from $\underline{Z}_k$, $0 \leq k \leq K$

In this thesis, the main problem analysed is of the filtering type. However, prediction techniques will also be used as an auxiliary tool in obtaining a reliable estimation (see Chapter VI).

An extension of the state estimation problem arises when some parameters of the model have also to be estimated. These parameters are treated as extra state variables and the problem thus defined is called an identification problem. In this thesis no identification problem will be studied but some comments about a possible application of this technique will be made in section 3.4.1.

The presentation of the state estimation methods will follow a sequence in which the degree of difficulty increases: it will start with the time invariant state vector and will end up with the solution of the problem given by (3.1) and (3.2).

3.3.1 STATIC STATE ESTIMATION

In this section the state vector is assumed to be time invariant. It is also assumed that no previous estimation ($\hat{\underline{x}}_o$) is available. Therefore, a certain degree of redundancy is required in order to allow the choice of a "best" estimate. In the linear case the problem can be modelled by (3.4), (3.7) and (3.8), which will then assume the form

$$\underline{Z} = \underline{H}\,\underline{x} + \underline{v} \tag{3.11}$$

$$E\{\underline{v}\} = 0 \tag{3.12}$$

$$E\{\underline{v}\,\underline{v}^T\} = \underline{R} \tag{3.13}$$

The solution to the above problem is usually obtained by the minimisation of the Weighted Least Squares (WLS) criterion

$$J = (\underline{Z} - \underline{H}\,\underline{x})^T\,\underline{R}^{-1}\,(\underline{Z} - \underline{H}\,\underline{x}) \tag{3.14}$$

The optimality condition on the WLS criterion is given by

$$\frac{\partial J}{\partial \underline{x}} = 2\,\underline{H}^T\,\underline{R}^{-1}\,(\underline{Z} - \underline{H}\,\underline{x}) = 0 \tag{3.15}$$

the solution of which gives the estimator

$$\hat{\underline{x}} = \underline{A}^{-1}\,\underline{H}^T\,\underline{R}^{-1}\,\underline{Z} \tag{3.16}$$

$$\underline{A} = \underline{H}^T\,\underline{R}^{-1}\,\underline{H} \tag{3.17}$$

In the above derivation the statistic information about the measurement error, given by (3.12) and (3.13), is not necessarily used. Matrix $\underline{R}$ can be chosen by simple "engineering judgement" of the accuracy of metering equipment. However, if (3.12) and (3.13) are used, some important properties can be attached to the solution. For instance, it can be shown[82] that (3.16) and (3.17) yield an unbiased estimate of $\underline{x}$ and that $\underline{A}^{-1}$ is the covariance matrix of the estimation error, i.e.

$$E\{\hat{\underline{x}}\} = \underline{x} \tag{3.18}$$

$$E\{(\hat{\underline{x}}-\underline{x})(\hat{\underline{x}}-\underline{x})^T\} = \underline{A}^{-1} \tag{3.19}$$

Moreover, if $\underline{v}$ is Gaussian, i.e. it has a normal distribution of probabilities, then (3.16)-(3.17) is a maximum likelihood estimator.

The non-linear problem defined by (3.2) can be solved using the results of the linear case by linearisation of $\underline{h}(\underline{x})$ around some point $\underline{x}_o$ as follows:

$$\underline{Z} = \underline{H}(\underline{x}_o)(\underline{x}-\underline{x}_o) + \underline{v} \tag{3.20}$$

$$\underline{H}(\underline{x}_o) = \left.\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_o} \tag{3.21}$$

If a linearisation point $\underline{x}_o$ close to the solution is not available, then the solution can be obtained by an iterative process where the result of each estimation is used as the linearisation point for the next estimation.

Many iterative procedures are available in the literature[24]. The one which seems to be preferred is the Gauss-Newton method which is given by:

$$\underline{x}^{i+1} = \underline{x}^{i} + \underline{A}^{-1}(\underline{x}^{i})\ \underline{H}^{T}(\underline{x}^{i})\ \underline{R}^{i}(\underline{Z} - \underline{h}(\underline{x}^{i})) \tag{3.22}$$

$$\underline{A}(\underline{x}^{i}) = \underline{H}^{T}(\underline{x}^{i})\ \underline{R}^{-1}\ \underline{H}(\underline{x}^{i}) \tag{3.23}$$

where i is the iteration counter.

3.3.2 STATIC STATE ESTIMATION USING A PRIORI ESTIMATES

Consider the same problem proposed in the previous section but suppose that before the present set of measurements was available, a state estimation was performed (a priori estimate). Let $\underline{x}_a$ be the result of this estimation and $\underline{P}$ the associated error covariance matrix. The a priori estimate can be considered an extra set of observations and an estimator derived from (3.16) and (3.17), provided the following substitutions are made

$$[\underline{Z}] \rightarrow \left[\begin{array}{c} \underline{x}_a \\ \hline \underline{Z} \end{array}\right] ;\ [\underline{H}] \rightarrow \left[\begin{array}{c} \underline{I} \\ \hline \underline{H} \end{array}\right] ;\ [\underline{R}] \rightarrow \left[\begin{array}{c|c} \underline{P} & \\ \hline & \underline{R} \end{array}\right] \tag{3.24}$$

where $\underline{I}$ is the nxn unity matrix. The new estimator will then be given by:

$$\hat{\underline{x}} = \underline{x}_a + (\underline{P} \quad + \underline{A})^{-1}\ \underline{H}^{T}\ \underline{R}^{-1}\ (\underline{Z} - \underline{H}\ \underline{x}_a) \tag{3.25}$$

and the associated error covariance matrix is

$$E\left\{ (\hat{\underline{x}} - \underline{x})(\hat{\underline{x}} - \underline{x})^T \right\} = (\underline{P} \quad + \underline{A})^{-1} \tag{3.26}$$

If various sets of measurements Z_j and associated error covariances R_j, j = 1,2,..., and an initial estimate as defined by (3.9) and (3.10) are available, then the estimator given by (3.25) and (3.26) can be generalised in a recursive form as:

$$\underline{x}_{j+1} = \underline{x}_j + \underline{P}_j^{-1} \, \underline{H}^T \, \underline{R}^{-1} \, (\underline{Z} - \underline{H} \, \underline{x}_j) \tag{3.27}$$

$$\underline{P}_j = \underline{P}_{j-1} + \underline{H}^T \, \underline{R}_j^{-1} \, \underline{H} \tag{3.28}$$

The same linearisation technique used in the previous section can be applied to the above estimator for the non-linear model. In order to truly minimise the WLS criterion in this case it would be necessary to use an iterative scheme like (3.22) and (3.23) in each of the recursions in (3.27) and (3.28). However, it is possible, in some situations, to obtain solutions with acceptable accuracy using only the linear estimator.

3.3.3 DYNAMIC STATE ESTIMATION

In the time varying problem a series of observations $\underline{Z}_k$ are performed at certain time intervals k = 1,2,..., and estimations of the state are required after each set of observations is available (filtering). Assume first the linear case modelled by (3-3) to (3-10). The solution can be obtained in a two-step procedure:

i. a prediction of the state at the next interval is made using the transition equation (3.3)

ii. the predicted value is used as an a priori estimate and the estimation is calculated using (3.25).

If an estimate $\underline{x}_k$ is available then the best prediction of the state at the time interval k+1, before the existence of $\underline{Z}_{k+1}$ is given by:

$$\underline{\tilde{x}}_{k+1} = E\ \{\underline{\Phi}_k\ \underline{\hat{x}}_k + \underline{w}_k\} = \underline{\Phi}_k\ \underline{\hat{x}}_k \tag{3.29}$$

with an error covariance

$$\underline{\tilde{P}}_{k+1} = \underline{\Phi}_k^T\ \underline{P}_k\ \underline{\Phi}_k + \underline{Q}_k \tag{3.30}$$

These results can be used as indicated in step ii. above to derive the dynamic estimator (Kalman filter), which in a recursive form is given by:

$$\underline{\hat{x}}_{k+1} = \underline{\tilde{x}}_{k+1} + \underline{K}_{k+1}(\underline{Z}_{k+1} - \underline{H}_k\ \underline{\tilde{x}}_{k+1}) \tag{3.31}$$

$$\underline{K}_{k+1} = \underline{\tilde{P}}_{k+1} + \underline{H}_k^T\ (\underline{R}_k + \underline{H}_k\ \underline{\tilde{P}}_{k+1}\underline{H}_k^T)^{-1} \tag{3.32}$$

$$\underline{\tilde{P}}_{k+1} = \underline{\Phi}_k^T\ \underline{P}_k\ \underline{\Phi}_k + \underline{Q}_k \tag{3.33}$$

$$\underline{P}_k = \underline{\tilde{P}}_k - \underline{K}_k\ \underline{H}_k\ \underline{\tilde{P}}_k \tag{3.34}$$

An extended version of the Kalman filter is obtained for the non-linear case defined in (3.1) and (3.2) by linearising $\underline{\phi}$ and $\underline{h}$ using the best available estimate of the state at each stage of the linearisation. The equations defining this filter are exactly the same as the ones presented above with $\underline{\Phi}_k$ and $\underline{H}_k$ defined as:

$$\underline{\Phi}_k = \left.\frac{\partial \underline{\phi}}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_k} \qquad (3.35)$$

$$\underline{H}_k = \left.\frac{\partial \underline{h}}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_k} \qquad (3.36)$$

3.3.4 USE AND ADVANTAGES OF STATIC AND DYNAMIC STATE ESTIMATION

The static state estimation approach is applicable to problems in which the state vector is time invariant or when the "dynamic" associated with it is so slow, when compared with the interval between sets of observations, that the system can be considered as changing states by discrete steps. In the first case, an estimation scheme like the one given by (3.27) can be used to process successive sets of information which have an overall amount of redundancy. In the second case each set of observations is associated with one particular value of the state vector. A model similar to the one given by (3.11) can then be used to estimate discrete values of the state. As no transition model enters in the formulation, it is not possible to use past information in the present state estimation. Therefore each set of information should contain a certain amount of redundancy to allow some form of error compensation.

For the general time-varying case, the dynamic approach should be preferred as it is the only one able to estimate the transient behaviour of the system. In the slow time-varying case referred to above, dynamic state estimation also has a great advantage: there is no need for redundancy in each individual set of observations. This is so because results of past estimation are projected into the future by the transition equation and can be used as extra observations. However, dynamic state estimation algorithms are usually more complicated than static ones and also require detailed models of the system dynamic

3.4 POWER SYSTEM STATE ESTIMATION

The power system load varies continually according to a daily pattern which suffers seasonal variations during the year and on special days (weekends, holidays, etc.). Apart from a small random fluctuation, the load varies smoothly. Sudden changes seldom occur and are the result of either a predictable event (e.g. disconnection of a large industrial load) or some abnormal state of operation (e.g. outages). System variables like generations, line flows, voltages, etc. are continually adjusted directly or indirectly to follow the load variation according to some operational strategy, control and network laws. In a very precise way, the system actually never achieves a steady-state. However, the amplitude of the oscillations caused by the transients are, in general, small compared with the overall change. Therefore, for many important classes of power system studies it is convenient to assume that variations in load are met instantaneously by variations in generation and other system variables. This hypothetical mode of operation is called the quasi-static mode[35]. In order to describe an operating point in the quasi-static mode it is usual to choose as state vector one in which the components are the complex nodal voltages in all system nodes. This vector is called the static state.

The power system state estimation problem analysed in this thesis, as already formulated informally before, is concerned with the estimation of the static state. The static state is a slow time varying vector whose variation shows a pattern similar to the one described above for the load. Its estimations are obtained from sets of measurements of system variables like active and reactive node injections, line flows and nodal voltages. The number of measurements usually exceeds the number of states, i.e. a certain degree of redundancy is available. The frequency in which measurements are taken and estimations are performed depends on particular operational requisites and varies in existent installations from seconds to several minutes.

3.4.1 TIME-VARYING STATE MODEL

An accurate and simple state transition model, in the form of the ones given by (3.1) or (3.3) is not available yet. The difficulties in developing such a model are:

i. To find a simple relationship ($\underline{\phi}$ or $\underline{\Phi}$) between the state vector ($\underline{x}_k$) and the load ($\underline{u}_k$, $\underline{w}_k$). This relationship would have to take into account the behaviour of generators, voltage regulators, governors, etc., and the network equations. Even for small systems such a relationship would involve a large number of differential and algebraic equations[5].

ii. To develop a model for the load variation ($\underline{u}_k$, $\underline{w}_k$). A large number of observations of the power system behaviour followed by the application of identification techniques would be required to obtain such a model. Even so, it is doubtful whether the result of such an experiment would lead to a practical model.

Therefore it should be concluded that, for the reasons stated above, no explicit model for the time varying state model will be available in the foreseeable future.

3.4.2 MEASUREMENT MODEL

A measurement model like the ones defined by (3.2), (3.4), (3.7) and (3.8) is made up of two components; a relationship between the measurement ($\underline{Z}_k$) and the state vector ($\underline{x}_k$) and a statistical model of the errors.

The relationship between measurements and state variables is obtained from information about the network, and metering system structure (connectivity of generators, lines and loads and location of measurements) and network parameters together with the network equations.

The structure of the network and metering system, which is usually referred to simply as the network configuration, is obtained in real-time from telemetered (or telephoned) switches and circuit breakers status. This information is usually condensed in two tables: the feeder and measurement tables[51]. The former is a description of the interconnection of the network nodes and the latter specifies the location and type of the measurements (measurement pattern). As a measurement is associated with a node or line, changes in the status of switches and breakers may lead to changes in the measurement pattern. The network parameters are obtained from off-line calculations and manufacturer information.

The measurement equations are derived from the load-flow equations[86] by expressing the measured variables as functions of the state variables. Using the elementary network model pictured in table 3.1, these equations are given by

$$P_{ik} = V_i^2 g_{ii} + V_i V_k (g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}) \qquad (3.37)$$

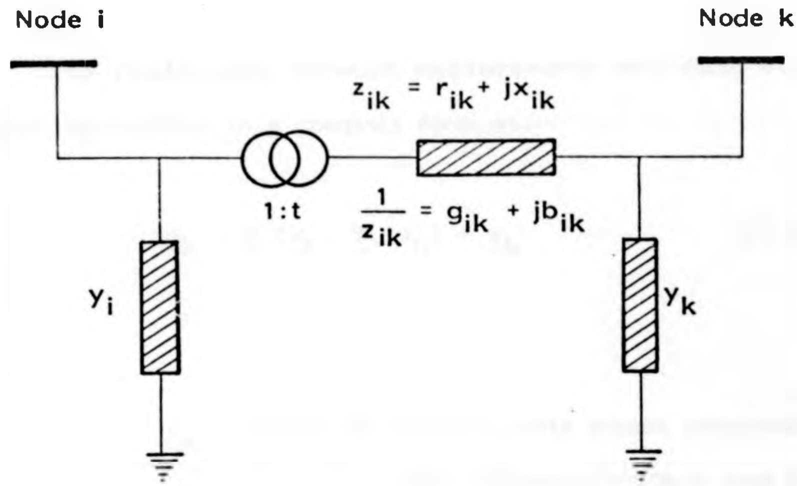
$$Q_{ik} = -V_i^2 b_{ii} + V_i V_k (g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}) + V_i^2 y_i \qquad (3.38)$$

$$P_i = \sum_{k \varepsilon \alpha_i} P_{ik} \qquad (3.39)$$

$$G_i = \sum_{k \varepsilon \alpha_i} Q_{ik} \qquad (3.40)$$

where:

- α_i : set of nodes directly connected to node i
- P_{ik}, Q_{ik} : active and reactive power flow in line i-k
- P_i, Q_i : active and reactive injection in node i
- V_i : voltage magnitude at node i
- $\theta_{ik} = \theta_i - \theta_k$: angular difference between voltages at nodes i and k



r_{ik} = transmission line or transformer resistance

x_{ik} = transmission line or transformer reactance

y_{ik} = total line susceptance

t = off-nominal tap ratio (t = 1 for lines)

Admittance	Transmission line	Transformer
g_{ik}	$\frac{r_{ik}}{\lvert z_{ik}\rvert^2}$	$\frac{r_{ik}}{\lvert z_{ik}\rvert^2}\ t$
b_{ik}	$\frac{-x_{ik}}{\lvert z_{ik}\rvert^2}$	$\frac{-x_{ik}}{\lvert z_{ik}\rvert}{}^2\ t$
y_i	$y_{ik}/2$	$\frac{(t^2 - t)}{\lvert z_{ik}\rvert}$
y_k	$y_{ik}/2$	$\frac{(1 - t)}{\lvert z_{ik}\rvert^2}$

Table 3.1 - Network branch elementary model

The relationship between measurements and state variables can then be written in a compact form as:

$$\underline{Z}_k = \underline{h}\,(\underline{x}_k,\ \underline{Y},\ s_k) + \underline{v}_k \qquad (3.41)$$

where

$\underline{Z}_k$: vector of measurements whose components are active and reactive injections and line flows and voltage magnitudes

$\underline{h}$: non-linear vector valued function whose components are the equation defined in (3.37) to (3.40) .

$\underline{Y}$: the network admittance matrix whose elements are the admittances $g_{ik} + jb_{ik}$

s_k : is a compact representation of feeder and measurement tables (network configuration)

$\underline{v}_k$: measurement error

k : time interval

The network configuration may contain errors caused by unreported switches or circuit breakers status changes or failure of the network configurator. No mathematical model is available for this type of error due to the difficulty in modelling equipment or software malfunction and human mistakes.

Errors in network parameters are usually modelled as zero mean random variables with a time structure of the bias type as they are constant in time[21,90].

The measurement error vector $\underline{v}_k$ is made up of two components as follows:

$$\underline{v}_k = \underline{v}'_k + \underline{b}_k \tag{3.42}$$

where:

$\underline{v}'_k$: measurement noise

$\underline{b}_k$: gross measurement error

The measurement noise is a sum of errors introduced by the various components of the telemetering system (transducers, TP's, TC's, etc.). Certain components of this sum are dependent on the actual value of the measurements that produce a correlation between the errors of different measurements and between measurements in different instants of time. Others are better modelled as bias type errors. Despite this complex structure, it is common practice in power system state estimation[78] to assume that $\underline{v}'_k$ is a "white noise"[82] and its components are uncorrelated. It is also usual to model the noise as a normally distributed random variable with zero mean and known standard deviation. These assumptions reduce the work of deriving estimation algorithms and do not have a large effect on the overall performance of the estimators due to the relative importance of the measurement noise in the final result, as pointed out in Chapter II.

However, it is important to keep these approximations in mind whenever statistical inferences are made from the results of estimations.

Under the above assumptions the measurement noise is modelled as follows:

$$\underline{v}'_k \text{ is } N\ (\underline{0},\ \underline{R}) \tag{3.43}$$

$$\underline{R} = \text{diagonal } (\sigma_1{}^2\ \sigma_2{}^2\ \ldots\sigma_m{}^2) \tag{3.44}$$

$$E\ \{(v'_j)^T\ \underline{v}_k\} = \underline{0} \qquad \text{for} \quad j \neq k \tag{3.45}$$

where σ_m is the standard deviation of the m-th measurement error.

Gross measurement errors are large, totally unpredictable, errors due to total or partial failure of the metering system or observation during transient swings. For the same reasons presented for configuration errors, no mathematical model can be derived for this type of error.

3.5 APPROACHES TO POWER SYSTEM STATE ESTIMATION

The power system state estimation problem modelled in the previous section presents three major problems when approached using the techniques described in sections 3.2 and 3.3:

i. absence of an adequate state transition model
ii. presence of gross measurements and topological errors
iii. high dimensionality

To overcome these difficulties a great deal of work was done and continues to be done in order to adapt and extend the existent estimation techniques to the problem at hand. The general philosophy behind this work, which is also followed in this thesis, is to decompose the problem in such a way as to make possible the application of the available estimation techniques to part of it, by the introduction of approximations, and by developing non-standard methods to deal with components of the problem not yet adequately covered by the standard estimation methods.

Three main approaches to the problem have been proposed in the literature[78] and in the next sections they are briefly described, compared and improvements in some of them, which are reported in the following chapters of this thesis, are pointed out.

3.5.1 STATIC STATE ESTIMATION

Static state estimation has been the most used approach to the power system estimation problem[1,12,27,44,49,51,78]. It avoids completely the problem of modelling the time behaviour of the state vector by performing isolated state estimations, at some specified periods of time, in which only one meter scan (snapshot) is used. The price to pay for the simplicity of this approach is a relatively high degree of redundancy required to guarantee a reliable estimation.

The model used is the one defined by (3.41) to (3.45). The decomposition technique mentioned earlier is used to deal with the different types of errors: logical checks are performed in the incoming data to detect large errors in the measurement or network configuration, and state estimation algorithm similar to the one given in (3.22) is used to filter out the measurement noise and post estimation statistical tests are performed to detect eventual large errors still present in the results (see Chapter III). A variation of this approach is the use of some algorithms which have the ability to reject bad data automatically. In Chapter IV one of these algorithms, called Bad Data Suppression, is presented and compared with the conventional WLS algorithms and in Chapter V an improved version of another of these algorithms, the one which uses as criteria the sum of the moduli of the residuals, is described.

Static state estimation is a practical approach to power system state estimation. It works well whenever a relatively large redundancy ratio is available and the interval between estimations is large enough to really make each estimation run completely uncorrelated to each other. However, if a more closely monitoring of the system is desired, i.e. if measurement scans are taken at small intervals, the use of static state estimation may not be the most efficient way of tackling the problem, as useful information obtained in previous estimations are wasted. Techniques which take into consideration the time evolution of the state vector may become faster and require a smaller redundancy ratio for the same level of reliability.

3.5.2 DYNAMIC STATE ESTIMATION

Despite the difficulties in finding a model for the static state dynamics mentioned earlier, some attempts to use the dynamic estimation technique described in section 3.3.3 can be found in the literature[16,22,23,54,69,70,85]. In these experiments the state change between two consecutive estimations is modelled simply by a random variable, i.e. the state is assumed time invariant and the only thing that changes is the uncertainty about the value of the state. This model, as given in reference (70), is given by:

$$\underline{x}_{k+1} = \underline{x}_k + \underline{w}_k \tag{3.46}$$

$$E\ \{\underline{w}_k\} = 0 \tag{3.47}$$

$$E\ \{\underline{w}_k\, \underline{w}_j\} = \delta_{kj}\ \underline{Q} \tag{3.48}$$

$$\underline{Q} = \alpha^2 (\Delta t)^2 \ \mathrm{diag}\ \{q_i^{\,2}\} \tag{3.49}$$

where

δ_{kj} is the Kroenecker delta

q_i is the maximum rate of change of the i-th state in the past estimations

Δt is the time between estimations

α is a parameter calculated off-line and used to "tune" the estimator

The application of the extended Kalman filter to the above model will produce the following estimator

$$\underline{\tilde{P}}_k = \underline{\tilde{P}}_{k-1} + \underline{Q} \tag{3.50}$$

$$\underline{G}_k = \underline{\tilde{P}}_k \, \underline{H}_k^T \, (\underline{R}_k + \underline{H}_k \, \underline{\tilde{P}}_k \, \underline{H}_k^T)^{-1} \tag{3.51}$$

$$\underline{\hat{x}}_k = \underline{\hat{x}}_{k-1} + \underline{G}_k \, (\underline{Z}_k - \underline{h} \, (\underline{\hat{x}}_{k-1})) \tag{3.52}$$

$$\underline{P}_k = [\underline{I} - \underline{G}_k \, \underline{H}_k] \, \underline{\tilde{P}}_k \tag{3.53}$$

In order to avoid the matrix inversion in (3.51) at every estimation, a sequential processing of the measurements is often used.

A technique for bad data detection and identification using the estimator described above has been recently proposed in reference (70). This method is based on the statistical analysis of the so-called "innovation process".

$$\underline{\nu}_k = \underline{Z}_k - \underline{h} \, (\underline{\hat{x}}_{k-1})$$

Their authors claim that by examining the shape of probability distribution function of $\underline{\nu}_k$, it is possible to say whether the measurement vector $\underline{Z}_k$ contains bad data or not.

Some criticism has been raised in the literature against the estimation method just described. The criticisms are based on the following points:

a. The estimator given by (3.4) to (3.5) is not truly a dynamic estimator as the simplified model used does not allow an accurate prediction of the future states of the system.

b. The tuning of matrix Q, i.e. the calculation of the q_i's and α is a laborious off-line process.

c. The sequential processing of measurements may become an unstable process requiring a time-consuming ordering of measurements to guarantee convergence.

d. The bad data detection technique, based only on the statistical analysis of the innovation process, may fail as it is based on a model that is "at best a very crude representation of the actual state behaviour"[78].

3.5.3 TRACKING STATE ESTIMATION

The practical limitations of the static state estimation approach pointed out in section 3.5.1 cannot be overcome by the use of dynamic estimation techniques as explained in the previous section. However, it is possible to develop a class of estimators, based on static models and simple assumptions about the time behaviour of the state vector, which may have a better general performance in following (or tracking) closely the time varying state vector. In these estimators the main objective is to improve computational efficiency and reliability rather than dynamic response. This class of estimators are usually referred to, in the power system industry, as tracking state estimators.

Some examples of tracking state estimators can be found in the literature[3,7,44,62]. The general form of these estimators is the same as the one given by (3.27) or (3.52) with the only difference that the gain matrix is chosen based on engineering or heuristic judgement rather than by a mathematical optimisation process. These algorithms are mainly orientated to the filtering of measurement noise. A simple procedure of checking large changes in measurements between consecutive snapshots is suggested in reference[23] for a tracking estimator which evolved from the simplified dynamic model described in the previous section.

In reference (51) a co-ordination scheme between a pre-estimation "data validation" stage, in which those changes in measurements of consecutive snapshots are taken into account, and a bad data suppression estimator is described. In Chapter VI of this thesis, this idea is developed into a full tracking state estimator by the introduction of a prediction stage, in which present estimate values of measured variables are extrapolated to the next snapshot, in order to take into account the time evolution of the system and to avoid discrepancies caused by transients. A similar method involving the estimator proposed in Chapter V of this thesis (piece-wise linear criterion) is also presented.

3.6 CONCLUSIONS

The power system state estimation problem cannot be solved straightforwardly by standard techniques. The absence of an adequate model for the time behaviour of the state vector, the presence of gross measurement and topological errors and the high dimensionality of the problem require the introduction of non-standard procedures in deriving practical power system state estimators.

Three approaches to the problem have so far been put forward in the literature: static, dynamic and tracking state estimation. Static state estimation, which has been the most widely studied of th e three, is adequate when the interval between estimations is large enough to make them really uncorrelated. If this interval is not so large, i.e. a closer monitoring of the system is required, the tracking state estimation approach may become more efficient and reliable. Dynamic estimators, developed using simplified models, have many inconveniences and do not seem to constitute a practical option.

CHAPTER IV

A CRITICAL REVIEW OF WLS-BASED STATE ESTIMATION METHODS

Most of the research work carried out so far in power system state estimation is based on the application of the WLS method to static models. Some important results achieved in this work are presented and compared in this chapter as they form the basis for the development of the algorithms presented in the next chapters of this thesis.

4.1 INTRODUCTION

The first suggested approaches to the power system state estimation problem used a static model and a method of solution based on the WLS criterion[79,80,81]. The static model was chosen in these early days, and is still the most widely used today, as a consequence of the unavailability of an adequate model for the time behaviour of the static state as discussed in the last chapter. The WLS method was preferred due to its excellent filtering capability demonstrated by the large experience available with the method in other applications[82,92]. No model of the measurement error statistics is necessary in this approach. The only information required by the WLS method, apart from the system parameters and the measurements themselves, are a set of "weights" which represent the relative accuracy of the measurements. Even these weights do not need to be known accurately as their influence in the results is small.

Later studies in power system state estimation[29,45] have suggested that rejection of gross measurement and topological errors is more important than the filtering of the small measurement noise, i.e. a reliable solution is more important than an accurate one. The WLS algorithm behaves very badly in the presence of these errors due to its quadratic criterion. Therefore, modifications in the approach initially used were necessary. A first improvement was the introduction of the assumption of Gaussian error. This assumption allows the use of statistical tests on the estimation results with the objective of bad data detection, identification and elimination[1,45]. A second type of improvement was introduced by changes in the WLS algorithm itself in order to render it less vulnerable to bad data. This can be achieved by altering the used performance criterion[64,67].

A large amount of work has also been carried out with the objective of improving the computational performance of the WLS estimators. These efforts have been conducted on two different lines. The first one makes use of transformation of variables to obtain a version of the algorithm which is easier to implement. An example of this approach is the Line-Only algorithm[27]. The second class of improvements is achieved by the introduction of approximations in the basic algorithm derived from physical characteristics of the power system. Examples of this approach are the Fast Decoupled estimators[12,19,20,41,47,74].

4.2 WEIGHTED LEAST SQUARES METHOD

The power system static state estimation problem can be formulated as the solution of the following overdetermined set of equations

$$\underline{Z} = \underline{h}\ (\underline{x}) + \underline{v} \tag{4.1}$$

where:

$\underline{Z}$ is the measurement vector (Mx1)

$\underline{x}$ is the state vector (Nx1)

$\underline{v}$ is the error vector (Mx1)

$\underline{h}(\cdot)$ is a non-linear vector function relating $\underline{Z}$ and $\underline{x}$ and defined in (3.37) and (3.40)

In the WLS approach the state estimate x is defined as the value of $\underline{x}$ which minimises

$$J(\underline{x}) = (\underline{Z} - \underline{h}(\underline{x}))^T\ \underline{R}^{-1}(\underline{Z} - \underline{h}(\underline{x})) \tag{4.2}$$

where $\underline{R}^{-1}$ is a diagonal matrix of weights proportional to the accuracy of the measurements. If statistical properties of the error are known, the use of weights equal to the inverse of the error covariance will produce a maximum likelihood estimate[25,82]. A necessary condition for the minimum of $J(\underline{x})$ is given by

$$\left.\frac{\partial J(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\hat{\underline{x}}} = 2\underline{H}^T(\hat{\underline{x}})\ \underline{R}^{-1}\ (\underline{Z} - \underline{h}(\hat{\underline{x}})) = 0 \tag{4.3}$$

where

$$H(\hat{\underline{x}}) = \left.\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\hat{\underline{x}}} \tag{4.4}$$

is the Jacobian matrix evaluated at $\hat{x}$. Sufficient conditions for the local minimisation of (4.2) can be derived but are not important in practice and will be ignored in this thesis.

Equation (4.3) represents a set of 2N-1 non-linear equations in an equal number of unknowns in which the measured variables are given as a function of the state variables. As these functions are a sum of sines and cosines, a general analytical solution is difficult, if not impossible. Therefore iterative solutions should be used. Several iterative algorithms are available in the literature[24]. All of them have the following general form:

$$\underline{x}^{i+1} = \underline{x}^{i} + \underline{G}^{i}\ \underline{H}^{T}(\underline{x}^{i})\ \underline{R}^{-1}\ (\underline{Z} - h(\underline{x}^{i})),\ i = 0,1,2,... \tag{4.5}$$

where $\underline{x}^{0}$ is an initial guess, $\underline{G}^{i}$ is a gain matrix and i is the iteration counter.

The only difference among the existent methods is the choice of the gain matrix $\underline{G}$. Great freedom exists in the choice of this matrix. Any full rank matrix used in (4.5) will produce the minimum of (4.3) provided convergence occurs.

4.2.1 BASIC WLS ALGORITHM

A linearised analysis of the convergence of (4.5) and some heuristic arguments[82] indicate that an "optimum" gain, in terms of the number of iterations, is given by:

$$\underline{G}^{i} = \underline{A}\ (\underline{x}^{i}) = \underline{H}^{T}(\underline{x}^{i})\ \underline{R}^{-1}\ \underline{H}(\underline{x}^{i}) \tag{4.6}$$

The above gain matrix was used in most of the initially proposed power system state estimators[78].

If $\underline{R}$ is chosen as the error covariance matrix, $\underline{A}(\underline{x})$ gives the covariance matrix of the state estimate error, which represents an extra advantage of this choice of gain.

Possible stopping rules for the algorithm defined by (4.5) and (4.6) are to stop the iterations when $J(\underline{x}^{i+1}) - J(\underline{x}^{i})$ or the magnitude of all components of $\underline{x}^{i+1} - \underline{x}^{i}$ are less than some predetermined value. The latter criterion is the most commonly used since the former needs a relatively large amount of calculation.

The implementation of the above algorithm consists of two main steps in each iteration:

a. calculation of matrices $\underline{H}(\underline{x}^{i})$ and $\underline{A}(\underline{x}^{i})$

b. solution of the system of linear equations

$$\underline{A}(\underline{x}^{i})(\underline{x}^{i+1} - \underline{x}^{i}) = \Delta\underline{b}^{i} \tag{4.7}$$

$$\Delta\underline{b}^{i} = \underline{H}^{T}(\underline{x}^{i})\ \underline{R}^{-1}\ (\underline{Z} - \underline{h}(\underline{x}^{i})) \tag{4.8}$$

Formulae to calculate the elements of the Jacobian matrix $\underline{H}(\underline{x}^{i})$ can be found in references[10,35,86]. The elements of $\underline{A}(\underline{x}^{i})$ and $\Delta\underline{b}^{i}$ are given by

$$a_{jk} = \underline{H}_{j}^{T}\ \underline{R}^{-1}\ \underline{H}_{k} \tag{4.9}$$

$$\Delta b_{j} = \underline{H}_{j}^{T}\underline{R}^{-1}\ (\underline{Z} - \underline{h}(\underline{x}^{r})) \tag{4.10}$$

where $\underline{H}_j$ and $\underline{H}_k$ are columns of $\underline{H}(\underline{x}^i)$. It is therefore convenient to store only the non-zero elements of $\underline{H}(\underline{x}^i)$ column by column using a simple list processing scheme. An alternative way is not to store explicitly the Jacobian by evaluating only one row of $\underline{H}(\underline{x}^i)$ at a time. The contributions of this row to $\underline{A}(\underline{x}^i)$ and $\Delta\underline{b}(\underline{x}^i)$ can then be calculated before evaluating the next row[51]. Matrix $\underline{A}(\underline{x}^i)$ is real, symmetrical and with a large proportion of zero elements. Its inverse, in an implicit form, can be obtained in an efficient way using one of the factorisation methods and ordering schemes described in reference (13).

4.2.2 FAST-DECOUPLED WLS ALGORITHM

The basic WLS algorithm described in the previous section is accurate and has good convergence characteristics but its computing time and storage requirements are excessive for on-line implementation, where a process control computer is likely to be used. These requirements are mainly due to the need for evaluating and factorising the gain matrix at every iteration. A possible way of alleviating this numerical burden is to use a constant or a piece-wise constant gain matrix. However, the number of iterations required by such algorithm will certainly be higher and the overall saving in computing time may be not that great.

A very much more efficient improvement in the WLS algorithm is achieved by the application of the fast decoupling techniques which have been used successfully in the load flow problem[87]. In order to simplify the understanding of the WLS decoupled algorithm, the measurement equations will be partitioned into active and reactive subsets as follows:

$$\underline{Z}_p = \underline{H}_p\ (\underline{\theta}, \underline{V}) + \underline{w}_p \tag{4.11}$$

$$\underline{Z}_q = \underline{h}_q\ (\underline{\theta}, \underline{V}) + \underline{w}_q \tag{4.12}$$

and consequently the Jacobian matrix will then be given by:

$$\underline{H}\ (\underline{\theta},\ \underline{V}) = \left[\begin{array}{c|c} \underline{H}_{p\theta} & \underline{H}_{pV} \\ \hline \underline{H}_{q\theta} & \underline{H}_{qV} \end{array}\right] \tag{4.13}$$

In normal steady state operation of a power system, with an EHV transmission network, the following approximations are usually accepted as reasonable:

$$V_i \cong 1.0 \text{ pu}$$

$$\cos\theta_{ik} \cong 1.0 \tag{4.14}$$

$$g_{ik} \sin\theta_{ik} << b_{ik} \sin\theta_{ik}$$

where V_i is the voltage magnitude at node i, θ_{ik} is the angular difference between nodes i and k and g_{ik} and b_{ik} are elements of the admittance matrix. Under these assumptions, the Jacobian submatrices defined in (4.13) maintain the following relations:

$$|\underline{H}_{q\theta}| << |\underline{H}_{p\theta}|$$

$$\tag{4.15}$$

$$|\underline{H}_{pV}| << |\underline{H}_{q\theta}|$$

where $|\underline{H}|$ indicates the moduli of the elements of $\underline{H}$. Moreover, the elements of the Jacobian vary little with changes in the state variables ($\underline{\theta}$ and $\underline{V}$). Therefore, the Jacobian matrix can be approximated, with good accuracy, by a block-diagonal and state independent matrix

$$\underline{H} = \left[\begin{array}{c|c} \underline{H}_p & \\ \hline & \underline{H}_q \end{array}\right] \qquad (4.16)$$

where $\underline{H}_p$ and $\underline{H}_q$ are obtained from $\underline{H}_{p\theta}$ and $\underline{H}_{q\theta}$ by the introduction of the approximations given in (4.14).

The above approximations on the calculation of the Jacobian matrix can be introduced in the WLS estimator in two ways:

a. using the approximated Jacobian only in the calculation of the gain matrix

b. extending the approximations also to the evaluation of $\underline{H}^T \underline{R}^{-1} (\underline{Z} - \underline{h}(\underline{x}))$.

In the first scheme, the resulting algorithm still produces the same solution as the basic WLS algorithm. In the second one, optimality is lost. However, simulation studies[11,41,51] with both algorithms have demonstrated that the solution given by scheme b is very close to the optimal solution and adequate for practical purposes. As the algorithm given by scheme b is more efficient, from a computational point of view, it should be the one to be chosen in practical applications.

A further improvement in the algorithm is achieved by dividing the reactive measurement equations by the magnitude of the voltage of the node in which the measurement is taken and also by ignoring series resistances in the computation of the elements of the sub-Jacobian $\underline{H}_p$ (11,51).

The final version of the Fast Decoupled State Estimator (FDSE) is then given by

$$\underline{\theta}^{i+1} = \underline{\theta}^{i} + \underline{A}_p^{-1}\ \underline{H}_p^{-1}\ \underline{R}_p^{-1}\ (\underline{Z}_p - \underline{h}_p\ (\underline{\theta}^{i},\ \underline{V}^{i})) \qquad (4.17)$$

$$\underline{V}^{i+1} = \underline{V}^{i} + \underline{A}_q^{-1}\ \underline{H}_q^{-1}\ \underline{R}_q^{-1}\ (\underline{Z}'_q - \underline{h}_q\ (\underline{\theta}^{i},\ \underline{V}^{i})) \qquad (4.18)$$

where

$$\underline{A}_p = \underline{H}_p^{T}\ \underline{R}_p^{-1}\ \underline{H}_p \qquad (4.19)$$

$$\underline{A}_q = \underline{H}_q^{T}\ \underline{R}_q^{-1}\ \underline{H}_q \qquad (4.20)$$

i is the iteration counter, $\underline{Z}'_q$ is the value of the measurement divided by the corresponding voltage magnitude and the elements of $\underline{H}_p$ and $\underline{H}_q$ are given in Table 4.1.

The computational work required by the FDSE is much smaller than the one by the basic WLS. Matrices $\underline{A}_p$ and $\underline{A}_q$ have to be calculated, factorised and stored only once at the beginning of the iterative process. As can be seen in Table 4.1, the elements of $\underline{H}_p$ and $\underline{H}_q$ are equal to, or a linear combination of, the line parameters. Therefore, no explicit calculation and storage of these elements is really required. Whenever necessary, they can be readily obtained from the arrays containing the line parameters.

$\dfrac{\partial P_i}{\partial \theta_j} = \sum\limits_{k \varepsilon \alpha_i} \dfrac{1}{x_{ik}}$	$\dfrac{\partial Q_i'}{\partial V_i} = \sum\limits_{k \varepsilon \alpha_i} b_{ik} - \sum\limits_{k \varepsilon \alpha_i} y_k$
$\dfrac{\partial P_i}{\partial \theta_k} = -\dfrac{1}{x_{ik}}$	$\dfrac{\partial Q_i'}{\partial V_i} = -b_{ik}$
$\dfrac{\partial P_{ik}}{\partial \theta_i} = \dfrac{1}{x_{ik}}$	$\dfrac{\partial Q_{ik}'}{\partial V_k} = b_{ik} - y_i$
$\dfrac{\partial P_{ik}}{\partial \theta_k} = -\dfrac{1}{x_{ik}}$	$\dfrac{\partial Q_{ik}'}{\partial V_k} = -b_{ik}$
$\dfrac{\partial V_i}{\partial \theta_i} = 0$	$\dfrac{\partial V_i}{\partial V_i} = 1$

$$Q_i' = \frac{Q_i}{V_i} \quad ; \quad Q_{ik}' = \frac{Q_{ik}}{V_i} \quad \text{or} \quad \frac{Q_{ik}}{V_k}$$

Table 4.1 - Elements of matrices $\underline{H}_p$ and $\underline{H}_q$

Simulation studies performed using the FDSE described above[11,31] have demonstrated its superiority over the basic WLS algorithm. For a convergence tolerance of 10^{-4} in the state variables, which produce results with an accuracy adequate for practical applications, the number of iterations required by the FDSE is about the same as the basic WLS and, in some cases, less. The storage requirement is largely reduced as only two block diagonals of the gain matrix have to be stored. The solution time for the FDSE increases linearly with the problem size while for the basic WLS algorithm this increase is more close to a quadratic curve. Convergence sensitivity to line X/R ratio is adequate for most of the situations likely to occur in practice.

4.2.3 LINE-ONLY ALGORITHM

The Line-Only algorithm developed by a team of the American Electric Power (AEP)[27,...,31] is also a WLS based algorithm. It achieves great computational efficiency by the use of an ingenious transformation of variables. However, it has the limitation of not allowing the processing of all types of measurements (e.g. injections).

Let $\underline{\dot{S}}$ denote the vector of complex line flow measurements. A relationship similar to (4.1) can be established relating $\underline{\dot{S}}$ to the state variables

$$\underline{\dot{S}} = \underline{f}(\underline{\dot{x}}) + \underline{v} \qquad (4.21)$$

where $\underline{\dot{x}}$ represents the complex nodal voltage, $\underline{v}$ the error in the line flow measurements and $\underline{f}(\cdot)$ is a function relating the complex line flows and state variables. The Line-Only algorithm is obtained minimising

$$J(\underline{\dot{x}}) = (|\underline{\dot{S}} - \underline{f}(\underline{\dot{x}})|)^T \underline{R}^{-1} (|\underline{\dot{S}} - \underline{f}(\underline{\dot{x}})|) \qquad (4.22)$$

where $\underline{R}^{-1}$ is a matrix of weights similar to the one used in (4.2).

The voltage drop across the network branches in which measurements are taken is given by

$$\underline{\dot{e}} = \underline{B}^{-1}\underline{\dot{S}} - \underline{\dot{U}} \qquad (4.23)$$

where

$\underline{\dot{B}}$: diagonal complex matrix of elements $\dot{B}_m = x_i^*(g_{ik}+jb_{ik})$

$\underline{\dot{U}}$: complex vector of elements : $\dot{U}_m = \dot{x}_i\, y_{ik}/(g_{ik}+jb_{ik})$

$\underline{\dot{e}}$: complex vector of elements $\dot{e}_m = \dot{x}_i - \dot{x}_k$

m : measurement number

i-k : branch of the network in which measurement m is taken.

Introducing (4.22) into (4.21) the performance criterion is reduced to

$$J(\underline{\dot{x}}) = (|\underline{\dot{e}} - \underline{C}\,\underline{\dot{x}}|)^T \underline{D} (|\underline{\dot{e}} - \underline{C}\,\underline{\dot{x}}|) \qquad (4.24)$$

where

$\underline{D}$: diagonal matrix of elements $D_m = R_m^{-1}/(|x_i||g_{ik} + jb_{ik}|)$

$\underline{C}$: measurement-to-node incidence matrix

The minimum of $J(\dot{x})$ is obtained by solving

$$\left.\frac{\partial J(\underline{x})}{\partial \dot{\underline{x}}}\right|_{\underline{x}=\dot{\underline{x}}} = 2\, \underline{C}^T \underline{D}\, (\dot{\underline{e}} - \underline{C}\, \dot{\underline{x}}) = 0 \qquad (4.25)$$

which in a rearranged form produces the estimator

$$\dot{\underline{x}} = (\underline{C}^T\ \underline{D}\ \underline{C})^{-1}\ \underline{C}^T\ \underline{D}\ \dot{\underline{e}} \qquad (4.26)$$

As the vector of voltage drops $\dot{\underline{e}}$ has to be obtained from (4.23), in which there is an explicit dependence on the state vector, the algorithm requires an iterative solution. To compute $\dot{\underline{e}}$ from (4.23) the voltage level of the system has to be set. This is done by specifying one of the nodal voltages in the system which is assumed to be known without error. The column and row corresponding to this voltage is then excluded from the $\underline{C}$ matrix which avoids the singularity of $\underline{C}^T\ \underline{D}\ \underline{C}$.

Matrices $\underline{C}$ and $\underline{D}$ are real and vectors $\dot{\underline{x}}$ and $\dot{\underline{e}}$ are complex and usually expressed in cartesian co-ordinates. Hence the problem is intrinsically decoupled and a block successive displacement technique can be used. As the voltages vary little in the iterative process, the dependence of matrix $\underline{D}$ in the state variables can be dropped, producing a constant gain algorithm similar to the FDSE described in the previous section.

Simulation studies comparing the Line-Only and FDSE algorithms[11,51] show that the former has a slightly superior performance in terms of computation time and storage requirements. However, the limitation of not being able to handle injection measurements (in its original and efficient version) represents a considerable drawback on the practical applicability of the method . For instance, it makes impossible the use of zero injection pseudo-measurements which enhances the redundancy ratio at no cost.

4.3 POST-ESTIMATION BAD DATA DETECTION AND IDENTIFICATION

The WLS estimators described in the previous sections have an adequate performance only when the network configuration is correct and the measurement error is small and random. Otherwise the accuracy of the estimates may be badly affected. This can be identified by the presence of large residuals (difference between measured and estimated values) which indicates that the "fitting" of the input data into the system model was not well performed.

In the case of gross measurement error, the large residuals may in some cases correspond to the measurement grossly in error. In other cases, due to the "smearing effect"[45] inherent to the WLS method, healthy measurements may be affected also. Topological errors affect the result of the estimation only when measurements are taken in the line which status is wrong. In that case the residuals corresponding to these measurements will probably be large in a similar way as if they contained gross error.

Techniques for post-estimation detection and identification of gross measurement and topological errors are mainly based in the information contained in the residuals. As both types of errors manifest in the same way (large residuals) a method to differentiate between them is required. The usual approach is to assume initially that the configuration is correct and eliminate the measurements with large residuals. Afterwards a retest is carried out in order to verify if the large residuals were due to gross error in the measurements or a wrong status of the line where the measurements are taken.

In the following sections some techniques to detect, identify and eliminate probable grossly wrong measurements are described. In section 4.6 a procedure to identify topological errors is also described.

4.3.1 LINEARISED ANALYSIS OF THE ESTIMATION RESULTS[45,82]

Assume that the measurement error is small, normally distributed, with zero mean and covariance matrix $\underline{R}$. Then the results of an estimator using the basic WLS algorithm given by (4.5) and (4.6) has the following statistical properties

$$E\{\underline{x} - \hat{\underline{x}}\} = \underline{0} \tag{4.27}$$

$$E\{(\underline{x} - \hat{\underline{x}})^T (\underline{x} - \hat{\underline{x}})\} = \underline{\Sigma}_x = \underline{H}^T(\hat{\underline{x}})\ \underline{R}^{-1}\ \underline{H}(\hat{\underline{x}}) \tag{4.28}$$

where $\underline{x}$ and $\hat{\underline{x}}$ are true and estimated values of the state.

The estimated residuals, defined as

$$\underline{\hat{r}} = \underline{Z} - \underline{h}(\underline{\hat{x}}) \tag{4.29}$$

where $\underline{Z}$ is the vector of measurements, is a normally distributed variable with statistics given by

$$\underline{E}\ \{\underline{\hat{r}}\} = \underline{0}$$

$$E\ \{\underline{\hat{r}}^T\ \underline{\hat{r}}\} = \underline{\Sigma}_r = \underline{R} - \underline{H}(\underline{\hat{x}})\ \underline{\Sigma}_x \underline{H}(\underline{x}) \tag{4.30}$$

Normalised residuals are defined as

$$\underline{\hat{r}}_N = \underline{\hat{r}}\ [\underline{D}^{-1}]^{\frac{1}{2}} \tag{4.31}$$

$$D = \text{diagonal}\ \underline{\Sigma}_r \tag{4.32}$$

and are vectors whose components are N(0,1) random variables.

The performance index $J(\underline{\hat{x}})$ calculated using (4.2) is a sum of squared normally distributed random variables. Therefore it follows a chi-squared distribution of probabilities with K = M-N degrees of freedom. The mean and variance of this distribution are given by

$$E\ \{J(\underline{\hat{x}})\} = K \tag{4.33}$$

$$V\ \{J(\underline{\hat{x}})\} = 2K \tag{4.34}$$

If $K \geq 30$ the chi-squared distribution approaches a normal distribution. In that case the standardised random variable

$$\bar{J}(\hat{\underline{x}}, K) = \frac{J(\hat{\underline{x}}) - K}{\sqrt{2K}} \tag{4.35}$$

becomes zero mean, unit variance Gaussian, i.e. N(0,1).

4.3.2 BAD DATA DETECTION

The bad data detection techniques used in state estimation are derived from the well-known subject of hypothesis testing of statistics[65,72]. In this technique, hypothesis are formulated about statistical properties of random variables involved in an experiment, for instance the type of distribution (normal, exponential, etc.) or the parameters of a specific distribution (mean, variance, etc.) from which statistical properties of other variables (outputs of the experiment) can be derived analytically. If the statistical properties of the outputs agree with the ones derived from the hypothesis, then this is a confirmation that the hypotheses are correct within certain probability limits.

The problem of bad data detection can be formulated as a hypothesis testing problem using the results of the previous section. Under the assumption of Gaussian error, it was concluded that the performance index $J(\hat{\underline{x}})$ is chi-squared distributed and the normalised residuals are normally distributed. If one or more measurements contain gross error or if the status of a line is not correct, some of the residuals used to calculate $J(\underline{x})$ will no longer be normally distributed. Therefore the calculated value of the performance index or of some normalised residuals will fall too far out on the "tails" of the respective distribution of probability functions.

The bad data detection test can be formulated as a hypothesis testing problem with two hypotheses:

H_o : no bad data exists

H_1 : H_o is not true

Depending whether the performance index or the residuals are used to test the hypothesis, three different detection methods are available in the literature[45]:

a. <u>The performance index or $J(\hat{\underline{x}})$-test</u>

The $J(\hat{x})$ test is formulated as

Accept H_o if $J(\hat{\underline{x}}) \leq \gamma$

Reject H_o if $J(\hat{\underline{x}}) > \gamma$

The detection threshold level γ is obtained from the chi-squared or normal distribution curves depending on the degrees of freedom of the particular problem. The choice of γ determines the so-called "false alarm probability" P_e, i.e. the probability of rejecting H_o when it is actually true. For example, if the normal distribution is used a value of γ = 1.65 corresponds to P_e = 0.05 for the standardised index.

b. The normalised residuals or $\hat{r}_N$-test

The normalised residuals given by (4.30) are N(0,1) random variables. Therefore, if all residuals fall in the ± 3.0 range, then there is a 99.97% chance that no bad data exists. The $\hat{r}_N$ test is formulated as

$$\text{Accept } H_o \text{ if } |\hat{r}_{N,i}| \leq 3.0 \;, \quad i = 1,\ldots,M$$

$$\text{Reject } H_o \text{ if } |r_{N,i}| > 3.0 \;, \quad i = 1,\ldots,M$$

The evaluation of the normalised residuals requires a considerable computational effort due to the need to calculate $\underline{\Sigma}_r$. As only the diagonal elements of $\underline{\Sigma}_r$ are required, this work can be reduced by the use of the sparse inverse technique[15]. In the FDSE the covariance matrix $\underline{\Sigma}_r$ is available only in an approximated form. Simulation studies performed using this approximated matrix in the calculation of the residuals have shown adequate results[41]. As $\underline{H}$ is constant in the FDSE, $\underline{\Sigma}_r$ needs to be recalculated only in cases of change in the network configuration.

c. The weighted residuals or $\hat{r}_W$-test

An alternative approach to the $\hat{r}_N$-test, which requires less computational effort, is a test based on the weighted residuals. These residuals are defined as:

$$\hat{\underline{r}}_W = \hat{\underline{r}}\,[\underline{R}^{-1}]^{\frac{1}{2}} \qquad (4.36)$$

The $\hat{r}_W$-test can then be stated as

$$\text{Accept } H_o \text{ if } |\hat{r}_{W,i}| \leq 3.0, \quad i = 1,\ldots,M$$

$$\text{Reject } H_o \text{ if } |\hat{r}_{W,i}| > 3.0, \quad i = 1,\ldots,M$$

It can be shown[45] that

$$\hat{r}_N \geq \hat{r}_W$$

which makes the $\underline{\hat{r}}_N$-test more effective than the $\hat{r}_W$-test.

Simulation studies using the above tests[45,51] have shown that $\underline{\hat{r}}_N$-test is more effective than the $\underline{J}(\underline{\hat{x}})$-test for a single gross measurement error while the $J(\underline{\hat{x}})$-test has a better performance for multiple gross measurement error or configuration error. Therefore, it is safer to implement both tests. In that case detection of bad data would be indicated if either one of the tests fails.

4.3.3 BAD DATA IDENTIFICATION

After the detection of bad data by one of the tests described above, the identification and subsequent elimination of the bad data effect in the estimation results, should be performed.

Bad data identification is a problem not yet well solved in static state estimation. The only information available that can be used for identification purposes are the measurement residuals. From an intuitive point of view, the large residuals should correspond to the measurement containing gross error. Unfortunately, this is not what usually happens in practice. Due to the smearing effect previously referred to, the straightforward comparison of the residuals magnitudes cannot produce a definitive answer to the identification problem.

A heuristic approach in which selected groups of suspected bad data are removed from the measurement set and re-estimation performed is available in the literature[45]. There exists two versions of this technique:

a. Ordered Residual Search

In this version the residuals are put into a descending order of magnitude, and the measurement corresponding to the largest residual is removed first. If bad data is still detected, a further estimation is carried out with the following measurement in the list removed, and so on.

b. Grouped Residual Search

Here the first few largest residuals are removed simultaneously and then put back one after the other until a bad data is detected.

The first version of the residual search is suitable for the case in which there is only one gross error while the second can be used for multiple gross errors or configuration errors.

In the identification method described above, it is necessary to perform successive re-estimations of the state using a set of measurements from which some measurements were deleted. This would require, in principle, the re-calculation and re-factorisation of the gain matrix. Less time-consuming approaches have been proposed in the literature. In the first one the factorised inverse is modified using the Shermann and Morrinson formula[15]. The second one, much faster, is to substitute the suspected measurement by a pseudo-measurement obtained from some results of the linearised analysis previously described in section 4.3.1[41]. A third and even simpler method is to assign zero weights to the suspected measurements[51]. This last method usually requires a few more iterations than the others in the cases which several bad data occurs in the same node.

4.4 BAD DATA SUPPRESSION ALGORITHMS

An adequate solution for an estimation problem in which some measurements are grossly in error would be one in which the residuals corresponding to these measurements were approximately equal to the error. This kind of solution can never be achieved using the WLS criterion as the sum of the square of such large residuals would certainly make that solution non-optimal. Estimators less vulnerable to bad data can be designed using criteria which assign less importance to large residuals. Examples of this kind of estimators are the Bad Data Suppression (BDS) algorithm[64,67] and the estimator using linear programming[48]. The first one, which can be considered as an extension of the WLS method, will be described in this section while the second one will be studied in the next chapter.

4.4.1 BASIC BAD DATA SUPPRESSION ALGORITHM[64]

The BDS estimator is obtained by minimising the performance criterion

$$\bar{J}(x) = \underline{\rho}^T(\underline{x})\ \underline{R}^{-1}\ \underline{\rho}(\underline{x}) \tag{4.37}$$

where each element of the vector $\underline{\rho}(\underline{x})$ is defined as

$$\rho_m(x) = \begin{cases} r_m & \text{if } \left|\dfrac{r_m}{\sigma_m}\right| \le \lambda \\ f(r_m) & \text{if } \left|\dfrac{r_m}{\sigma_m}\right| > \lambda \end{cases} \tag{4.38}$$

where r_m is the measurement residual as defined in (4.29) and $f(\cdot)$ is a function chosen in such a way to assign less weight to large residuals. Some of the functions suggested in the literature are given in Figure 4.1 and Table 4.2.

The minimisation of $J(x)$ can be achieved by an iterative process similar to the one used for the basic WLS algorithm

$$\underline{x}^{i+1} = \underline{x}^i + \Lambda^{-1}(x^i)\ \underline{H}^T(\underline{x}^i)\ \underline{G}(\underline{x}^i)\ \underline{R}^{-1}\ \underline{\rho}(\underline{x}^i) \tag{4.39}$$

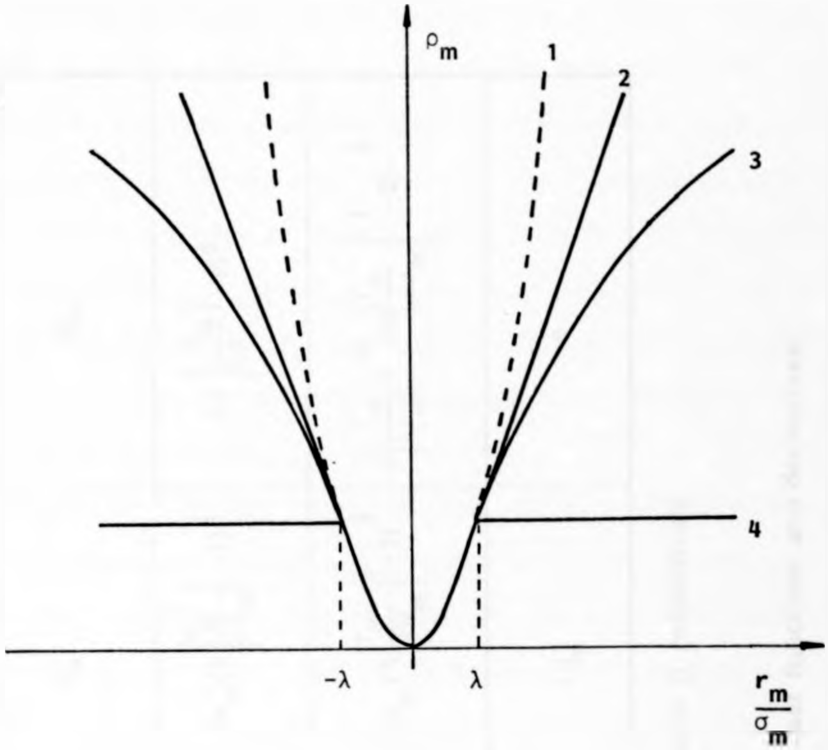
where

$$\underline{\Lambda}(x^i) = \underline{H}^T(\underline{x}^i)\ \underline{G}^T(\underline{x}^i)\ \underline{R}^{-1}\ \underline{G}(\underline{x}^i)\ \underline{H}(\underline{x}^i) \tag{4.40}$$

$$\underline{H}\ (\underline{x}^i) = \left.\frac{\partial h(\underline{x})}{\partial x}\right|_{\underline{x}=\underline{x}^i} \tag{4.41}$$

$$\underline{G}\ (\underline{x}^i) = \left.\frac{\partial\, \underline{\rho}(\underline{x})}{\partial x}\right|_{\underline{x}=\underline{x}^i} \tag{4.42}$$

and i is the iteration counter.



1 - Quadratic (plain WLS)

2 - Quadratic-tangent

3 - Quadratic-square-root

4 - Quadratic-constant

Figure 4.1 - Non-quadratic cost functions

	$\left\|\frac{r_m}{\sigma_m}\right\|$		$\left\|\frac{r_m}{\sigma_m}\right\| > \lambda$	
	ρ_m	g_m	ρ_m	g_m
Quadratic-tangent	r_m	1	$\mathrm{sign}\,(r_m)\, \lambda\sigma_m \left(2 \left\|\frac{r_m}{\lambda\sigma_m}\right\| - 1\right)^{\frac{1}{2}}$	$\left(2 \left\|\frac{r_m}{\lambda\sigma_m}\right\| - 1\right)^{\frac{1}{2}}$
Quadratic-square-root	r_m	1	$\mathrm{sign}\,(r_m)\, \lambda\sigma_m \left(4 \left\|\frac{r_m}{\lambda\sigma_m}\right\|^{\frac{1}{2}} - 3\right)^{\frac{1}{2}}$	$\left(\left\|\frac{r_m}{\lambda\sigma_m}\right\|\right)^{-\frac{1}{2}} \left(4 \left\|\frac{r_m}{\lambda\sigma_m}\right\|^{-\frac{1}{2}} - 3\right)^{-\frac{1}{2}}$
Quadratic-constant	r_m	1	$\lambda\sigma_m$	0

ρ_m and g_m are elements of the vector $\underline{\rho}$ and matrix $\underline{G}$ respectively

Table 4.2: Non-quadratic cost functions and derivatives

The choice of the breakpoint value λ affects the convergence and bad data suppression properties of the BDS estimator. If λ is chosen too small, the convergence is slow and the risk of local minima is increased as healthy measurements may be taken as bad data. As λ increases the bad data suppression effect is decreased and finally disappears. A value of $\lambda = 5.0$ was reported[51] to produce good results. If all the residuals are below the breakpoint λ, the $\underline{G}$ matrix is equal to the unit matrix and the BDS algorithm reduces to the WLS algorithm.

A variant of the algorithm described above is obtained if in (4.38) the weighted residuals are substituted by the normalised residuals in the definition of the breakpoint. The algorithm thus obtained is reported to have a better performance in bad data suppression[45].

If several bad data occur in the vicinity of a node, it may happen that many of the $\underline{G}$ matrix elements are small and placed in such a position as to impair the diagonal dominance of the $\underline{\Lambda}$ matrix. If this occurs, numerical instability or slow convergence is likely to occur. Such difficulties with the BDS algorithm have been reported in the literature[67].

A program written for the WLS algorithm can be easily transformed into one for the BDS algorithm by the simple introduction of a routine which modifies the large residuals according to the chosen criterion. The storage requirements of such a program are virtually the same as the WLS but the time necessary for a solution is usually larger due to the slower convergence of the BDS method.

4.4.2 FAST DECOUPLED BAD DATA SUPPRESSION ALGORITHM[51]

The basic BDS algorithm, in the same way as the basic WLS, is not adequate for on-line applications due to its computational inefficiency. A further drawback of the algorithm is its numerical instability. These problems are mainly due to the recalculation of the $\underline{\Lambda}$ matrix at every iteration. In a similar manner to the WLS method, fast decoupling techniques can also be applied to the BDS algorithm to improve its computational performance.

A first difficulty found in fast decoupling the BDS algorithm is the presence of the $\underline{G}$ matrix in the definition of the gain matrix $\underline{\Lambda}$. The elements of $\underline{G}$ depend upon the magnitude of the weighted (or normalised) residuals and consequently change considerably from iteration to iteration. An efficient way of avoiding this problem is to use as gain matrix the same as used for the FDSE algorithms. The algorithm thus obtained produces the same solution given by the basic BDS method requiring a few more iterations. The BDS estimator can then be decoupled in a similar way to the WLS estimator, producing the following algorithm

$$\underline{\theta}^{i+1} = \underline{\theta}^{i} + A_p^{-1} \, \underline{H}_p^T \, \underline{R}_p^{-1} \, \underline{G}_p \, \underline{\rho}_p(\underline{\theta}^i, \underline{V}^i) \qquad (4.43)$$

$$\underline{V}^{i+1} = \underline{V}^{i} + \underline{A}_q^{-1} \, \underline{H}_q^T \, \underline{R}_q^{-1} \, \underline{G}_q \, \underline{\rho}_q(\underline{\theta}^i, \underline{V}^i) \qquad (4.44)$$

where $\underline{G}_p$, $\underline{G}_q$ and $\underline{\rho}_p$, $\underline{\rho}_q$ are submatrices of $\underline{G}$ and $\underline{\rho}$ respectively.

In the algorithm above, the following inequalities are implicitly assumed

$$\underline{H}_{p\theta}^{T}\ \underline{R}_{p}^{-1}\ \underline{G}_{p} \gg \underline{H}_{q\theta}^{T}\ \underline{R}_{q}^{-1}\ \underline{G}_{q}$$

$$\underline{H}_{qV}^{T}\ \underline{R}_{q}^{-1}\ \underline{G}_{q} \gg \underline{H}_{pV}^{T}\ \underline{R}_{p}^{-1}\ \underline{G}_{p} \qquad (4.45)$$

These inequalities remain valid only if $\underline{R}_{p}^{-1}\ \underline{G}_{p}$ and $R_{q}^{-1}\ G_{q}$ are of the same order of magnitude. Therefore, in order to satisfy (4.45) throughout the iterative process, whenever a bad data is encountered in either the active or reactive part of a complex measurement, the corresponding term in matrices $\underline{G}_{p}$ and $\underline{G}_{q}$ are set to equal values.

The use of the constant gain matrices $\underline{A}_{p}$ and $\underline{A}_{q}$ eliminates the numerical instability reported in the basic algorithm and reduces considerably time and storage requirements making the fast decoupled BDS algorithm adequate for on-line applications.

Extensive simulation studies reported in reference (51) demonstrated that the fast decoupled BDS algorithm has a performance very similar to the fast decoupled WLS together with the post-estimation bad data detection and identification technique described in section 4.3. Both methods are able to detect, identify and eliminate bad data provided the local redundancy in the region of trouble is high enough.

4.5 IDENTIFICATION OF TOPOLOGICAL ERRORS

Both the residual search analysis and the BDS estimator, described earlier in this chapter, are able to detect and eliminate gross measurement and topological errors in order to produce a good state vector in most of the situations likely to occur in practice. However, they cannot distinguish whether the detected bad data corresponds to a gross measurement or topological error.

This important problem has been in some ways neglected by the researchers. Only minor references to it can be found in the literature until very recently[1,45,78]. In fact, reference (59) is the only one found by the author of this thesis in which the problem was analysed more carefully and a procedure outlined for its solution.

The proposed method is based on the fact that the inclusion in the configuration of a line that is actually off, or vice-versa, affects primarily the estimate of the injections at the end nodes. These injections, if measured, will probably be identified as bad data and therefore suppressed. The state vector obtained after the suspected measurement suppression is independent of the topological error. This healthy state vector can then be used to test whether the suppressed measurements contain gross error or are a result of topological error. This is done by a simulation process in which the status of suspected lines are changed and the injections at their ends recalculated and compared with the rejected ones.

4.6 CONCLUSIONS

The WLS based algorithms reviewed in this chapter constitute a powerful tool for the implementation of on-line power system state estimation. The fast decoupled versions of the plain WLS and BDS algorithms have adequate filtering capability and storage and computing time requirements acceptable for use on a process control computer. However, some problems remain not completely solved. One of the most important of these is the difficulty in the detection and identification of gross measurement and topological errors in systems with not very high redundancy ratio. Both the residual search technique used with the plain WLS method and the BDS algorithm either fail completely or require a large computing time to find the right solution in this case. Another important problem, which has not yet been fully appreciated by the researchers, is the inability of the algorithms to differentiate automatically between topological and gross measurement errors. An improvement in these two and other weak points, using the static approach, seems difficult to be achieved. The answer to those problems may be found in the use of extra information to increase the redundancy ratio, for instance the information available in past estimations.

CHAPTER V

AN EFFICIENT PIECEWISE-LINEAR CRITERION (PLC) DECOUPLED STATE ESTIMATOR

In this chapter an improved version of a state estimator using as performance criterion the sum of the moduli of the residuals is described. This estimator has the ability of rejecting automatically topological and gross measurement errors in most of the situations likely to occur in practice. Storage and computing time requirements of the algorithm were reduced considerably in relation to a previously proposed formulation, by exploiting certain particular characteristics of the problem which allowed the use of fast decoupling and advanced linear programming techniques.

5.1 INTRODUCTION

In the previous chapter of this thesis the inadequacy of the basic WLS method to cope with bad data was pointed out. In that chapter, it was also described how modifications can be introduced in the original WLS criterion in order to render the estimator less vulnerable to bad data. All these modifications were introduced with the objective of reducing the WLS undesirable effect of assigning great importance to measurements with large residuals which correspond most probably to bad data. This type of estimator may be included in the general category of non-quadratic estimators.

A completely different type of non-quadratic estimator, which has very good bad data rejection properties, may be derived by using as criterion the sum of the moduli of the residuals. This new type of estimator, whose basic formulation was initially proposed in reference (48), can be formulated as a sequence of linear programming (LP) problems, which can be solved using any of the available LP techniques[43,57]. This estimator will be called in this thesis the Piece-wise Linear Criterion (PLC) estimator, which seems to be a designation which more appropriately characterises the method.

The version of the estimator which will be presented in this chapter has been considerably improved in terms of storage and computing time requirements. The improvements introduced are of two kinds: The decomposition of the LP problems into two smaller ones by the application of decoupling techniques, similar to the ones used for the WLS algorithm in the previous chapter, and the use of advanced LP techniques like the combined use of the revised simplex and dual simplex with inverse basis in compact form and reinversion.

In power system engineering, particularly in on-line applications, there was a tendency of avoiding methods involving LP algorithms based on the fear that these algorithms always require large storage and computing time. More recently, this tendency has been reversed, partially due to many problems faced when using non-linear optimisation and also, as it appears, to a better understanding of the efficiency and reliability of the LP techniques, provided the algorithms are well adapted to the problem structure[88,89]. The state estimator to be presented in this chapter may well be analysed from this point of view when compared to equivalent ones based on the WLS criterion.

5.2 PROPERTIES OF THE PLC ESTIMATOR SOLUTION

The state estimation problem, in a more general formulation, can be viewed as the solution of a redundant set of algebraic equations. This set is usually inconsistent due to the presence of error in the measurements. Let this set of equations be represented by

$$\underline{Z} = \underline{h}\,(\underline{x}) + \underline{v} \tag{5.1}$$

where

$\underline{Z}$: M x 1 vector of independent terms (measurements)

$\underline{x}$: N x 1 vector of unknowns (state)

$\underline{h}(\cdot)$: function relating $\underline{Z}$ and $\underline{x}$

$\underline{v}$: M x 1 error vector

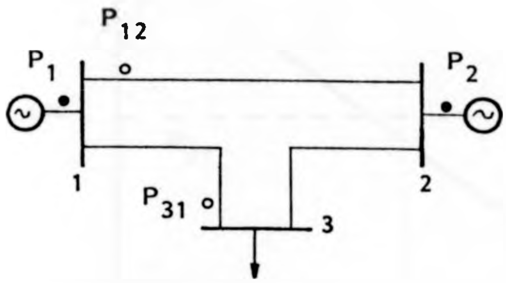
M > N

A solution for the above set of equations, in the usual sense of a vector which satisfies simultaneously all the equations, does not exist due to its inconsistency. The problem has been usually solved by the WLS method in which a solution is obtained by minimising a quadratic function of the residuals as shown previously in this thesis. The method can be interpreted geometrically as minimising the sum of the squares of the distances from the solution point to the hyperplanes representing the measurement equations. In the case of small and random errors, this solution is usually close to the ideal solution of the free of error ($\underline{v} = \underline{0}$) set of equations. If gross measurement or topological error is present in the data, the hyperplanes corresponding to the affected measurements or status will be at a great distance from the ideal solution and will attract the WLS solution, affecting strongly the accuracy of the estimate.

Another possible way of establishing a solution for the set of equations defined by (5.1) is to choose one of the "partial solutions" obtained by solving a non-redundant system formed by any N of the M equations. These solutions are more or less close to the ideal solution depending on the size of the error of the chosen equations (measurements). This approach was attempted without success in the early days of power system state estimation[7,78]. The failure of the method was probably due to the absence of a rule to choose the "best" partial solution. In reference (48) a completely new formulation of this basic idea was proposed. The method, referred to in this thesis as the PLC estimator, is based on the formulation of the estimation problem as a sequence of LP problems in which the objective function is the sum of the moduli of the residuals and the constraints are the linearised measurement equations. The property of an LP problem solution of always being in an extreme point of the region determined by the constraints guarantee that the chosen solution is one of the partial solutions referred to above, while the minimisation of the criterion pushes the solution towards the ideal one. Assuming that the measurement error is comparable with the degree of accuracy required for the estimates, i.e. that a high filtering capability is not essential, the estimates produced by this technique are adequate for practical applications.

The M-N equations (or measurements) left out when obtaining a partial solution in the method described above, have obviously no effect on the estimation result. This fact is the reason for the good bad data rejection property of the PLC estimator. As explained before, the hyperplanes corresponding to bad data are very far from the ideal solution. All the partial solutions in which such a hyperplane is involved will also be at a great distance from the other hyperplanes. Therefore, these solutions will almost certainly be rejected by the algorithm producing an estimate free of the bad data effect.

In order to illustrate the ideas exposed above, an example involving a three busbar system is presented in figures 5.1 to 5.4. In figure 5.1 the system diagram, parameters, measurement placement and equations are presented. In figure 5.2 the measurement equations for both the cases with and without error are plotted. In figure 5.3 the measurement equations, corresponding to the case in which the measurements contain error, are plotted again and the six possible partial solutions ($s_1, \dots, s_6$) are shown. The ideal solution (S_I) and the one given by the WLS method (S_W) are also shown. The solution given by the PLC algorithm can be any one of s_1 to s_6 (in fact it is s_4 in that case) which are all farther from the ideal solution than the one given by the WLS method. Nevertheless, the accuracy of any of these solutions, perhaps with the exception of s_1, is adequate for practical purposes. In figure 5.4, the same problem is shown, the only difference being that the measurement of the line flow P_{31} is set to zero



System Parameters

$b_{12} = 4.167$ pu

$b_{13} = 5.556$ pu

$b_{23} = 6.250$ pu

100 MVA base

• active injection

o active line flow

Linearised Measurement Equations : $\underline{Z} = \underline{H}\ \underline{x}$

$$\underline{x} = \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} \qquad \underline{Z} = \begin{bmatrix} P_1 \\ P_2 \\ P_{12} \\ P_{31} \end{bmatrix} \qquad \underline{H} = \begin{bmatrix} -b_{12} & -b_{13} \\ b_{12} + b_{23} & b_{23} \\ -b_{12} & 0 \\ 0 & b_{13} \end{bmatrix}$$

True Values

$\theta_2 = -\ 3° = -.052$ rad.

$\theta_3 = -\ 8° = -.140$ rad.

$P_1 = 99.5$ MW

$P_2 = 32.7$ MW

$P_{12} = 21.7$ MW

$P_{31} = 77.8$ MW

Corrupted Value of the Measurements (± 3% error)

$P'_1 = 102.5$ MW

$P'_2 = 31.7$ MW

$P'_{12} = 22.4$ MW

$P'_{31} = 75.5$ MW

Figure 5.1 - Example to illustrate the properties of the PLC estimator.

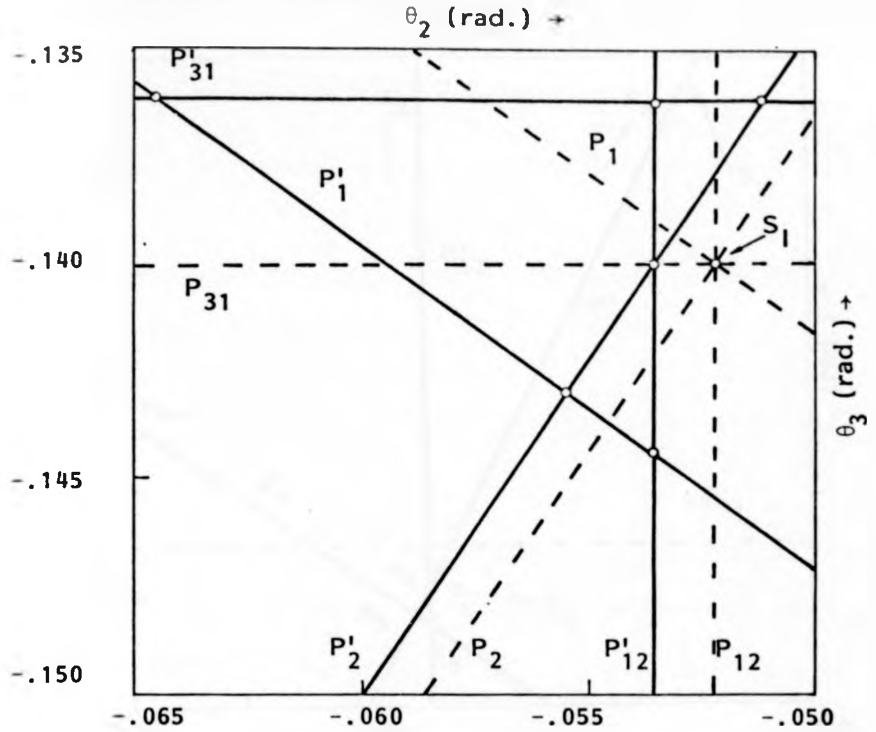


Figure 5.2 - Measurement equations with (full line) and without error

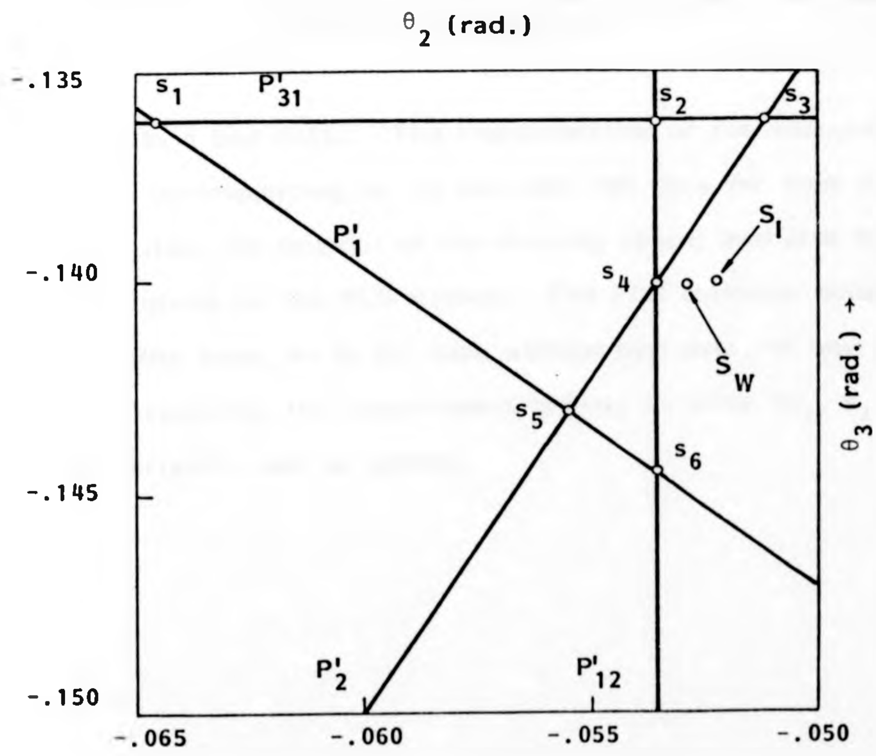


Figure 5.3 Measurement equations and WLS and PLC solutions

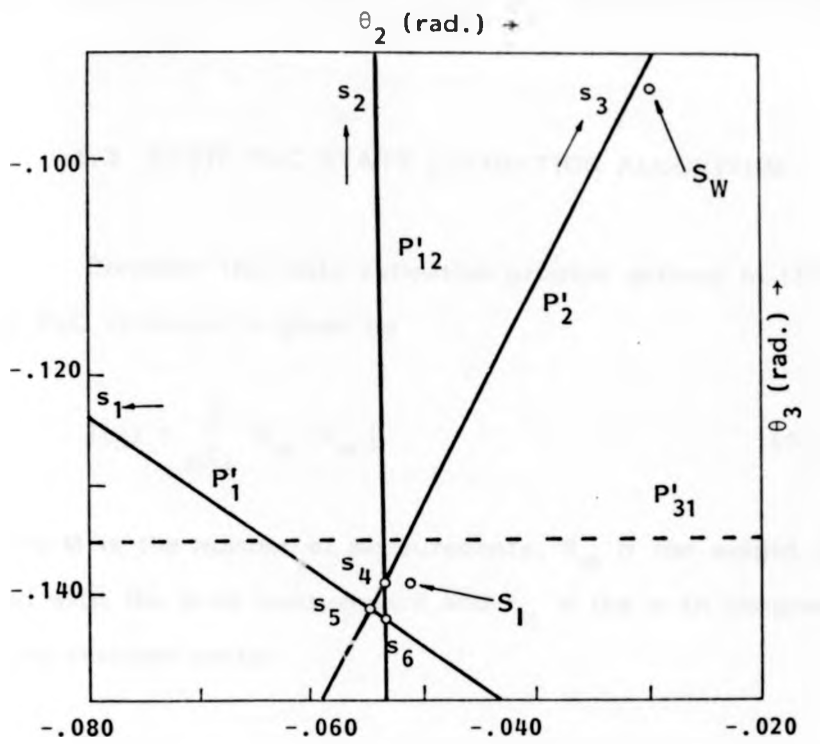


Figure 5.4 Measurement equations for the case in which measurement P'_{31} contains a gross error and WLS and PLC solutions

to simulate a bad data. The representation of the measurement equation corresponding to the bad data fall very far from the ideal solution (in fact out of the drawing space) and also the solution giver by the WLS method. The PLC estimator solution remains the same, as in the case without bad data, as any partial solution involving the measurement grossly in error (s_1, s_2, s_3) would certainly not be optimal.

5.3 BASIC PLC STATE ESTIMATION ALGORITHM

Consider the state estimation problem defined in (5.1). The PLC criterion is given by

$$J(\underline{x}) = \sum_{m=1}^{M} R_m \, |r_m| \tag{5.2}$$

where M is the number of measurements, R_m is the weight associated with the m-th measurement and r_m is the m-th component of the residual vector

$$\underline{r} = \underline{Z} - \underline{h}\,(\underline{x}) \tag{5.3}$$

As shown in Chapters III and IV, a usual procedure to solve non-linear state estimation problems is an iterative process in which successive linear approximations of the given problem are solved. This technique will be followed here to derive the PLC estimator. The Taylor expansion of (5.3) around an initial guess $\underline{x}^{\circ}$ gives

$$\underline{r}^{o} = \Delta\underline{Z}^{o} - \underline{H}(\underline{x}^{o})\, \Delta\underline{x}^{o} \tag{5.4}$$

where

$$\Delta\underline{Z}^{o} = \underline{Z} - h(\underline{x}^{o}) \tag{5.5}$$

$$\Delta\underline{x}^{o} = \underline{x} - \underline{x}^{o} \tag{5.6}$$

$$\underline{H}(\underline{x}^o) = \left.\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}^o} \qquad (5.7)$$

The non-linear estimation problem can then be formulated as a sequence of minimisation problems in which the objective function is the criterion given by (5.2) and the constraints are the set of equations defined in (5.4). This formulation in compact form can be written as follows:

$$\underset{\Delta \underline{x}^i}{\text{Minimise}} \qquad J^i = \sum_{m=1}^{M} R_m \, |r_m{}^i| \qquad (5.8)$$

subject to $\quad \underline{H}(\underline{x}^i) \, \Delta \underline{x}^i + \underline{r}^i = \Delta \underline{Z}^i$

where i = 1,2,... is the iteration counter. The iterative process stops when the values of $\Delta \underline{x}^i$ is less than a specified tolerance.

The minimisation problem defined above cannot be solved straightforwardly as the moduli function is piece-wise linear and non-differentiable at r = 0, as can be seen in figure 5.5(a). However, it is possible to define a transformation of variable which produces an equivalent linear objective function. The transformation is obtained by substituting the terms r_m by the difference of two slack variables as follows:

$$r_m = s_{2m-1} - s_{2m} \quad , \; m = 1, \ldots, M \qquad (5.9)$$

such that

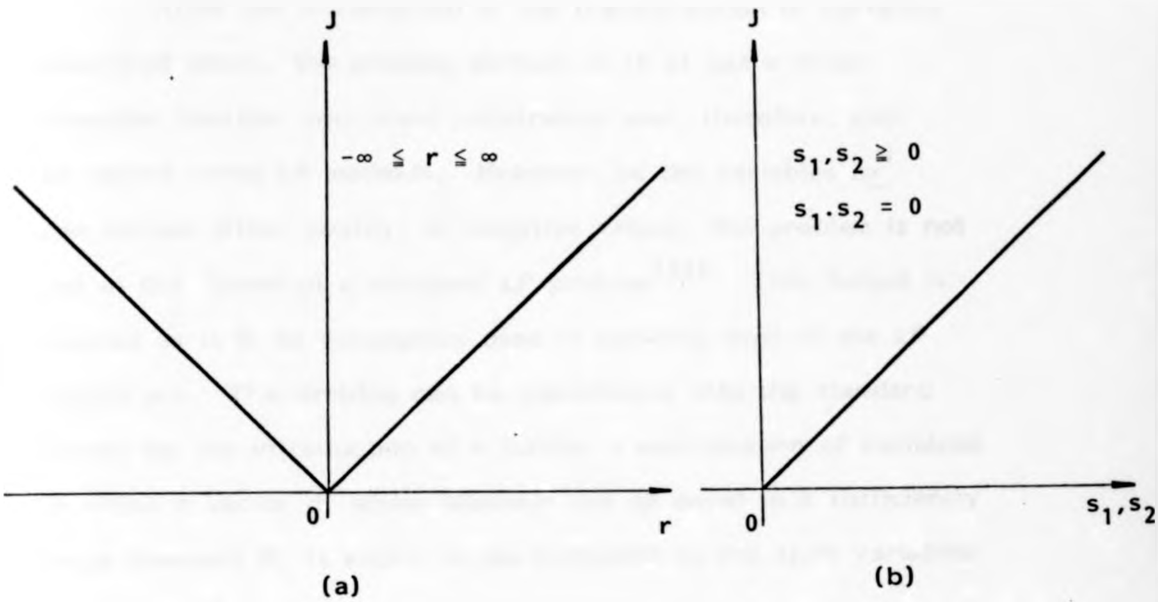


Figure 5.5 Original and transformed criteria of the PLC state estimator.

$$s_{2m-1},\ s_{2m} \geq 0 \tag{5.10}$$

$$s_{2m-1} \cdot s_{2m} = 0 \tag{5.11}$$

which implies that

$$|r_m| = |s_{2m-1} - s_{2m}| = s_{2m-1} + s_{2m}\ ,\ m = 1, \ldots, M \tag{5.12}$$

as can be seen in figure 5.5(b).

After the introduction of the transformation of variables described above, the problem defined in (5.8) has a linear objective function and linear constraints and, therefore, can be solved using LP methods. However, as the variables $\Delta \underline{x}^i$ can assume either positive or negative values, the problem is not yet in the format of a standard LP problem[43]. This format is desired as it is an assumption used in deriving most of the LP algorithms. The problem can be transformed into the standard format by the introduction of a further transformation of variables in which a vector $\underline{d}$, whose elements are all equal to a sufficiently large constant D, is added to the increment in the state variables as follows:

$$\Delta \underline{x}' = \Delta \underline{x} + \underline{d} \qquad (5.13)$$

which will lead to the redefinition of $\Delta \underline{Z}$ as

$$\Delta \underline{Z}' = \Delta \underline{Z} + \underline{H}\ \underline{d} \qquad (5.14)$$

As a result of this transformation, whatever the positive or negative value of the elements of $\Delta \underline{x}^i$ (provided they are between certain limits determined by the size of D), the transformed vector $\Delta \underline{x}_i'$ will certainly have all components greater than or equal to zero.

The PLC estimator can then be formulated as a sequence of LP problems as follows:

$$\underset{(\Delta\underline{x}')^i}{\text{Minimise}} \quad J^i = \sum_{m=1}^{M} R_m \, (s_{2m-1} + s_{2m})$$

$$\text{subject to} \quad \left[\underline{H}(\underline{x}^i) \quad \underline{U} \right] \begin{bmatrix} (\Delta \underline{x}')^i \\ \\ \underline{S} \end{bmatrix} = (\Delta \underline{Z}')^i \qquad (5.15)$$

$$(\Delta\underline{x}')^i \, , \; \underline{S} \geq 0$$

where

$$\underline{U} = \begin{bmatrix} 1 & -1 & & & & & \\ & & 1 & -1 & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & \cdot \\ & & & & & 1 & -1 \end{bmatrix}_{(M \times 2M)} \qquad (5.16)$$

$$\underline{S} = (s_1, \, s_2 \, \ldots . \, s_{2M-1} \;\; s_{2M})^T \qquad (5.17)$$

In the final solution of the LP problem defined in (5.15) to (5.17) at least one of the slack variables included in each constraint will be null (non-basic). Therefore, the condition given by (5.11) is automatically enforced.

A flow chart of the algorithm just described is shown in figure 5.6.

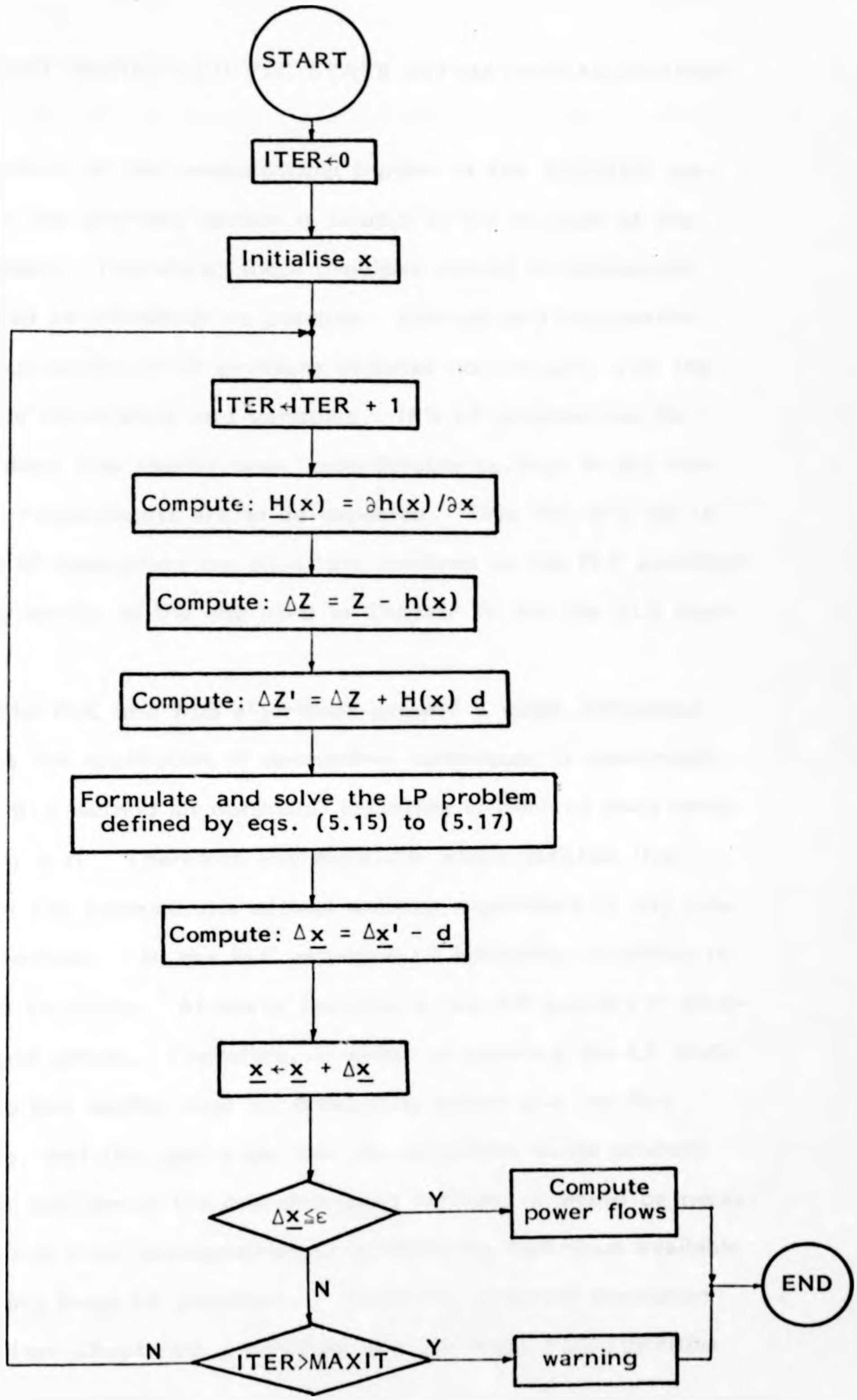


Figure 5.6 Flow-chart of the basic PLC algorithm

5.4 FAST DECOUPLED PLC STATE ESTIMATION ALGORITHM

Almost all the computational burden of the algorithm described in the previous section is located in the solution of the LP problems. Therefore, these problems should be formulated and solved as efficiently as possible. Storage and computation time requirements of LP problems increase non-linearly with the number of constraints and variables. If a LP problem can be broken down into smaller ones, considerable savings in the computation requirements are to be expected. This fact has led to the idea of decoupling the equations involved in the PLC estimator in a way similar to the one used in Chapter IV for the WLS algorithm.

The PLC and WLS algorithms present a major difference as far as the application of decoupling techniques is concerned. In the WLS method an optimality condition is derived analytically (equation 4.3). Therefore any algorithm which satisfies this condition will produce the optimal solution regardless of any later approximations. In the PLC estimator no optimality condition is available explicitly. At every iteration a new LP problem is formulated and solved. Therefore, in order to separate the LP problems into two smaller ones by decoupling active and reactive variables, and still guarantee that the algorithm would produce the same solution of the non-decoupled version, it would be necessary to use some decomposition or partitioning technique available for solving large LP problems. However, practical simulation studies (see Chapter VII) performed with the basic PLC algorithm and a sub-optimal fast decoupled version, which will be presented below, show that the results of both algorithms are about the same, particularly with respect to the rejection of bad data. This result, which in some ways confirms a previous one performed for the WLS method[41], indicates that there is an inherent decoupling in the way active and reactive measurement errors contribute to the result of the state estimator.

A fast decoupled PLC state estimator can then be derived adopting a sub-optimal decoupled criterion and a coefficient matrix obtained using the same fast decoupled Jacobian sub-matrices used in the fast decoupled WLS estimator. The sequence of LP problems to be solved are as follows:

$$\text{minimise } J_p^i = \sum_{m=1}^{M_p} R_m^p \left(s_{2m-1} + s_{2m}\right)$$

$$\text{subject to } \begin{bmatrix} \underline{H}_p & \underline{U}_p \end{bmatrix} \begin{bmatrix} (\Delta\underline{\theta}')^i \\ \\ \underline{S}_p \end{bmatrix} = (\Delta\underline{Z}_p)^i \qquad (5.18)$$

$$\Delta\underline{\theta}', \ \underline{S}_p \geq 0$$

and

$$\text{minimise } J_q^i = \sum_{m=1}^{M_q} R_m^q \left(s_{2m-1} + s_{2m}\right)$$

$$\text{subject to } \begin{bmatrix} \underline{H}_q & \underline{U}_q \end{bmatrix} \begin{bmatrix} (\Delta\underline{V}')^i \\ \\ \underline{S}_q \end{bmatrix} = (\Delta\underline{Z}_q)^i \qquad (5.19)$$

$$\Delta\underline{V}', \ \underline{S}_q \geq 0$$

where

p, q	:	sub/superscripts indicate active and reactive variables
$\underline{V}$, $\underline{\theta}$	:	voltage magnitude and phase angle vectors
$\underline{U}_p$, $\underline{U}_q$, $\underline{S}_p$, $\underline{S}_q$	:	are matrices and vectors similar to the ones defined in (5.16) and (5.17) with the appropriate dimensions.

A flow chart of the fast decoupled PLC estimator is given in Figure 5.7.

5.5 EFFICIENT IMPLEMENTATION OF THE PLC STATE ESTIMATOR

Standard library routines to solve LP problems like the ones given by (5.18) and (5.19) are largely available. Although some of these "packages" are very efficient to solve ordinary LP problems, their use in connection with the PLC state estimator would not be adequate for two reasons:

i. In a real-time environment the state estimator will most certainly be implemented in a process control computer with insufficient computational capability to accommodate a general purpose package.

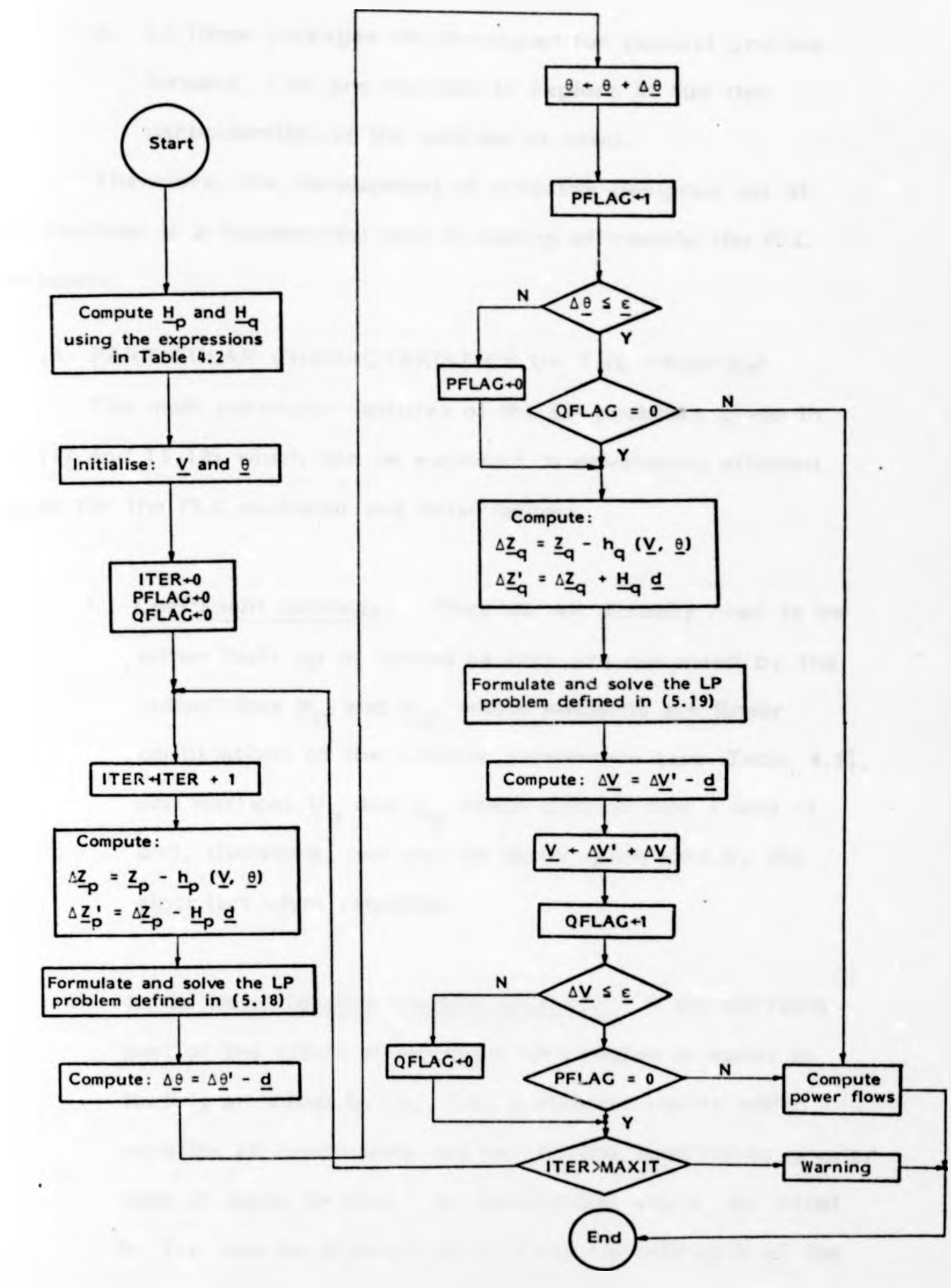


Figure 5.7: Flow-chart of the fast-decoupled PLC estimator

ii. as these packages are developed for general problem formats, they are not able to explore in full the particularities of the problem at hand.

Therefore, the development of a custom-designed set of LP routines is a fundamental step in coding efficiently the PLC estimator.

5.5.1 PARTICULAR CHARACTERISTICS OF THE PROBLEM

The main particular features of the LP problems given in (5.18) and (5.19) which can be exploited in developing efficient codes for the PLC estimator are listed below:

i. Coefficient matrices: They do not actually need to be either built up or stored as they are composed by the submatrices $\underline{H}_p$ and $\underline{H}_q$, whose elements are linear combinations of the network parameters (see Table 4.1), and matrices $\underline{U}_p$ and $\underline{U}_q$ which contain only 1 and -1 and, therefore, can also be easily generated by the algorithm when required.

ii. Initial basic feasible solution (b.f.s.): A considerable part of the effort of solving a LP problem is spent in finding an initial b.f.s., i.e. a solution vector which satisfies all constraints and has all the components greater than or equal to zero. In the problem above, an initial b. f.s. can be obtained very easily by setting M of the existent slack variables equal to the r.h.s. vector and the remaining variables equal to zero.

iii. Sparsity: The coefficient matrices, and in consequence any basis matrix derived from them, are very sparse. For a system of 100 nodes and 200 lines, with an average of 3 lines connected to each node, in which all injections and line flows at one end of the lines are measured, the number of non-zero elements in the Jacobian submatrices ($\underline{H}_p$ or $\underline{H}_q$) would be only 400 (approximately). This will produce densities of 0.7% and 1.0% for the coefficient and basis matrices respectively.

iv. Repeated solutions: The iterative process given in Figure 5.7 consists basically of repeated solutions of two LP problems (active and reactive) in which only the r.h.s. vector (weights) and the matrix of coefficients remain unchanged during the whole process.

5.5.2 METHOD OF SOLUTION OF THE LP PROBLEMS

Several algorithms are available in the literature to solve LP problems like the ones given by (5.18) and (5.19)[43,57]. Some of them, like the simplex method, require an initial b.f.s. to start with. Others, like the dual-simplex and primal-dual methods, can start with a non-feasible solution and iteratively force it to become feasible in such a way that, when it does, it is also optimal. In all methods, at each iteration a new extreme point of the feasible region (or a new b.f.s.) is obtained by pivot operations on the original coefficient matrix augmented by the r.h.s. vector (tableau). In the revised simplex method, instead of actually performing the pivot operations in all elements of the tableau, at each iteration only the relevant elements are updated by the inverse basis matrix.

This procedure, of always accessing the original data, is more efficient from the point of view of speed, storage and accuracy. This technique can also be extended to other LP methods. The basis matrix can be stored in full or only the part corresponding to structural variables (reduced basis). If the problem has a relatively high degree of sparsity the full or partial inverse basis should be stored using a product form.

A readily available initial b.f.s., the availability in core of the original data (coefficient matrix, cost vector, etc.) and the degree of sparsity of the coefficient matrix, point towards the use of the revised simplex method with inverse basis in compact form to solve the LP problems given by (5.18) and (5.19). In this method the b.f.s. at each iteration is obtained from the original data by multiplying the r.h.s. vector by the inverse basis matrix, which is a square matrix obtained from the coefficient matrix by selecting some variables as basic according to some rule. The inverse basis is stored in compact form through a representation as a product of elementary matrices. Only one column of each elementary matrix has to be stored. These columns are usually called eta-vectors and the whole set is also called the eta-file. A fairly detailed description of this method is given in Appendix A.

The algorithm presented in Figure 5.7 usually converges in four or five iterations from a flat start (unit voltage magnitudes and null phase angles). The result of the first iteration is usually close to the final solution, the remaining iterations performing only small adjustments on the state variables in order to achieve the overall specified accuracy. Therefore, the r.h.s. vectors $\Delta \underline{Z}_p$

and $\Delta\underline{Z}_q$ change very little from the second iteration onwards. This is especially so for the transformed variables $\Delta\underline{Z}'_p$ and $\Delta\underline{Z}'_q$ which have absolute values much greater than the original ones, provided the constant D used in (5.13) is large enough. On the other hand, most of the work of a simplex iteration cycle for any one of the sequence of LP problems that have to be solved by the algorithm, if they are started from a b.f.s. like the one described in section 5.5.1(ii),is spent on bringing the state variables (or structural variables) into the basis. These operations are usually repeated in almost the same order in all the problems of the sequence. Also the optimal basis does not change very much from one iteration to the other. Therefore, if a full simplex iterative cycle is performed for every LP problem in the sequences defined in (5.18) and (5.19), a large amount of repeated operations would be performed without any extra improvement in the final result and increasing largely the algorithm computing time requirement.

There are two ways in which the observations above can be used to reduce the time requirement of the PLC estimator:

i. to solve the first problem of the sequence (active or reactive) by the revised simplex algorithm and to use the same optimal basis for the next problems.

ii. instead of using the same basis for all problems of the sequence, actually obtain a new optimal basis for each problem using the dual-simplex method (see Appendix A) which usually requires only a few iterations in this case.

Method i. is very fast but does not guarantee optimality. As will be shown in Chapter VII, it usually gives good results for problems in which no bad data is present, but has a rate of failure greater than the original method for the cases with bad data. Method ii. produces the same solution of the original algorithm with a very small increase in computing time over method i. This is so because, as mentioned earlier, only a few variables change in the optimal basis in two consecutive LP problems in the sequences given by (5.18) and (5.19) and, therefore, only few iterations of the dual simplex algorithm would be required.

5.5.3 STORAGE CONTROL OF THE INVERSE BASIS COMPACT REPRESENTATION

As can be seen in Appendix A, at every iteration of the simplex or dual simplex algorithm, using the inverse basis in compact form, a new eta vector is formed. This increases the number of elements (and storage) required to represent the inverse basis. Reinversion techniques exist to recalculate these elements, from time to time in order to compact the representation. One of these techniques, which is called Pre-assigned Pivot Procedure[46] was used in the development of a program to study the performance of the PLC estimator and is also described in Appendix A.

There is no definite rule to determine how many times and at what particular instant in the process, the reinversion should take place. If storage is the major problem, then a possible rule would be to perform the reinversion whenever the space required to store the eta-vectors exceeds a certain limit. If computing time is the major issue, then a compromise between the time spent in the reinversion and the time saved by having a more compact representation of the eta-vectors should be arrived at.

5.5.4 EFFICIENT ALGORITHM

Figure 5.8 shows a flow chart of an efficient procedure to solve each one of the LP problems which appear in the fast decoupled version of PLC estimator. This procedure uses the ideas explained in the last sections. The flow chart should be used together with Appendix A, in which each of its steps related to the iterative cycle of the revised simplex and dual simplex algorithms, with inverse in compact form, is described.

Although two different algorithms are used simultaneously in that procedure, the amount of extra programming required is small as both the revised simplex and dual-simplex algorithms use the same basic operations (forward and backward multipliction by the inverse basis, calculation of reduced price, etc.) only differing in the rules to choose the pivots and, therefore, could share most of the developed routines.

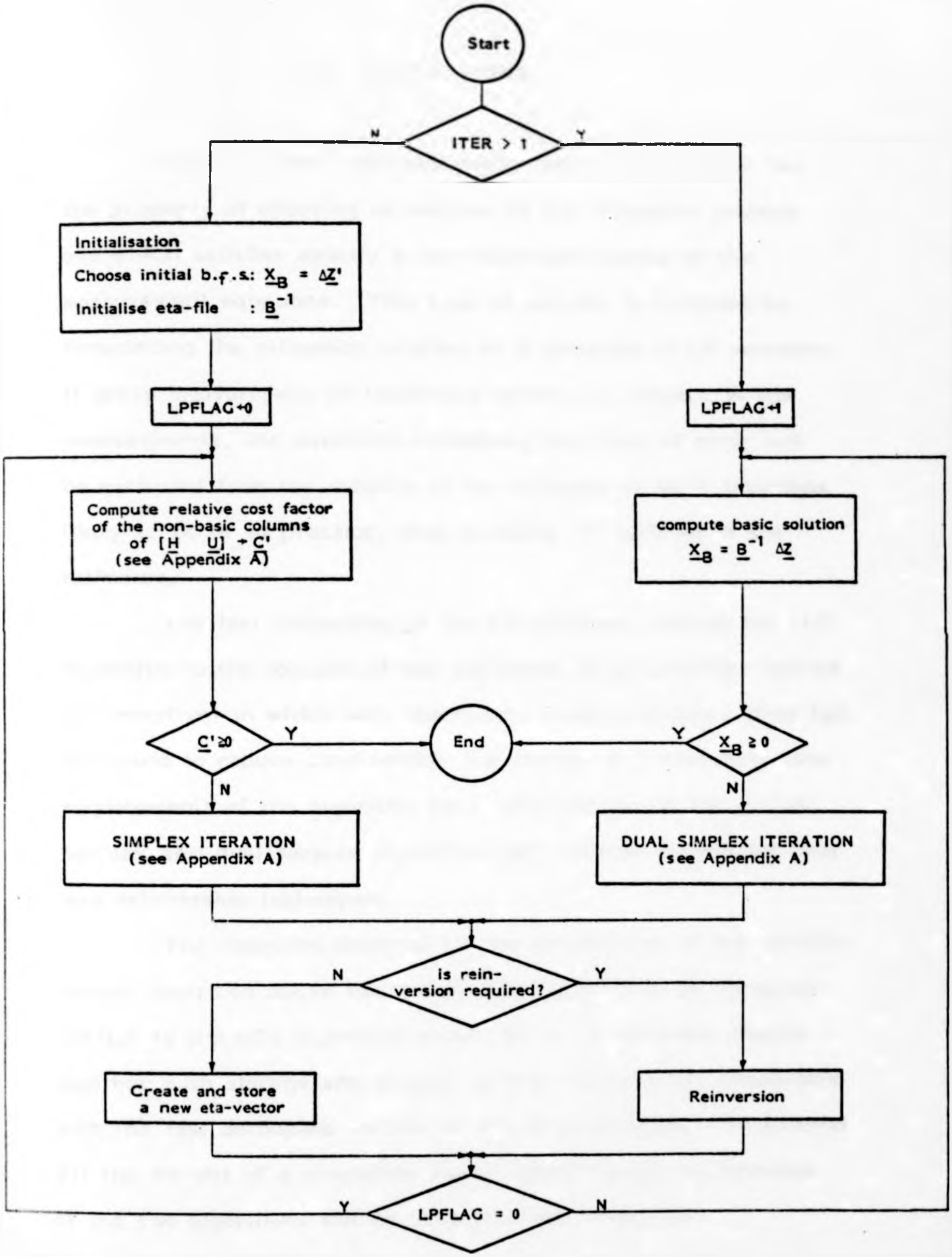


Figure 5.8 Flow chart of the efficient procedure to solve each LP in the PLC estimator

5.6 CONCLUSIONS

The PLC state estimator presented in this chapter has the property of choosing as solution to the estimation problem one which satisfies exactly a non-redundant subset of the measurement equations. This type of solution is obtained by formulating the estimation problem as a sequence of LP problems. If gross measurement or topological errors are present in the measurements, the equations containing this type of error will be excluded from the solution of the estimator in most situations likely to occur in practice, thus avoiding the spoiling of the estimates.

The fast decoupling of the LP problems reduces the PLC algorithm to the solution of two sequences of LP problems (active and reactive) in which only the r.h.s. vector changes. This fact was used to reduce considerably the storage and computing time requirements of the algorithm by a combined use of the revised simplex and dual simplex algorithms with inverse in compact form and reinversion techniques.

The algorithm obtained by the introduction of the modifications described above has a bad data suppression performance similar to the BDS algorithm presented in the previous chapter together with storage and computing time requirements comparable with the fast decoupled version of the same algorithm. In Chapter VII the results of a simulation study comparing the performance of the two algorithms will be presented and discussed.

CHAPTER VI

GROSS MEASUREMENT AND TOPOLOGICAL ERROR SUPPRESSION IN TRACKING STATE ESTIMATION

In this chapter a class of tracking state estimation algorithm obtained by the combined use of time-series prediction techniques and non-quadratic state estimation algorithms is presented. The estimators achieve a more efficient bad data suppression capability than the equivalent static approach, by making use of the information obtained in recent estimations. An overview of the way in which the tracking estimators interact with other components of the on-line data processing system is also presented.

6.1 INTRODUCTION

The main objective of a power system state estimator is to maintain an updated data base which is used for security monitoring, economic dispatch, etc. Some elements of this data base (voltages, power flows, etc.) follow the variation in demand and, therefore, should be re-estimated from time to time. The frequency of this updating depends on particular operational requirements of the system and is usually higher in periods when rapid changes in system variables take place (e.g. morning load pick-up, peak periods, immediately after an outage, etc) in which the system is submitted to severe stress and emergency situations are more likely to occur, which requires a closer monitoring.

The conventional procedure to schedule a static state estimator is to perform estimations at relatively long time intervals (5 to 30 mins) or when a change in the network topology takes place. In order to take into account possible rapid changes in the state between the scheduled estimations, measurement scans are taken at a much shorter interval (every 30 seconds, for instance) and compared with the results of the previous estimation. If an overall "large change" is detected, then a new estimation is performed[30,49,53].

The above procedure is adequate in most of the practical situations as far as the updating of the data base is concerned, i.e. the "age" of the information available for security monitoring, economic dispatch, etc., is acceptable. However, from the point of view of the estimation efficiency, it is not the best way of tackling the problem. By allowing relatively large intervals between estimations and isolating each of these estimations from the previous ones, some information is wasted. This affects mainly the task of bad data detection and identification which, as seen in previous chapters of this thesis, requires a relatively large redundancy ratio. Still due to the large interval between estimations, a non-linear formulation of the estimation problem is necessary which requires an iterative algorithm to produce a solution. This requirement may become a severe limitation in certain systems in which response times for relatively large estimation problems, in a process control computer, as fast as 10 seconds are expected[49].

An improvement in the scheme described above can be obtained by the use of tracking state estimators like the ones introduced in Chapter III. In that approach, instead of only testing whether the values of a new measurement scan are sufficiently different from the previous estimation to justify a new one, the algorithm would actually update the value of the state variables by performing a run of the tracking estimator. Of course, a new estimation would not necessarily occur at every scan. Depending on the delay imposed by the telemetering system, the capability of the processor in which the estimator will be run and the input-output facilities, a compromise solution should be achieved which may determine that estimations should be performed only after a few scans, provided that the total interval between two consecutive estimations is kept small enough.

In this chapter, an incremental static model of the system state time evolution and a predictive technique which extrapolates the values of past estimations, together with the non-quadratic state estimation algorithms described in Chapters IV and V, are used to derive tracking state estimators which are more efficient from the point of view of suppression of bad data than the equivalent static approaches.

The predictive technique referred to above is based on the assumption that in normal operation the power system loads, and consequently all the other system variables, change smoothly during a load cycle. Therefore a reasonable prediction of the future behaviour of the system can be obtained using only past observations and time-series techniques[17,42,52]. Whenever the system behaviour departs largely from this predicted path, some trouble in the form of bad data or sudden change in the system operating point is assumed and communicated to the estimator which, by a careful weighting of the importance of these changes, will be able to produce a correct estimate of the current state.

6.2 STATIC INCREMENTAL MODEL OF THE STATE TIME EVOLUTION

The practical impossibility of deriving a true dynamic model for the power system static state (voltage magnitude and angles at all busbars) was explained in Chapter III. In that chapter it was also pointed out that tracking state estimators can be derived by extending the static state estimation techniques to the time varying case. This approach often leads to much more simple and efficient alogorithms than the consideration of elaborated dynamic estimators based on simplified models[78].

Following the above ideas, the problem of tracking the time evolution of the static state will be considered here as the solution of a sequence of static estimation problems (or the solution of a sequence of redundant sets of algebraic equations), in which each problem differs from the previous one by small changes in the coefficient of the equations and in the r.h.s. or independent variables. Therefore, after obtaining the solution of the first problem of the sequence, using any one of the methods described in Chapters IV and V, the solution of the remaining problems will be obtained by calculating the changes in the dependent variables (states) caused by changes in the independent variables (measurements).

Let the measurement and state vectors be related by the equation

$$\underline{Z}_k = \underline{h}\,(\underline{x}_k) + \underline{w}_k \tag{6.1}$$

where

k : time interval

$\underline{Z}_k$: measurement vector (Mx1)

$\underline{x}_k$: state vector (Nx1)

$\underline{w}_k$: error vector (Mx1)

$\underline{h}(\cdot)$: non-linear function defined in (3.37) to (3.40)

Assume that a state estimation was performed at instant k-1 and let $\hat{\underline{x}}_{k-1}$ be the result of this estimation. Then define

$$\hat{\underline{Z}}_{k-1} = \underline{h}\,(\hat{x}_{k-1}) \tag{6.2}$$

Let $\underline{Z}_k$ be the vector of measurements at k and

$$\Delta\underline{Z}_k = \underline{Z}_k - \hat{\underline{Z}}_{k-1} \tag{6.3}$$

$$\Delta\underline{x}_k = \underline{x}_k - \hat{\underline{x}}_{k-1} \tag{6.4}$$

From (6.2) to (6.4) and using Taylor's series expansion the following relationship can be derived

$$\Delta\underline{Z}_k = \underline{H}\,(\hat{\underline{x}}_{k-1})\,\Delta\,\underline{x}_k + \underline{\omega}_k \tag{6.5}$$

where

$$\underline{H}\,(\hat{\underline{x}}_{k-1}) = \left.\frac{\partial \underline{h}}{\partial \underline{x}}\right|_{\underline{x}=\hat{\underline{x}}_{k-1}} \tag{6.6}$$

The error vector $\underline{\omega}_k$ is a sum of the present snapshot measurement error with the previous estimation error and the error introduced by the linearisation. Under the assumption that the estimations are performed at relatively small intervals, the error of linearisation is comparable with the measurement error.

6.2.1 DECOUPLED MODEL

As already pointed out in Chapters IV and V, the sensitivity of voltage phase angles (magnitudes) to changes on reactive (active) variables in an EHV network is small. In those chapters, it was also indicated that errors in active (reactive) variables have little influence on the estimates of voltage magnitudes (phase angles). Therefore, the incremental model given by (6.5) can be reasonably approximated by two independent sets of equations as follows:

$$\Delta \underline{Z}_k^p = \underline{H}_p\,(\underline{x}_k)\,\Delta \underline{\theta}_k + \underline{\omega}_k^p \tag{6.7}$$

$$\Delta \underline{Z}_k^q = \underline{H}_q\,(\underline{x}_k)\,\Delta \underline{V}_k + \underline{\omega}_k^q \tag{6.8}$$

where

$\underline{\theta}$, $\underline{V}$: voltage phase angle and magnitude vectors

$\underline{Z}^p$, $\underline{Z}^q$: subvectors of $\underline{Z}$ corresponding to active and reactive measurements

$\underline{\omega}^p$, $\underline{\omega}^q$: subvectors of $\underline{\omega}$ corresponding to active and reactive measurement errors

$\underline{H}_p(\cdot)$, $\underline{H}_q(\cdot)$: main diagonal submatrices of $\underline{H}(\cdot)$.

6.2.2 DECOUPLED MODEL WITH CONSTANT COEFFICIENT MATRICES

The elements of the Jacobian submatrices $\underline{H}_p(\underline{x}_k)$ and $\underline{H}_q(\underline{x}_k)$ vary little with changes in the state $\underline{x}_k$ (see Chapter IV). The error introduced in the state estimation if these matrices are made state independent by calculating its elements around nominal values, are acceptable as shown by results of simulation studies using fast decoupled static estimators[11,41,51].

The successful results obtained with the FDSE described in reference (11) and reviewed in Chapter IV, make the decoupled matrices used in that method a good choice for the static incremental model. In fact, these matrices seem to represent fairly accurately the relationship between the measured and state variables over a large range of operating conditions. Using these matrices the final incremental model will be given by:

$$\Delta \underline{Z}_k^p = \underline{H}_p \Delta \underline{\theta}_k + \underline{\omega}_k^p \tag{6.9}$$

$$(\Delta Z')_k^q = \underline{H}_q \Delta \underline{V}_k + \underline{\omega}_k^q \tag{6.10}$$

where

$$(\Delta Z')_k^q = \Delta \underline{Z}_k^q / \underline{V}_k \tag{6.11}$$

and the elements of H_p and H_q are calculated as indicated in Table 4.1.

6.3 PREDICTION OF THE VALUES OF MEASURED AND STATE VARIABLES IN THE NEXT INTERVAL

The power system load changes daily according to a predictable pattern. Sudden variations are not frequent and are either the result of a predictable event or an indication of some abnormal state of operation. The other system variables, like voltages, power flows, etc., follow these variations in load according to the adopted operational and control strategy and network constraints. Apart from eventual transients, these variables present a variation pattern similar to the demand. This behaviour of the power system has already been described in previous chapters to characterise the so-called quasi-static mode of operation.

A simple relationship between changes in load and corresponding changes in network variables is not available. Neither is a model for the daily load variation (see section 3.4.1). However, due to the relatively simple (in view of the application in mind) behaviour of the power system described above, it is possible to obtain reasonably accurate predictions of the system load and other variables, based on the statistical analysis of previous observation.

Time-series techniques are particularly useful in predicting the future behaviour of a process like the one described above, in which a relatively simple process is to be observed but an adequate model is not available. Several applications of these techniques to the subject of short-term load forecast, which has many similarities with the problem analysed in this section, can be found in the literature[75].

Techniques used to forecast time-series can be used to calculate predicted values of the measured and state variables in the next measurement scan. These values are then used in the tracking estimators to be described in the next sections for the following purposes:

i. to provide a better starting point in the algorithm whenever it is required.

ii. to "smooth out" fluctuations in the measured variables caused by transients

iii. to increase the volume of information (redundancy) available to the estimator, which will make easier the task of bad data detection and identification.

The purpose given in iii. is the most important of the three and is the one responsible for making the tracking approach more efficient than the static one from the point of view of the overall redundancy required.

6.3.1 EXPONENTIAL SMOOTHING OF STATE AND MEASURED VARIABLES

The values of state and measured variables obtained in each run of the estimator constitutes a family of time-series. In a rigorous sense, those series are correlated to a degree which depends on the coupling existent between each pair of corresponding variables. However, due to theoretical and practical difficulties existent in dealing with correlated time series, and also because high accuracy is not required, these time-series will be assumed to be independent. Provided a relatively short period is considered (say up to 30 mins), each of these series can be reasonably modelled as being made up of a trend component plus a random change[93]. As only one step ahead predictions are required (next scans), the use of a linear trend is adequate.

Several prediction (or forecasting) techniques are available to be used with univariate time-series like the ones described above[17,42,52]. Among these, exponential smoothing was chosen to be used in this application, in place of more sophisticated methods like the Box-Jenkins approach for instance, based on the three points below:

i. it produces result of sufficient accuracy

ii. it is fully automatic

iii. it is fast and requires very little extra storage

Items ii. and iii. above are particularly important in view of the on-line application.

The version of exponential smoothing which will be used here is the one devised by Holt and Winters for series modelled as above and which is described in Appendix B. Suppose that measurement scans and estimations are performed at regular intervals: let $\bar{y}_{k-1}$ denote the predicted value of the variable being modelled by the series (state or measured variable) and T_{k-1} the trend component (i.e. the expected increase or decrease per interval in the current value) at instant k-1. Then, after a run of the estimator is performed at the time interval k, producing the value y_k for the variable in question, the predicted values of the series are updated by the following formulae:

$$\bar{y}_k = \alpha\, y_k + (1 - \alpha)(\bar{y}_{k-1} + T_{k-1}), \quad 0 < \alpha < 1 \tag{6.12}$$

$$T_k = \beta(\bar{y}_k - \bar{y}_{k-1}) + (1 - \beta)\, T_{k-1}, \quad 0 < \beta < 1 \tag{6.13}$$

and the predicted value in the next scan is given by

$$y^*_{k+1} = \bar{y}_k + T_k \tag{6.14}$$

where α and β are the smoothing constants.

The choice of the smoothing constants can be made in two ways: in the first one, they are chosen according to an empirical assessment of the particular characteristics of the series under consideration. In general, the larger the random change associated with the series the lower the values of the optimal smoothing constant. A more objective approach is to select these values based on a minimisation of the error that would be obtained in "forecasting" the previous observed values of the series (see Appendix B). The first approach is obviously much faster and produces results adequate for practical applications as will be shown in Chapter VII.

The recursive process defined by equations (6.12) and (6.13) can be started by setting

$$\bar{y}_1 = y_1 \tag{6.15}$$

$$T_1 = y_1 - y_0 \tag{6.16}$$

and then using the equations recursively for $k = 2,3,\ldots,n$.

6.4 TRACKING STATE ESTIMATORS

A possible way of using the prediction technique described in the previous section, to improve the overall redundancy ratio in the model given by (6.9) and (6.10), is to use the predicted value of the state as an a priori estimate for an algorithm like the ones described in section 3.3.2. However, two major disadvantages arise in this approach: a. it is difficult to obtain reasonably accurate values for the error covariance matrix of the predicted values (matrix P in (3.25)), as several sources of errors are involved and no model for the time structure of these errors is available; b. the need to refactorise the gain matrix at every interval would probably be excessive, impairing the computational performance of the tracking algorithm.

Another possible approach, which is adopted in this thesis, is to use a two-step procedure: first the values of the measurements obtained in the present scan are compared with the predicted values in order to check whether large discrepancies exist; afterwards an estimation is performed in which suspected measurements receive special attention. In this approach, the increase in the amount of information available to the estimator is achieved indirectly by indicating possible bad data. An efficient way of transferring this information to the estimator can be achieved using the non-quadratic estimators described in Chapters IV and V, which have an automatic facility for discriminating against these suspected measurements. This approach does not have any one of the disadvantages of the previous one and has the advantage of using algorithms which have been previously tested successfully.

6.4.1 BAD DATA DISCRIMINATION TECHNIQUE

In normal conditions of operation (no outage, no sudden change in load) and if the data acquisition system is working well (no meter communication failure), the differences between the predicted values of the measured variables, obtained using the technique described in section 6.3, and the actual value of the measurements, should be small. These differences reflect all kinds of errors involved in the process of estimation-prediction, like the measurement noise, error in modelling the time-series, random fluctuation in the load, etc. Although impossible to prove mathematically, due to the almost empirical nature of the prediction technique, it is not unreasonable to assume that this set of differences is a random variable with an approximated normal distribution of probabilities[69,70]. The result of simulation studies reported in Chapter VII indicates that this assumption is acceptable when the measurement error and the load random variation are modelled as Gaussian white noise[82] which is a usual assumption in power system state estimation studies[62,78]. This distribution has a zero mean and its variance can be determined approximately by off-line analysis of previously obtained data.

Two types of events can disturb the process described above:

i. a sudden change in the system operating point due to loss of a big load, outage, etc.

ii. gross measurement error

In the case of transmission line outage, the change in status of the line may be reported or not. In the negative case a wrong model of the network would be used by the estimator.

Both types of events described above are indicated by some of the differences between predicted and measured values being larger than the maximum expected (for instance three times the standard deviation of the prediction error). Moreover the mean and standard deviation of the sample of differences will also be far from the expected values. Therefore, by calculating these parameters of the sample, it is posible to detect the two types of abnormality. The problem that remains is how to distinguish between the two situations.

In the case of sudden change in the operating state, the large differences between predicted and measured values are correlated as they represent actual changes in the power flow pattern. In the case of gross measurement noise, these differences are completely uncorrelated, as failure on one meter does not have influence in other meters. Therefore, by analysing the extent of the correlation between these large differences, a distinction between the two abnormal situations can be achieved.

A possible approach for this discrimination test is to use a method similar to the one described in references (69) and (70) in which the "skewness" of the distribution of probability of the differences between predicted and measured values is calculated and compared with a threshold limit. As already pointed out in Chapter III, this technique relies too much on a simplified model of the problem.

In this thesis an approach based on the spatial (or topological) correlation of the measured variables was preferred. The method is based on the principle that if a disturbance like the ones described in event i. above occurs in a region of the system, all the measurements taken in that region will experience a sudden change. Therefore, if a large difference between predicted and measured values of a specific variable occurs in isolation, i.e. no other large difference is observed in measured variables in neighbouring nodes, that is a strong indication of a gross measurement error.

The above principle can easily be developed into an automatic routine to discriminate between sudden change and gross measurement error situations. One possible procedure is as follows: a suspected measurement is checked against other measurements in the same node and in neighbouring nodes; for each flagged measurement found in these lists a certain number of points is added to the measurement flag. Depending on the final scores the suspected measurements are confirmed as containing gross error or not. If high scores are obtained by most of the flagged measurements, then it is an indication of a real sudden change in the system state. In practice, such a procedure should incorporate certain particular characteristics of the power system in which it is going to be implemented, such as existence of radial lines, data concentrator in some regions, etc. The thresholds below which a score should be for a suspected measurement to be confirmed as bad data also depends on the particular system and should be worked out beforehand. In figure 6.1 a flow-chart of one possible routine to produce the above scores is presented.

6.4.2 QUADRATIC-SQUARE ROOT CRITERION (QSR) ALGORITHM

In this first tracking algorithm the problem defined in (6.9) and (6.10) is solved using the Bad Data Suppression technique described in section 4.4. The information obtained, using the bad data discrimination procedure described in the last section, is used initially to force the application of the non-quadratic correction only to measurements suspected of containing gross error. This improves the suppression effectiveness, speed and reliability of the algorithm[51].

Among the various non-quadratic functions suggested in section 4.4, the quadratic-square root was the one chosen due to its reported better performance[45,51,67]. Even though the problem considered here is linear, due to the "extra" nonlinearity

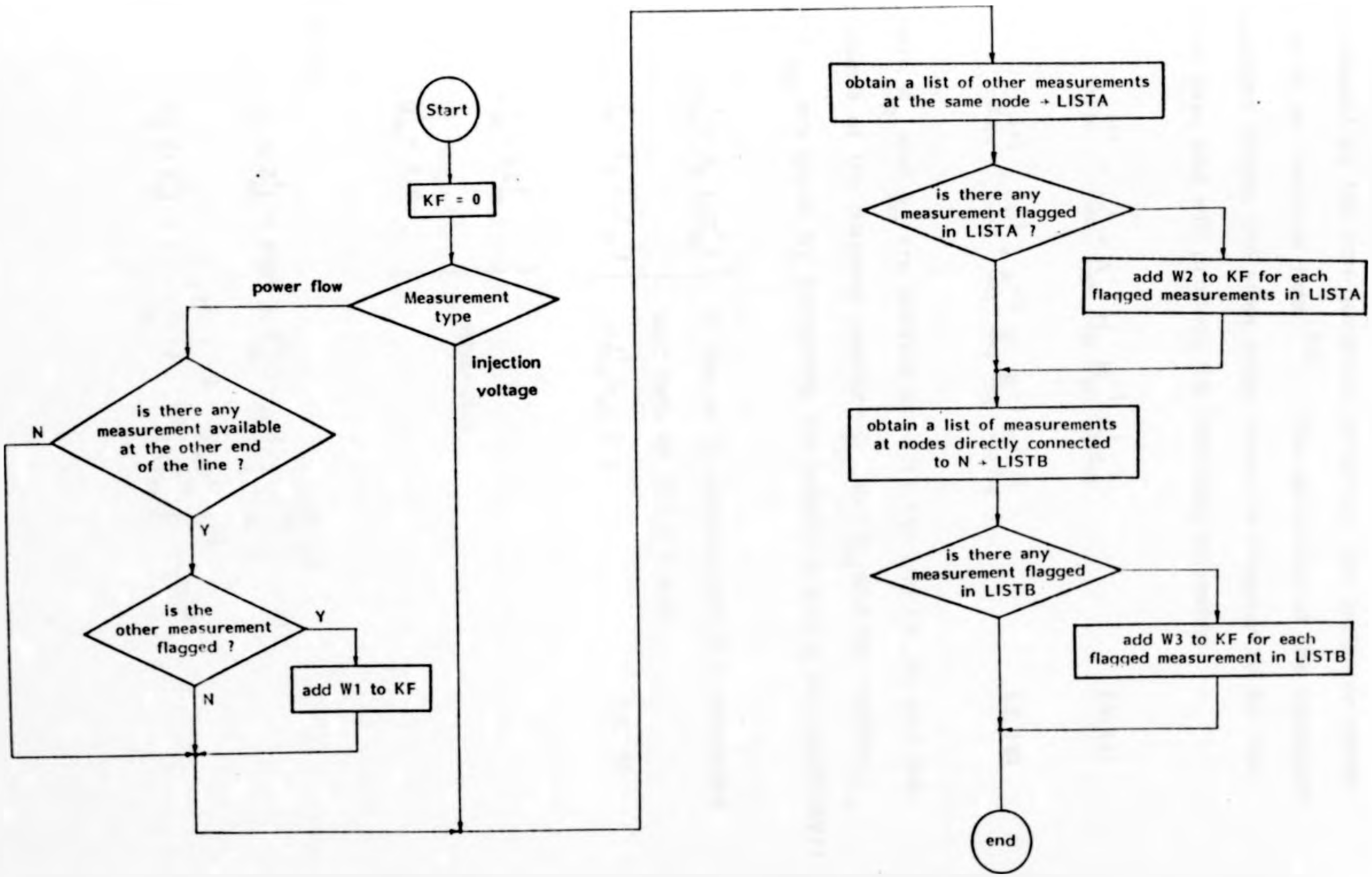


Figure 6.1 Flow chart of a routine to produce scores indicating whether a large change between predicted and measured values corresponds to a gross measurment error or a sudden change in the states (the measurement is taken at node N).

introduced by the non-quadratic criterion, the estimator needs to be in an iterative form[45]. The derivation of the estimation equations follows the same steps shown in Chapter IV for the static case and will produce the following estimator

$$\Delta \underline{\theta}_k^{i+1} = \Delta \underline{\theta}_k^{i} + \underline{A}_p^{-1} \underline{H}_p \underline{R}_p^{-1} \underline{G}_p^{i} \underline{\rho}_p^{i} \tag{6.17}$$

$$\Delta \underline{V}_k^{i+1} = \Delta \underline{V}_k^{i} + \underline{A}_q^{-1} \underline{H}_q \underline{R}_q^{-1} \underline{G}_q^{i} \underline{\rho}_q^{i} \tag{6.18}$$

where $\underline{A}_p$ and $\underline{A}_q$ are defined as in (4.19) and (4.20) and the elements of the diagonal matrices $\underline{G}_p$ and $\underline{G}_q$ and the vectors $\underline{\rho}_p$ and $\underline{\rho}_q$ are given by (dropping the indices p and q for simplicity):

$$\left.\begin{array}{l} \rho_m^i = f_1\,(\Delta Z_m^i) \\[2ex] g_m^i = f_2\,(\Delta Z_m^i) \end{array}\right\} \begin{array}{l} \text{if the m-th measurement is a suspected} \\ \text{bad data or if } i > 1 \text{ and} \\ |\Delta Z_m^i/\rho_m| > \lambda \end{array} \tag{6.19}$$

$$\left.\begin{array}{l} \rho_m^i = \Delta Z_m^i \\[2ex] g_m^i = 1.0 \end{array}\right\} \text{otherwise}$$

where

$$f_1\,(\Delta Z_m^i) = \text{sign}\,(\Delta Z_m^i)\,\lambda\sigma_m\,\left(4\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|^{\frac{1}{2}} - 3\right)^{\frac{1}{2}}$$

$$f_2\,(\Delta Z_m^i) = \left(\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|\right)^{-\frac{1}{2}}\left(4\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|^{-\frac{1}{2}} - 3\right)^{-\frac{1}{2}}$$

and

i = iteration counter

m = measurement number

λ = threshold

σ_m = assumed standard deviation of m-th measurement error

If no measurement is flagged as suspected bad data, the algorithm above reduces to a plain linear WLS estimator and, therefore, no iterative process is required. Otherwise, the iterative process will use as starting point the one obtained by the difference between the predicted values and the estimated values in the last interval, which should be very close to the solution and, therefore, reduce the number of required iterations.

The choice of the matrices $\underline{R}_p$ and $\underline{R}_q$ (whose elements are usually the inverse of the measurement error covariance in the static case) is not simple in the tracking mode. Rigorously these matrices should be time variant and include some information about the accumulated error of past estimations. However, as will be confirmed by the results of simulation studies presented in Chapter VII, constant matrices whose elements are the covariance of the measurement error for "half-of-the-scale"[31] values produce acceptable results.

6.4.3 PIECEWISE-LINEAR CRITERION (PLC) ALGORITHM

The application of PLC method described in Chapter V to the problems given by (6.9) and (6.10) is straightforward. As the model and criterion are both linear, the resulting estimator will also be linear and, therefore, not requiring an iterative approach. Each step of the tracking estimator is defined by the following LP problems:

$$\text{Minimise} \sum_{m=1}^{M_p} R_m^p \, (s_{2m-1} + s_{2m})$$

$$\text{subject to } [\underline{H}_p \quad \underline{U}_p] \begin{bmatrix} (\Delta \underline{\theta}_k)' \\ \underline{S}_p \end{bmatrix} = (\Delta \underline{Z}_k^p)' \qquad (6.20)$$

$$(\Delta \underline{\theta}_k)', \ \underline{S}_p \geq 0$$

and

$$\text{Minimise} \sum_{m=1}^{M_q} R_m^q \, (s_{2m-1} + s_{2m})$$

$$\text{subject to } [\underline{H}_p \quad \underline{U}_p] \begin{bmatrix} (\Delta \underline{V}_k)' \\ \underline{S}_q \end{bmatrix} = (\Delta \underline{Z}_k^q)' \qquad (6.21)$$

$$(\Delta \underline{V}_k)', \ \underline{S}_q \geq 0$$

where

M_p, M_q : are the number of active and reactive measurements

$(\Delta\underline{\theta}_k)'$, $(\Delta\underline{V}_k)'$: are the transformed values of the increments in the voltage angles and magnitude as defined in (5.13)

$(\Delta\underline{Z}_k^p)$, $(\Delta\underline{Z}_k^q)$: are the transformed values of the increments in the active and reactive measurements as defined in (5.14)

$\underline{U}_p$, $\underline{U}_q$, $\underline{S}_p$, $\underline{S}_q$: are as defined in (5.16) and (5.17).

The same remarks about the choice of matrices $\underline{R}_p$ and $\underline{R}_q$ made in the last section apply to the algorithm above.

If no measurement is flagged as bad data, the problems defined by (6.20) and (6.21) can be solved using the dual simplex technique described in Chapter V, in which the optimal solution of the last estimation is used as an initial basic solution. As the estimations are performed at short and regular intervals, the difference between the measurement increments $(\Delta\underline{Z}_k^p, \Delta\underline{Z}_k^q)$ from one interval to the other should be small and, therefore, by reasons already explained in Chapter V, only few iterations of the dual simplex will be required.

If at least one measurement is flagged as a suspected bad data, then a different procedure should be adopted, as explained below:

i. if the suspected measurements correspond to a slack variable in the optimal basis of the previous estimation, i.e. if it had no influence on the result of that estimation, then that optimal basis is maintained in the present estimation, which guarantees that the suspected measurement is certainly excluded from the solution. This procedure slightly impairs the optimality of the estimator, but is convenient from the point of view of computational efficiency.

ii. if the suspected measurement corresponds to slack variables not in the basis, then two procedures can be followed:

a) to perform a brand new solution of the LP problem in which these measurements are excluded by receiving a zero weight;

b) to forget about the suspected bad data and perform an ordinary dual-simplex cycle in the hope that the algorithm will by itself detect the bad data, as it usually does in the static mode of operation.

6.5 OVERALL APPROACH

In a practical on-line implementation, the algorithms described in this chapter will work together with many others like the network configurator, data validator, etc., as already pointed out in previous chapters. Also, they will interface with a database from which all information about parameters, network and metering system configuration, etc. will be obtained and to which the results of the state estimator will be directed. Finally, a certain interaction between the state estimator and the system's operators is required to sort out situations which the estimator is not able to solve by itself.

Another important aspect to be considered in an on-line implementation, is the way in which the various components of the state estimator and correlated programs are scheduled in order to perform the consecutive estimations. This aspect is particularly important if the tracking approach is used, as in that case an almost continuously updating of the state is performed.

In the next sections, these and a few other aspects of an integrated data processing scheme will be analysed. The analysis will be limited to general aspects of the problem, particularly the ones closely associated with the algorithms presented in this thesis. In a practical development, some particular characteristics of the problem, like the computer system configuration, processor features (word length, etc), operational system, data base structure, etc., should also be taken into consideration.

6.5.1 INITIALISATION

Power system monitoring is a twenty-four hour per day job. As a consequence of this requirement, state estimations should be performed in a continuous fashion. However, in periods of light load (overnight, for instance) or for maintenance purposes, the state estimator may be "switched-off" for relatively long periods of time. This may also happen as a consequence of a computer or telemetering system breakdown. Therefore, from time to time, a re-initialisation of the state estimator is required.

The main characteristic of the re-initialisation operation is that no adequate information about the recent past of the system is available. Therefore the estimator should rely solely on one measurement and status scan to produce a new estimation. This situation requires the use of the static state estimation approach.

Both tracking algorithms described in this chapter have the same basic formulation of their corresponding static algorithms. Therefore they can be easily rescheduled to operate as a static estimator by changing the input variables (the increment in the measurement should be replaced by the measurements themselves) and corresponding outputs, and allowing some iterations. This procedure should be preceded by a pre-estimation stage in which thorough checks on the income measurements and status are performed in order to identify any bad data. Possible approaches to this pre-estimation stage are the data validators proposed in references (45,51). A past-estimation bad data detection and identification module, using the tests described in section 4.3, is also required. This whole process should be repeated as many times as necessary to guarantee that this first estimation is free of any gross measurement or status error.

6.5.2 PREDICTION AND ESTIMATION SCHEDULE

Although primarily designed to produce estimations at short time intervals, it may happen that in a particular application a new estimation at every measurement scan is either not desired or not convenient, during a certain period of the estimation operation. In that case at least the predictive stage and the analysis of the changes between scans should be performed at the arrival of every new measurement scan.

A possible scheduling scheme would be to run the estimator at regular intervals (say every five scans) or when the prediction/change analysis stage detects some abnormality in the new scan. In the latter case, a tracking estimation would be performed if the abnormality turns out to be a suspected bad data, or a complete static state estimation is run if a major change in the operating point is detected.

As the time series considered for the prediction technique uses as observations the estimated values (instead of the measured values which are not reliable), the predicted parameters (level and trend) are not adjusted between estimations and, therefore, only an extrapolation of the present values are performed until new estimated values become available.

In figure 6.2 a hypothetical situation showing the main aspects of the problem discussed above is depicted. An estimated value obtained at interval zero is extrapolated until interval 5, in which another estimation and prediction are performed. At interval 9 a large deviation between the measured and predicted value is observed, which the change analysis module interprets as a bad data, and a new tracking estimation is performed before the scheduled interval.

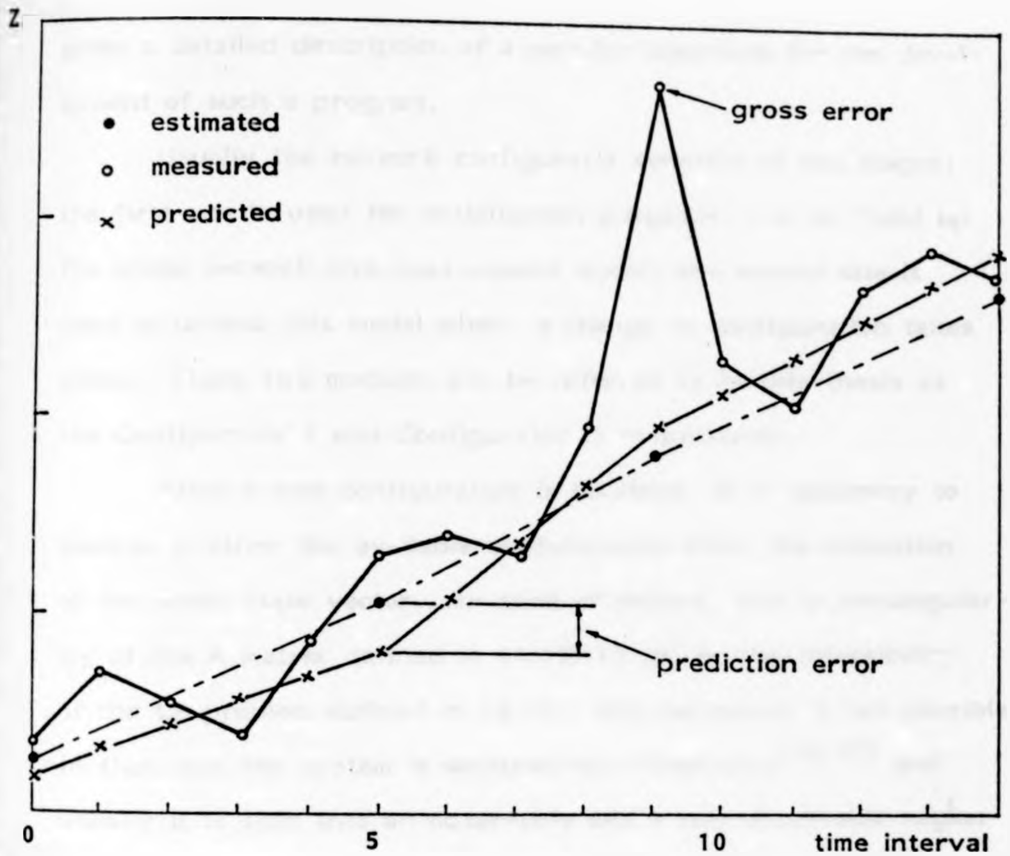


Figure 6.2: Example of prediction and estimation scheduling

6.5.3 CONFIGURATOR AND OBSERVABILITY

Whenever the estimator is initialised or a change in the status of a circuit breaker or switch takes place, a redefinition of the network topology and measurement pattern is required. This updating is performed by a program called the network configurator which processes logical variables associated with the status of circuit breakers and switches. Reference (51) gives a detailed description of a possible approach for the development of such a program.

Usually the network configurator consists of two stages: the first one is used for initialisation purposes, i.e. to build up the whole network and measurement model; the second one is used to update this model when a change in configuration takes place. These two modules will be referred to in this thesis as the Configurator I and Configurator II respectively.

After a new configuration is obtained, it is necessary to analyse whether the available measurements allow the estimation of the whole state vector. In some situations, due to nonsingularity of the A matrix defined in (4.19)-(4.20) or the infeasibility of the LP problem defined in (5.15), this estimation is not possible. In that case the system is declared non-observable[38,66] and usually it is split into an observable and a non-observable region and the estimation is then performed only for the observable one.

6.5.4 POST ESTIMATION BAD DATA ANALYSIS

The results of every tracking estimation should be tested, using one of the detection tests described in section 4.3.2, in order to assess if it is free from the effect of possible bad data. This test is necessary, even when non-quadratic estimators are used, because in some circumstances the estimators may fail in suppressing a bad data, for instance, if this bad data occurs in a region in which the local redundancy ratio is low.

If bad data is detected by the above test, the safest procedure is to switch to the static estimator as the resources of the tracking approach have been fully used at this stage. However, this situation is bound to occur very infrequently as it requires the bad data escaping a very hard suppression procedure.

6.5.5 CONFIGURATION ERROR

If a sudden change in the system state was detected by the change analysis module and, after a static state estimation is performed, bad data is still detected by the procedure described in the previous section, that is a strong indication that there is an error in the configuration.

As a result of the bad data analysis performed after every static estimation, some measurements may be deleted. A procedure like the one proposed in reference (59) and reviewed in section 4.5 can then be used to sort out whether these deleted measurements really contain gross error or are the result of a configuration error. In the last case, a warning should be given to the operator in order to check the suspected status and rerun the network configurator.

6.5.6 GENERAL FLOW CHART

In figure 6.3 a general flow chart of an on-line state estimator, incorporating all the features described in this and previous chapters, is presented. In that flow chart the flow of information, i.e. the way in which the data is used by the various modules of the program is stored and exchanged, is omitted for reasons of simplicity. This aspect of the problem is often highly dependent on the particular computer system in which the estimator is going to be implemented and, therefore, requires special attention in each particular application.

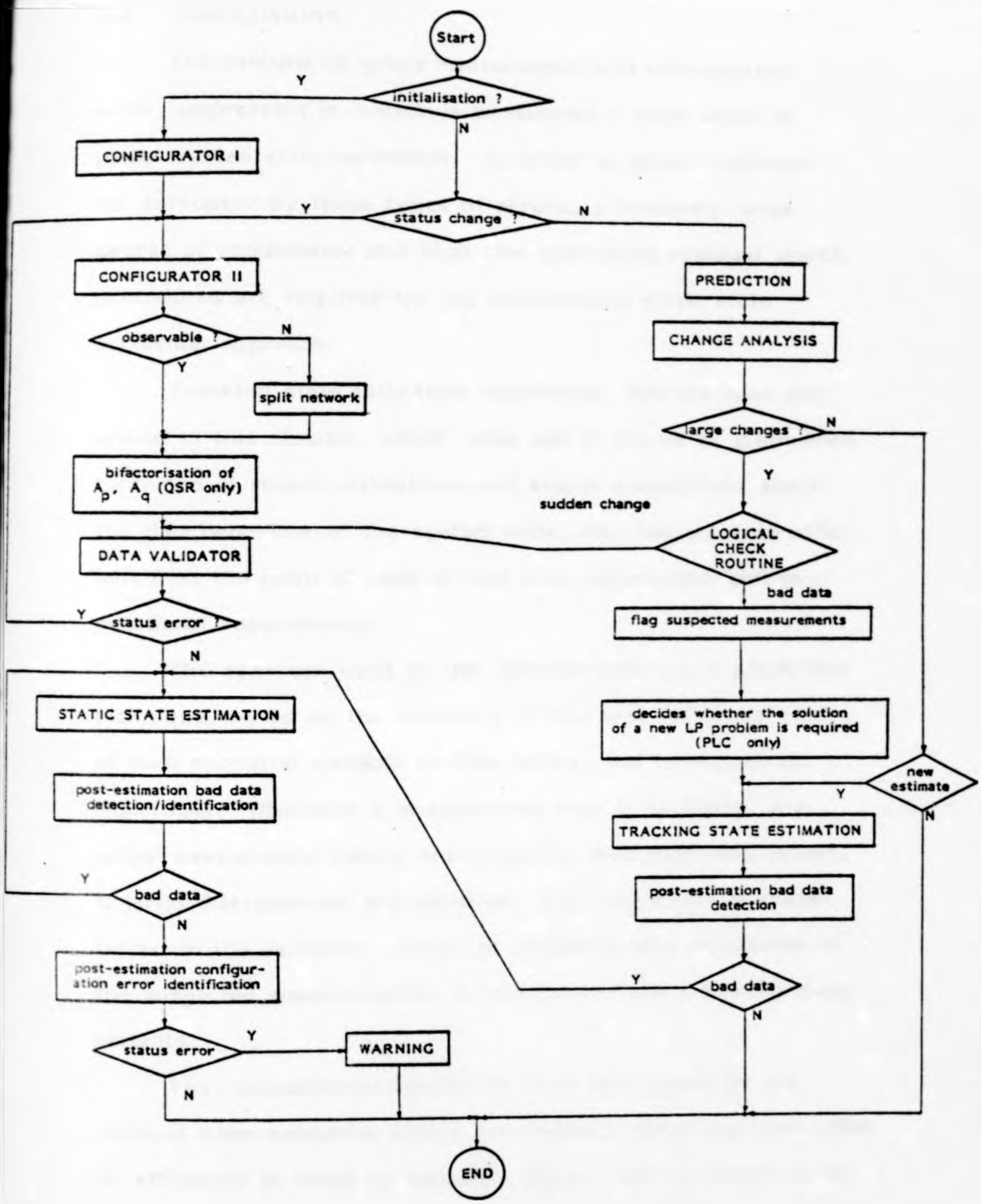


Figure 6.3: Flow chart of an integrated static/tracking state estimator

6.6 CONCLUSIONS

The problem of gross measurement and configuration error suppression is nowadays considered a major issue in power system state estimation. In order to obtain estimates not corrupted by these types of errors, a relatively large degree of redundancy and high time-consuming residual search procedures are required by the conventional static state estimation approach.

Tracking state estimation algorithms, like the ones described in this chapter, which make use of the extra information contained in recent estimations and simple assumptions about the time behaviour of the system state, may become more efficient from the point of view of bad data suppression and computational requirements.

The approach used in this chapter combines a predictive technique, based on the modelling of the past estimated values of each measured variable as time series, and non-quadratic algorithms. Whenever a measurement scan is available, the actual measurement values are compared with predicted values. If large discrepancies are detected, this information is transferred to the estimator, which by assigning less importance to the suspected measurements, is able to produce a healthy state estimate.

The computational requirements of the algorithms are reduced when compared with a conventional static approach while its efficiency in terms of bad data suppression is increased, as can be checked by the results of simulation studies reported in Chapter VII.

CHAPTER VII

RESULTS OF THE SIMULATION STUDIES

In this chapter the results of a comprehensive performance study of the algorithms developed in the research project reported in this thesis, which were described in Chapters V and VI, in comparison with algorithms previously existent, which have been reviewed in Chapter IV, is presented. The study was performed using simulated data and the approach used was the Monte Carlo technique. General comments about the overall results are presented together with samples of the numerical values obtained.

7.1 INTRODUCTION

A definitive test for a power system state estimation method can only be made in an on-line environment. This is so because, as pointed out in previous chapters, no adequate model for the load-generation matching process is available. Also some additional elements, like the statistical properties of the measurement noise, frequency of occurrence and size of gross measurement error, the probability of occurrence of unreported status change, etc. are not yet well established. Therefore, it is not possible to reproduce accurately in a simulated environment the situations that the estimator will face in practice. However, as a first step in assessing the potentialities of a newly developed technique, it is reasonable to perform some tests using simulated data. Moreover, if two different methods are tested using the same set of data, a fair comparison of the performance of the methods is likely to be achieved.

The most used method to assess the performance of a state estimator algorithm using simulated data is the Monte Carlo approach[82]. In this method, a random numbers generator is used to produce the measurement error which is added to the results of an exact calculation of the measured values corresponding to an arbitrary"true" state. Afterwards a state estimation is performed using as input the "corrupted" measurements and the resulting state estimate is then compared with the true values. The process is repeated a relatively large number of times (20 or 30), with different sets of random numbers, to ensure that the results have some statistical significance.

The choice of the parameters of the simulation, like the size of the measurement noise, the rate of change of system variables, etc., is difficult as relatively little practical experience is available and a lot of intuition should be used. Very severe situations will probably eliminate methods which would have an adequate performance in average conditions while mild situations could hide possible weaknesses of the method.

Another difficult point in a study like the one reported in this chapter, is the choice of the sample of results to be shown in a limited space among the large amount of results obtained throughout the study. The procedure adopted in this thesis is to concentrate the results in the form of general comments and to exhibit some numerical values with the only objective of illustrating these comments rather than as proof or conclusive evidence.

7.2 SIMULATION OF THE MEASUREMENT SYSTEM

The input data to a state estimator is the network parameters and configuration, the measurement pattern (type and location of the measurements) and the values of telemetered variables. In an on-line environment these data would be available in the data-base which is updated from time to time by the network configurator, the data acquisition system and the operators. In a simulated study these data have to be generated by an auxiliary program (simulator) and stored in some files which play the role of a data base.

The central piece of the simulator is a load flow program which produces the true values of the state and measured variables. Other components are routines to specify the required measurement pattern, to add errors (noise and bad data) to the true values of the measurement and to simulate the time evolution of the system. A flow chart of the simulator is given in Figure 7.1 and in the following sections some of the parameters used in the simulation study reported in this chapter are also given.

7.2.1 Simulation of the system time evolution

The time evolution of the system static state is simulated by the calculation of successive load flows in which load and generation vary from initially given values. Each one of these load flows corresponds to an operating point of the system at the moment in which a measurement scan is taken. The load curve at each busbar is composed of a trend (linear, quadratic, etc.) plus a random fluctuation. The rate of change of the load

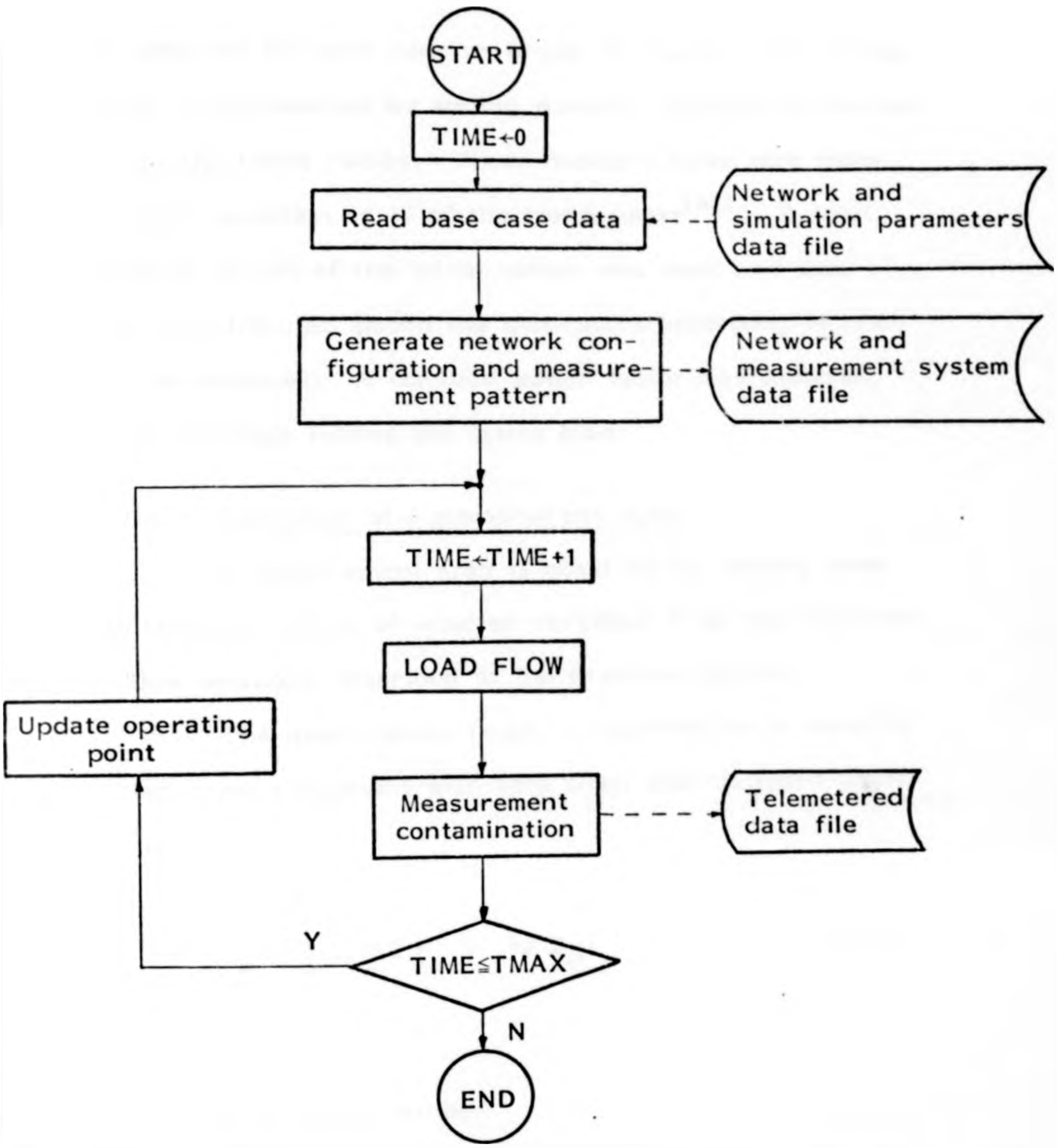


Figure 7.1: Flow chart of the simulation program

is made different for each node or group of nodes. The random fluctuation is represented by adding normally distributed random numbers to the trend values. These numbers have zero mean and standard deviation of 1% of the trend value[62]. A total load variation of 50% of the initial values was used and this variation is distributed among the generators according to pre-specified percentages. A constant power factor was assumed, so the reactive load follows the active load.

7.2.2 Simulation of a measurement scan

A measurement scan is obtained by adding some error to the true values of selected variables from the sequence of load flow solutions described in the previous section.

The measurement noise is simulated by a normally distributed error component with zero mean and standard deviation given by

$$\sigma_m = \frac{1}{3}\,(.005\ F_m + .02\ Z_m) \tag{7.1}$$

where

- m : measurement number
- F_m: meter full-scale
- Z_m: actual value of the measurement

The coefficients in equation (7.1) were set taking into consideration reported error bounds for telemetering equipment[28].

Gross measurement error is simulated by setting some chosen measurement to values which differ from the true values by 50% to 100%. The number and location of the bad data was chosen based on a judgement of the relative difficulty of detection. Three standard situations were always included in the studies: a) one isolated gross error; b) more than one uncorrelated gross errors, i.e. gross error in measurements electrically far away from each other; c) correlated gross errors.

Configuration errors were simulated in the estimation programs themselves by altering the values of network parameters in such a way as to simulate the addition or subtraction of a line.

7.3 PERFOMANCE ASSESSMENT

The performance of the algorithms in the simulation studies as already commented in the introduction to this chapter, is assessed by comparing the estimated values of the measured variables and the respective true values. However, the difference between estimated and true values alone is not enough to characterise the performance of the estimator. A comparison of the size of this difference with the error present in the measurements is necessary in order to assess whether the result of the estimation constituted an improvement over the raw data.

In order to quantitatively measure the quality of the estimator results, the following performance indicators were used:

$$R^{E}_{ave} = \frac{1}{M} \sum_{i=1}^{M} \frac{z^{E}_{m} - z^{T}_{m}}{\sigma_{m}} \qquad (7.2)$$

$$R^{M}_{ave} = \frac{1}{M} \sum_{i=1}^{M} \frac{z^{M}_{m} - z^{T}_{m}}{\sigma_{m}} \qquad (7.3)$$

$$R^{E}_{max} = \max_{m=1,M} \frac{z^{E}_{m} - z^{T}_{m}}{\sigma_{m}} \qquad (7.4)$$

$$R^{M}_{max} = \max_{m=1,M} \frac{z^{M}_{m} - z^{T}_{m}}{\sigma_{m}} \qquad (7.5)$$

where

M : number of measurements

z^{E}_{m}, z^{M}_{m}, z^{T}_{m} : estimated, measured and true values of the m-th measurement

σ_{m} : standard deviation of the m-th measurement error

The above quantities give an absolute measure of the measurement and estimation errors. Relative indicators of the estimator performance can be obtained dividing corresponding quantities as follows:

$$R_{ave} = \frac{R^{E}_{ave}}{R^{M}_{ave}} \qquad (7.6)$$

$$R_{max} = \frac{R^{E}_{max}}{R^{M}_{max}} \qquad (7.7)$$

The values obtained for these indicators can be interpreted as follows:

R_{ave}, R_{max}:

- $\gg 1.0$: the result of the estimation is worse than the original data
- $\cong 1.0$: the result of the estimation is about the same as the original data
- $\ll 1.0$: the result of the estimation is better than the original data. If bad data were present in the measurements, it means that the bad data were suppressed.

7.4 COMPUTATIONAL FACILITIES

The simulation studies reported in this chapter were performed in a general purpose 65K/265K, 60-bit word, CDC 7600 computer. This machine does not have integer-variable packing facilities, i.e. an integer or a real variable use the same word length. Therefore, storage requirements reported for the algorithms should be viewed as an upper limit, as in machines which have the packing facility it is bound to be smaller. The installation operates in a time share mode which implies that the CPU times recorded for the algorithms should be considered as average values.

7.5 TEST SYSTEMS AND MEASUREMENT PATTERNS

The systems used in the simulation studies were the IEEE 14, 30, 57 and 118 busbar standard load flow test systems.

Several measurement patterns were defined for each system by choosing different combinations of voltage magnitudes, injections and line-flows as measurements, with redundancy ratios varying in the range 1.6 to 3.3, which are limit values likely to be found in practice[78]. Table 7.1 shows some information about the test systems as well as two different measurement patterns for each system, one corresponding to a low and the other to a high redundancy ratio. These two measurement patterns are the ones associated with the results that will be shown in later sections of this chapter.

System	No.of lines	Max. R/X	Number of measured voltages	injections	flows	(*) Redundancy
14 BB	20	.51	2 2	4 4	40 80	1.7 3.2
30 BB	41	1.11	6 6	12 12	82 164	1.7 3.1
57 BB	80	1.09	7 7	14 14	160 320	1.6 3.0
118 BB	186	.47	10 10	18 18	372 744	1.7 3.3

Table 7.1: Test systems and measurement patterns

(*) The redundancy ratio is obtained by dividing the number of measurements by the number of state variables.

7.6 PERFORMANCE COMPARISON OF THE PLC, BDS AND WLS ALGORITHMS

The simulation studies involving the improved version of the Piece-wise Linear Criterion (PLC) estimator described in Chapter V were performed with the following objectives:

i. to assess the bad data suppression capability of the algorithm, both in its basic formulation and in the decoupled version, relatively to the BDS estimator which is the only other similar approach available in the literature.

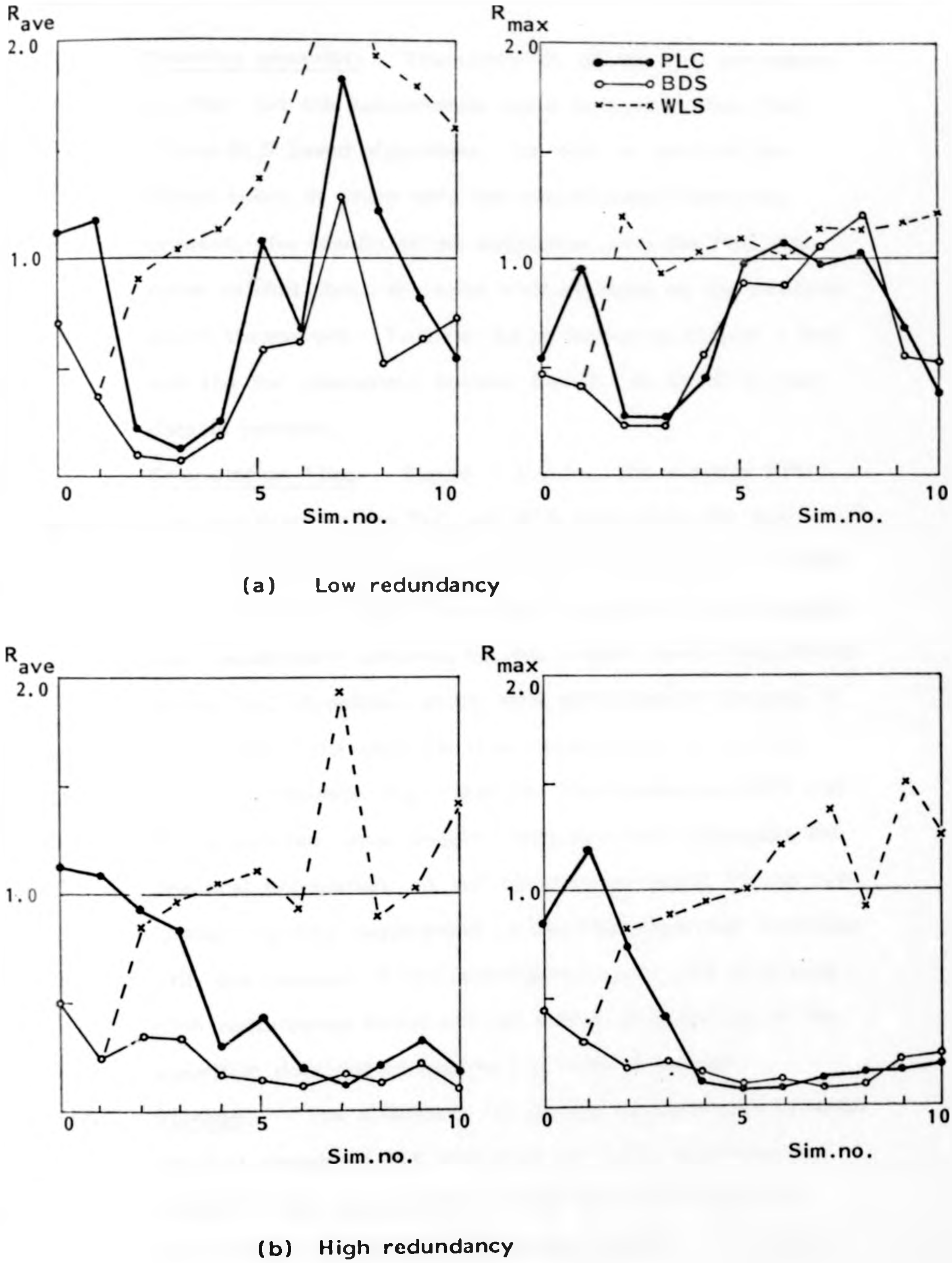
ii. to compare the computational performance of the algorithm in terms of time and storage requirements and reliability with other state estimation approaches.

For each of the test systems and measurement patterns shown in Table 7.1, a total of twenty simulations were performed, each one corresponding to a different set of measurement errors (noise and bad data). In each one of these simulations, estimations were performed using the improved version of the PLC estimator, the basic PLC, the basic WLS, the fast decoupled WLS and the fast decoupled BDS. From the results of these studies, the following conclusions were drawn:

i. <u>Bad data suppression capability</u> Both basic and improved versions of the PLC algorithm showed results similar to BDS estimator. Both approaches have an adequate performance for the cases of high redundancy (say over 2.0) in which the estimators were able to suppress bad data in more than 90% of the simulated situations. In the cases of low redundancy, the rate of total or partial failure was about 50%, most of the cases corresponding to interactive bad data. Almost no difference was noticed between the performance of the basic and decoupled versions of the PLC estimator which confirms assumptions made in Chapter V. In Figure 7.2, the results of some simulations performed with the IEEE 30-busbar system are shown. These results are typical of the ones found throughout the study. Table 7.2 below gives an indication of the type and quantity of bad data associated with each of the simulations depicted in Figure 7.2.

Simulation	Bad Data
0	None
1	None
2	Single
3	Single
4	Two non-interactive
5	Two non-interactive
6	Four non-interactive
7	Two interactive
8	Four interactive
9	Four interactive
10	One line with wrong status

Table 7.2: Type and quantity of bad data present in the simulations for which results are shown in Figure 7.2.



(a) Low redundancy

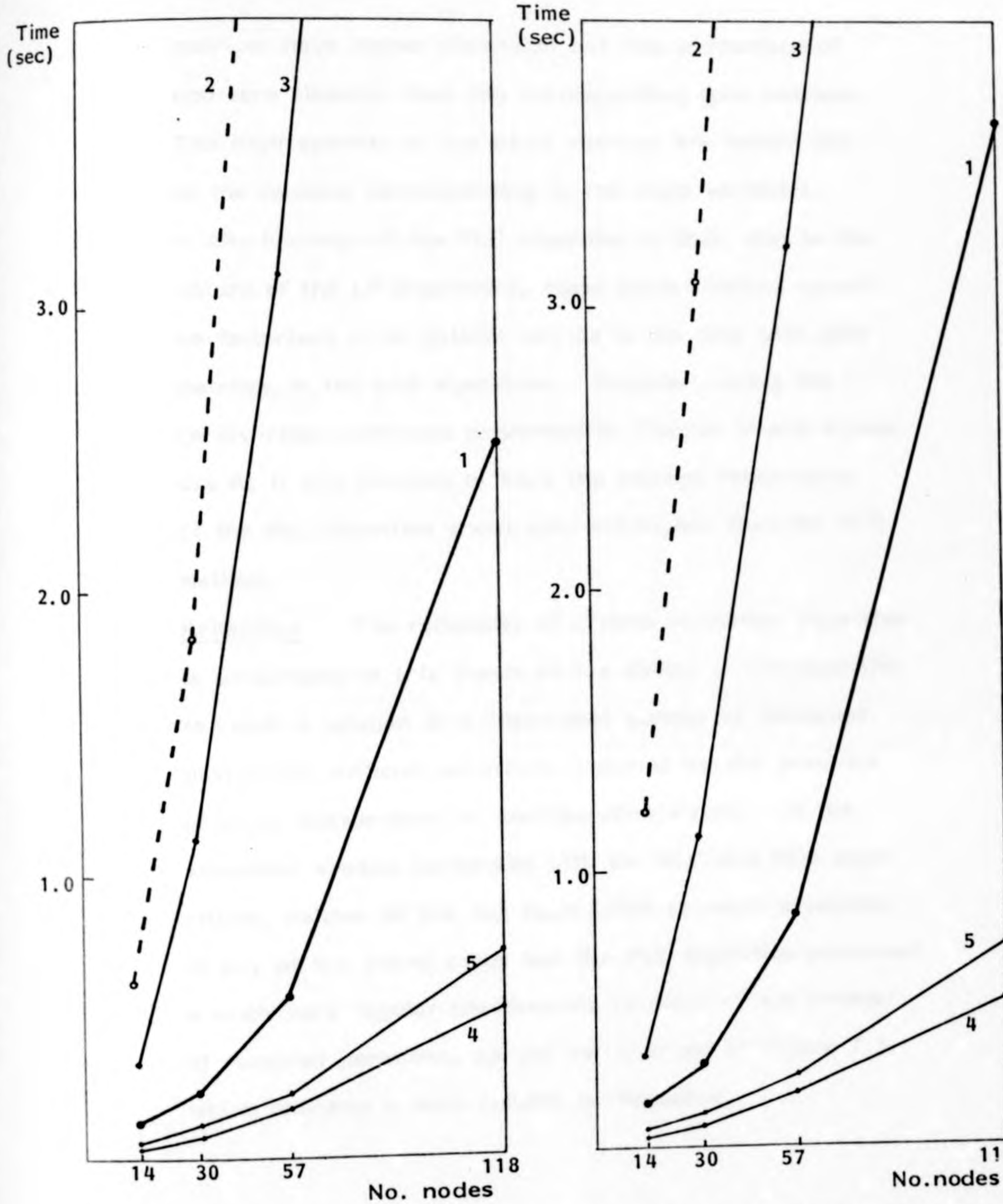
(b) High redundancy

Figure 7.2 : Comparison of the performances of the LPC, BDS and WLS estimators. Test system IEEE 30-busbar.

ii. Filtering capability The capability of the PLC estimators to filter out the measurement noise is weaker than that of the WLS based algorithms. In fact, in most of the tested cases in which only the measurement noise was present, the results of the estimation with the PLC algorithm exhibit about the same level of noise as the measurements themselves. This can be observed in Figure 7.2(a) and (b) for simulations number 0 and 1 in which no bad data is present.

iii. Computation Time Figure 7.3 shows the average CPU time required by the PLC and WLS algorithms for the test systems and measurement patterns contained in Table 7.1. From this figure the large reduction in the execution time requirement achieved by the improvements introduced in the PLC algorithm, which were described in Chapter V, is evident. Although the time requirement of the PLC algorithm remains larger than the fast decoupled BDS and WLS algorithm, these requirements are now acceptable for practical application. A last observation about Figure 7.3 is that the time requirement of the PLC algorithm increases with the increase in the redundancy ratio. As in practice high redundancy ratios are not usual, this feature of the algorithm does not constitute a serious drawback.

iv. Storage The difference in storage requirements between the fast decoupled PLC and WLS (or BDS) algorithm is located in the space used to store the basis matrices in the first and the gain matrix in the second. The basis

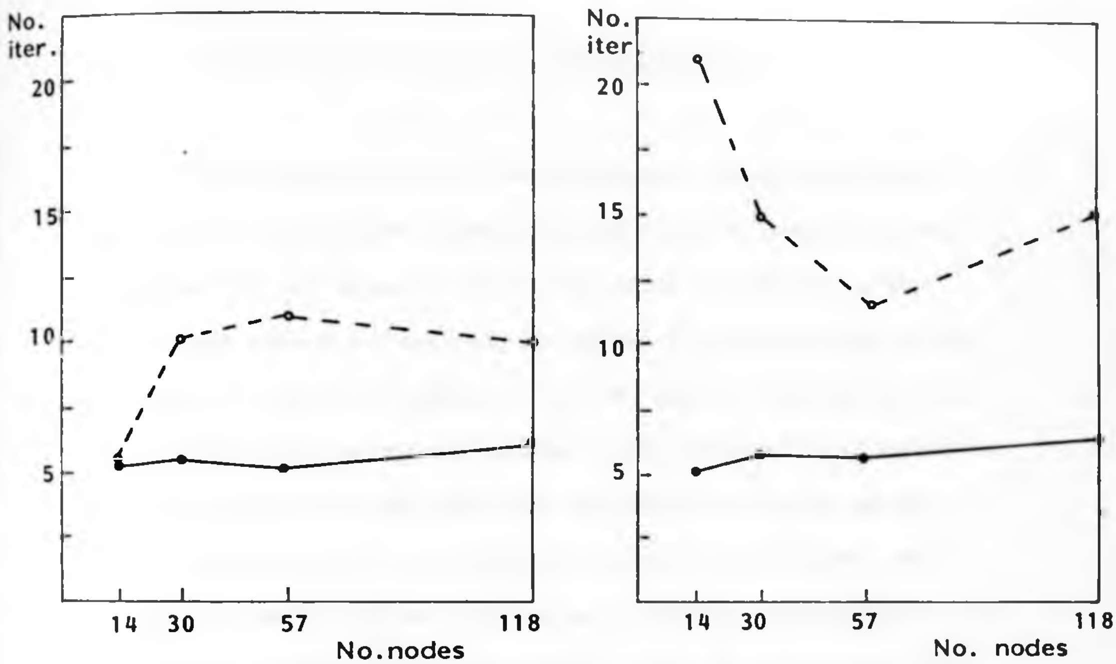


(a) Low redundancy (b) High redundancy

Figure 7.3: Execution time comparison for the improved PLC(1) basic PLC(2), basic WLS(3), fast decoupled WLS(4), fast decoupled BDS(5).

matrices have higher dimension but less percentage of non-zero elements than the corresponding gain matrices. The high sparsity of the basis matrices are mainly due to the columns corresponding to the slack variables. A disadvantage of the PLC algorithm is that, due to the nature of the LP algorithms, these basis matrices cannot be factorised in an optimal way as is the case with gain matrices in the WLS algorithms. However, using the re-inversion technique presented in Chapter V and Appendix A, it was possible to keep the storage requirement of the PLC algorithm about only 40% higher than the WLS method.

v. Reliability The reliability of a state estimation algorithm is understood in this thesis as the ability of the algorithm to reach a solution in a reasonable number of iterations, despite the difficult conditions imposed by the presence of gross measurement or configuration errors. In the simulation studies performed with the PLC and BDS algorithms, neither of the two have failed to reach a solution in any of the tested cases but the PLC algorithm presented a much more regular performance in terms of the number of required iterations, as can be observed in Figure 7.4, which indicates a more reliable performance.



(a) Low redundancy (b) High redundancy

Figure 7.4: Comparison of the number of iterations required by the PLC and BDS algorithms in cases in which there are a relatively large number of bad data (convergence tolerance: 10^{-4} pu)

7.7 TRACKING STATE ESTIMATOR RESULTS

The main objective of the simulation studies performed with the tracking state estimators described in Chapter VI was to assess the performance of the approach in relation to the static state estimation method, in terms of bad data suppression capability. Secondary objectives of the study were the comparison of the performance of the two developed tracking algorithms (PLC and BDS) and evaluation of computational requirements.

As shown by the results of the previous section, and as already commented on in previous chapters, the difficulty in bad data detection and elimination using the static approach occurs mainly in the case of low redundancy ratio system. Therefore the simulation studies reported in this chapter concentrated on the low redundancy system described in Table 7.1.

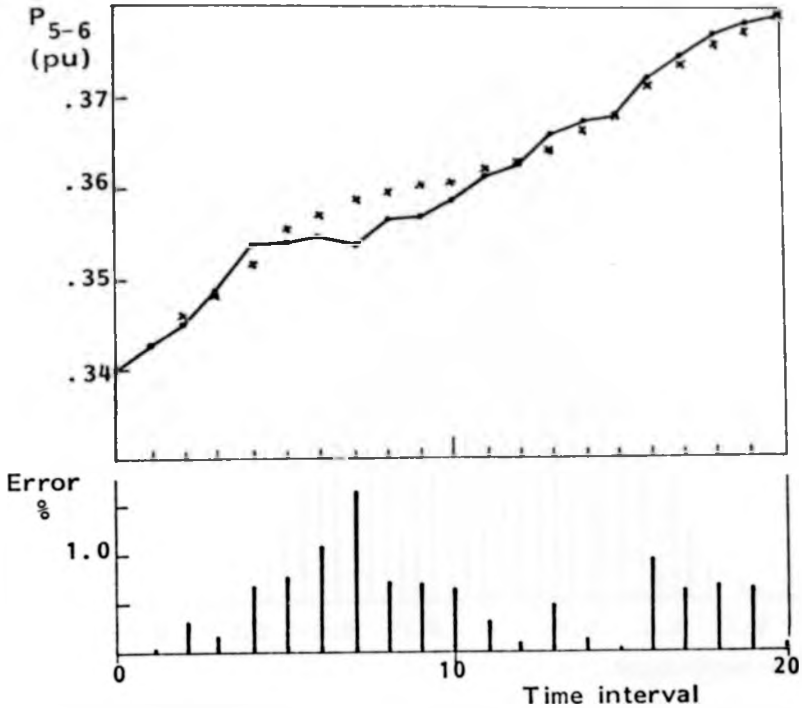
In sections 7.7.1 and 7.7.2 some comments about the prediction and logical search procedure are presented in order to individually analyse their performance and in section 7.7.3 the overall performance of the tracking state estimators is discussed.

7.7.1 Prediction technique results

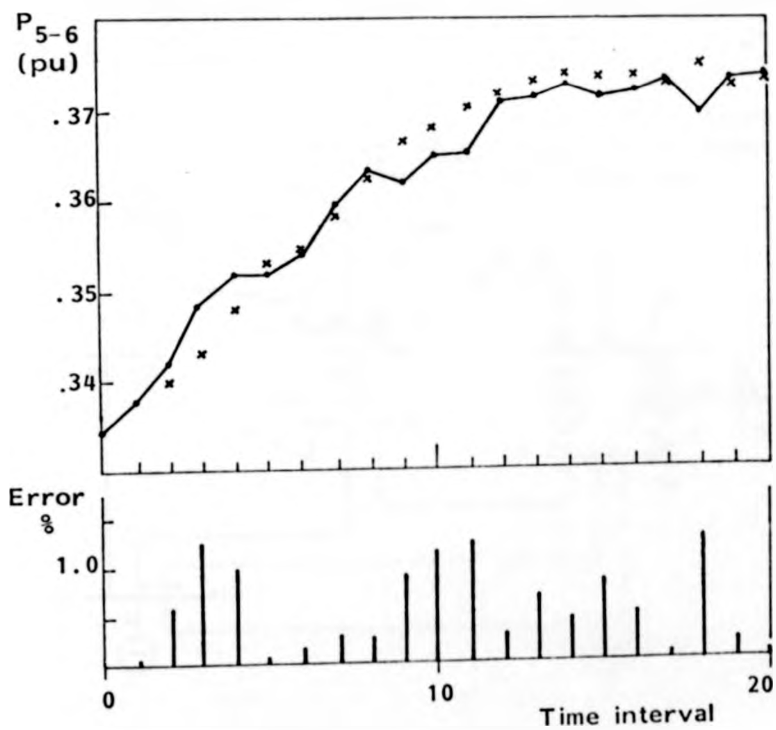
The results obtained using the predictive technique described in section 6.3 can be considered adequate for the purposes of bad data detection and identification. In all the simulations performed, errors greater than 1% (which is the size of the random component of the load) seldom occurred and the maximum error observed was less than 5%. Therefore the simulated bad data (which was set between 50% and 100% of the measurement size) were easily spotted. These results apply to all the shapes of the load trend component tested (linear, quadratic, sinusoidal and exponential). Several values of the smoothing constants α and β in (6.12) and (6.13) were tested and the value of 0.3 for both constants was found to produce the best results. The difference between measured and estimated variables, in the cases in which no bad data was present, has a distribution of probability close to the normal in most of the observed cases which confirms the assumption used in Chapter VI. In Figures 7.5 and 7.6 typical samples of the results obtained using the prediction technique are shown.

7.7.2 Bad Data discrimination results

The bad data discrimination technique described in section 6.4.1, which has the function of differentiating between a situation of sudden change in the system state and gross error in the measurements, worked in almost all the cases of single or multiple non-interactive bad data, but has a fairly high rate of failure in the cases of multiple interactive bad data, particularly



(a) Linear trend



(b) Sinusoidal trend

Figure 7.5: Estimated and predicted values of power flow in line P_{5-6} of the IEEE-14 busbar system and corresponding percentage error.

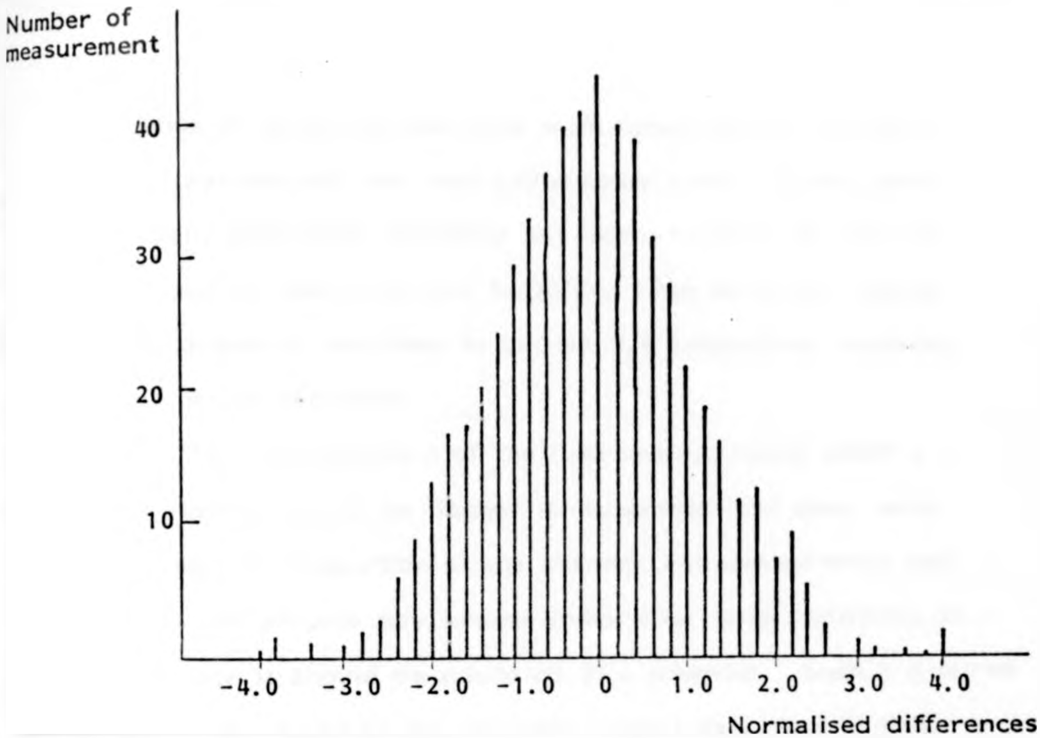


Figure 7.6: Statistical distribution of normalised differences between measured and predicted values in a sample of a simulation using the IEEE-118 system, low redundancy, linear trend.

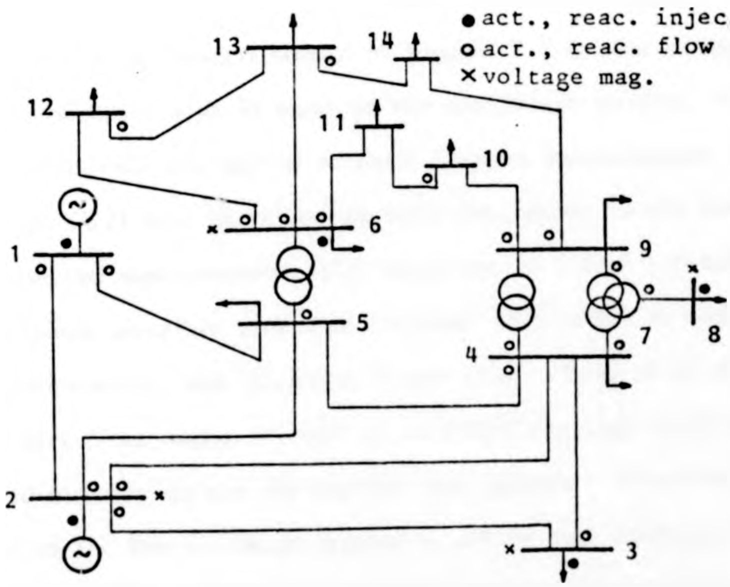


Figure 7.7 IEEE 14-busbar system showing the measurement pattern used in the example of the logical search routine

the ones in which the bad data were concentrated in regions of the system with low local redundancy ratio. These cases, however, constitute situations not likely to occur in practice and some of them could not be solved even using the logical residual search described in section 4.4 followed by repeated runs of the estimator.

The determination of the limit scores, below which a measurement should be flagged as suspected bad data, were performed by inspection of the network and measurement pattern. The process may become tedious for large networks, in which case it should be automated in a program. Such a program was not developed in the research project described in this thesis and is left as a suggestion for further research.

In order to illustrate the behaviour of the bad data discrimination routine, as well as the determination of the limit scores, an example involving the IEEE 14-busbar system, with the measurement pattern shown in Figure 7.7 is now presented. For this example, and in most of the simulation studies, the number of points attributed to each flagged measurement at the same node (W2) and in nodes directly connected to the node of the suspected measurement (W3) were set to 2 and 1 respectively. The maximum possible scores associated with nodes in which there are measurements, are given in Table 7.3. This table indicates that, apart from nodes 11 and 12, a relatively high degree of local redundancy exists throughout the system. Therefore it was decided to fix the maximum scores based on the situation in which at least one other measurement in the same node and two in neighbouring nodes are flagged, i.e. a maximum score of 4 to characterise a bad data except in nodes 11 and 12 in which this maximum score was made equal to 2.

Node	Number of measurements		Maximum possible scores
	at the same node	at neighbouring node	
1	2	5	9
2	3	9	15
3	1	7	9
4	2	11	15
5	0	14	14
6	3	3	9
7	1	5	7
9	1	6	8
11	0	2	2
12	0	2	2
13	0	5	5

Table 7.3: Maximum scores for the example using the bad data discrimination technique

Two situations in which large discrepancies between predicted and measured values were observed, are included in this example. In the first one, gross measurement equal to 50% of the true value was introduced in the measurements of the power flows of lines 5-6 and 6-13. In the second one, a sudden change in the system operating point was simulated by setting the load at busbar 3 to zero. The results of the discrimination routine is shown in table 7.4, in which N1 corresponds to the number of flagged measurements in the same node and N2 is the corresponding number for neighbouring nodes.

Case	Large changes	N1	N2	Score	Decision
1	flow in 5-6	0	1	1	Bad data
	flow in 6-13	0	1	1	
2	injection in 1	2	2	6	sudden change
	injection in 2	1	5	7	
	injection in 3	1	2	4	
	flow in 1-2	2	2	6	
	flow in 1-5	2	2	6	
	flow in 2-3	1	5	7	
	flow in 3-4	1	2	4	

Table 7.4: Results of the example using the bad data discrimination procedure (only active measurements are considered)

7.7.3 Overall approach results

For each one of the four low redundancy measurement systems shown in Table 7.1, a total of 10 simulations were performed using different sets of measurement noise and bad data. Different shapes of the load trend (linear, quadratic, sinusoidal, etc.) were also used. In each simulation a total of 30 measurement sets, corresponding to equal number of measurement scans, were generated. The bad data were introduced in the time intervals 10 and 20. The system state was estimated for the whole 30 interval period using the PLC and QSR (or BDS) tracking estimators described in section 6.4. At the time intervals 0, 10, 20 and 30 complete static state estimation (see Figure 6.3) was performed using the fast decoupled state estimator and logical residual search described in sections 4.3 and 4.5. From the results of these studies, the following conclusions were drawn:

i. Bad data suppression capability Both PLC and QSR tracking estimators showed a percentage of success in suppressing bad data superior to 90% of the tested cases against 75% for the plain static approach. Some of the cases in which the tracking estimators succeeded and the static one failed, correspond to relatively simple cases of interactive bad data which have a fair chance of occurrence while the 10% of the cases in which both approaches failed corresponds to situations less likely to occur in practice. No substantial difference was observed in the performance of the PLC and QSR tracking estimators.

In Figures 7.8 and 7.9, two examples of the results obtained in the studies are shown. These results correspond to the most interesting group of results, i.e. the ones in which the tracking algorithms succeeded while the static estimator failed. The type, size and place of the bad data simulated in these examples are shown in Tables 7.5 and 7.6.

Time interval	Bad data
10	± 50% error in act./react. injection at node 1 and act./react. flows in line 1-3
20	± 50% error in act. /react. power flows in lines 1-3 and 1-2

Table 7.5: Simulated bad data in the example shown in Figure 7.8.

Time interval	Bad data
10	± 50% in act./react. injections at nodes 1 and 3
20	± 50% in act./react. injection at node 7 and in flows of lines 6-7 and 7-8

Table 7.6: Simulated bad data in the example shown in Figure 7.9.

ii. Tracking and filtering capability Both algorithms were able to follow the time evolution of the systems for all the types of trends experimented. As can be seen in Figures 7.8 and 7.9, in the intervals in which no bad data is present, a filtering performance more or less similar to the one shown by the corresponding static approach was observed. This fact indicates that no substantial loss in that respect should be expected from the use of the tracking approach.

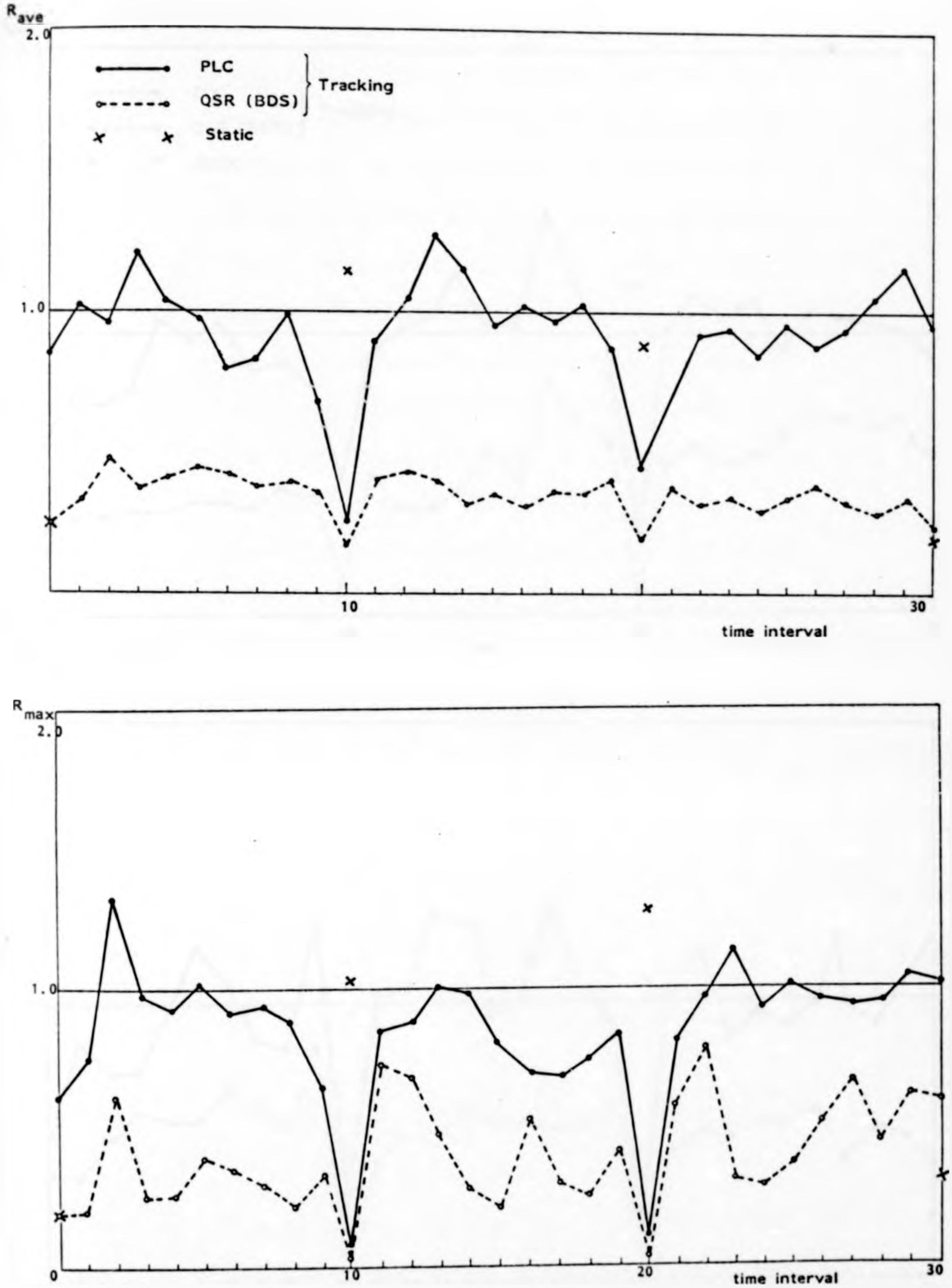


Figure 7.8: Comparison of the performance of the QSR and PLC tracking estimators with the static estimators. The static estimation is performed at intervals 0, 10, 20 and 30. Bad data is introduced in the measurement scans at intervals 10 and 20. Test system: IEEE 30-busbar, linear trend.

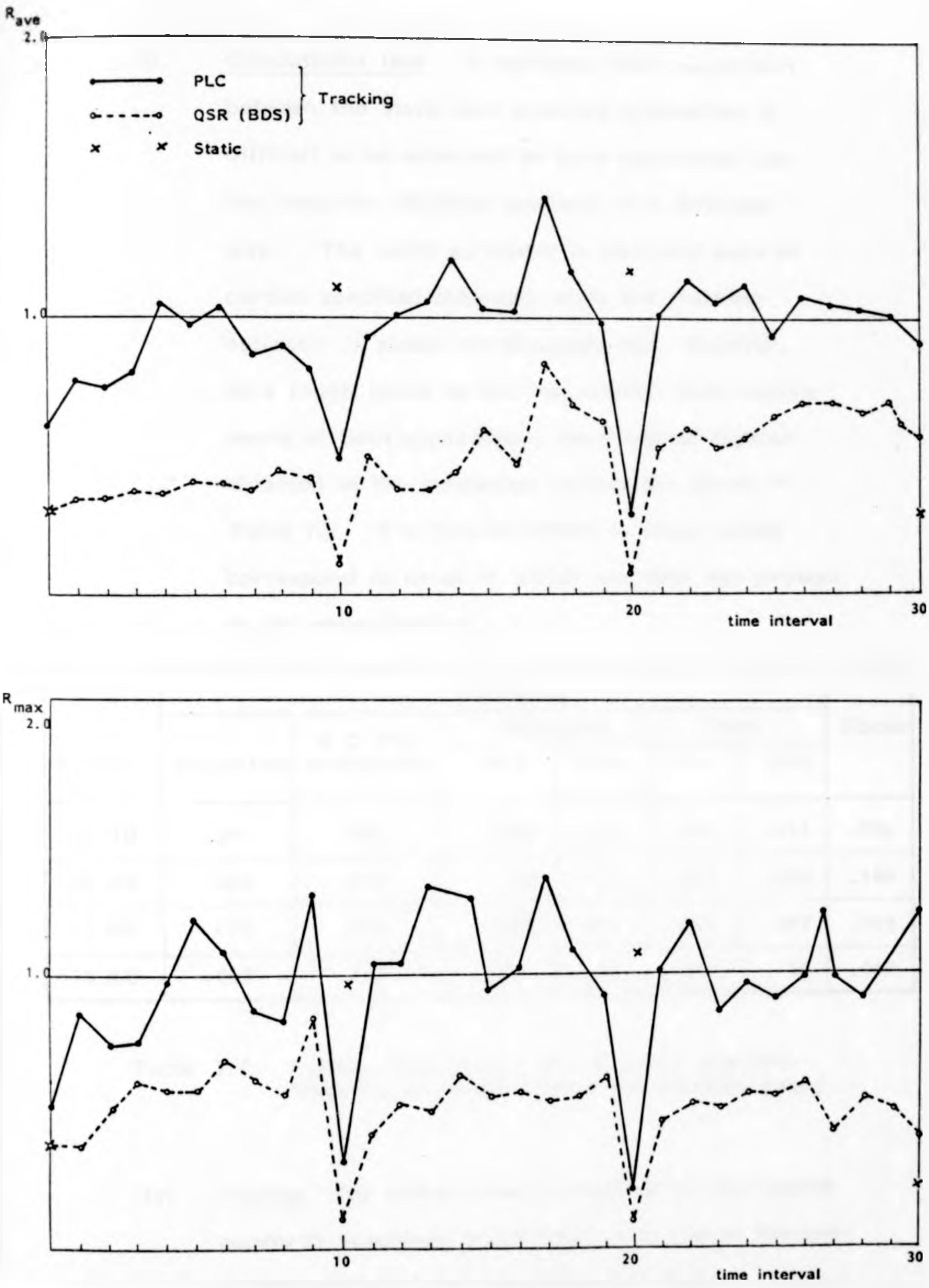


Figure 7.9: Comparison of the performance of the QSR and PLC tracking estimators with the static estimator. The static estimation is performed at intervals 0, 10, 20 and 30. Bad data is introduced in the measurement scans at intervals 10 and 20. Test system IEEE 57-busbar, sinusoidal trend.

iii. Computation time A definitive time comparison between the static and tracking approaches is difficult to be achieved as both approaches use the computer facilities available in a different way. The static estimator is executed once at certain specified intervals while the tracking estimator is almost continuously run. However, as a rough guide to the computation time requirements of both approaches, some typical figures obtained in the simulation studies are given in Table 7.7. The results shown in these tables correspond to cases in which bad data was present in the measurements.

System	Tracking						Static
	Prediction	B.D.Discrimination	Estimation		Total		
			PLC	QSR	PLC	QSR	
14 BB	.001	.005	.009	.009	.015	.015	.080
30 BB	.002	.010	.020	.013	.032	.025	.190
57 BB	.004	.023	.039	.030	.066	.057	.310
118 BB	.009	.057	.214	.114	.280	.180	.987

Table 7.7: Typical computation time for PLC and QSR tracking estimators and state estimator (sec.)

iv. Storage The extra storage required by the overall approach described in section 6.5 is due to the prediction and bad data discrimination routines. The requirements of the prediction routine is small as only the result of the last prediction is required by the exponential smoothing technique. The requirement of the bad data discrimination routine is also small as it requires few extra arrays for the measurement flags. The basic data used by the discrimination routine (feeder and measurement tables) have to be used by other components of the overall estimation approach and therefore does not increase the overall requirement. In the programs developed the total extra storage does not exceed 10%.

v. Reliability The PLC tracking estimator is linear. Therefore no iterative process is required which guarantees a high degree of reliability. The iterative process used by the QSR estimator is also very reliable as very good initial conditions (given by the prediction routine) are available. Usually only one iteration is enough to obtain a solution with a 10^{-4} tolerance on the state variables.

7.8 CONCLUSIONS

Two sets of simulation results were reported in this chapter: the first one is a comparison of the performance of the improved version of the PLC state estimator described in Chapter V with the static state estimator algorithms reviewed in Chapter IV; the second one is a comparison between the tracking state estimation approach presented in Chapter VI and the static approach described in Chapter IV and V.

The results obtained with the PLC estimator indicate that the method has a large potential applicability in power system state estimation mainly due to its good bad data suppression capability. The feature of the method of being able to reject bad data automatically allows a direct comparison with the BDS algorithm which is the only other state estimator with this property. The application of the two methods (PLC and BDS) to a large number of cases of single, multiple, interactive and non-interactive bad data produced results which demonstrates that the two methods have about the same performance in bad data suppression which is also equivalent to the one obtained by the logical residual search procedure described in Chapter IV. This suppression effect is adequate in systems with relatively high redundancy ratio but become less effective in system (or regions of a system) in which this redundancy ratio is smaller.

The PLC algorithm has shown a more reliable convergence characteristic than the BDS method, reaching the solution in almost the same number of iterations for a specific convergence tolerance regardless of the number, size and place of the bad data. This is a characteristic particularly important for a method which is going to be used on-line.

A great improvement in the computational requirements of the PLC algorithm was achieved by the introduction of the modifications suggested in Chapter V. The time and storage requirements of the algorithm in this version, although still longer than the equivalent WLS algorithms, was found to be acceptable for on-line implementation.

The tests performed with the tracking state estimators (PLC and QSR) show that both estimators are able to follow (or track) the time evolution of the system state vector with an accuracy comparable with the uncertainty of the system's operational limits. This performance was observed in periods of relatively large load variation and for different shapes of the load curve. The QSR tracking algorithm usually produces results more accurate than the PLC tracking algorithm which is a result similar to the one observed in the corresponding static algorithms.

Both tracking algorithms have shown an improved bad data suppression effect in relation to the static estimation algorithms, particularly in the case of systems with not very high redundancy ratios. Several cases of interactive bad data in such systems, which were unidentified by the static estimators, have been suppressed by the tracking estimators. These results confirm the effectiveness of the prediction-discrimination technique in increasing the amount of information available to the estimator and consequent improvement of its detection capability. No substantial difference in the bad data suppression effectiveness was observed between the PLC and QSR tracking estimators.

The storage requirements of the tracking estimators are practically the same as the ones of the equivalent static approaches as most of the information required by the prediction-discrimination procedure is also used by the static algorithms. The time requirements of tracking algorithms if a single scan is considered, is much less than the one required by the equivalent static algorithm, but this cannot be considered in absolute terms as the two approaches use the computer facilities in different ways. The QSR algorithm is slightly faster than the PLC algorithm.

The PLC tracking estimator, being a non-iterative algorithm, is theoretically a more robust estimator than the QSR tracking estimator. However, in the cases studied no convergence problems were observed for the QSR estimator probably due to the availability of an excellent starting point for the algorithm produced by the prediction routine.

CHAPTER VIII

GENERAL CONCLUSIONS

The use of digital computers for on-line control and supervision of power systems has become common practice nowadays. The main objective of this control is to keep the system operating point in a secure region. This preventive or security control is achieved by the use of sophisticated algorithms which simulate a set of contingencies likely to occur and detects possible emergency situations caused by these contingencies. These algorithms require as starting point the knowledge of the system present operating condition or state. State Estimation has become one of the most accepted methods of obtaining these operating conditions from telemetered values of some system variables due to its advantages in terms of generality, efficiency, reliability and economy.

At least three approaches have been proposed for the power system state estimation problem: static, dynamic and tracking. In the first one, isolated measurement scans (snapshots) are processed at some prespecified instants of time using a WLS algorithm. In the second one the Kalman filtering technique is used to produce a continuous processing of consecutive scans by a method in which the model of the system dynamic is used to project into the future the information obtained in previous estimations. The third approach may be understood as a compromise between the first two in which static state estimation algorithms are adapted to take into consideration the time varying characteristics of the system state. The disadvantage of the first approach is the requirement of a relatively high redundancy ratio and/or computation time to produce reliable estimations. Dynamic estimators could overcome this problem if adequate models for the power system dynamics and the measurement uncertainty were available. As this is not so, the estimators derived using very simplified models do not exhibit the properties of the true dynamic estimators, apart from some undesirable computational drawbacks. Tracking state estimators may become more efficient than the other two approaches by combining some advantages of the two approaches and avoiding some of their disadvantages.

Most of the work carried out in power system state estimation so far has been concentrated on improving the performance of the WLS based algorithms for static estimation. Some of this effort was directed towards the development of faster algorithms such as the fast decoupled and line-only estimators. Another important part of the work was concentrated on trying to overcome the inability of the original WLS algorithm to deal with gross measurement and topological error. Some success has been obtained by the use of logical residual search procedures based on statistical properties of the measurement error. Another attempt to solve the problem was made by a modification in the original WLS criterion itself in order to make the algorithm less vulnerable to bad data (BDS method). Both approaches have a similar performance in eliminating bad data which is adequate in a large number of practical situations with the exception of the ones in which the bad data occurs in areas of low local redundancy. Both approaches are not able to distinguish whether a suppressed bad data corresponds to a gross measurement or configuration error. This distinction can only be made by a further stage in which the results of the estimation are examined.

A completely different attempt to produce an estimator with good performance in the presence of bad data is the Piecewise Linear Criterion estimator which represents the estimation problem as a series of LP problems. The original formulation of this method has a bad data performance similar to the BDS algorithm referred to above, but very large storage and computing time requirements. In Chapter V of this thesis, an improved version of this algorithm was proposed, based on the fast decoupling of the LP problems and the simultaneous use of the simplex and dual simplex algorithms to solve them. Simulation studies reported in Chapter VII showed that this improved version of the algorithm has much less storage and computing time requirements than the original formulation while maintaining its bad data suppression qualities. Although the time and storage requirements of the improved version of the PLC algorithm are still larger than the corresponding WLS algorithm, these requisites are acceptable for on-line implementation. The algorithm is extremely reliable in terms of the number of iterations required to reach the solution even in cases of large numbers of bad data in which the BDS algorithm usually shows a slight instability.

As pointed out above, the static estimation algorithms have some difficulty in suppressing bad data in regions of the system with low local redundancy. In those cases either a time consuming search for the bad data is required, or, even worse, the methods are not able to identify the bad data at all. To increase the redundancy ratio by the installation of extra measurements points, a large investment in meter and telecommunication equipment would be required. An alternative to this physical improvement of the redundancy ratio can be achieved by the use of the tracking state estimator approach referred to above. In Chapter VI of this thesis, a class of tracking state estimators developed according to this idea have been reported. The estimators are based on a linear static incremental model of the static state time evolution. The increase in the effective redundancy ratio is achieved by a combination of a predictive stage based on time-series forecasting techniques and an estimation stage using non-quadratic criterion estimators. Under the assumption of smooth load variation, an abnormality is detected whenever the measured values of a quantity deviates largely from its predicted value. This abnormality can be either caused by a sudden change in the operating state (loss of a large load, line outage, etc.) or gross error in the measurements. A logical check routine based on the topology of the network and measurement system is used to differentiate between these two situations. The information produced by this routine is then transmitted to the estimator in order to decrease or even eliminate the influence of the bad data in the estimation. Simulation studies comparing the performance of the tracking and static methods, which were reported in Chapter VII of this thesis, show a more reliable bad data suppression capability of the tracking approach in the case of systems with redundancy ratios in the range likely to be found in practice.

The result of the tests performed with the state estimation algorithms described in this thesis, in which a wide variety of measurement patterns and error types and sizes were simulated, indicates that the state estimation approach may have a high degree of reliability and computational efficiency provided these algorithms are combined adequately and the measurement system has a reasonable amount of redundancy. The integrated use of tracking algorithms to follow the slow time variation of the system state with static algorithms for initialisation purposes or in cases of sudden change in the system state, was found to be the most efficient way of obtaining consecutive state estimations from the point of view of the redundancy ratio required for a given degree of reliability in the estimates and the better utilisation of the computational resources. Although a definitive evidence of the suitability of the state estimation approach can only be achieved by tests performed in an on-line environment, the results obtained with simulated data show a performance of the algorithms high enough to recommend the state estimation approach for use in power system monitoring and control.

Although the theoretical background for power system state estimation is at pressent well established and efficient algorithms are available, some improvements in the existent methods are still possible and desirable. One of these, which has already been discussed above, is the effective on-line identification of topological errors which may have been treated as gross measurement errors by the state estimator. Another point which requires some investigation is the error analysis of non-quadratic estimator results in which case a similar analysis existent for the WLS algorithms is not valid. An interesting point for further investigation, which arises from the prediction module used in the tracking estimators described in Chapter VI of this thesis, is a possible association of this module of the estimator with the one responsible for the short-term load forecasting. Finally, as a result of increasing use of new computer hardware facilities, like computer networks and vector and parallel processing, an investigation of the influence of this new technology in the power system state estimation methods should be considered carefully.

APPENDIX A

SOME LINEAR PROGRAMMING TECHNIQUES

In this appendix a brief review of the linear programming techniques used in developing an efficient PLC estimator is presented. The aims of the appendix are to complement the information contained in Chapter V and to establish a common notation and terminology rather than to present a comprehensive review of the subject.

A.1 THE REVISED SIMPLEX METHOD(13, 43, 57)

The linear programming problems which have to be solved in each iteration of the PLC estimator can be written in a general form as

$$\text{minimise} \quad J = \underline{c}\,\underline{x} \tag{A.1}$$

$$\text{subject to} \quad \underline{A}\,\underline{x} = \underline{b} \tag{A.2}$$

$$x \geq 0 \tag{A.3}$$

where:

J : objective function

$\underline{c}$: cost coefficient vector (nx1)

$\underline{x}$: vector of unknown variables, including the main and slack variables (nx1)

$\underline{A}$: constraints coefficient matrix (nxm)

$\underline{b}$: known r.h.s. vector (mx1)

$n > m$

A basic solution for the above problem is a vector obtained by setting n-m unknown variables equal to zero and solving the resulting square system of equations obtained from (A.2). If this basic solution satisfies (A.3) it is called a basic feasible solution (b.f.s). The simplex algorithm is a procedure which searches for the optimum by discrete changes from one b.f.s. to the other, starting from any b.f.s., and improving (decreasing) the objective function at every step.

At each step of the algorithm, the unknown variables are divided into a set of m basic variables ($\underline{x}_b$) and m-n unknown non-basic variables ($\underline{x}_a$). The elements of the cost vector and coefficient matrix corresponding to the basic variables are called the basic cost vector ($\underline{C}_b$) and the basis matrix ($\underline{B}$). Equations (A.1) and (A.2) then become

$$[\underline{A}' \quad \underline{B}] \begin{bmatrix} \underline{x}_a \\ \\ \underline{x}_b \end{bmatrix} = b \qquad \text{(A.3)}$$

$$J = \underline{c}_a \, \underline{x}_a + \underline{c}_b \, \underline{x}_b \qquad \text{(A.4)}$$

As the non-basic variables are made equal to zero, the b.f.s. at each step of the algorithm is given by

$$\underline{x}_b = \underline{B}^{-1} \, \underline{b} \qquad \text{(A.5)}$$

with the associated cost

$$\underline{J} = \underline{c}_b \, \underline{x}_b \qquad (A.6)$$

The simplex algorithm, in its revised version, is as follows:

1. Find an initial b.f.s. and corresponding initial basis matrix

2. Calculate the reduced cost vector $\underline{c}'$:

$$\underline{c}' = \underline{c}_a - \underline{c}_b \, \underline{B}^{-1} \, \underline{A}' \qquad (A.7)$$

3. Determine: $\min_j c'_j = c'_s$, j = non-basic variables

   where c'_j is an element of $\underline{c}'$.

   If $c'_s \geq 0$, stop. The current basic solution is optimal.

4. Calculate

$$\underline{P} = \underline{B}^{-1} \, \underline{A}_s \qquad (A.8)$$

   where $\underline{A}'_s$ is the s-th column of $\underline{A}$.

5. Determine: $\min\limits_{j} \frac{p_j}{b_j} = \frac{p_r}{b_r}$

where p_j and b_j are the j-th elements of $\underline{P}$ and $\underline{b}$ respectively.

6a. Interchange x_r by x_s as basic variable.

b. Update $\underline{B}^{-1}$ by a pivot operation on the element at column s and row r.

c. Compute the new basic solution : $\underline{x}_b = \underline{B}^{-1}\,\underline{b}$

Return to step 2.

A.2 INVERSE BASIS IN THE PRODUCT FORM[8,9,13,40,57,71]

In step 6b of the revised simplex algorithm presented in the previous section, the inverse basis matrix B^{-1} is updated by a pivot operation. An equivalent operation is performed by multiplying the previous basis by the following elementary matrix.

$$E_i = \begin{bmatrix} 1 & & n_1 & & \\ & 1 & n_2 & & \\ & & \ddots \vdots & & \\ & & n_r & \ddots & \\ & & \vdots & & \\ & & n_m & & 1 \end{bmatrix} \qquad (A.9)$$

where

$$
n_i = \begin{cases} -\dfrac{a_{is}}{a_{rs}} & , \; i = 1, \ldots, m \; , \; i \neq r \\[2ex] \dfrac{1}{a_{rs}} & , \; i = r \end{cases} \qquad \text{(A.10)}
$$

a_{ij} = element of A.

After k pivot operations, the inverse basis is given by:

$$
\underline{B}^{-1} = E_k \, E_{k-1} \, \cdot \cdot \cdot \; E_1 \, \underline{B}_{initial} \qquad \text{(A.11)}
$$

The elementary matrices can be stored by recording only the non-zero elements (and their row position) of the non-unit column and its position in the matrix. These columns are often called "eta-vectors".

If the basis matrix is sparse, great savings in storage and computing time can be obtained by the representation of its inverse in compact form provided some precautions are taken as explained in the next section.

A.3 REINVERSION TECHNIQUES[40,46,57,71]

If the inverse basis in product form is used, at each iteration of the revised simplex algorithm a new eta-vector is generated and added to the current set. Therefore the number of elements required to represent the inverse basis increases very rapidly with the number of iterations. The problem is aggravated by the fact that eta-vectors generated in late iterations tend to contain more non-zero elements. Apart from obvious large storage requirements, this problem also affects the computing time as more elements enter in the product defined in (A.11).

The representation of the inverse basis as a product form is not unique. It depends on the chosen sequence of pivot operations. This sequence also has a drastic influence in the generation of non-zero elements. Therefore after some iterations it may become worthwhile to compact the representation of the current inverse basis by a regeneration of the eta-vectors, using the original problem data and an optimal sequence of pivot operations. This has also the advantage of eliminating cumulative errors.

The basic procedure in developing a set of eta-vectors is as follows:

1. select a column of the basis not already selected for pivoting.
2. transform this column by applying the current set of eta-vectors.
3. choose a pivot element for the transformed column in a row where no column has pivoted previously.
4. form a new eta-vector from the transformed column using (A.10) and add to the eta-vector set.
5. repeat steps 1 through 4 until all columns have been pivoted.

Many procedures to optimise the process of generating the eta-vectors just described are available in the literature. The improvements sought by these procedures are:

a. to minimise the time required to generate a new set of eta-vectors by reducing the number of "column transformations" (step 2 of the above procedure).

b. to reduce the number of created zero elements.

An analysis of the generation process shows that both objectives can be achieved if the rows and columns ordering of the matrix is re-arranged in such a way as to keep the matrix as close as possible to a lower triangular representation. Most of the basis matrices found in practical problems cannot be fully triangularised and attempts to do so result in the matrix in the form shown in Figure A.1, in which the dashed area contains only zero elements. Pivot operations down the main diagonal in section A of the matrix will neither require the execution of step 2 of the procedure described above nor create non-zero elements.

Section B of the matrix shown in Figure A.1 is often called the "bump" and the sorting out of an optimal sequence for the pivoting of its elements is the main step in a reinversion method. Many techniques to find this sequence are available in the literature. The one which was used in the program developed

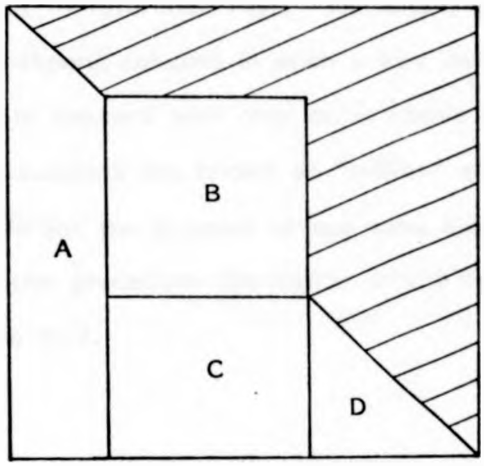


Figure A.1

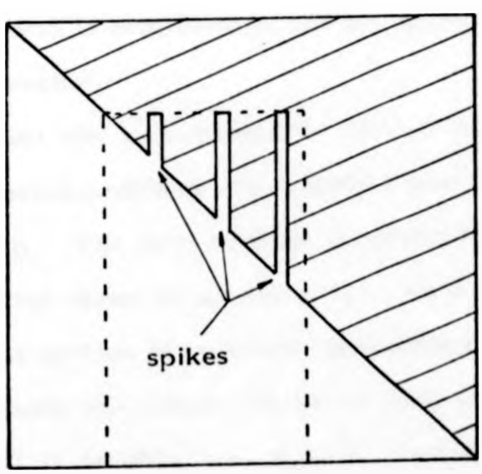


Figure A.2

to test the PLC estimator is called Pre-assigned Pivot Procedure and is described in reference (46). The basic idea of this method is to postpone columns in such a way as to have as many as possible columns with only zeros above the main diagonal. The postponed columns are known as "spikes" and are the only ones responsible for the creation of non-zero elements.
At the end of the procedure the matrix would have the form given in Figure A.2.

A.4 DUAL-SIMPLEX ALGORITHM[43,57,71]

The test of optimality in the revised simplex algorithm requires that all components of $\underline{C}'$ should be positive. As can be observed in (A.7), the value of $\underline{C}'$ does not depend on the r.h.s. vector $\underline{b}$. Therefore, any basic solution which produces all $\underline{C}'$ positive and is also feasible will be optimal independently of the r.h.s. vector.

This fact can be exploited for the solution of repeated linear programming problems which differs one from the other only by the vector $\underline{b}$. The first problem is solved by the (primal) simplex algorithm given in section A.1. As a subproduct of this solution an optimal basis matrix is available. For the next problem this basis will remain optimal as long as the basic solution given by (A.5) is feasible, i.e. $\underline{x}_b \geq 0$. Otherwise a procedure based on the duality theory can be developed to move from this basic solution to another in such a way that $\underline{C}'$ is kept positive and eventually reaches the optimal solution. This procedure is called the dual simplex algorithm.

Assuming that a basis matrix for which the corresponding $\underline{C}' \geq 0$ is available, the dual simplex algorithm is as follows:

1. Calculate : $\underline{x}_b = \underline{B}^{-1}\,\underline{b}$

   If $\underline{x}_b \geq 0$, stop. The current basic solution is optimal.

2. Choose row r as

$$\underline{x}_{b,r} = \min_i \underline{x}_{b,i} < 0$$

   where $\underline{x}_{b,i}$ are the elements of $\underline{x}_b$.

3. Choose column s as

$$\frac{C_s}{-P_s} = \min \frac{C'_j}{-P_j}$$

   where C'_j and P_j are the elements of $\underline{C}'$ and $\underline{P}$ respectively as defined in (A.7) and (A.8).

4. Obtain the new basis by a pivot operation on the element r-s and go back to step 1.

APPENDIX B

PREDICTION USING EXPONENTIAL SMOOTHING

B.1 BASIC METHOD

A time-series is a collection of observations made sequentially in time[17]. The problem of forecasting future values of a time-series can be summarised as follows: given a series of observations $x_1, x_2, \ldots, x_n$, it is required to forecast $\hat{x}_{n+h}$ for some positive integer h. The earliest version of exponential smoothing, which was proposed by C. C. Holt in 1978[17,42,52], applies only to the case in which the time-series is stationary and non-seasonal. In that case a reasonable estimate of the value of the series in the next interval is given by a weighted sum of past observations

$$\bar{x}_{n+1} = c_o x_n + c_1 x_{n-1} + c_2 x_{n-2} + \ldots \qquad (B.1)$$

where c_i, $i = 0,1,\ldots,n$, are weights. If more weight is to be given to recent observations, a possible set of weights would be a series of geometric weights which decrease by a constant ratio like

$$c_i = \alpha(1-\alpha)^i \qquad , \; i = 0,1,\ldots \qquad (B.2)$$

where α is a constant such that $0 < \alpha < 1$. In that case (B.1) becomes:

$$\bar{x}_{n+1} = \alpha\, x_n + \alpha\,(1-\alpha)\, x_{n-1} + \alpha(1-\alpha)^2\, x_{n-2} + \ldots \qquad (B.3)$$

The value given by (B.3) depends on all the previous observations. In order to obtain an expression in which this value is made dependent only on the last observation, equation (B.3) is usually re-written as:

$$\bar{x}_{n+1} = \alpha\, x_n + (1-\alpha)(\alpha\, x_{n-1} + \alpha(1-\alpha)\, x_{n-2} + \ldots) \qquad \text{(B.4)}$$

which produces the following recursive form

$$\bar{x}_{n+1} = \alpha\, \bar{x}_n + (1-\alpha)\, \alpha\, \bar{x}_{n-1} \qquad \text{(B.5)}$$

Equation (B.5) represents a basic algorithm for basic exponential smoothing which replaces the original series x_i by a "smoothed" series $\bar{x}_i$. A starting point to the algorithm can be chosen simply by setting $\bar{x}_1 = x_1$. The forecast of future values of the series are given by the latest smooth value, so that

$$\hat{x}_{n+n} = \bar{x}_n \qquad \text{(B.6)}$$

The constant α is termed the "smoothing constant" and an optimal value for this constant can be obtained by minimising the sum of the squared error which would be obtained if past values of the series were calculated using (B.5). The minimisation is carried out as follows[17]: the sum of squared prediction errors is computed for different values of α between 0 and 1, say in steps of 0.1, and a value is chosen which minimises this sum. Usually the sum of squares is quite flat near the minimum and so the choice of α is not critical.

B.2 HOLT-WINTERS METHOD

The prediction procedure described in the previous section can be generalised to apply to time-series containing a trend. One of the possible generalisations available is the method devised by Holt and Winters. This method, for the case of linear trend, is given by:

$$\bar{x}_n = \alpha x_n + (1-\alpha)(\bar{x}_{n-1} + T_{n-1}) \qquad \text{(B.7)}$$

$$T_n = \gamma (\bar{x}_n - \bar{x}_{n-1}) + (1-\gamma)\, T_{n-1} \qquad \text{(B.8)}$$

where α and γ are constants such that $0 < \alpha, \gamma < 1$.

A simple way to obtain "starting up" values for the algorithm given above is to set

$$\bar{x}_1 = x_1 \qquad \text{(B.9)}$$

$$T_1 = x_1 - x_o \qquad \text{(B.10)}$$

The forecast of future values of the series is given by

$$\hat{x}_{n+h} = \bar{x}_n + h\, T_n$$

Optimal values for the smoothing constants α and γ can be obtained by a minimisation process similar to the one described in the previous section.

APPENDIX C

USER'S MANUAL OF THE DEVELOPED PROGRAMS

C.1 INTRODUCTION

The following three programs have been developed in association with the research work reported in this thesis:

Simulator

BDS state estimator

PLC state estimator

The programs were written in FORTRAN IV (ANSI Standard) and each one of them is an "all in core" program,i.e. they do not use any auxiliary means of storage during the execution phase. The only machine dependent feature of the programs is the two-level storage feature of the computer in which the programs were developed and tested (CDC 7600). If the programs are required to run on other machines the statements corresponding to this mode of storage (LEVEL 2 statements) should simply be removed.

Apart from the programs referred to above, others were used in the research project for the performance comparisons reported in Chapter VII. However, these programs were developed in association with other projects and are well documented in references (11) and (51).

C.2 SIMULATOR

The simulator is used to generate the data (parameters, network and measurement system configuration, measurement values, etc), according to the procedure described in section 7.2. In Figure 7.1 of that section a flow chart of the program is presented in which the input and output files used by the program are shown.

The main subroutines of the program are:

LFINP - reads and writes the input data

LFSOL - solves a load flow problem using the fast decoupled method

LFOUT - writes the result of the load flow

FD1SEL - generates network configuration and measurement pattern (this subroutine simulates the network configurator)

TRSIM - updates the operating point by changing the values of loads and generations

TRMEAS - assigns the values for the measured variables and introduces the measurement noise (this subroutine simulates the telemetering system)

This program also uses a set of subroutines for the solution of a system of linear equations by the Bifactorisation method[13] land subroutines from the NAG library (which are described in a manual edited by UMRCC) for the generation of random numbers.

C.2.1 INPUT DATA STRUCTURE

1 - One title card of 80 alphanumeric characters

2 - One general information card with the following specification:

Columns	Description	Type
1-10	MVA Base	Real
11-20	Convergence tolerance	Real
21-25	Maximum number of iterations	Integer
26-33	Slack busbar name	Alphanumeric
34-44	Standard deviation of load random component	Real

3 - One control card with the following specifications:

Columns	Description	Type
1-5	Maximum number of simulations	Integer
6-10	Maximum number of time intervals	Integer
11-15	Type of load trend component	Integer

4 - Busbar data cards. One card for each network data with the following specification:

Columns	Description	Type
1-8	Busbar name	Alphanumeric
9-13	Specified voltage	Real
14-18	Active generation	Real
19-23	Reactive generation	Real
24-28	Active load	Real
29-33	Reactive load	Real
34-38	Nominal voltage	Real
49-58	Load rate of change	Real
59-68	Generator share of total load (%)	Real

5 - Blank card

6 - Branch data cards. One card for each network branch with the following specification:

Columns	Description	Type
1-8	Sending end name	Alphanumeric
9-16	Receiving end name	Alphanumeric
17-24	Resistance (pu)	Real
25-33	Reactance (pu)	Real
34-41	Shunt susceptance (pu)	Real
42-47	Initial tap position	Real
49-53	Minimum tap position	Real
54-57	Tap step	Real
58-63	Maximum tap position	Real
64-69	Specified voltage	Real

7 - Blank card

The specification of the measurement pattern (place and type of the measurement) is obtained by changing the values of a few variables in the subroutine FD1SEL according to comment cards in that routine.

C.2.2 OUTPUT DATA STRUCTURE

The simulator generates two data files to be used by the estimator programs. The first one which is called FBASE contain information about network parameters, configuration, meter-full scale, etc. The second one contains the measured and true values of measurement and the true values of state variables and is called TLMDT.

FBASE Structure

1 - One title card of 80 alphanumeric characters

2 - Node cards. One card for each busbar with the following specification:

Columns	Description	Type
5-8	Busbar name	Alphanumeric
10-13	Blank if not the reference busbar. Any character otherwise	Alphanumeric

3 - Blank card

4 - Branch cards. One card for each branch with the following specification:

Columns	Description	Type
5-8	Sending end name	Alphanumeric
13-16	Receiving end name	Alphanumeric
17-26	Resistance (pu)	Real
27-36	Reactance (pu)	Real
37-46	Shunt susceptance (pu)	Real
47-56	Tap position	Real

5 - Blank card

6 - Active measurement pattern cards. One card for each measurement with the following specification:

Columns	Description	Type
1-4	Measurement type	Integer
5-8	Measurement location	Integer

7 - Termination card : 99999 (columns 1 to 5)

8 - Reactive measurement pattern cards. One card for each measurement with the following specification:

Column	Description	Type
1-4	Measurement type	Integer
5-8	Measurement location	Integer

9 - Termination card: 99999 (columns 1 to 5)

10 - Meter full-scale cards. One card for each meter, in the same order as the measurement patterns, with the following specifications:

Columns	Description	Type
1-10	Meter full-scale	Real

TLMDT Structure

1 - Measurement cards. One card for each measurement, in the same order as the measurement patterns in FBASE, with the following specifications:

Columns	Description	Type
1-15	Measured value	Real
16-30	True value	Real
31-45	Error standard deviation	Real

2 - Transformer tap. One card for each TCUL transformer with the following specification:

Columns	Description	Type
1-15	Tap position (pu)	Real

3 - State variables cards. One card for each busbar with the following specifications:

Columns	Description	Type
1-15	Voltage magnitude	Real
16-30	Voltage phase angle	Real

The conventions used for the measurement patterns specification are as follows:

	= 0	injection
Type	= 999	voltage magnitude
	≠ 0 or 999	line flow (line number)

Location	injection or volt. mag : node number
	line flow : line number

C.3 BDS STATE ESTIMATOR

The BDS state estimator program uses the estimation algorithms described in section 6.4.2 and the bad data detection technique described in section 6.4.1. The input data to this program is the one contained in the files FBASE and TLMDT described in the previous section and a one-card-file of control variables which is specified as follows:

Columns	Description	Type
0-5	AC/DC key (= 1 full AC estimator; = 0 active estimator only)	Integer
6-10	Simulation key (= 1 simulated data; = 0 real-time data)	Integer
11-15	Number of time intervals	Integer
16-20	Number of simulations	Integer

The main subroutines of this program are:

TRINP - reads and writes (part of) the input data

TRSOL - main control routine for the BDS algorithm

TRFAC - build up and factorise gain matrices

BDSUP - applies bad data suppression effect

TRLDT - reads telemetered data (TLMDT)

TROUT - writes the result of the estimator

TRHP - calculates active Jacobian

TRHQ - calculates reactive Jacobian

TRZCP - calculates $\underline{h}_p(\underline{x})$

TRZCQ - calculates $\underline{h}_q(\underline{x})$

SEA - calculates the gain matrices

SEDB - calculates the product $\underline{H}^T \underline{R}^{-1} \Delta \underline{Z}$

C.4 PLC STATE ESTIMATOR

The PLC state estimator uses the estimation algorithm described in section 6.4.3, and the bad data detection technique described in section 6.4.1. The input data for this program is the same as the one described for the BDS estimator in the last section.

Some of the subroutines used in this program are identical to the ones used in the BDS estimator. These common subroutines are: TRINP, TROUT, TRLDT, TRHP, TRHQ, TRZCP and TRZCQ.

The remaining main subroutines of the program are:

TRSOL - main control routine of the PLC algorithm

LPROG - main control routine for the solution of the LP problems

PRSPX - primal simplex algorithm

DUSPX - dual simplex algorithm

PVSEL - selection of an optimal pivot ordering for reinversion

REINV - reinversion

C.5 SAMPLE OF THE PROGRAMS' OUTPUT

In the next pages a sample of the output of the simulator and the state estimator programs for the case of the IEEE 14-busbar system is presented.

STATE ESTIMATION SIMULATION PROGRAM

IEEE 14-BUSBAR SYSTEM

MVA BASE = 100.0
CONVERGENCE TOLERANCE = .00100
MAX. NO. OF ITERATIONS =100
SLACK BUSBAR =NA01

BUSBAR DATA

NODE	VOLTAGE P.U.	P.GEN MW	Q.GEN MVAR	P.LOAD MW	Q.LOAD MVAR	NOM VOLTAGE KV
NA01	1.06000	0.00000	0.00000	0.00000	0.00000	1.00000
NA02	1.04000	80.00000	0.00000	21.70000	12.70000	1.00000
NA03	0.00000	50.00000	20.00000	94.20000	19.00000	1.00000
NA04	0.00000	0.00000	0.00000	47.80000	3.90000	1.00000
NA05	0.00000	0.00000	0.00000	7.60000	1.60000	1.00000
NA06	0.00000	20.00000	44.00000	11.20000	7.50000	1.00000
NA07	0.00000	0.00000	0.00000	2.00000	1.00000	1.00000
NA08	0.00000	20.00000	35.00000	0.00000	0.00000	1.00000
NA09	0.00000	0.00000	0.00000	29.50000	16.60000	1.00000
NA10	0.00000	0.00000	0.00000	9.00000	5.80000	1.00000
NA11	0.00000	0.00000	0.00000	3.50000	1.80000	1.00000
NA12	0.00000	0.00000	0.00000	6.10000	1.60000	1.00000
NA13	0.00000	0.00000	0.00000	13.50000	5.80000	1.00000
NA14	0.00000	0.00000	0.00000	14.90000	5.00000	1.00000

NETWORK DATA

NODE	TO NODE	RESISTANCE	REACTANCE	SHUNT SUSC.	INIT.TAP	BOT.TAP	STEP	TOP TAP	VOLTAGE
NA01	NA02	.01938	.05917	.02640					
NA01	NA05	.05403	.22304	.02460					
NA02	NA03	.04699	.19797	.02190					
NA02	NA04	.05811	.17632	.01870					
NA02	NA05	.05695	.17388	.01700					
NA03	NA04	.06701	.17103	.01730					
NA04	NA05	.00335	.04211	.00640					
NA13	NA14	.17093	.34802	0.00000					
NA06	NA11	.00498	.19890	0.00000					
NA06	NA12	.12291	.25581	0.00000					
NA06	NA13	.06615	.13027	0.00000					
NA07	NA08	0.00000	.17615	0.00000					
NA07	NA09	0.00000	.11001	0.00000					
NA09	NA10	.03181	.08450	0.00000					
NA09	NA14	.12711	.27038	0.00000					
NA10	NA11	.08205	.19207	0.00000					
NA12	NA13	0.00000	.19988	0.00000					
NA04	NA09	0.00000	.55618	0.00000	1.0320	.9000	.01000000	1.1000	0.0000
NA05	NA06	0.00000	.25202	0.00000	1.0730	.9000	.01000000	1.1000	0.0000
NA04	NA07	0.00000	.20912	0.00000	1.0220	.9000	.01000000	1.1000	0.0000
NA09	NA09	0.00000	-5.26000	0.00000					
NA01	NA01	0.00000	-5.00000	0.00000					
NA14	NA14	0.00000	5.00000	0.00000					
NA05	NA05	0.00000	-5.00000	0.00000					
NA09	NA09	0.00000	5.00000	0.00000					

TOTAL NUMBER OF ELECTRICAL NODES = 14
TOTAL NUMBER OF LINES INCLUDING TRANSFORMERS = 20
TOTAL NUMBER OF IN-PHASE TRANSFORMERS = 3
NUMBER OF CONTROLLABLE IN-PHASE TRANSFORMERS = 0
NUMBER OF FIXED-TAP IN-PHASE TRANSFORMERS = 3

ACTIVE MEASUREMENT CONTAMINATION

TIME INTERVAL 1
SIMULATION NO 1

M.NUMBER	TYPE	ST.DEV	ERROR	SPECIFIED	IDEAL
1	2	.005100	-.005026	1.185053	1.190079
2	2	.003050	.001427	.716382	.714955
3	4	.002952	.002806	.688407	.685601
4	4	.002215	.000279	.506756	.506477
5	4	.002143	.002683	.505467	.502784
6	4	.002116	.002355	.496609	.494253
7	4	.001717	-.004135	.391075	.395209
8	4	.000255	.000013	-.055807	-.055820
9	4	.001933	-.000625	-.440650	-.440026
10	4	.000473	-.000213	.101592	.101805
11	4	.000496	.000234	.109097	.108863
12	4	.000452	.000442	.096089	.095647
13	4	.001095	.001075	.249700	.248625
14	4	.001088	.000539	-.245921	-.246460
15	4	.001628	.001794	.370126	.368333
16	4	.000215	-.000050	.044486	.044536
17	4	.000426	-.000114	.087668	.087782
18	4	.000282	.000050	-.064613	-.064663
19	4	.000136	.000140	.021035	.020895
20	4	.000556	.000005	.126760	.126756
21	4	.001472	.001332	.343064	.341732
22	4	.000689	-.000133	.146475	.146608

REACTIVE MEASUREMENT CONTAMINATION

TIME INTERVAL 1
SIMULATION NO 1

M.NUMBER	TYPE	ST.DEV	ERROR	SPECIFIED	IDEAL
1	3	.000218	.000219	-.045187	-.045406
2	3	.000930	.001066	-.197864	-.198931
3	5	.000566	.000336	.130172	.129837
4	5	.000232	.000334	.049812	.049477
5	5	.000048	-.000012	.010483	.010494
6	5	.000180	-.000236	-.034271	-.034035
7	5	.000208	.000219	-.042082	-.042300
8	5	.000020	-.000019	-.001996	-.001977
9	5	.000511	-.000304	-.113700	-.113396
10	5	.000748	-.000512	.169915	.170427
11	5	.000829	.001162	.189904	.188742
12	5	.000368	.000284	.076696	.076411
13	5	.000864	.000888	.199941	.199053
14	5	.001691	-.001593	-.388795	-.387203
15	5	.001427	-.000841	.327221	.328062
16	5	.000415	-.000656	-.085133	-.084477
17	5	.000461	.000325	.098538	.098213
18	5	.000716	.000091	-.154068	-.154159
19	5	.000246	-.000193	.053497	.053690
20	5	.000300	-.000011	.070033	.070044
21	5	.000235	-.000143	.050402	.050545
22	5	.000209	-.000296	-.043067	-.042771
23	1	.001060	-.001774	1.058226	1.060000
24	1	.001040	.000178	1.040178	1.040000

ACTIVE MEASUREMENT CONTAMINATION

TIME INTERVAL 0
SIMULATION NO 1

M.NUMBER	TYPE	ST.DEV	ERROR	SPECIFIED	IDEAL
1	2	.005005	.002684	1.164264	1.161580
2	2	.002999	.003877	.703677	.699800
3	4	.002896	.004572	.672656	.668084
4	4	.002178	-.000058	.493419	.493477
5	4	.002097	.003181	.492336	.489155
6	4	.002078	-.000896	.482366	.483262
7	4	.001691	-.001302	.385893	.387195
8	4	.000239	.000056	-.051605	-.051661
9	4	.001890	.000386	-.426626	-.427012
10	4	.000464	-.000673	.098546	.099219
11	4	.000487	.000029	.106047	.106017
12	4	.000450	.000696	.095725	.095029
13	4	.001089	.002995	.249613	.246618
14	4	.001067	.000647	-.239311	-.239957
15	4	.001609	-.000948	.361708	.362656
16	4	.000222	-.000035	.046580	.046615
17	4	.000426	.000594	.088286	.087692
18	4	.000272	-.000235	-.061925	-.061691
19	4	.000136	.000055	.020323	.020269
20	4	.000552	.000182	.125807	.125625
21	4	.001473	.000132	.342065	.341934
22	4	.000689	-.000798	.145800	.146598

REACTIVE MEASUREMENT CONTAMINATION

TIME INTERVAL 0
SIMULATION NO 1

M.NUMBER	TYPE	ST.DEV	ERROR	SPECIFIED	IDEAL
1	3	.000210	.000224	-.042891	-.043115
2	3	.000987	.000619	-.215461	-.216080
3	5	.000583	-.000192	.134735	.134927
4	5	.000222	.000481	.047159	.046678
5	5	.000036	.000013	.006193	.006180
6	5	.000191	.000080	-.037135	-.037215
7	5	.000219	.000105	-.045590	-.045695
8	5	.000022	.000006	-.002486	-.002492
9	5	.000509	.000177	-.112661	-.112839
10	5	.000771	.001179	.172450	.171270
11	5	.000830	.000286	.189164	.188878
12	5	.000387	-.000369	.075811	.076180
13	5	.000863	.000026	.199017	.198991
14	5	.001692	.002706	-.384760	-.387465
15	5	.001423	-.001787	.325185	.326972
16	5	.000413	-.000505	-.084488	-.083986
17	5	.000461	.000085	.098451	.098367
18	5	.000715	-.001106	-.155511	-.154406
19	5	.000246	.000124	.053806	.053682
20	5	.000298	.000106	.069499	.069393
21	5	.000235	.000032	.050505	.050473
22	5	.000213	-.000332	-.044223	-.043891
23	1	.001060	-.000100	1.059900	1.060000
24	1	.001040	-.001391	1.038609	1.040000

```
*********************
* STATE ESTIMATION *
*     RESULTS       *
*********************
```

OPERATING MODE= ITERATIVE (INITIALIZATION)
CONVERGENCE TOLERANCE= .00010
NUMBER OF ITERATIONS= 5
SIMULATION NUMBER= 1
TIME INTERVAL= 0

STATE VARIABLES (PU/DEG)

BUSBAR NAME	TRUE VOL MAG	TRUE VOL PH ANG	ESTIMATED VOL MAG	ESTIMATED VOL PH ANG	ERROR(%) VOL MAG	ERROR(%) VOL PH ANG
NA01	1.0600	0.0000	1.0594	0.0000	.1	0.0
NA02	1.0400	-1.9041	1.0394	-1.9110	.1	.4
NA03	1.0187	-7.1027	1.0180	-7.1245	.1	.3
NA04	1.0210	-6.5920	1.0204	-6.6133	.1	.3
NA05	1.0271	-5.6295	1.0264	-5.6489	.1	.3
NA06	1.0933	-9.7309	1.0926	-9.7515	.1	.2
NA07	1.0527	-8.1913	1.0521	-8.2092	.1	.2
NA08	1.1182	-6.1335	1.1172	-6.1534	.1	.3
NA09	1.0192	-10.3223	1.0185	-10.3425	.1	.2
NA10	1.0247	-10.6850	1.0241	-10.7062	.1	.2
NA11	1.0586	-10.7283	1.0581	-10.7491	.1	.2
NA12	1.0649	-10.4665	1.0641	-10.5042	.1	.4
NA13	1.0548	-10.6731	1.0540	-10.7119	.1	.4
NA14	.9822	-10.9637	.9814	-10.9899	.1	.2

ERROR IN ESTIMATED STATES (%)

	MAXIMUM	AVERAGE
MAGNITUDE	.1	.1
PH. ANGLE	.4	.3

ACTIVE MEASUREMENTS (MW)

MEAS NUMBER	MEAS TYPE	LOCA (BB)	LINE END	STAND DEVIA	WEIGHT	TRUE VALUE	MEASURED VALUE	ESTIMATED VALUE	MEAS-TRUE (X 1/SD)	MEAS-ESTM (X 1/SD)	ESTM-TRUE (X 1/SD)
1	INJE	NA01	----	.501	.40E+01	116.156	116.424	116.418	.536	.012	.524
2	INJE	NA02	----	.300	.11E+02	69.960	70.348	70.071	1.293	.923	.370
3	FLOW	NA01	NA02	.289	.12E+02	66.808	67.266	66.959	1.580	1.056	.521
4	FLOW	NA01	NA05	.218	.21E+02	49.348	49.342	49.459	.027	.537	.511
5	FLOW	NA02	NA03	.210	.23E+02	48.916	49.234	48.996	1.517	1.131	.386
6	FLOW	NA02	NA04	.208	.23E+02	48.326	48.237	48.416	.431	.862	.431
7	FLOW	NA02	NA05	.169	.35E+02	38.720	38.589	38.805	.770	1.277	.507
8	FLOW	NA03	NA04	.024	.18E+04	-5.166	-5.161	-5.165	.234	.175	.060
9	FLOW	NA04	NA05	.189	.28E+02	-42.701	-42.663	-42.730	.204	.357	.153
10	FLOW	NA13	NA14	.046	.46E+03	9.922	9.855	9.871	1.456	.356	1.093
11	FLOW	NA06	NA11	.049	.42E+03	10.602	10.605	10.590	.062	.298	.236
12	FLOW	NA06	NA12	.045	.49E+03	9.503	9.573	9.616	1.547	.963	2.509
13	FLOW	NA06	NA13	.109	.84E+02	24.662	24.961	24.897	2.750	.593	2.157
14	FLOW	NA07	NA08	.107	.86E+02	-23.996	-23.931	-23.937	.605	.053	.552
15	FLOW	NA07	NA09	.161	.39E+02	36.266	36.171	36.259	.589	.547	.042
16	FLOW	NA09	NA10	.022	.20E+04	4.662	4.658	4.659	.158	.066	.091
17	FLOW	NA09	NA14	.043	.55E+03	8.769	8.829	8.816	1.394	.301	1.093
18	FLOW	NA10	NA11	.027	.14E+04	-6.169	-6.193	-6.187	.880	.188	.673
19	FLOW	NA12	NA13	.013	.56E+04	2.027	2.032	2.035	.403	.168	.571
20	FLOW	NA04	NA09	.055	.33E+03	12.563	12.581	12.542	.330	.698	.368
21	FLOW	NA05	NA06	.147	.46E+02	34.193	34.207	34.161	.089	.309	.220
22	FLOW	NA04	NA07	.069	.21E+03	14.660	14.580	14.611	1.158	.449	.709

ACTIVE MEAS. RESIDUALS

	MAXIMUM	AVERAGE
MEAS-TRUE/SD	2.75	.82
MEAS-ESTM/SD	1.28	.51
ESTM-TRUE/SD	2.51	.63

REACTIVE MEASUREMENTS (MVAR/PU)

MEAS NUMBER	MEAS TYPE	LOCA (BB)	LINE END	STAND DEVIA	WEIGHT	TRUE VALUE	MEASURED VALUE	ESTIMATED VALUE	MEAS-TRUE (X 1/SD)	MEAS-ESTM (X 1/SD)	ESTM-TRUE (X 1/SD)
1	INJE	NA01	----	.021	.23E+04	-4.312	-4.289	-4.284	1.067	.222	1.289
2	INJE	NA02	----	.099	.10E+03	-21.608	-21.546	-21.581	.627	.356	.271
3	FLOW	NA01	NA02	.058	.29E+03	13.493	13.474	13.481	.329	.136	.193
4	FLOW	NA01	NA05	.022	.20E+04	4.668	4.716	4.682	2.167	1.534	.632
5	FLOW	NA02	NA03	.003	.87E+05	.618	.619	.621	.382	.414	.796
6	FLOW	NA02	NA04	.019	.27E+04	-3.722	-3.714	-3.728	.419	.736	.318
7	FLOW	NA02	NA05	.022	.21E+04	-4.570	-4.559	-4.567	.479	.365	.114
8	FLOW	NA03	NA04	.002	.21E+06	-.249	-.249	-.248	.273	.240	.512
9	FLOW	NA04	NA05	.051	.39E+03	-11.284	-11.266	-11.239	.350	.524	.874
10	FLOW	NA13	NA14	.077	.17E+03	17.127	17.245	17.162	1.530	1.078	.452
11	FLOW	NA06	NA11	.083	.15E+03	18.888	18.916	18.816	.345	1.205	.860
12	FLOW	NA06	NA12	.039	.67E+03	7.618	7.581	7.586	.955	.139	.814
13	FLOW	NA06	NA13	.086	.13E+03	19.899	19.902	19.848	.030	.627	.597
14	FLOW	NA07	NA08	.169	.35E+02	-38.747	-38.476	-38.476	1.599	.000	1.599
15	FLOW	NA07	NA09	.142	.49E+02	32.697	32.519	32.777	1.256	1.816	.560
16	FLOW	NA09	NA10	.041	.59E+03	-8.398	-8.449	-8.436	1.220	.271	.950
17	FLOW	NA09	NA14	.046	.47E+03	9.837	9.845	9.867	.162	.472	.654
18	FLOW	NA10	NA11	.072	.20E+03	-15.440	-15.551	-15.472	1.548	1.109	.439
19	FLOW	NA12	NA13	.025	.17E+04	5.368	5.381	5.382	.504	.044	.548
20	FLOW	NA04	NA09	.030	.11E+04	6.939	6.950	6.943	.356	.222	.136
21	FLOW	NA05	NA06	.024	.18E+04	5.047	5.051	5.037	.136	.587	.451
22	FLOW	NA04	NA07	.021	.22E+04	-4.389	-4.422	-4.412	1.559	.501	1.058
23	VOLT	NA01	----	.001	.89E+06	1.060	1.060	1.059	.094	.446	.541
24	VOLT	NA02	----	.001	.92E+06	1.040	1.039	1.039	1.337	.760	.578

REACTIVE MEAS. RESIDUALS

	MAXIMUM	AVERAGE
MEAS-TRUE/SD	2.17	.78
MEAS-ESTM/SD	1.82	.58
ESTM-TRUE/SD	1.60	.63

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*********************
* STATE ESTIMATION *
*      RESULTS      *
*********************
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OPERATING MODE= TRACKING
SIMULATION NUMBER= 1
TIME INTERVAL= 1

STATE VARIABLES (PU/DEG)

BUSBAR NAME	TRUE VOL MAG	TRUE VOL PH ANG	ESTIMATED VOL MAG	ESTIMATED VOL PH ANG	ERROR(%) VOL MAG	ERROR(%) VOL PH ANG
NA01	1.0600	0.0000	1.0594	0.0000	.1	0.0
NA02	1.0400	-1.9041	1.0394	-1.9647	.1	3.2
NA03	1.0187	-7.1027	1.0168	-7.3241	.2	3.1
NA04	1.0210	-6.5920	1.0193	-6.7718	.2	2.7
NA05	1.0271	-5.6295	1.0255	-5.7720	.2	2.5
NA06	1.0933	-9.7309	1.0917	-9.8860	.1	1.6
NA07	1.0527	-8.1913	1.0508	-8.3716	.2	2.2
NA08	1.1182	-6.1335	1.1167	-6.2497	.1	1.9
NA09	1.0192	-10.3223	1.0172	-10.5519	.2	2.2
NA10	1.0247	-10.6850	1.0229	-10.9055	.2	2.1
NA11	1.0586	-10.7283	1.0569	-10.9186	.2	1.8
NA12	1.0649	-10.4665	1.0629	-10.6377	.2	1.6
NA13	1.0548	-10.6731	1.0529	-10.8533	.2	1.7
NA14	.9822	-10.9637	.9802	-11.1907	.2	2.1

ERROR IN ESTIMATED STATES (%)

	MAXIMUM	AVERAGE
MAGNITUDE	.2	.2
PH. ANGLE	3.2	2.1

ACTIVE MEASUREMENTS (MW)

MEAS NUMBER	MEAS TYPE	LOCA (BB)	LINE END	STAND DEVIA	WEIGHT	TRUE VALUE	MEASURED VALUE	ESTIMATED VALUE	MEAS-TRUE (X 1/SD)	MEAS-ESTM (X 1/SD)	ESTM-TRUE (X 1/SD)
1	INJE	NA01	----	.301	.40E+01	119.008	118.505	119.056	1.006	1.101	.097
2	INJE	NA02	----	.300	.11E+02	71.496	71.638	71.888	.476	.832	1.307
3	FLOW	NA01	NA02	.289	.12E+02	68.360	68.641	68.335	.969	1.055	.086
4	FLOW	NA01	NA05	.218	.21E+02	50.448	50.476	50.521	.128	.208	.336
5	FLOW	NA02	NA03	.210	.23E+02	50.278	50.547	50.393	1.279	.735	.544
6	FLOW	NA02	NA04	.208	.23E+02	49.425	49.661	49.563	1.134	.469	.665
7	FLOW	NA02	NA05	.169	.35E+02	39.521	39.108	39.620	2.445	3.029	.585
8	FLOW	NA03	NA04	.024	.18E+04	-5.582	-5.561	-5.570	.054	.457	.512
9	FLOW	NA04	NA05	.189	.28E+02	-44.003	-44.065	-44.183	.330	.626	.956
10	FLOW	NA13	NA14	.046	.46E+03	10.181	10.159	10.102	.459	1.232	1.692
11	FLOW	NA06	NA11	.049	.42E+03	10.886	10.910	10.927	.480	.365	.845
12	FLOW	NA06	NA12	.045	.49E+03	9.565	9.609	9.643	.982	.767	1.749
13	FLOW	NA06	NA13	.109	.84E+02	24.863	24.970	25.027	.987	.525	1.512
14	FLOW	NA07	NA08	.107	.88E+02	-24.646	-24.592	-24.664	.505	.676	.170
15	FLOW	NA07	NA09	.161	.39E+02	36.833	37.013	36.965	1.114	.298	.816
16	FLOW	NA09	NA10	.022	.20E+04	4.454	4.449	4.402	.225	2.121	2.346
17	FLOW	NA09	NA14	.043	.55E+03	8.778	8.767	8.734	.268	.762	1.030
18	FLOW	NA10	NA11	.027	.14E+04	-6.466	-6.461	-6.426	.184	1.310	1.494
19	FLOW	NA12	NA13	.013	.56E+04	2.090	2.104	2.107	1.045	.268	1.313
20	FLOW	NA04	NA09	.055	.33E+03	12.676	12.676	12.684	.007	.146	.154
21	FLOW	NA05	NA06	.147	.46E+02	34.173	34.306	34.193	.904	.768	.136
22	FLOW	NA04	NA07	.069	.21E+03	14.661	14.648	14.615	.193	.477	.670

ACTIVE MEAS. RESIDUALS

	MAXIMUM	AVERAGE
MEAS-TRUE/SD	2.44	.69
MEAS-ESTM/SD	3.03	.83
ESTM-TRUE/SD	2.35	.86

REACTIVE MEASUREMENTS (MVAR/PU)

MEAS NUMBER	MEAS TYPE	LOCA (BB)	LINE END	STAND DEVIA	WEIGHT	TRUE VALUE	MEASURED VALUE	ESTIMATED VALUE	MEAS-TRUE (X 1/SD)	MEAS-ESTM (X 1/SD)	ESTM-TRUE (X 1/SD)
1	INJE	NA01	----	.021	.23E+04	-4.541	-4.519	-4.505	1.043	.655	1.698
2	INJE	NA02	----	.099	.10E+03	-19.893	-19.786	-19.869	1.081	.836	.245
3	FLOW	NA01	NA02	.058	.29E+03	12.984	13.017	12.979	.575	.655	.081
4	FLOW	NA01	NA05	.022	.20E+04	4.948	4.981	4.962	1.509	.876	.633
5	FLOW	NA02	NA03	.003	.87E+05	1.049	1.048	1.061	.324	3.060	3.336
6	FLOW	NA02	NA04	.019	.27E+04	-3.404	-3.427	-3.407	1.236	1.066	.170
7	FLOW	NA02	NA05	.022	.21E+04	-4.230	-4.208	-4.223	.995	.683	.312
8	FLOW	NA03	NA04	.002	.21E+06	-.198	-.200	-.201	.664	.628	1.491
9	FLOW	NA04	NA05	.051	.39E+03	-11.340	-11.370	-11.308	.597	1.215	.617
10	FLOW	NA13	NA14	.077	.17E+03	17.043	16.992	17.012	.664	.271	.393
11	FLOW	NA06	NA11	.063	.15E+03	18.874	18.990	18.916	1.400	.896	.504
12	FLOW	NA06	NA12	.039	.67E+03	7.641	7.670	7.685	.736	.400	1.136
13	FLOW	NA06	NA13	.080	.13E+03	19.905	19.996	19.952	1.029	.484	.545
14	FLOW	NA07	NA08	.169	.35E+02	-38.720	-38.880	-38.832	.941	.278	.663
15	FLOW	NA07	NA09	.142	.49E+02	32.806	32.722	32.772	.591	.349	.242
16	FLOW	NA09	NA10	.041	.59E+03	-8.448	-8.513	-8.493	1.588	.485	1.103
17	FLOW	NA09	NA14	.046	.47E+03	9.621	9.654	9.633	.705	.455	.250
18	FLOW	NA10	NA11	.072	.20E+03	-15.416	-15.407	-15.343	.127	.888	1.015
19	FLOW	NA12	NA13	.025	.17E+04	5.369	5.350	5.348	.785	.055	.840
20	FLOW	NA04	NA09	.030	.11E+04	7.004	7.003	6.983	.037	.674	.711
21	FLOW	NA05	NA06	.024	.18E+04	5.055	5.040	5.000	.609	1.725	2.334
22	FLOW	NA04	NA07	.021	.22E+04	-4.277	-4.307	-4.308	1.390	.060	1.450
23	VOLT	NA01	----	.001	.69E+06	1.060	1.058	1.059	1.674	1.088	.586
24	VOLT	NA02	----	.001	.92E+06	1.040	1.040	1.039	.171	.771	.600

REACTIVE MEAS. RESIDUALS

	MAXIMUM	AVERAGE
MEAS-TRUE/SD	1.67	.66
MEAS-ESTM/SD	3.66	.80
ESTM-TRUE/SD	3.34	.87

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POWER SYSTEM TRACKING STATE ESTIMATION AND BAD DATA PROCESSING

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Abstract – In this paper the tracking of the time varying power system static state is analysed. The relative importance of the small measurement noise and the eventual occurrence of gross error in the measurements is used as a design criterion for the development of fast tracking estimators. The basic approach uses a pre-estimation bad data detection/elimination scheme based on the exponential smoothing of past estimations and logical checks followed by an estimation stage. Three different estimation algorithms were tested: plain weighted least squares, quadratic square-root and linear criterion. Numerical results showing the performance of the three estimators in a simulated test case are also presented.

INTRODUCTION

The power system static state (voltage magnitude and phase angles at all nodes) is a slow time varying vector. It follows the daily cycle of electric energy demand. A state estimator is a set of programs which obtains estimates of the static state at some required instants of time from telemetered values of network variables (line flows, node injections, voltage magnitudes etc.) and topological information. The measurements contain a certain amount of error which can be of two types, either a small statistically "well behaved" error due to instrument inaccuracy, interference, miscalibration, etc. (measurement noise) or a large, unpredictable error due to some sort of partial or total failure of the telemetering system, transients, etc. (bad data). The error introduced by the measurement noise is comparable with the uncertainty of most of the operational constraints (e.g. transmission overload limits) against which the results of the estimation will be checked. Therefore high filtering capacity is not a necessary requirement of the estimator. Bad data, however, can seriously distort the results of the estimation, producing completely unreliable state estimates. Therefore a practical power system state estimator should be designed bearing in mind that it should be able to detect and eliminate efficiently the grossly wrong measurements eventually present in a snapshot and from the bad data free set of measurements it should obtain an estimate of the state with the accuracy required by the application programs that will use the estimation results. Most of the state estimation methods proposed in the literature[1,2,3] use the static approach in which each snapshot is considered separately. An algorithm based on the Weighted Least Squares (WLS) method is normally used to filter the measurement noise and a post estimation residual analysis is carried out to detect and identify bad data. However, some attempts have been made to explore the time varying characteristics of the state. They can be divided into two categories: dynamic state estimation approach based on Kalman filtering techniques and using a simplified model of the state dynamic behaviour[4,5], and tracking state estimation which extends the techniques developed for static estimation to the time varying case without explicit definition of the dynamic model[6,7].

Measurement scans are normally taken at short intervals (up to one minute). Assuming that a process control computer is used, a complete static state estimation could not possibly be performed for each scan. The conventional procedure is to store the data and only when a substantial change is detected (or after some specified period of time, whichever comes first), a new estimation is performed[8]. However, in some situations (e.g. during an unusual load pick up period) a more close monitoring of the system state would be desirable. Moreover, a large interval between estimations weaken the correlation between consecutive estimations making detection and identification of bad data even more difficult.

Tracking state estimator algorithms can be designed in order to give results with a delay which allow the estimator to "keep up" with the rate of incoming data. In this paper three tracking estimator schemes are analysed. The algorithms are based on a step by step linearisation of the network equations and on the assumption of a smooth load variation. The principle of decoupling active and reactive variables is also used. The filter elements of the estimators are based on a quadratic (WLS), quadratic-square root and linear (sum of the moduli of the residuals) criterion respectively. All of them use a pre-estimation detection/identification of bad data procedure based on an exponential smoothing of previous estimations. The main purpose of the study was to show the viability of tracking estimation either on its own or as a complement to other methods of state estimation.

FORMULATION OF THE PROBLEM

The measurement and state vectors at an instant of time k are related by the equation

$$\underline{Z}(k) = \underline{h}(\underline{x}(k)) + \underline{W}(k) \tag{1}$$

where

$\underline{Z}(k)$ - measurement vector (mx1)
$\underline{x}(k)$ - state vector (nx1)
$\underline{w}(k)$ - measurement error vector (mx1)
$\underline{h}(\cdot)$ - nonlinear function given by network laws (see Appendix A)

Assume that a state estimation has been performed at an instant of time k-1 and let $\hat{\underline{x}}(k-1)$ be the result of that estimation. Then define

$$\hat{\underline{Z}}(k-1) = \underline{h}(\hat{\underline{x}}(k-1)) \tag{2}$$

Let $\underline{Z}(k)$ be the vector of measurements at time k and define

$$\Delta\underline{Z}(k) = \underline{Z}(k) - \hat{\underline{Z}}(k-1) \tag{3}$$

$$\Delta\underline{x}(k) = \underline{x}(k) - \hat{\underline{x}}(k-1) \tag{4}$$

From (2), (3) and (4) and using Taylor series expansion

$$\Delta \underline{Z}(k) = H(\hat{\underline{x}}(k-1))\Delta \underline{x}(k) + \underline{W}'(k) \tag{5}$$

where

$$H(\underline{x}(k-1)) = \left.\frac{\partial \underline{h}}{\partial \underline{x}}\right|_{\underline{x} = \hat{\underline{x}}(k-1)} \tag{6}$$

$\underline{W}'(k)$ is a vector which components are the sum of the measurement errors plus the error introduced by the linearisation of $\underline{h}(\underline{x})$. Under the assumption that the system state varies little between two consecutive snapshots, the error of linearisation is comparable to the measurement noise.

Decoupled Model

The sensitivity of voltage phase angles (magnitudes) to changes on reactive (active) variables in an EHV network is small. This property has been exploited in the development of very efficient static state estimators[9,10,11]. The application of this decoupling technique to the tracking estimation model given by (5) will produce two independent sets of equations as follows:

$$\Delta \underline{Z}_p(k) = \underline{H}_p(\hat{\underline{x}}(k-1)\Delta\underline{\theta}(k) + \underline{W}'_p(k) \tag{7}$$

$$\Delta \underline{Z}_q(k) = \underline{H}_q(\hat{\underline{x}}(k-1)\Delta\underline{V}(k) + \underline{W}'_q(k) \tag{8}$$

where

$\underline{x}(k) = (\underline{\theta}(k) \mid \underline{V}(k))^T$; $\underline{\theta}, \underline{V}$ vectors of voltage phase angle and magnitude respectively

$\underline{Z}(k) = (\underline{Z}_p(k) \mid \underline{Z}_q(k))^T$; $\underline{Z}_p$, $\underline{Z}_q$ vectors of active and reactive measurements.

$\underline{W}'(k) = (\underline{W}'_p(k) \mid \underline{W}'_q(k))^T$; $\underline{W}'_p$, $\underline{W}'_q$ vector of error terms corresponding to active/reactive measurements.

$$H(\underline{x}(k)) = \begin{bmatrix} H_p(\underline{x}(k)) & H_{pq}(\underline{x}(k)) \\ H_{pq}(\underline{x}(k)) & H_q(\underline{x}(k)) \end{bmatrix}$$

Constant Jacobian Matrices

The changes in the elements of $H_p(\underline{x})$ and $H_q(\underline{x})$ due to changes in the state $\underline{x}$ is not significant. As observed in 9 and 11 the error introduced in the estimation by making these matrices independent of the state are acceptable. In Appendix A the approximations introduced in the calculations of the Jacobian elements in order to produce constant matrices are described. A further improvement in the model can be achieved by dividing each of the equations of the reactive set by the voltage at the busbar in which the measurement is taken. The final model is then given by

$$\Delta \underline{Z}_p(k) = \underline{H}_p\ \Delta\underline{\theta} + \underline{W}'_p(k) \tag{9}$$

$$\Delta \underline{Z}'_q(k) = \underline{H}'_q\ \Delta\underline{V} + \underline{W}''_q(k) \tag{10}$$

The error introduced by the decoupling procedure can also be considered as being incorporated in the error terms $\underline{W}'_p$ and $\underline{W}''_q$.

METHODS OF SOLUTION

Equations (9) and (10) represent two independent sets of overdetermined linear equations. Due to the presence of the error terms these systems are inconsistent, i.e., there is no solution that satisfies them exactly. A "solution" for such a system can be characterised by the choice of a criterion to be optimised. This criterion is usually defined as a function of the residual terms.

The most used criterion for the solution of problems like the one referred to above is the Weighted Least Squares (WLS). If the error is normally distributed the solution obtained with the WLS criterion can be proved to give the minimum covariance estimate. Even if the normality assumption cannot be guaranteed the WLS can still give good results provided the error terms are all small. However, if bad data is present the WLS method will give incorrect results due to excessive weight assigned to bad data.

The methods of solution adopted in this paper will consist of a two step procedure: first an analysis of the incoming measurements will be made in order to detect possible grossly wrong measurements (see next section); afterwards an estimation will be performed in which the suspected measurements will receive special attention.

Plain WLS[1]

In this method the ordinary WLS criterion is used. The suspected measurements are substituted by predicted values obtained by an exponential smoothing of the past estimation (see next section). The algorithm is given by

$$\Delta\underline{\theta}(k) = A_p^{-1} H_p^T R_p^{-1} \Delta \underline{Z}_p(k) \tag{11}$$

$$\Delta\underline{V}(k) = A_q^{-1} H_q^T R_q^{-1} \Delta \underline{Z}'_q(k) \tag{12}$$

where

$A_p = H_p^T R^{-1} H_p$; $A_q = H_q^T R^{-1} H_q$ are the constant gain matrices

$R = \mathrm{diag}\,(R_1, R_2, \ldots)$; R_1, R_2, ... are weights chosen according to the relative accuracy of the measurements (usually made equal to the inverse. of the assumed error covariance).

Modified WLS (Bad Data Suppression)[2]

The performance of the WLS method in the presence of bad data can be improved if a non-quadratic criterion which assigns less weight to measurements with large residuals is used. This technique has been used in static state estimation with reasonable success. It can be extended to the tracking state estimation problem by applying the nonquadratic correction term to the suspected measurements. Even though the models given by (9) and (10) are linear the resulting estimator has to be solved iteratively (for details see Appendix B). The algorithm is given by

$$\Delta\underline{\theta}^{i+1}(k) = \Delta\underline{\theta}^{i}(k) + A_p^{-1} H_p^T G_p(\Delta \underline{Z}_p(k))\underline{\rho}(\Delta \underline{Z}_p(k)) \tag{13}$$

$$\Delta\underline{V}^{i+1}(k) = \Delta\underline{V}^{i}(k) + A_q^{-1} H_q^T G_q(\Delta \underline{Z}'_q(k))\underline{\rho}(\Delta \underline{Z}'_q(k)) \tag{14}$$

where the elements of the diagonal matrices $G_p(\cdot)$ and $G_q(\cdot)$ and the vectors $\underline{\rho}_p(\cdot)$ and $\underline{\rho}_q(\cdot)$ are given by (using a simplified notation):

$$\left.\begin{aligned} \rho_m &= \Delta Z_m^i \\ g_m &= 1.0 \end{aligned}\right\} \quad \text{if the m-th measurement is not a suspected bad data}$$

$$\left.\begin{aligned} \rho_m &= \mathrm{sign}(\Delta Z_m^i)\lambda\sigma_m \left(4\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|^{\frac{1}{2}} - 3\right)^{\frac{1}{2}} \\ g_m &= \left(\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|^{-\frac{1}{2}} \left(4\left|\frac{\Delta Z_m^i}{\lambda\sigma_m}\right|^{-\frac{1}{2}} - 3\right)^{-\frac{1}{2}}\right) \end{aligned}\right\} \quad \text{otherwise}$$

m = measurement number
i = iteration counter
λ = chosen threshold
σ_m = assumed standard deviation of the measurement error

Linear Criterion (Linear Programming)[3,12]

In this estimator the criterion used is the sum of the moduli of the residuals. The filtering capacity of this estimator is not as high as the WLS based estimators. However it has a much better performance in the presence of bad data. If the linear criterion is used the problem can be formulated as the solution of the following Linear Programming problems (see Appendix C for details):

$$\underset{\Delta\underline{\theta}}{\text{Min}} \sum_{i=1}^{m_p} R_{p,i}(s'_{2i-1} + s'_{2i})$$
$$\text{s.to } (H_p \vdots U_p) \begin{bmatrix} \Delta\underline{\theta} \\ \text{-----} \\ \underline{S}_p \end{bmatrix} = \Delta\underline{Z}_p \qquad (15)$$

and

$$\underset{\Delta\underline{V}}{\text{Min}} \sum_{i=1}^{m_q} R_{q,i}(s''_{2i-1} + s''_{2i})$$
$$\text{s.to } (H'_q \vdots U_q) \begin{bmatrix} \Delta\underline{V} \\ \text{-----} \\ \underline{S}_q \end{bmatrix} = \Delta\underline{Z}'_q \qquad (16)$$

where

m_p, m_q are the number of active/reactive measurements
s'_i, s''_i are slack variables representing the residuals

$$U_p, U_q = \begin{bmatrix} 1 & -1 & & & & \\ & & 1 & -1 & & \\ & & & & \ddots & \\ & & & & 1 & -1 \end{bmatrix}_{(m_p x 2m_p),(m_q x 2m_q)}$$

If no measurement is suspected of being a bad data the weights R_p and R_q are chosen in the same way as in the WLS method. In the case of a suspected measurement the corresponding weight is reduced accordingly to the degree of suspicion.

BAD DATA REJECTION TECHNIQUE

The power system load varies daily according to a predictable pattern. Sudden variations are not frequent and when it occurs it is either the result of a predictable event (e.g. disconnection of a large load) or an indication of some abnormal state of operation (e.g outages). Network variables follow these variations according to the adopted control strategy and network constraints. Apart from eventual transients, these variables present variation patterns similar to demand. A simple relationship between changes in load and network variables is not available. The same occurs with a model for the daily variation in load. However, it is possible to obtain reasonably accurate prediction of the behaviour of these variables, within certain conditions, based on previous observations. Time series techniques are particularly appropriate to use in situations like that in which a relatively simple process (in view of the application in mind) is to be observed but an adequate model is not available.

Exponential Smoothing of Measured Variables[13]

The recent past estimated values of a single measured variable can be considered as a time series. Provided a relatively short period is considered (up to 1 hour), this time series can be reasonably modelled as being made up of a trend component plus a random change. The trend component varies according to the time of day but it usually repeats every day for corresponding periods of time. For instance, the morning load pick-up period in most systems can be observed to follow a linear pattern. The most suitable model for the trend component as well as information about the random change can be obtained from off-line studies of previous days' estimations.

Assuming a linear trend the predicted value of a measured variable in the next snapshot can be obtained through an exponential smoothing function like

$$\bar{Z}_i(k) = a_i(k-1) + 2b_i(k-1) + 2(1-\beta)\varepsilon_i(k-1) \qquad (17)$$

$$a_i(k) = a_i(k-1) + b_i(k-1) + (1-\beta^2)\varepsilon_i(k-1) \qquad (18)$$

$$b_i(k) = b_i(k-1) + (1-\beta)^2\varepsilon_i(k-1) \qquad (19)$$

$$\varepsilon_i(k-1) = \hat{Z}_i(k-1) - \bar{Z}_i(k-1) \qquad (20)$$

where

i : measurement number
$\hat{Z}_i(k)$: estimated value at instant k
$\bar{Z}_i(k)$: predicted value at instant k
β : time series parameter

The smoothing parameter β regulates the relative weight given to more recent estimations on the calculation of the predicted value. It can also be obtained by off-line calculations based on previous estimations.

Bad Data Discrimination

The expected values of the measured variables in the next snapshot are calculated at the end of each estimation using expression (17) to (20). When the present snapshot is available, the values of its components are checked against the predicted values. In normal conditions of operation the difference between measured and expected values should not exceed a certain threshold value determined by the parameters of the random component of the series. If one or more measurements do not pass the above test then an abnormal situation is detected. This abnormality can be due to either bad data or sudden change in the system state due to loss of a large load, unreported outage of line, etc. If a real sudden change occurs in the system then various correlated variables should be affected in the region near the abnormality.

In order to differentiate the situations described above an analysis of the intercorrelation between suspected bad data points should be performed. A logical check routine to perform this analysis should take into consideration particular characteristics of the power and telemetering systems such as existence of radial lines, data concentration in some regions, etc. In the tracking mode the required speed of response demands a fast algorithm. Furthermore, due to the small interval between estimations a bad data can be more easily spotted. A simple routine can be programmed as follows: a suspected data point is checked against other measurements in the same node and neighbouring nodes. For each measurement not flagged a certain number of points is added to the measurement flag. Depending on the final scores the suspected measurements are flagged as bad data or not.

Comments

The bad data discrimination method described above, even though producing good results in the majority of cases, may fail in certain situations. For instance, in the case of highly interactive bad data points, very low local redundancy, etc. If this failure occurs, two situations can arise: (a) a bad data goes into the estimator, (b) a good measurement is flagged as a suspected bad data. In the first case the plain WLS estimator would probably give a bad result depending on the size of the bad data. The two other algorithms, with a "built-in" way of rejecting bad data, could in most cases produce an accurate solution. In the second case the rejection (or weighting down) of good measurements would not cause much damage to the estimation provided the number of rejected measurements is not high enough to alter substantially the system redundancy.

COMPUTATIONAL ASPECTS

For initialisation purposes or when a major change in the system state (or network structure) takes place the algorithms described earlier should be used as a conventional iterative static state estimator. A network configurator[1] and a post-estimation bad data detection/identification routine[2] should also be incorporated in the estimator to be used when a static state estimation is performed. Figure 1 shows a basic flow chart of the complete estimator. At an initialisation mode or when a change in the network is reported the elements of the system admittance matrix and of the matrices H_p, H_q, A_p^{-1} and A_q^{-1} defined previously are calculated and stored in compact form using sparsity techniques[14] in a file of processed network data. This data is then used on the subsequent estimations until a new change occurs in the network. In normal tracking operation only the routines needed for this kind of estimation should be loaded which saves core space. In particular situations in which only active flows (and phase angles) are required, the reactive part of the algorithm can easily be "switched-off" and all voltage magnitudes be set to 1.0 per unit with considerable reductions in computer requirements.

SIMULATION RESULTS

The algorithms presented in the previous section were tested using simulated data from different power systems, measurement patterns, types of abnormality (bad data, sudden change of load or generation, etc.) and the pattern of load change. Due to space limitations only the simulation study performed for the IEEE 14-busbar system with the measurement pattern showed on Fig. 2 will be presented. This study was found to represent a typical behaviour of the algorithms among all the tested cases.

Description of the Simulation

The time evolution of the system static state was simulated by the calculation of successive load flows in which load and generation varied from initial given values. Each of those load flows correspond to the system operating point at the moment in which a snapshot is taken. The load curve at each busbar is composed of a linear trend plus a random fluctuation. The slope of the trend variation was made different for groups of nodes. The random fluctuation was represented by a normally distributed random number with zero mean and a standard deviation of 2% of the value of the trend component. A constant power factor was assumed so the reactive load follows the active load. The increase or decrease of the total load is distributed among the generators according to pre-specified percentages. The simulation study was made over a period

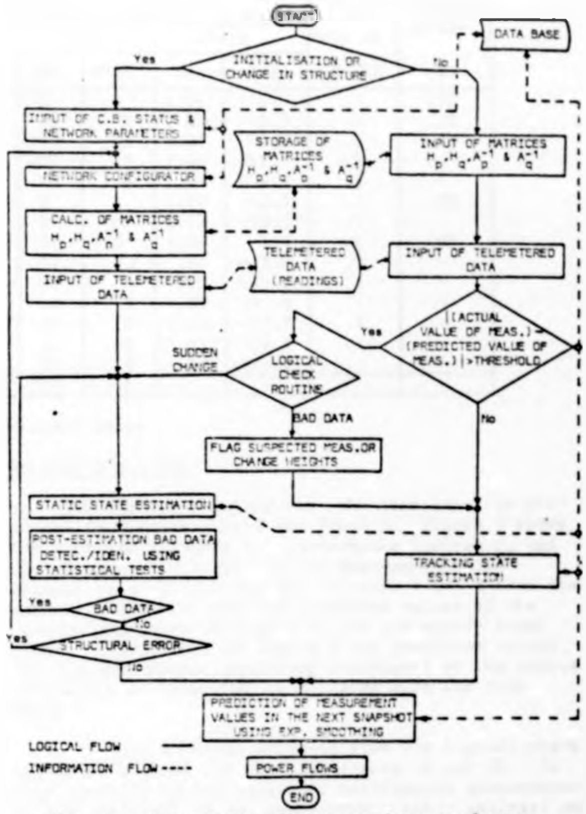

Fig. 1: Flow-chart of the complete estimator

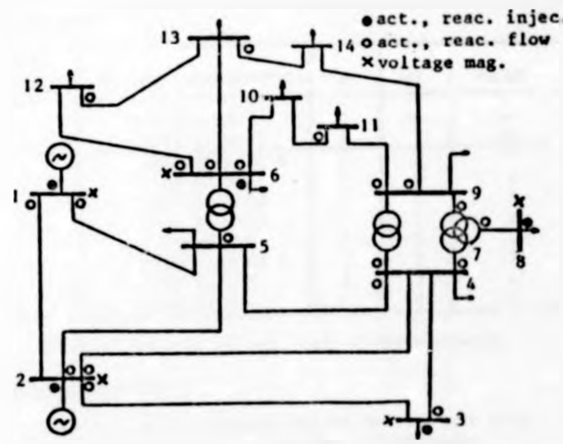

Fig. 2: Measurement pattern used in the simulation

of 30 time sample intervals. The initial and final values of load, generation and state variables for each busbar as well as the values of the rates of change of loads and generation are shown on Table 1.

The measurement noise was simulated by adding a normally distributed error to the values obtained from the load flows. The error bounds were set to .35% of the meter full-scale plus 2% of the actual value of the measured quantity.

BUSBAR	INITIAL VALUES						FINAL VALUES						LOAD RATE OF CHANGE	GENERATION %
	GEN.		LOAD		STATE		GEN.		LOAD		STATE			
	MW	MVAR	MW	MVAR	pu	deg.	MW	MVAR	MW	MVAR	pu	deg.		
1	116	-1	0	0	1.06	0	204	-13	0	0	1.06	0	.3	.36
2	96	8	26	15	1.04	-1.9	155	44	49	29	1.04	-3.9	.3	.28
3	60	19	113	23	1.01	-7.0	98	73	215	43	1.01	-14.2	.3	.18
4	0	0	57	5	1.01	-6.5	0	0	107	9	1.00	-11.1	.3	-
5	0	0	9	2	1.02	-5.5	0	0	18	4	1.01	- 9.1	.3	-
6	24	44	13	9	1.07	-9.7	43	52	0	0	1.07	-12.7	.2	.09
7	0	0	0	0	1.03	-8.2	0	0	0	0	1.02	-12.3	.2	-
8	24	35	0	0	1.09	-6.0	43	47	0	0	1.09	-8.4	.2	.09
9	0	0	35	20	1.00	-10.4	0	0	57	32	.97	-15.6	.2	-
10	0	0	10	7	1.00	-10.7	0	0	14	9	.98	-15.7	.1	-
11	0	0	4	2	1.04	-10.7	0	0	5	3	1.03	-14.8	.1	-
12	0	0	7	2	1.04	-10.5	0	0	10	2	1.03	-13.9	.1	-
13	0	0	16	7	1.03	-10.7	0	0	21	9	1.02	-14.3	.1	-
14	0	0	18	6	.96	-11.1	0	0	25	8	.93	-16.2	.1	-
TOTALS	[illegible]	107	309	98	-	-	543	203	[illegible]	148	-	-	-	-

Table 1: Initial and final values of the simulation study

At time sample 10 an error of 50% was introduced in the measured values of the active and reactive flows in lines 5-6 and 6-13 in order to simulate bad data. A sudden change in the system operating point was simulated at time sample 20 by setting the values of the active and reactive load at busbar 6 to zero.

In the bad data detection/elimination scheme a measurement was considered suspected of being a bad data whenever its value deviated more than 20 times the value of its assumed standard deviation from the predicted value. In the logical check routine the number of points attributed for each non-flagged measurement at the same node or in neighbouring nodes were set to 2 and 1 respectively. A score greater than 8 was required to differentiate a possible bad data from a real change in the network variables. In the exponential smoothing and quadratic-square root the values of the parameters β and λ used were $\beta = 0.6$ and $\lambda = 4.0$.

Performance Assessment

The performance of the algorithms in the simulation studies was assessed by checking the estimated values against the available true values. The information obtained from this check is summarised in the three following performance indicators:

$$J_M(k) = \sum_{i=1}^{m} (z_i^M(k) - z_i^T(k))^2/\sigma_i^2 \tag{21}$$

$$J_E(k) = \sum_{i=1}^{m} (z_i^E(k) - z_i^T(k))^2/\sigma_i^2 \tag{22}$$

$$R_E(k) = \max_{i=1,m} \left\{ \left| z_i^E(k) - z_i^T(k) \right| / \sigma_i \right\} \tag{23}$$

where m: number of measurements

z_i^T, z_i^M, z_i^E: are the true, measured and estimated values of the i-th measurement

The performance index J_M indicates the level of uncertainty on the measurements. The performance index J_E shows how close are the estimated values to the true values. In the case of a good filtering performance of the estimator J_E should be always smaller than J_M. The maximum value of the weighted residue R_E is used to complement the general information of the indicators J_M and J_E.

Numerical Results

The results obtained for the test case are presented on Figures 3 to 7 and Table 2. Figure 3 shows the relative value of the performance indices J_E and J_M. Figure 4 is a plot of the maximum weighted residue in each time sample. Figures 5 and 6 show the time evolution of true and estimated values of the voltage magnitude at busbar 13 and the active power flow in line 5-6 . In figure 7 the predicted values (by the exponential smoothing technique) of the active power flow in line 5-6 is compared with its true value.

In Table 2 values obtained from the logical check routine are shown for the time sample 10 and 20. In this table N_1 is the number of non-flagged measurements in the same node of the measurement being analysed and N_2 is the corresponding number for neighbouring measurements. The numbers shown correspond to active measurements only.

TIME SAMPLE	FLAGGED MEASUREMENTS	N_1	N_2	SCORE
10	flow in 5-6	0	16	16
	flow in 6-13	4	2	10
20	injection in 6	1	2	3
	flow in 6-11	1	0	2
	flow in 6-12	1	1	2
	flow in 6-13	1	1	2
	flow in 5-6	0	6	6
	injection in 1	2	3	7
	flow in 1-5	2	2	6
	flow in 2-5	2	8	12

Table 2: Results of the logical check routine

Analysis of Results

The following comments relate to the test case shown in this paper and also other simulations.

i. Filtering Capability The values of J_E/J_M and R_M around 1.0 and 3.0 shown on Figures 3 and 4 for the time samples not containing bad data or not corresponding to a sudden change show that the three algorithms produce results with a degree of uncertainty similar to that existing in the raw data. These results indicate that the product of the estimation is only a consistent set of network variables without any improvement in terms of accuracy compared to the available measurements. Under the assumption of small amplitude measurement noise this result is adequate for practical applications.

ii. Tracking Capability The ability of the algorithms based on a constant decoupled linear approximation of the network model to follow the state evolution can be seen by the same order of magnitude of J_E/J_M and R_M at the beginning and end of the simulation. The algorithms were even able to recover from a sudden change in the network operating point at the time sample 20.

iii. Exponential Smoothing The assumption of a linear trend for the exponential smoothing of past measurements produced good general results even in cases where other trends were used to simulate the system evolution provided the rate of change was not high. No investigation was made for the determination of optimal values for the parameter β but values between 0.6 and 0.8 were found to produce good results.

iv. Logical Check The simple logical check routine used performed well in the case of one bad data only or non-interactive multiple bad data points (particularly on relatively large systems). In the case of interactive bad data it can produce useful information only if the local redundancy is high (as is the case in the example shown).

v. Bad Data Rejection The values of J_E/J_M and R_M for time sample 10 shows a similar performance by the Quadratic-Square Root and Linear Programming estimators and poorer performance by the Plain WLS. This was due to the difficulty of finding a correct weight for the predicted value used in place of the rejected bad data. In general, in the cases which the pre-estimation procedure failed, the Plain WLS has always produced wrong results and the other two estimators have performance similar to the example shown here, except for cases of highly interactive bad data or low local redundancy.

CONCLUSIONS

In this paper an approach for the tracking estimation of the power system static state was presented. The combined use of a pre-estimation detection and identification scheme, with algorithms using some form of automatic bad data rejection, were shown to have an adequate performance under the assumed designed criteria.

The only situations in which the algorithms did not perform adequately was in cases of multiple interactive bad data in nodes with low level of local redundancy. However, this situation represents an enormous challenge for almost all of the existing state estimation techniques.

The performance of the presented algorithms can be improved with a more elaborate pre-estimation detection and identification of bad data routine incorporating particular characteristics of the power and telemetering systems.

ACKNOWLEDGEMENTS

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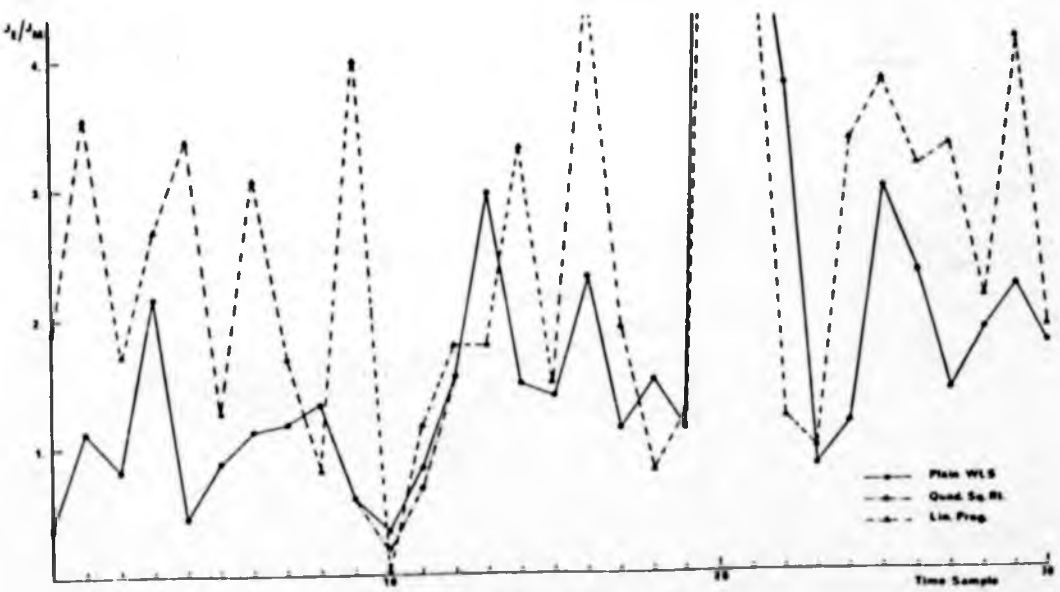

Figure 3

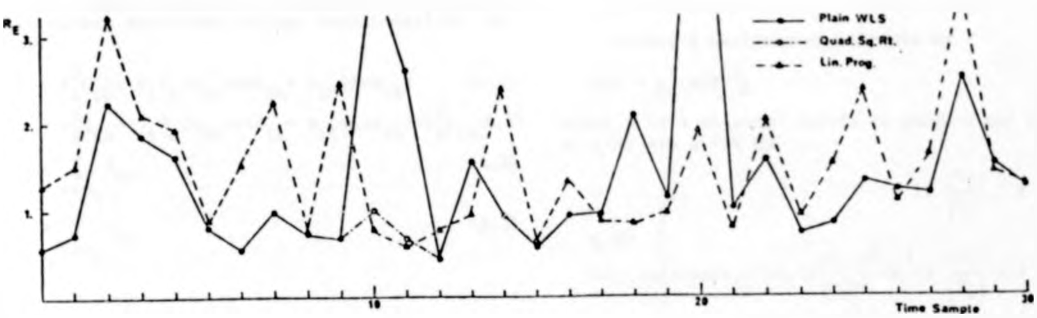

Figure 4

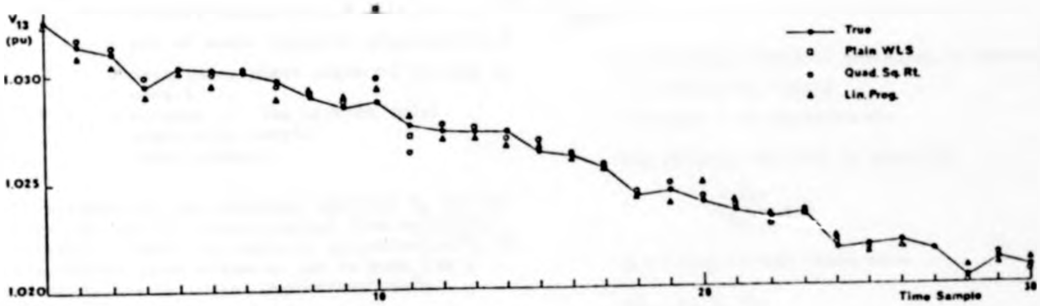

Figure 5

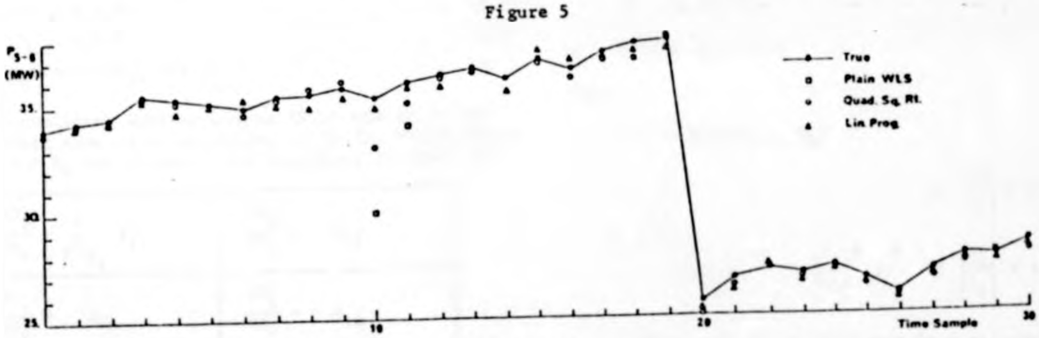

Figure 6

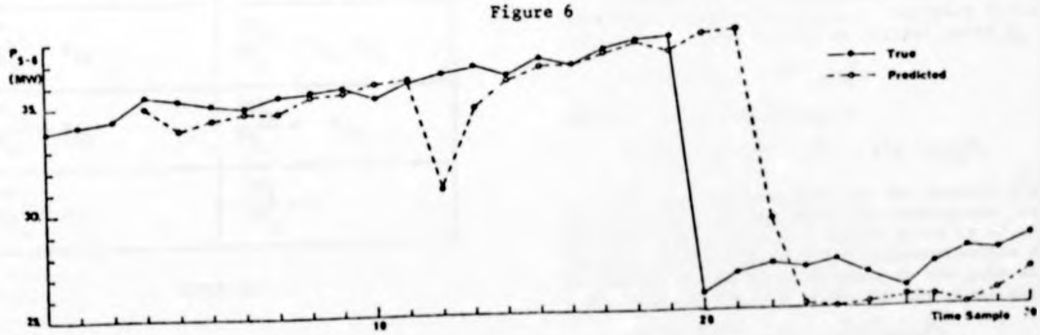

Figure 7

APPENDIX A

MEASUREMENT EQUATIONS AND JACOBIAN ELEMENTS

The power system variables normally measured for the purpose of state estimation are the active and reactive load injections and line flows and voltage magnitudes. Those variables are related to the state variables (phase angle and voltage magnitudes) by the equations:

$$P_{ik} = V_i^2 G_{ik} + V_i V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \qquad (A.1)$$

$$Q_{ik} = -V_i^2 B_{ik} + V_i V_k (G_{ik}\cos\theta_{ik} - B_{ik}\sin\theta_{ik}) + V_i^2 y_{ik} \qquad (A.2)$$

$$P_i = \sum_{k\in\alpha_i} P_{ik} \qquad (A.3)$$

$$Q_i = \sum_{k\in\alpha_i} Q_{ik} \qquad (A.4)$$

where

P_i, Q_i	: active, reactive injection at node i
P_{ik}, Q_{ik}	: active, reactive flow at line i-j
α_i	: set of nodes directly connected to i
V_i, θ_i	: magnitude, phase angle of voltage at node i
$G_{ik} + jB_{ik}$	: element of the network nodal admittance matrix
y_{ik}	: shunt element

The elements of the Jacobian matrices H_p and H_q' defined in (9) and (10) are obtained from equations (A.1) to (A.4). Based on physical characteristics of the EHV networks these elements can be made state independent by the following approximations[9]:

$$\cos\theta_{ik} = 1.0 \qquad (A.5)$$

$$V_i V_k = 1.0 \qquad (A.6)$$

$$G_{ik}\sin\theta_{ik} << B_{ik}\cos\theta_{ik} \qquad (A.7)$$

Introducing approximations (A.5) and (A.6) and neglecting some terms according to (A.7), the elements of H_p and H_q are given by the equations in table A.1.

$\frac{\partial P_i}{\partial\theta_i} = \sum_{k\in\alpha_i} B_{ik}$	$\frac{\partial Q_i'}{\partial V_i} = -B_{ii}$
$\frac{\partial P_i}{\partial\theta_k} = -B_{ik}$	$\frac{\partial Q_i'}{\partial V_k} = -B_{ik}$
$\frac{\partial P_{ik}}{\partial\theta_i} = B_{ik}$	$\frac{\partial Q_{ik}'}{\partial V_i} = B_{ik} - y_{ik}$
$\frac{\partial P_{ik}}{\partial\theta_k} = B_{ik}$	$\frac{\partial Q_{ik}'}{\partial V_k} = -B_{ik}$
$\frac{\partial V_i}{\partial\theta_i} = 0$	$\frac{\partial V_i}{\partial V_i} = 1$

TABLE A.1

APPENDIX B

QUADRATIC-SQUARE ROOT ESTIMATOR

Consider a linear estimation problem defined by the model equation

$$\underline{Z} = H\underline{x} + \underline{W} \qquad (B.1)$$

Define a performance criteria by

$$J(x) = \underline{\rho}^t(x) R^{-1} \underline{\rho} \qquad (B.2)$$

where R is a diagonal matrix of weights and the elements of $\underline{\rho}(x)$ are given by

$$\rho_i(\underline{x}) = \begin{cases} r_i & \text{if } \left|\frac{r_i}{\sigma_i}\right| \le \lambda \\ \operatorname{sign}(r_i)\,\lambda\sigma_i\left(4\left|\frac{r_i}{\lambda\sigma_i}\right|^{\frac{1}{2}} - 3\right)^{\frac{1}{2}} & \text{if } \left|\frac{r_i}{\sigma_i}\right| > \lambda \end{cases} \quad i = 1,\ldots m \qquad (B.3)$$

$$\underline{r} = \underline{Z} - H\underline{x} \qquad (B.4)$$

where

σ_i = assumed standard deviation of measurement error

λ = chosen threshold

m = number of measurements

The minimum of J(x) is given by

$$\frac{dJ(\underline{x})}{d\underline{x}} = 0 \qquad (B.5)$$

According to the chain rule

$$\frac{dJ}{d\underline{x}} = \frac{dJ}{d\underline{\rho}}\frac{d\underline{\rho}}{d\underline{r}}\frac{d\underline{r}}{d\underline{x}} \qquad (B.6)$$

or

$$2H^T G^T R^{-1} \underline{\rho}(\underline{r}) = 0 \qquad (B.7)$$

where

$$G = \operatorname{diag}(g_1 g_2 \ldots g_m)$$

$$g_i = \begin{cases} 1 & \text{if } \left|\frac{r_i}{\sigma_i}\right| \le \lambda \\ \left(\left|\frac{r_i}{\lambda\sigma_i}\right|\right)^{-\frac{1}{2}} \left(4\left|\frac{r_i}{\lambda\sigma_i}\right|^{\frac{1}{2}} - 3\right)^{-\frac{1}{2}} & \text{if } \left|\frac{r_i}{\sigma_i}\right| > \lambda \end{cases} \qquad (B.8)$$

Equation (B.7) is nonlinear in $\underline{x}$ and can be solved only by an iterative process. Applying Taylor series expansion to $\underline{\rho}(x)$ around an initial guess $\underline{x}_o$

$$\rho(\underline{x}) \cong \rho(\underline{x}_o) - GH(\underline{x} - \underline{x}^o) \qquad (B.9)$$

which leads to the iteration

$$H^T G^T R^{-1} GH(\underline{x}^{k+1} - \underline{x}^k) = H^T R^{-1} G\rho(\underline{x}^k) \qquad (B.10)$$

The gain matrix given by the product $H^T G^T R^{-1} GH$ has influence on the speed of convergence but not on the optimality of the solution given by (B.10). In order to avoid the need for refactorisation at every iteration it can be made equal to the gain matrix of a plain WLS estimator producing the final algorithm

$$H^T R^{-1} H(\underline{x}^{k+1} - \underline{x}^k) = H^T R^{-1} G\rho(\underline{x}^k) \qquad (B.11)$$

APPENDIX C

LINEAR PROGRAMMING ESTIMATOR

Consider the same estimation problem as in Appendix B but with a performance criteria given by

$$J = \sum_{i=1}^{m} R_i |r_i| \qquad (C.1)$$

$$r_i = Z_i - \underline{H}_i \underline{x} \qquad i=1, \ldots, m \qquad (C.2)$$

where $\underline{H}_i$ is the i-th column of H

Using the above criteria the estimation problem can be formulated as a linear programming problem provided some changes of variables are introduced. Let

i. substitute the residue terms r_i by the difference of two positive slack variables

$$r_i = s_{2i-1} - s_{2i} \qquad (C.3)$$

ii. add a large enough constant d to the elements of the state vector

$$x_i' = x_i + d \qquad i=1, \ldots, n \qquad (C.4)$$

which will lead to a redefinition of the measurement variables as

$$\underline{Z}' = \underline{Z} + H\underline{d} \qquad (C.5)$$

The estimation problem can now be formulated as a linear programming problem as

$$\text{Min } J = \sum_{i=1}^{m} R_i (s_{2i-1} + s_i) \qquad (C.6)$$

subject to $[H \vdots U] \begin{bmatrix} \underline{x} \\ \cdots \\ \underline{s} \end{bmatrix} = \underline{Z}'$

where

$$U = \begin{bmatrix} 1 & -1 & & & & \\ & & 1 & -1 & & \\ & & & & \ddots & \\ & & & & 1 & -1 \end{bmatrix}_{m \times 2m}$$

$$S = (s_1 s_2 \ldots s_{2m-1}\, s_{2m})$$

As in the final solution of the problem above at least one of the two slack variables defined by (C.3) will be null (nonbasic) the objective function given by (C.6) is equivalent to the performance criteria defined in (C.1).

The estimate given by (C.6) will always satisfy exactly (within the calculation accuracy) n of the m equations of the problem, i.e. it will lie on the point of intersection of n hyperplanes representing the n chosen equations. If the number of bad data is less than m-n the criterion given by (C.1) will almost always choose a solution point in which the equations corresponding to the bad data are excluded.

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A FAST DECOUPLED STATE ESTIMATOR USING LINEAR PROGRAMMING

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INTRODUCTION

The objective of a power system state estimator is to obtain a reliable estimate of the system state (voltage magnitude and phase angles at all nodes) from a set of telemetered information (usually analogue values of injections, line flows, voltage magnitudes and switches status). The measurements contain a certain degree of noise introduced by metering and communication devices and sometimes completely wrong information due to partial or total failure of the telemetering system (bad data).

Most of the existent state estimation algorithms[1], which are based on the Weighted Least Squares (WLS) method, are able to deal quite well with the measurement noise levels of modern telemetering systems. However, they usually fail in the presence of bad data. An acceptable solution can only be obtained by repeated runs of the estimator from which some suspected measurements are excluded[2].

State estimation algorithms with a better performance in the presence of bad data can be achieved by the use of other criteria than the WLS. An example of this technique is the Bad Data Suppression method[2]. If the level of noise of the measurements is not high (as is usual in practice) the sum of the moduli of the residuals can be used as a criterion to develop an estimator with a good bad data rejection performance[3]. In this estimator each iteration can be formulated as a Linear Programming (LP) problem. The property of the solution of a LP problem of always lying in one corner of the polyhedron made up by the constraints is the key feature of the method for the rejection of bad data

In this paper an improved version of the LP estimator in terms of speed and storage requirements is presented. Each of the LP problems is decomposed into two smaller ones by decoupling the equations corresponding to active and reactive measured variables. The coefficients of each LP problem are made constant during the iterative process by the introduction of some approximations. The LP problems are then solved using a simplified Revised Simplex algorithm adapted to the particular characteristics of the problem.

FORMULATION OF THE PROBLEM

The measurements and state vectors are related by the equation

$$Z = h(X) + W \tag{1}$$

where

- Z - measurement vector (mx1)
- X - state vector (nx1)
- W - measurement error vector (mx1)
- $h(.)$ - non-linear functions given by network laws[1]

Given a set of measurements Z the state estimation problem can be seen as the calculation of a vector $\hat{x}$ which "best fits" the given measurements. The estimate is obtained by optimising a chosen criterion which is usually a function of the residuals. The residuals are given by

$$r = Z - h(X) \tag{2}$$

In the WLS method the criterion is defined as

$$J_{WLS} = \sum_{i=1}^{m} w_i r_i^2 \tag{3}$$

where

r_i are the components of r

w_i are chosen weights (usually the inverse of the assumed covariance of the measurement error).

If the measurement errors are small and random (preferably if normally distributed) their effect in an estimate obtained by minimising (3) will be compensatory, i.e. the method will work as an averaging process producing an estimate with a level of uncertainty smaller than the one present in the measurements. In the case of bad data an adequate solution would be one in which the residuals corresponding to the grossly wrong measurements are approximately equal to the error. This kind of solution can never by achieved using the WLS criteria because the sum of the square of such large residuals would certainly make that solution not optimal.

Estimators less vulnerable to bad data can be designed using criteria which assigns less importance to large residuals. An extreme case would be the criteria given by

$$J_{LP} = \sum_{i=1}^{m} w_i |r_i| \tag{4}$$

An algorithm based on (4) will always choose an estimate that satisfies exactly n of the m equations of (1), i.e. the solution of the algorithm will lie in one of the vertices of the polyhedron defined by the measurement equations. Provided the number of bad data is smaller than m-n the algorithm will exclude the equations corresponding to the bad data from the final solution in most of the practical situations. The estimates will normally contain a degree of uncertainty of the same level of the given measurements (low filtering capability).

In Figure 1 a hypothetical 2-dimensional problem is depicted and the probable solutions obtained by minimising the criteria defined in (3) and (4) are shown. In this figure the full lines represent a set of four measurement equations corresponding to measurements Z_1 to Z_4. The dotted line represents the fourth equation in the case of large error (bad data). S_1 and S_2 are the solutions given by the LP criterion and S_4 and S_3 by the WLS criteria for the cases with and without bad data respectively.

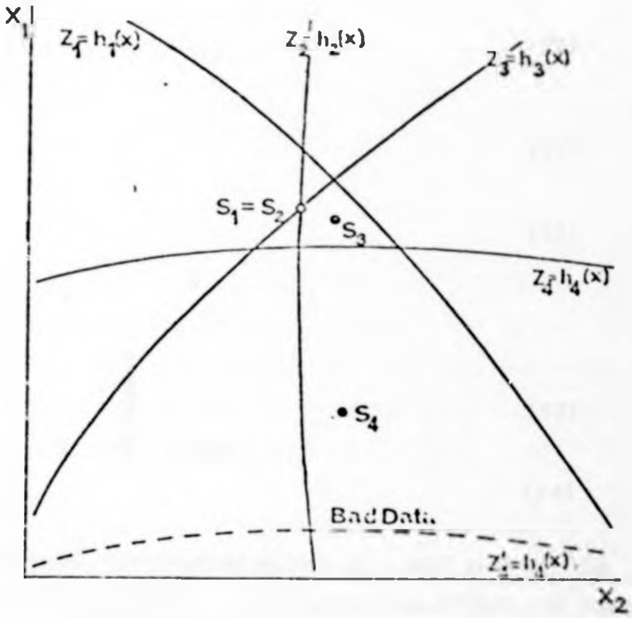

Figure 1. Two-dimensional example

LP DECOUPLED ALGORITHM

The usual procedure for the solution of non-linear state estimation problems is by successive linearisation[1]. The Taylor's expansion of (2) in the neighbourhood of X^o gives

$$r^o = \Delta Z^o - H(X^o)\Delta X^o \tag{5}$$

where

$$\Delta Z^o = Z - h(X^o)$$

$$\Delta X^o = X - X^o$$

$$H(X^o) = \left.\frac{\partial h(X)}{\partial X}\right|_{X=X^o}$$

The solution of the minimisation problem defined by (4) and (5) will give a first approximation to the estimate. The final solution will be given by solving the sequence of minimisation problems defined by

$$\text{minimise } J^k = \sum_{i=1}^{m} w_i |r_i^k| \tag{6}$$

$$\text{subject to} \quad H(X^k)\Delta X^k + r^k = \Delta Z^k \tag{7}$$

where k = 1, 2, ... is the iteration counter.

Each of the problems defined by (6) and (7) can be formulated as a LP problem provided some changes of variables are introduced to guarantee that the variables involved will remain positive. These changes are as follows:

1. substitute the residual terms r_i^k by the difference of two positive slack variables

$$r_i^k = s_{2i-1}^k - s_{2i}^k \qquad i = 1, \ldots, m \tag{8}$$

ii. add a large constant d to the elements of x

$$x_i' = x_i + d \qquad i = 1, \ldots, n \tag{9}$$

which will lead to a redefinition of Z as

$$Z' = Z + H(X^k)d \tag{10}$$

The LP problems can now be formulated as

$$\text{Min } J^k = \sum_{i=1}^{m} w_i(s_{2i-1} + s_i) \tag{11}$$

$$\text{Subject to } (H(X^k) \vdots U) \begin{bmatrix} X' \\ --- \\ S \end{bmatrix} = \Delta Z' \tag{12}$$

$$X', S \geq 0$$

where

$$U = \begin{bmatrix} 1 & -1 & & & & \\ & & 1 & -1 & & \\ & & & & 1 & -1 \end{bmatrix}_{m \times 2m} \tag{13}$$

$$S = (s_1 \; s_2 \; \ldots \; s_{2m-1} \; s_{2m})^T \tag{14}$$

In the optimal solution of the LP problems formulated above at least one of the slack variables defined by (8) will be null (non-basic). Therefore the objective function given by (11) is equivalent to (6).

The decoupling of active and reactive equations in the state estimation problem has been applied successfully to algorithms based on the WLS method. It reduces the computing time per iteration and storage by the use of constant gain matrices without affecting the reliability and accuracy of the algorithms. The same technique can be applied to the LP estimator by decoupling the set of constraint equations (12) into active and reactive subsets. A further simplification can be obtained by also assuming a sub-optimal decoupled criterion function. In this decoupled version the sequence of optimisation problems leading to the solution will be given by

$$\text{Min} \sum_{i=1}^{mp} w_{i,p}(s'_{2i-1} + s'_{2i})$$

subject to

$$(H_p \vdots U_p) \begin{bmatrix} \Delta\theta' \\ --- \\ S_p \end{bmatrix} = \Delta Z_p \tag{15}$$

and $\Delta\theta'_p, S_p = 0$

$$\text{Min} \sum_{i=1}^{mq} w_{i,q}(s''_{2i-1} + s''_{2i})$$

subject to

$$(H_q \vdots U_q) \begin{bmatrix} \Delta V' \\ --- \\ S_q \end{bmatrix} = \Delta Z_q \tag{16}$$

$$\Delta V', S_q \geq 0$$

where

θ, V :	vectors of voltage phase angles and magnitude
Z_p, Z_q :	vectors of active and reactive measurements
$W_{i,p}, w_{i,q}$:	active and reactive weights
U_p, U_q, S_p, S_q:	as defined in (13) and (14) with appropriate dimensions
mp, mq :	number of active, reactive measurements
H_p, H_q :	submatrices of the Jacobian matrix $H(X_o)$; the elements of H_p and H_q are made state independent by the introduction of some approximations as defined in[4].

COMPUTATIONAL ASPECTS

All purposes "packages" for the solution of LP problems like the ones given by (15) and (16) using different forms of the simplex method are largely available. These programs are very efficient and can be used with good results in off-line applications of the LP estimator. However, for on-line applications, in which a modest size process control computer is normally used, a smaller and dedicated LP routine will certainly be required.

In the development of the LP routine the following observations about particular characteristics of the LP problems will help to achieve efficient programming:

i. Matrix of coefficients On the submatrices corresponding to the Jacobian (H_p and H_q) should be stored. The elements of U_p and U_q can be generated easily by the algorithm when required. As H_p and H_q are sparse a compact form of storage[6] should be used.

ii. Initial basic feasible solution It can be obtained directly by choosing m slack variables as basic variables. This choice has as an extra advantage a unity initial basis matrix.

iii. Sparsity The coefficient matrix as a whole is very sparse due to the form of submatrices U_p and U_q. A typical value of percentage of non-zero elements is 1%. Therefore the Revised Simplex algorithm with the inverse basis in product form[5,6] should be used with considerable savings in storage and computing time.

NUMERICAL RESULTS

The Fast Decoupled Linear Programming (FDLP) state estimator just described was tested on a variety of simulated estimations corresponding to different networks, meter placement, number and size of bad data, etc. In each of the cases studied, estimations using the Full WLS (FWLS)[1], Fast Decoupled WLS (FDWLS)[4] and the Full Linear Programming (FLP)[3] were also performed in order to produce a comparative assessment of the algorithm studied.

The networks used for the study were the 5-busbar system given in [3] and the standard 14, 30 and 57 busbar IEEE test system. Table 1 shows the metering pattern adopted for each system. In this table NK = no. of nodes; NL = no. of lines; NI = no. of measured injections; NF = no. of measured flows; NV = no. of measured voltages and η = redundancy ratio.

The data for the simulations (measurements) were obtained by adding a normally distributed error to the values obtained from load flow calculations. The bounds of the error were set to .3% of the full scale plus 1.5% of the actual value of the measured quantity.

SYS	NK	NL	NI	NF	NV	n
A	5	7	2	14	1	1.9
B	14	20	4	40	2	1.7
C	30	21	12	82	6	1.7
D	57	80	14	160	7	1.6

Table 1 - Test Systems

The quality of the estimation was measured by the performance index

$$J_p = \frac{1}{m} \sum_{i=1}^{m} \frac{(Z_i^E - Z_i^T)^2}{\sigma_i^2} \tag{17}$$

where Z_i^E and Z_i^T are the estimated and true values of the i-th measurement and σ_i is the assumed standard deviation of the error. The expected value of J_p for an optimal estimator is given by the inverse of the redundancy ratio[4] n. For the systems given in Table 1 this value varies from .526 to .625. Values of J_p much larger than these indicate a poor performance of the estimator.

In Table 2 the results obtained in the simulations using the systems in Table 1 are summarised. The values shown for systems A,B,C and D correspond to the cases in which only the normally distributed error whose bounds are defined above is present. Systems B* and C* are the same as B and C with the only difference that the error in two measurements (in each system) were made equal to 50% of the actual value of the measurement (bad data).

METHOD	SYS.	A	B	B*	C	C*	D
FLP	No.Ite.	3	3	3	4	4	5
	Time	.090	1.088	1.120	8.845	9.101	17.237
	J_p	1.416	.910	1.125	1.236	1.401	1.191
FDLP	No.Ite.	4	3	4	4	5	5
	Time	.020	.196	.201	.818	.807	1.348
	J_p	1.534	.981	1.087	1.325	2.118	1.237
FWLS	No.Ite.	3	3	4	4	4	5
	Time	.018	.165	.166	.644	.670	1.227
	J_p	.254	.381	25.1	.297	15.7	.354
FDWLS	No.Ite.	4	4	5	4	5	5
	Time	.008	.023	.023	.058	.060	.147
	J_p	.230	.318	28.30	.320	18.3	.308

Table 2 - Summary of Results

The values of J_p in Table 2 show that the FDLP estimator has a performance similar to the FLP estimator. When no bad data is present in the measurements both LP estimators have a poorer response than the WLS estimators. However, in cases with bad data (B* and C*) the value of J_p for the LP estimators is practically unaffected which indicates a rejection of the bad data while showing unacceptable results for the WLS algorithms. A big improvement in terms of speed was obtained by the application of the decoupling technique to the LP estimator as can be seen by comparing the times fo the FDLP and FLP. The time requirements of the FDLP was found to be approximately equal to the FWLS which is acceptable for some practical applications. Storage requir ments of FDLP are about the same as the FDWLS provided an efficient programming is use for the Simplex algorithm.

CONCLUSIONS

In this paper a fast decoupled version of a state estimator using linear programming was presented. The results of simulation studies showed that the estimator i able to automatically reject bad data eventually present in the measurements avoiding the need for a post-estimation residual search and re-runs of the estimator as it is the case with the weighted least square estimator. This property of the linear programming estimator compensates its higher time requirements making the presented metho a practical option for on-line implementation.

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NONQUADRATIC STATE ESTIMATION: A COMPARISON OF METHODS

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In this paper a comparative performance study of two nonquadratic state estimators is presented. The first one uses a fast decoupled version of the bad data suppression method which combines numerical efficiency with a capability to eliminate bad data. The second one is based on a recently proposed estimator using linear programming which is also presented in a decoupled formulation. The two methods are compared in terms of the ability to detect and identify bad data, reliability of convergence, time and storage requirements. Results are given for networks of different sizes and characteristics with different metering patterns.

INTRODUCTION

One of the main problems encountered in developing power system state estimation algorithms for real-time applications is how to cope with the eventual presence of large and totally unpredictable errors in the measurements (bad data). Algorithms based on a quadratic weighting criterion (weighted least squares) fail in the presence of bad data due to unduly large weights assigned to measurements corresponding to large residuals which are probable bad data. Nonquadratic weighting criteria can be chosen to render the state estimator less vulnerable to bad data by minimising the influence of large residual measurements in the results of the estimation.

In this paper a performance analysis of two classes of nonquadratic estimators is presented. In the first one, the bad data suppression estimator (BDS), the weight of a measurement with a residual larger than a preset threshold is modified according to a nonquadratic criterion (e.g. quadratic-square root). This basic BDS algorithm, suggested in the literature [1,2], is presented in a fast decoupled version which possesses many of the numerical advantages of the fast decoupled weighted least squares estimator [3] and has in addition the useful function of bad data detection and identification. The second one uses as criterion the sum of the moduli of the residuals [4]. Each iteration of the algorithm is formulated as a linear programming (LP) problem. In the present version the LP problem is reduced to two smaller ones by decoupling active and reactive variables.

FAST DECOUPLED BAD DATA SUPPRESSION ESTIMATOR

In the BDS estimation problem [1,2], it is required to find the estimate of the state vector x such that

$$J(x) = \rho^t(x) R\, \rho(x) \tag{1}$$

is minimum, where $\rho(x)$ is a vector of non-linear functions, the m-th element of which is

$$\rho_m(x) = \begin{cases} r_m/\sigma_m & \text{if} \quad |r_m/\sigma_m| \le \lambda \\ f(r_m/\sigma_m) & \text{if} \quad |r_m/\sigma_m| > \lambda \end{cases} \tag{2}$$

where f is a non-linear function of r_m. In this paper the following function (quadratic-square root) is used:

$$f(r_m/\sigma_m) = \lambda\sigma_m \text{ sign } (r_m)(4\left|\frac{r_m}{\sigma_m}\right|^{\frac{1}{2}} - 3)^{\frac{1}{2}} \tag{3}$$

An approximate value for the solution can be found through the iterative process

$$\theta^{k+1} = \theta^k + A_p^{-1} H_p R_p G_p \rho_p(\theta^k, V^k) \tag{4}$$

$$V^{k+1} = V^k + A_q H_q R_q G_q \rho_q(\theta^k, V^k) \tag{5}$$

where $A_p = H_p^t R_p H_p$; $A_q = H_q^t R_q H_q$

$G_p = \partial\rho_p(x)/\partial r_p$, $G_q = \partial\rho_q(x)/\partial r_q$

The algorithm uses constant coefficient matrices A_p and A_q whose diagonal dominance is preserved even though more than one large error is present in the vicinity of a node. Hence the numerical instability of the basic BDS algorithm reported for such cases is overcome.

The breakpoint λ affects convergence and the bad data suppression effect: if λ is too small (e.g. ≈ 1) convergence is slow and the risks of local minima are increased as healthy measurements will be taken as bad data. However, if λ is too large (≈ 20) then the usual bad data ($>5\sigma$) will not be recognised. A good choice was found to be $\lambda = 5$.

When bad data is detected in the active or reactive part of a complex measurement the corresponding terms in matrices G_p and G_q are set to equal values such that the following inequalities assumed in the derivation of equations (4) and (5) are maintained.

$$H_p^t R_p G_p >> H_{q\theta}^t R_q G_q \text{ and } H_q^t R_q G_q >> H_{pV}^t R_p G_q$$

FAST DECOUPLED LINEAR PROGRAMMING ESTIMATOR

In this method the state estimates are obtained by minimising

$$J(x) = \sum_{m=1}^{M} R_m |r_m| \tag{6}$$

Using the above criterion the estimation problem can be formulated as a sequence of LP problems which in a decoupled version will be given by

$$\text{Minimise} \sum_{m=1}^{M_p} R_m p\,(s_{2m-1} + s_{2m})$$

$$\text{subject to } (H_p, U_p)\begin{bmatrix} \theta^k - \theta^{k-1} \\ s_p \end{bmatrix} = r_p^k \tag{7}$$

$$\theta^k - \theta^{k-1}, s_p \geq 0$$

and

$$\sum_{m=1}^{M_q} R_m, q\,(s_{2m-1} + s_{2m})$$

$$\text{subject to } (H_q, U_q)\begin{bmatrix} V^k - V^{k-1} \\ s_q \end{bmatrix} = r_q^k \tag{8}$$

$$V^k - V^{k-1}, s_q \geq 0$$

where

$$U_p, U_q = \begin{bmatrix} 1 & -1 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & 1 & -1 & \cdots & \cdots \\ \vdots & & \vdots & & & \vdots \\ \cdots & \cdots & \cdots & \cdots & 1 & -1 \end{bmatrix}$$

The state estimates given by (7) and (8) will always satisfy (within the calculation accuracy) N of the M measurement equations, i.e. it will lie on the point of intersection of N hyperplanes representing the N chosen equations. If the number of bad data is less than M-N in most of the practical situations the estimator will choose a solution point in which the equations corresponding to the bad data are excluded.

In order to obtain an algorithm suitable for real-time application a customised routine exploiting the particular characteristics of the LP problems given in (7) and (8) should be used. An improvement in speed, while maintaining the bad data elimination characteristics of the method, is obtained by solving completely the LP problems only on the first iteration. In the following iterations the same optimal basis is maintained.

SIMULATION STUDIES

Simulation results for the 14, 30 and 57 busbars IEEE standard test systems, with the measurement patterns shown in table 1, are presented in this section. The measurements were simulated by adding a normally distributed random perturbation to the results of a load-flow. The standard deviation of the error was calculated as .4% of the meter full-scale plus 1-3% of the true values of the measurements. In some cases larger errors (bad data) were also added to the measurements. The location of the bad data was chosen at random and the size was fixed as 25% of the true value.

The validity of the estimations was assessed by calculating the performance indices J_1, J_2 and J_3 as follows

$$J_i = \frac{1}{M} \sum_{m=1}^{M} (Z'_m - Z''_m)^2/\sigma_m^2, \quad i = 1,2,3$$

where Z'_m and Z''_m are measured and true values for J_1, measured and estimated values for J_2 and estimated and true values for J_3, respectively. The expected values of J_1, J_2 and J_3 are shown in table 1.

Table 1. Test systems and measurement patterns

System	No. lines	Max. R/X	Measurement Pattern A No. Meas	Redun-dancy	Expected values J_1	J_2	J_3	Measurement Pattern B No. Meas.	Redun-dancy	Expected values J_1	J_2	J_3
14BB	20	.6	58	2.1	1.0	.53	.47	98	3.6	1.0	.72	.28
30BB	41	1.1	106	1.8	1.0	.46	.56	188	3.2	1.0	.69	.31
57BB	80	1.1	213	1.9	1.0	.47	.53	369	3.3	1.0	.69	.31

Tables 2, 3 and 4 contain the results of three of the studies carried out. For each system three out of ten simulations are shown. The simulation studies were run in a general purpose 65K/265K 60-bit word CDC 7600 computer. The time shown is average CPU time in seconds. In those tables NBD = no. of bad data and NIT = no. of iterations.

The bad data capability of both estimators was observed to be adequate for the case of high redundancy ratio (table 4) in which, for all ten simulations, the bad data were detected. In the cases of low redundancy ratios (tables 2 and 3) both methods failed partially or totally in 50% of the cases containing bad data. The effect of the measurement noise level was found to be of little importance in the final results for both estimators. The LP method has less filtering capability than the BDS method as can be observed by values of J_3 larger than the expected values. This effect is less pronounced in the case of high redundancy ratio (table 4).

Table 2. Results for measurement pattern A and 3% measurement noise

System	Error		BDS				LP			
	NBD	J_1	NIT	Time	J_2	J_3	NIT	Time	J_2	J_3
	0	.72	5	.037	.43	.30	5	.077	1.78	1.72
14BB	4	28.54	8	.046	28.54	1.16	5	.088	27.07	1.21
	6	56.16	9	.050	14.94	15.15	5	.113	62.28	48.45
	0	.75	5	.070	.29	.46	5	.374	1.13	1.69
30BB	4	23.53	10	.099	4.90	.77	5	.465	15.90	1.74
	6	38.39	13	.121	8.37	20.42	5	.353	49.49	54.47
	0	.82	5	.201	.39	.47	6	1.021	1.13	.94
57BB	4	16.27	8	.199	3.01	.68	6	.987	18.37	2.01
	10	19.65	12	.207	4.70	5.70	6	1.123	21.16	22.11

Table 3. Results for measurement pattern A and 1% measurement noise

System	Error		BDS				LP			
	NBD	J_1	NIT	Time	J_2	J_3	NIT	Time	J_2	J_3
	0	.72	6	.039	.43	.28	5	.077	.91	.97
14BB	4	263.78	7	.040	24.31	1.06	5	.088	259.81	1.39
	6	522.35	25	.105	44.9	76.30	5	.115	521.74	65.89
	0	.75	5	.073	.29	.47	5	.365	2.32	2.54
30BB	4	168.36	15	.130	11.20	.58	5	.347	170.63	2.67
	6	195.72	17	.145	19.05	132.80	7	.412	667.50	744.03
	0	.81	6	.187	.48	.51	6	1.734	1.95	1.87
57BB	4	324.25	8	.213	16.17	.88	6	1.001	2.18	2.16
	10	410.18	16	.207	25.16	257.16	6	.995	287.17	341.47

Table 4. Results for measurement pattern B and 1% measurement noise

System	Error		BDS				LP			
	NBD	J_1	NIT	Time	J_2	J_3	NIT	Time	J_2	J_3
	0	.79	6	.045	.61	.18	13	.164	.78	.40
14BB	4	67.89	7	.050	18.75	.88	5	.192	66.74	.42
	6	410.70	22	.113	28.90	.48	5	.209	410.27	1.11
	0	.85	6	.081	.70	.30	5	1.503	.92	.38
30BB	4	44.55	8	.121	5.93	.35	5	1.798	40.50	.73
	6	105.42	21	.162	12.84	.48	5	1.563	103.59	.75
	0	.94	7	.174	.67	.28	6	2.753	1.06	.82
57BB	4	22.49	9	.203	3.00	.34	6	3.088	18.34	.81
	10	81.67	13	.264	9.53	.78	6	3.187	56.31	1.91

The time and storage requirements of the LP method are larger than that of the BDS method, particularly for a system with high redundancy ratio. As the routines used to obtain the implicit inverse of the matrices A_p and A_q in the BDS method are more efficiently programmed than the routine used to solve the LP problems, it is believed that the performance of the LP method can be improved in terms of time and storage requirements.

The number of iterations required by the BDS method in some cases was relatively large compared with the LP method which indicates that the LP method is slightly more reliable. No cases of non-convergence happened in the simulation studies for any method.

CONCLUSIONS

A performance study of two non-quadratic state estimators was reported in this paper. The first method is a fast decoupled version of the bad data suppression algorithm and the second method is a decoupled estimator using linear programming. Both methods were found to behave adequately in the presence of bad data provided the redundancy ratio is high enough. The bad data suppression algorithm has smaller time and storage requirements than the linear programming method while the latter is slightly more reliable.

LIST OF SYMBOLS

N	= number of state variables
M	= number of measurements
θ, V	= voltage angle and magnitude vectors
x	= state vector; $x = (\theta, V)$
Z	= measurement vector
$h(x)$	= non-linear vector-function relating Z and x
k	= iteration counter
p, q	= subscripts indicating active, reactive quantities
m	= subscript indicating the m-th component of a vector
σ_m	= standard deviation of the m-th measurement error
R^m	= diagonal matrix of weights ($R_m = 1/\sigma_m^2$)
r	= residual vector; $r = Z - h(x)^m$
H_p, H_q, $H_{p\theta}$, H_{pV}	= submatrices of $\partial h(x)/\partial x$ as defined in reference [3] for the FDA method
S	= vector of slack variables; $S = (s_1 \; s_2 \; \ldots \; s_{2M})$

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