DEVELOPMENT AND DESIGN OF ADJUSTABLE SPEED INDUCTION MACHINES

Thesis presented to the Victoria University of Manchester for the award of the degree of Ph.D. by

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TABLE OF CONTENTS

Page No.

CHAPTER I.

POLE CHANGE MOTORS USING PHASE-MIXING TECHNIQUES.

1.	Introduction.	1
2.	Short-Stator Machines.	4
3.	The New Pole-Changing System.	5
4.	Development of the Basic System.	9
	4. 1. Winding Arrangements.	10
	4. 2. Machines with no inactive arcs.	11
5.	The 3k System.	12
	5. 1. Winding arrangements in a 3k block system.	13
	5. 2. Chorded coils.	13
	5. 3. A second basic coil arrangement.	14
6.	An Experimental Machine.	17
7.	A General Scheme for any Block Shape or Pole Number.	20
8.	Some two-speed Switching Arrangements.	21
9.	The Second Experimental Machine.	27
10.	Conclusions.	29
11.	References.	31

CHAPTER II.

WINDING DESIGN FOR POLE-CHANGE MACHINES USING PHASE-SHIFT TECHNIQUES.

1.	Introduction.	32
2.	The Datum Machine.	33

TABLE OF CONTENTS (contd)

		Page No.
3.	Magnitude of Harmonic Pole Numbers.	35
	3.1. 3 and 5-speed machines.	36
	3.2. Two-speed machines.	38
	3.3. Application of Chording to Harmonic Reduction.	39
4.	'Two Layer' Segments.	41
5.	The Effect of Using Practical Windings.	44
	5.1. The effect of mechanical shift on winding harmonics.	45
	5.2. Effect of Electrical Phase Shift on winding harmonics.	47
6.	Terminal Voltage.	48
	6.1. Reverse Phase Sequence Voltages.	48
	6.2. Zero Phase Sequence Components.	50
	6.3. Magnitude of the Supply Voltage.	52
7.	Pole Change combined with Reversal.	53
8.	Winding Examples.	55
9.	An Experimental Machine.	61
	9.1. An Industrial Machine.	63
10.	Conclusions.	64
11.	Appendices.	

CHAPTER III.

COMPLETELY BALANCED TWO-SPEED MACHINES.

1.	Introduction.	65
2.	Machines using Interleaved Blocks.	65
3.	Conditions for Voltage Balance.	65

TABLE OF CONTENTS (contd)

		Page No.
4.	Balanced Winding Segments.	66
	4.1. A machine which is balanced on one pole number.	67
	4.2. A completely balanced machine.	67
5.	Winding Examples.	72
6.	An Experimental Machine.	75
7.	Conclusions.	77

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SUMMARY.

The thesis describes the development of a new type of pole-change motor giving 2, 3 or 5 different speeds. The development is an extension of the principles of "phase-mixing" which were explained in a recent paper [1], which described a continuously-variable-speed brushless induction motor.

In the earlier machine a fraction of the periphery could not be used owing to the continuously variable pole-pitch requirement, and the stator windings in the active arc were fed from phase-shifting regulators.

The thesis shows how acceptance of a finite number of specific speeds may be exploited to enable the whole of the periphery of the machine to be used, and, at the same time, to replace the phase-shifting regulators by switches. The complexity of the switch arrangement depends on the number of speeds required, two speed machines requiring six machine terminals.

Theoretical investigation of the harmonic effects is considered and the material is supported by experimental results from several machines. High values of copper utilization and low harmonic content are shown to be possible with the phase-mixing technique.

CHAPTER I.

POLE-CHANGE MOTORS USING PHASE-MIXING TECHNIQUES

1. INTRODUCTION

Fundamentally, the stator of an induction motor may be regarded as a slotted iron structure with a series of current-carrying conductors in the slots. Whatever the arrangement of conductors in any one slot, their effects may be regarded, so far as the rotor is concerned, as being equivalent to that of a single conductor in the slot carrying alternating current.

The question of the speed of the fundamental travelling field produced by the whole series of currents in slots is determined entirely by the average phase difference between adjacent slot currents around the periphery.* Thus a 2-pole motor having 12 stator slots has an average phase displacement of 30° per slot. This may be obtained by having a phase difference of 60° between alternate pairs of slots and zero difference between the others, as in Figure 1(a), or by having a uniform 30° difference, as in Figure 1(b). The difference between the windings shown in Figures 1(a) and (b) will affect the harmonic content of the field but not the fundamental speed. Figure 1(a) shows a fully pitched, 3-phase winding. In such a winding 60° is the minimum phase difference which can be obtained in adjacent slots, but with a chorded winding such as that shown in Figure 1(b), a 30° phase increment is obtained by the technique of combining currents of two different phases within the same slot.

This technique of 'phase mixing' was recently extended to enable a continuously-variable-speed motor to be developed [1]. Since the publication

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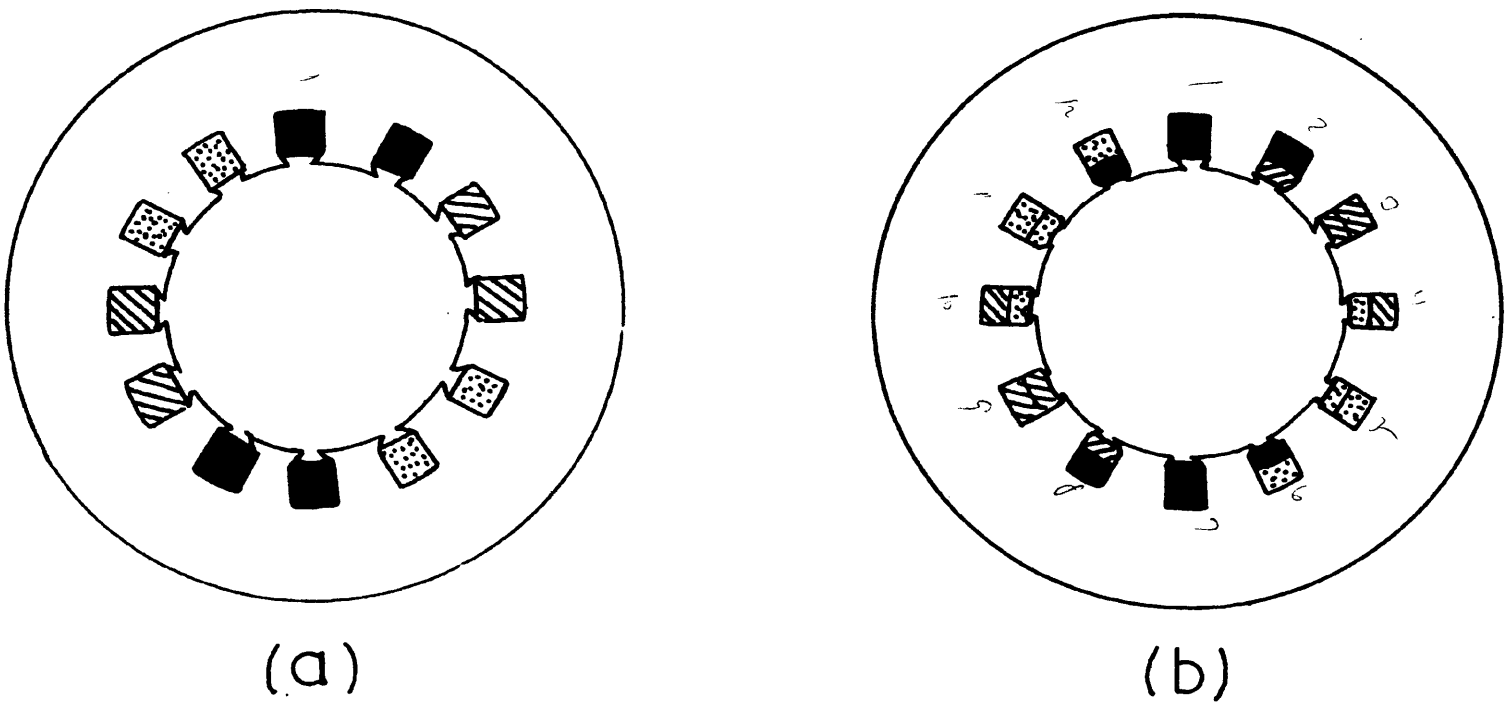
* The question of space-harmonic content is determined both by the relative amplitudes and phases of the particular slot current arrangement.



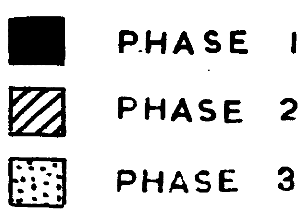


Figure 1. Winding arrangements in conventional induction motor stators.

(a) full pitch.
(b) 5/6 chorded.

of the original paper on this machine the name 'phase-mixer' motor has been applied to it. The principle is illustrated in Figure 2. Each of the three windings is a polyphase arrangement for producing a travelling field in the conventional manner, except that the numbers of conductors in successive slots are varied so that, theoretically, the current contribution of any winding in any slot is proportional to the appropriate shaded area. If, for example, winding A consists of a 2-layer winding having 2 slots per pole and phase, 5/6 chorded, the vector diagram for the slot currents in slots 1-12 is as shown in Figure 3(a). Winding B also consists of a 2-layer winding of the same pitch as A, and its vector diagram for the currents in slots 2-13 is shown in Figure 3(b). If the effective currents from windings A and B are in phase with each other in any one slot, then they are in phase with each other in every slot and the complete vector diagram for the total slot currents for slots 1-13 is as shown in Figure 3(c). The contributions from winding B are shown as continuous lines and those from winding A as broken lines.

In such a condition, the current distribution is identical with that of a conventional winding of 2 slots per pole and phase, 5/6 chorded, the slot currents from slots 1-12 corresponding exactly with those of Figure 1(b).

If now winding A is fed with a current whose phase is such that in each slot the contributions from windings A and B differ in phase by 60°, the vector diagram of the effective currents for slots 1-13 will be as shown in Figure 3(d). Slot 1, which was filled entirely with winding A, will carry a current advanced in phase, relative to the condition shown in Figure 3(c),

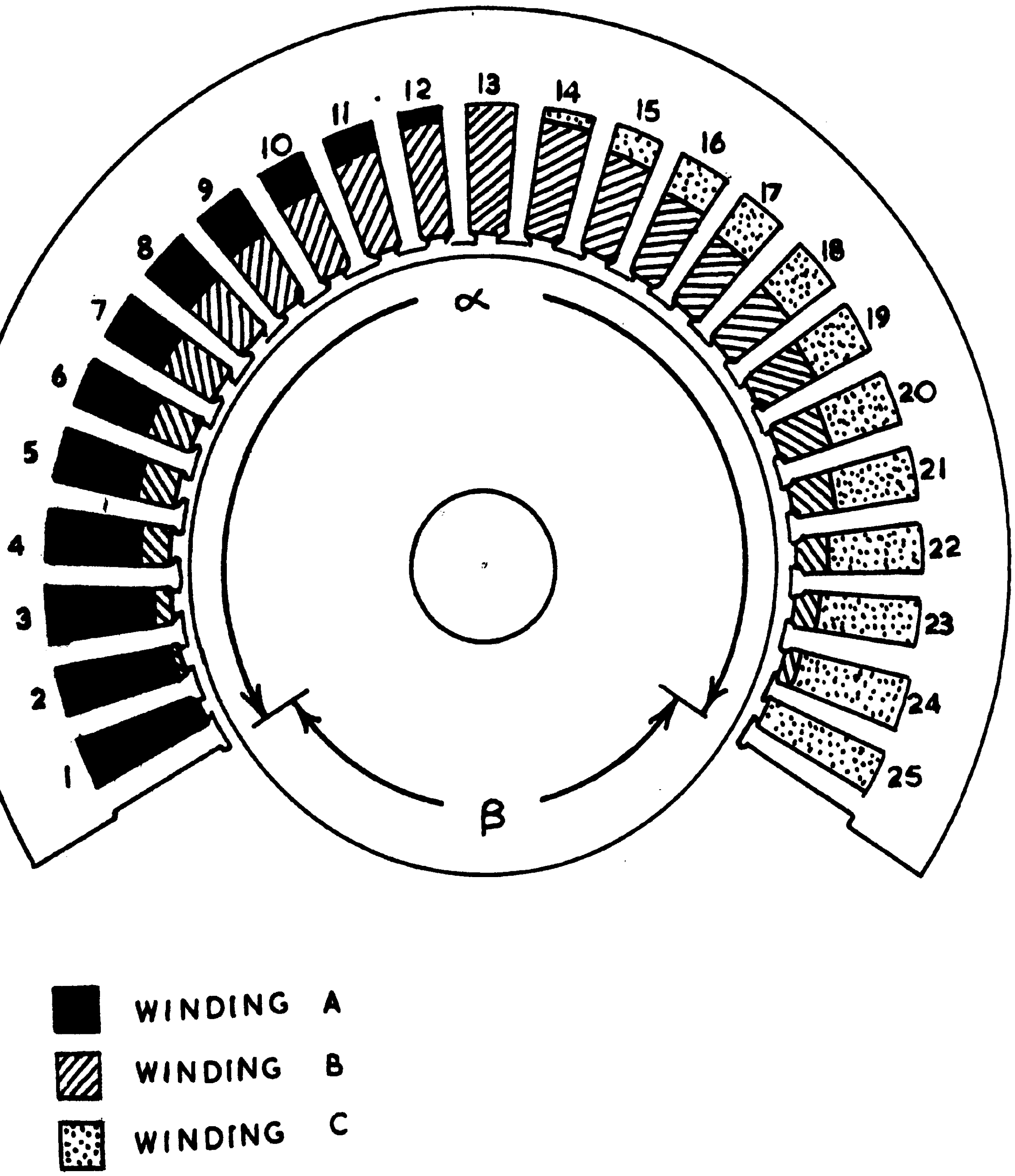


Figure 2. Winding arrangements in a 'phase mixer' motor.

by the full 60°, while slot 13, which carries only winding B, will have the same vector as in Figure 3(c); thus the 12 intervals of 30° between the current vectors in slots 1-13 in Figure 3(c) have been compressed into a total of 300°, and it will be appreciated that, by suitable choice of the number of turns, the phase of the currents in intermediate slots can be arranged to be equally spaced for any particular phase shift. The degree of error for any other angle of phase shift is shown in Reference 1 to be very small. Windings A, B and C are fed from three sources of current whose relative phases may be varied. The practice generally consists of feeding windings A and C from the secondary windings of phase-shifting regulators whose primary windings are in series with winding B and the mains supply. If each of the phase-shifters is set so as to produce zero phase shift, the active arc α of the machine shown in Figure 2 is identical with the corresponding arc of a conventional motor, except that the number of poles, n, on the active arc need not necessarily be such that $n360/\alpha$ is an even whole number, where α is the angle, in degrees, subtended by the arc at the centre of the machine. The break in the magnetic circuit, represented by the arc subtending an angle β at the centre, enables the rotor flux to decay virtually to zero between leaving one edge of the stator and entering the other. On re-entry, each rotor tooth is free to accept whatever phase of flux and current is appropriate, so that speeds intermediate between those associated with conventional induction motors are possible.

When the phase-shifters are set to an angle other than zero - it being

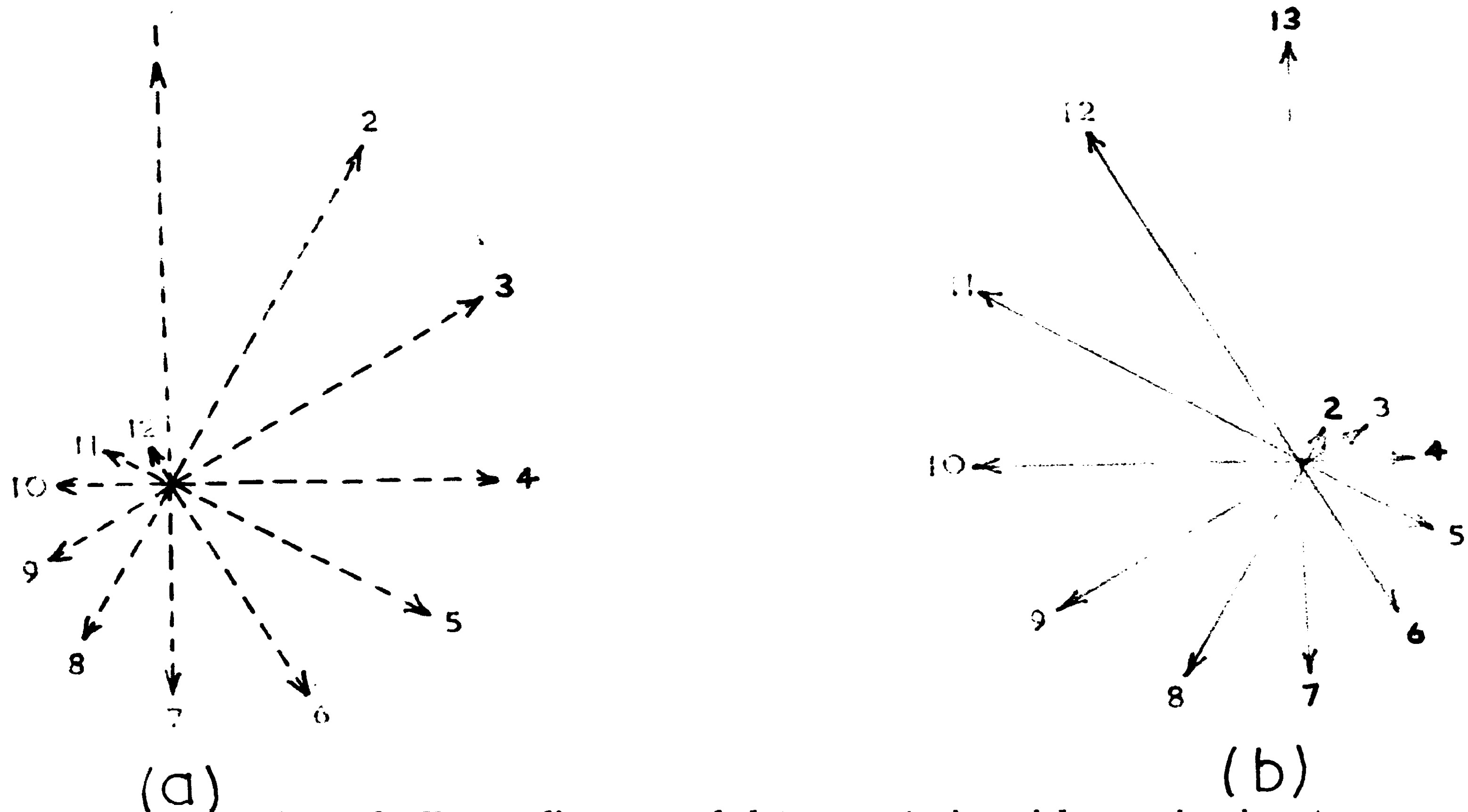


Figure 3. Vector diagrams of slot currents in a 'phase mixer' motor.

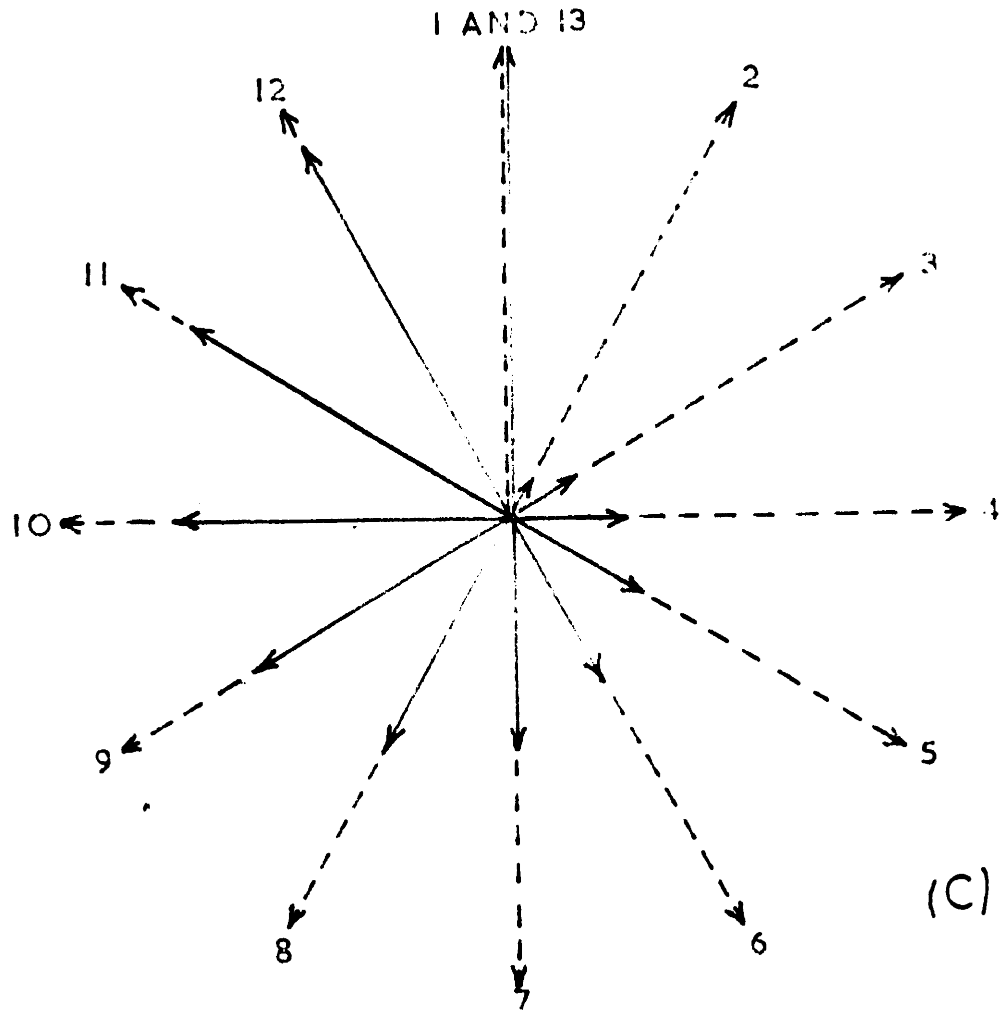


Figure 3. Vector diagrams of slot currents in a 'phase mixer' motor.

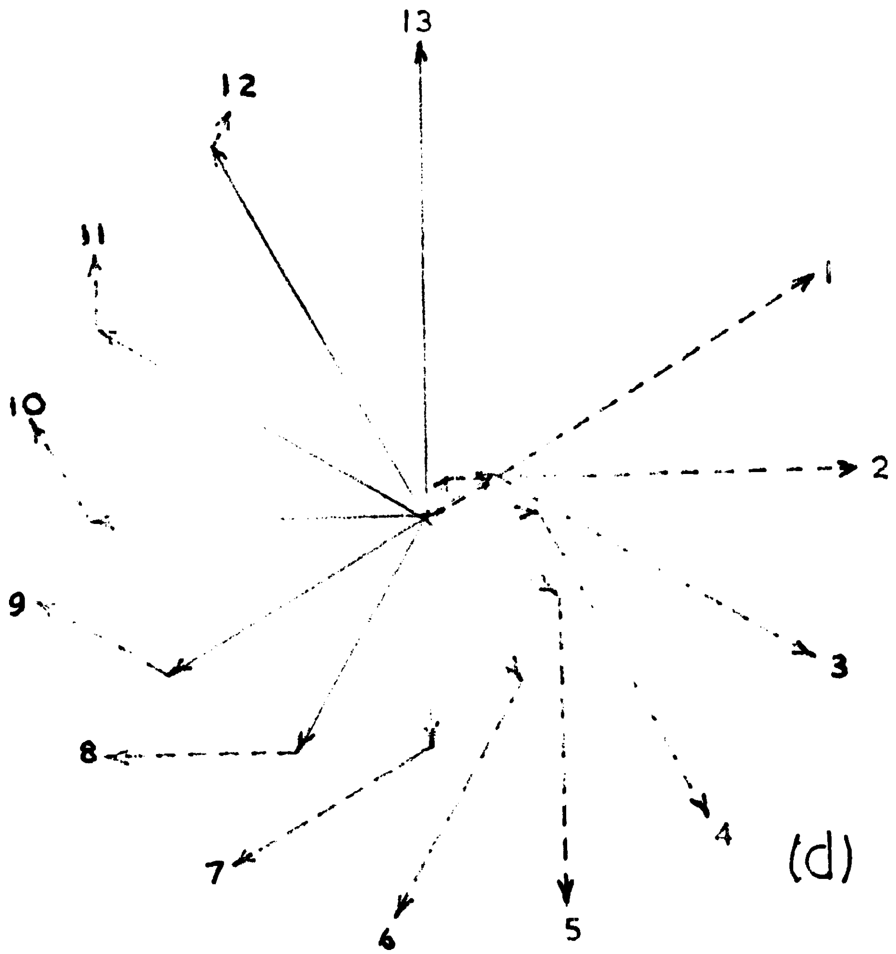


Figure 3. Vector diagrams of slot currents in a 'phase mixer' motor.

understood that if one is set to produce an angular shift of θ, the other is set to produce a shift of $-\theta$ - the current mixing in the slots is equivalent to 'compressing' the poles. With a break in the magnetic circuit making the motor a 'short-stator' machine, continuous speed control is therefore possible, the effective number of poles on the machine being given by $n360/\alpha$, even though this quantity is not an even whole number.

2. SHORT-STATOR MACHINES

Short-stator machines have inherent limitations which have been discussed at some length in earlier papers [2, 3]. To summarize their properties, maximum efficiency is obtained at a value of slip between $1/(n + 1)$ and $2/(n + 2)$, so that the peak efficiency cannot theoretically exceed $n/(n + 1)$, excluding stator losses. Not all the iron is usefully employed, owing to the non-uniform flux distribution inherent in short-stator machines, and the power factor is lower than that of a comparable conventional machine. The efficiency limit restricts the usefulness of short-stator machines to low-speed drives, which are only economically acceptable in fairly large sizes.

During experiments on the helical motor which is described in Section 8 of Reference 1 an investigation was carried out into the behaviour of short-stator machines in which the pole pitch did not correspond to the speed of a conventional induction motor (i.e. $n360/\alpha$ was not equal to an even integer) but in which the iron structure in the inactive arc β was not cut away. It was found that rotor flux was carried around the arc β with very little decay and on re-entry reacted either favourably or unfavourably, depending on its phase, with the stator current. If such a machine is wound and fed so as to produce the speed of a 7-pole motor, for example, it will be found to run either as a 6-pole motor or as an 8-pole motor, depending on whichever speed is nearer to that of the rotor. In neither case will the machine run at a high efficiency; excessive rotor losses will occur, and the machine may run noisily. When a pole-stretching system is used with a continuous iron structure, the rotor speed changes in jumps only, each speed corresponding to that of a conventional machine with a multiple of 2 poles.

If, however, a short-stator machine is wound and fed so that $n360/\alpha$ equals 2 or 4 or 6, etc., the carried-over flux reacts favourably, and the characteristics of the motor resemble very closely those of the corresponding conventional machine.

3. THE NEW POLE-CHANGING SYSTEM

It was pointed out in the earlier paper that, if it is proposed to accept a number of discrete speeds in place of continuous control, as in the phase-mixer motor, the phase-shifting equipment can be replaced by switches. Figure 4(a) shows how specified amounts of effective phase shift may be imparted to the currents in windings A and C of Figure 2 by means of switches. When all the switches are set to the upper position, the current system is as shown in Figure 4(b). Moving all switches to the centre contact results in a current system as shown in Figure 4(c). Comparison between the winding B in Figures 4(b) and (c) shows that an effective phase shift of 120° has been introduced in relation to winding C,

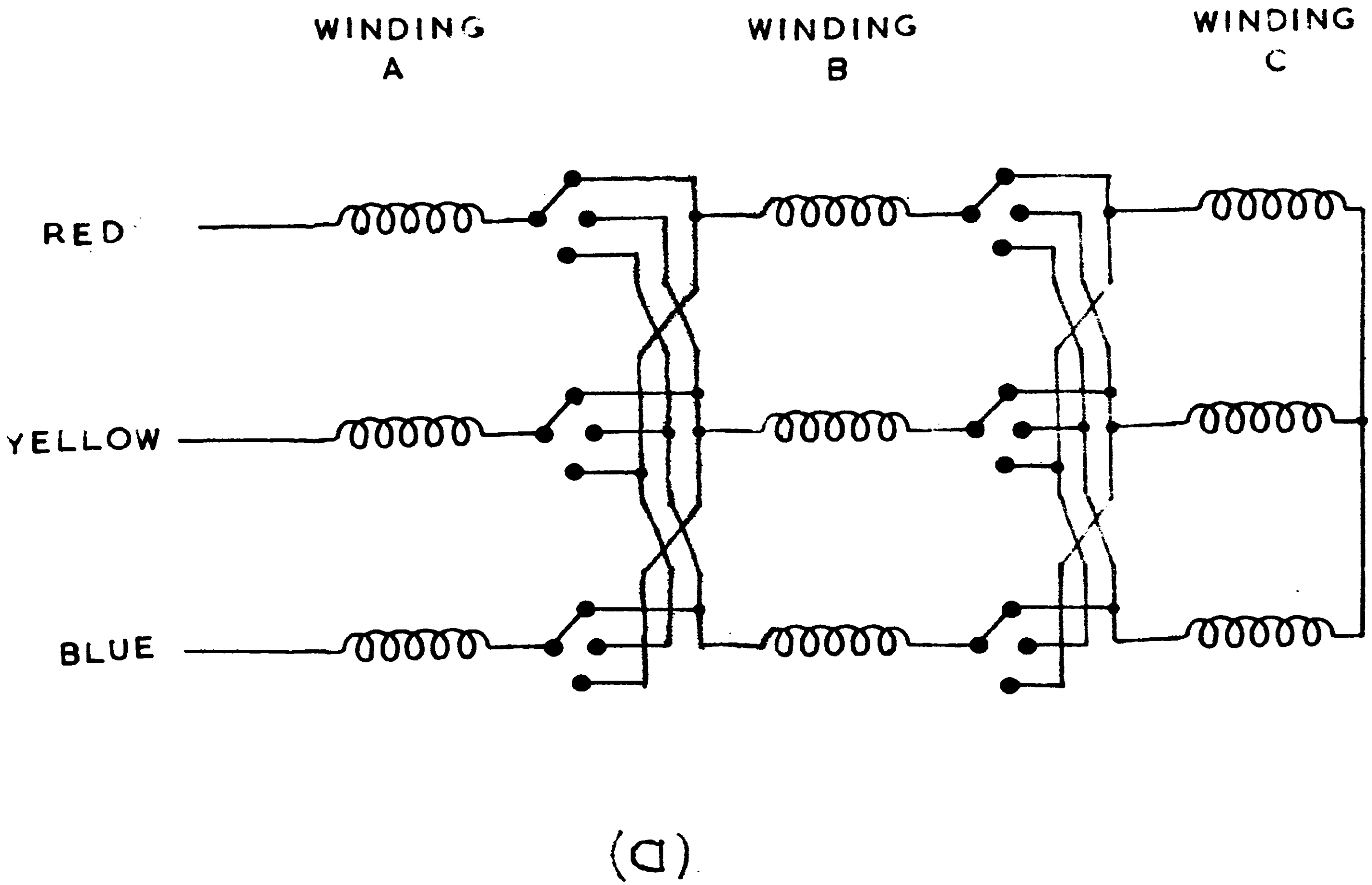


Figure 4. (a). **Switch connections for a 3-speed motor.**

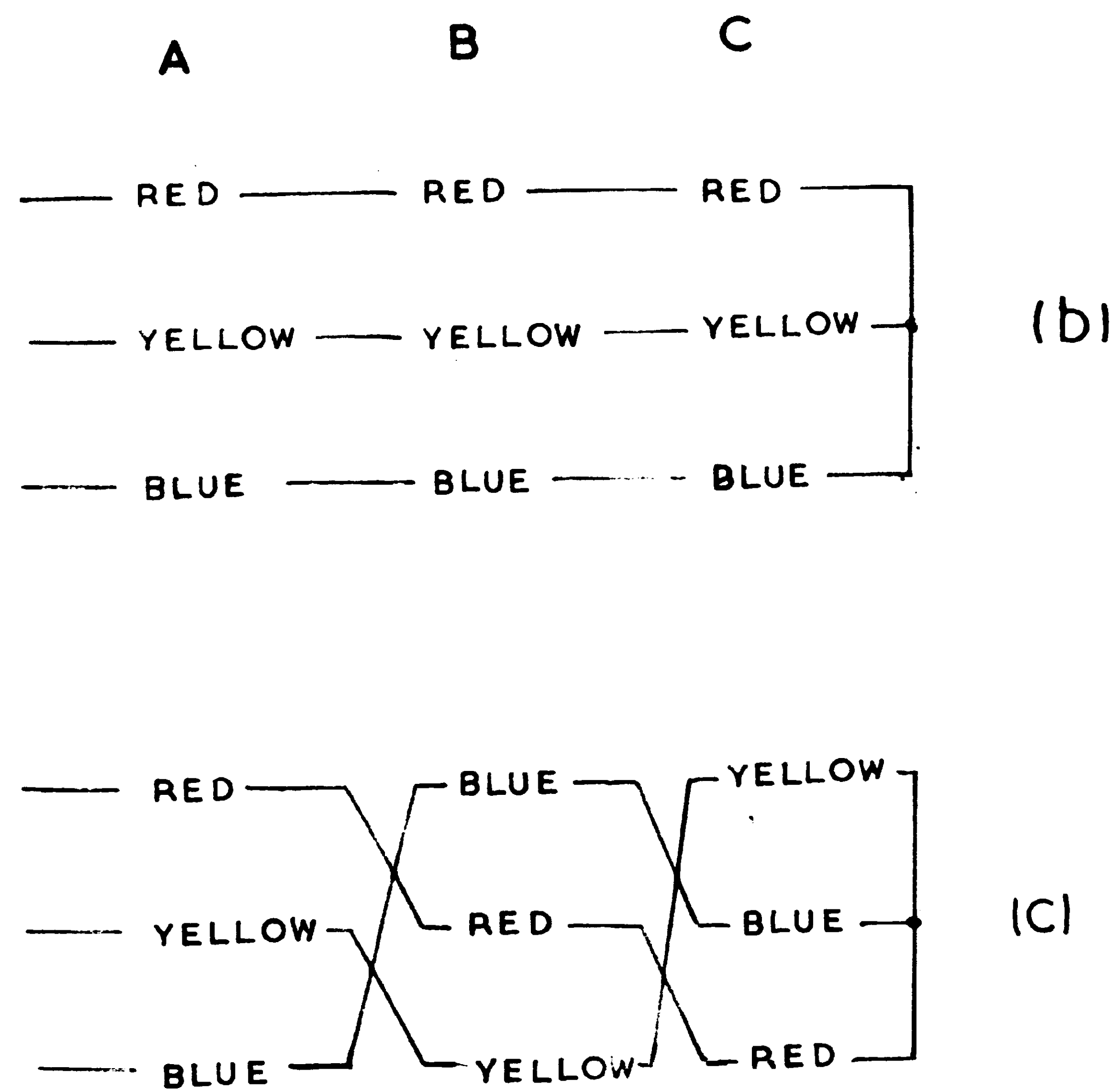


Figure 4 (b) and (c). Current phases for particular switch positions.

while for winding A the phase shift is 240° (or -120°). Setting all the switches to the lower position produces a phase shift of 240° ($\equiv -120^{\circ}$) in winding B and 480° ($\equiv 120^{\circ}$) in winding A. The use of such an arrangement produces a 3-speed motor. If, in addition to the above switches, winding B is fed through a reversing switch, additional phase shifts of $\pm 60^{\circ}$ are possible, and a 5-speed motor results.

It is now proposed to develop this method of switching to produce 3- or 5-speed arrangements so that at any of the three or five settings the number of poles in the active arc is such as to favour carry-over of flux and yield a conventional even-pole-number machine in which short-stator limitations no longer apply. Such machines are effectively 3- or 5-speed pole-change motors in which the whole of the stator winding is used at each speed.

The equations which must be solved to satisfy the above conditions are derived as follows. Figure 2 shows a motor with three windings. Reference 1 designates these windings as '$+\theta$', 'mains' and '$-\theta$' and goes on to show that the speed range of such a system is limited by the maximum phase shift which can be applied in the end slots. For example, slot 1 of Figure 2 can be advanced in phase by 120° and slot 25 retarded by 120°, a total phase change around the arc α of 240°. Further phase shift results in uneconomic use of stator copper, since a slot such as slot 7 may contain approximately equal quantities of copper from two windings, a situation which, for a 120° phase mix, gives a resultant effective current in the slot no larger than either of its constituents. Reference 1 goes on to describe how the total phase around the arc α may be increased by using five windings, the centre winding occupying a smaller arc and being fed directly from the mains and mixed with phase-shift windings $+\theta$ and $-\theta$. The latter windings are continued beyond the slots containing 100% of phase-shift winding in decreasing quantities and are mixed with windings carrying double-phase-shift currents designated $+2\theta$, -2θ. Such a system allows each of the end slots to be shifted by 240°, since the 2θ winding is never mixed with the mains one, and a total shift around the active arc of 480° is possible. Figures 5(a) and (b) show diagrammatic representations of the system using three windings in an active arc, and Figure 5(c) shows a similar representation of five windings on an active arc.

With the switching arrangements described, the values of θ available are $\pm 60^\circ$ and $\pm 120^\circ$. Thus a motor carrying five windings is able to accept overall phase-shift angles of $\pm 240^\circ$ (for $\theta = 60^\circ$) or $\pm 480^\circ$ (for $\theta = \pm 120^\circ$); this, when converted into pole numbers, means adding or subtracting $^4/3$ of a pole (for a 60° shift) or $^8/3$ of a pole (for 120° shift) from end to end of the block.

The requirement that the carried-over flux shall react favourably for $\theta = 0$ is

$$n\frac{360}{\alpha} = 2p \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

where p is an integer.

The other conditions with phase shift applied are

$$(n + 1\tfrac{1}{3})360/\alpha = 2(p + 1) \text{ for } \theta = 60^\circ \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

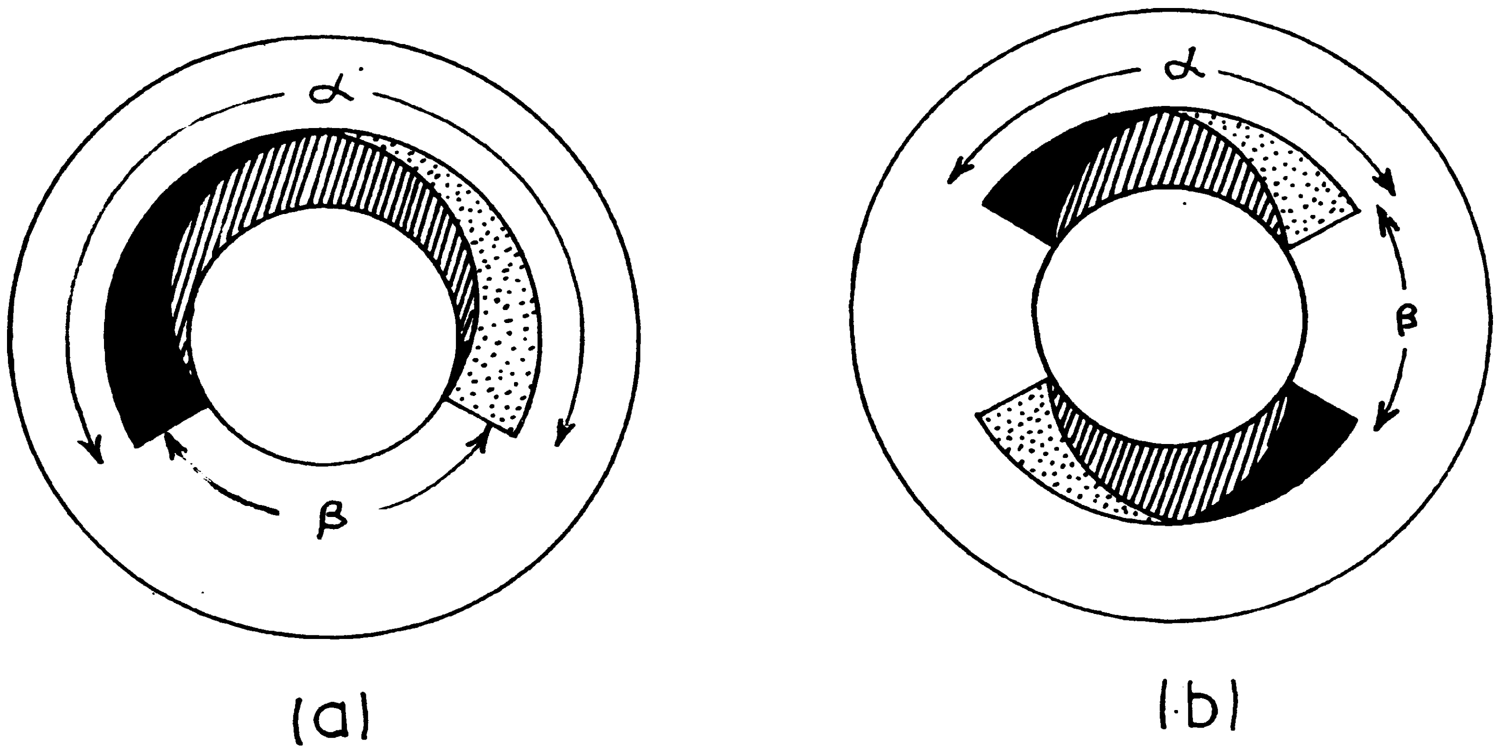
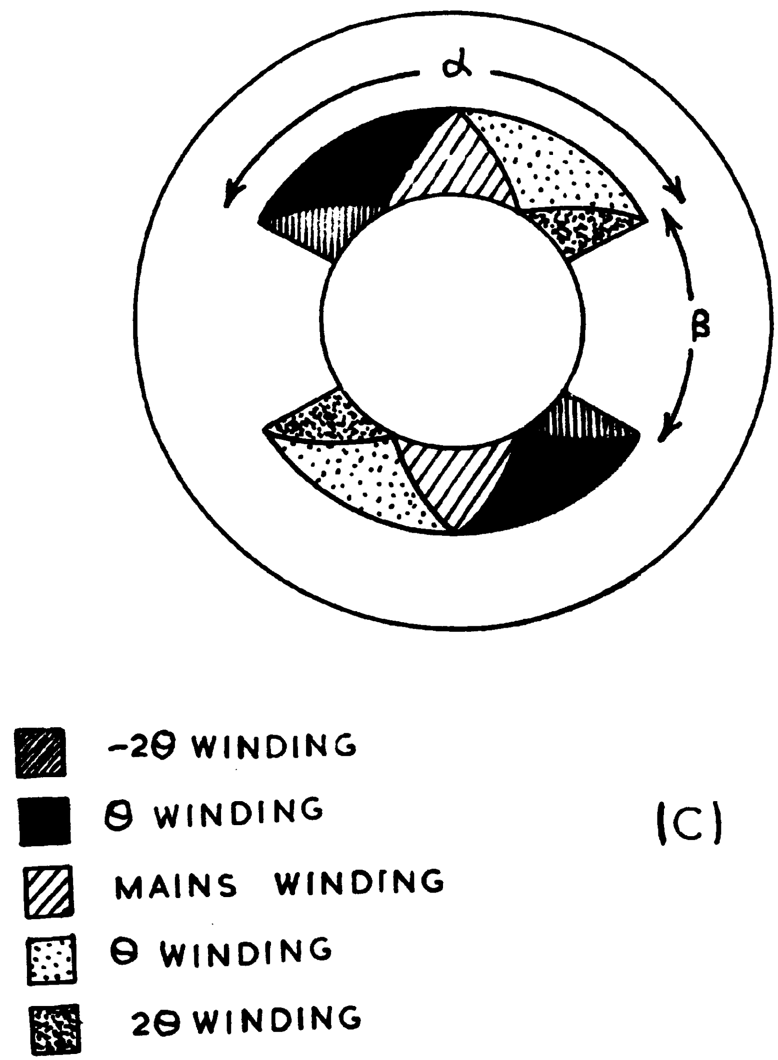


Figure 5. Partially energised pole-change stators.

(a) 240^o active arc -0, M, 0
(b) 120^o active arc -0, M, 0

Figure 5. Partially energised pole-change stators.

(c) 120° active arc -2θ, $-\theta$, M, θ, 2θ.



$$(n + 2\tfrac{2}{3})360/\alpha = 2(p + 2) \text{ for } \theta = 120^{\circ} \quad .... \quad (3)$$

$$(n - 1\tfrac{1}{3})360/\alpha = 2(p - 1) \text{ for } \theta = -60^{\circ} \quad .... \quad (4)$$

$$(n - 2\tfrac{2}{3})360/\alpha = 2(p - 2) \text{ for } \theta = -120^{\circ} \quad .... \quad (5)$$

Although there are five equations with only two variables, the unique solution for all five equations merely demands that

$$1\tfrac{1}{3} \times 360/\alpha = 2$$

or

$$\alpha = 240^{\circ}$$

All five equations are then satisfied by $n = 4p/3$. The minimum number of poles is clearly two, so that $p - 2 = 1$, or $p = 3$ and $n = 4$. This arrangement gives speeds of 3,000, 1,500, 1,000, 750 and 600 r.p.m. for 50 c/s

If $p - 2 = 2$ is chosen, $p = 4$ and $n = 5\frac{1}{3}$ and the resulting speeds are 1,500, 1,000, 750, 600 and 500 r.p.m., and any five adjacent speeds are possible.

If the motor is wound with three windings only, $\pm\frac{2}{3}$ pole or $\pm 1\frac{1}{3}$ poles are possible, as total phase shift and the above procedure results in a value of α of 120°.

Three-speed drives, which naturally involve fewer switches, can be made with $\pm\theta$ windings only or with both $\pm\theta$ and $\pm 2\theta$ windings. If only $\pm\theta$ windings are used in conjunction with $\pm 60^{\circ}$ phase shifts, the relevant equations are

$$n360/\alpha = 2p$$

$$(n + \tfrac{2}{3})\, 360/\alpha = 2(p + 1)$$

$$(n - \tfrac{2}{3})\, 360/\alpha = 2(p - 1)$$

so that $\alpha = 120^{\circ}$ and the speeds are 3,000, 1,500, 1,000 or any three adjacent speeds.

If 120° switching is used, any three adjacent speeds are possible with $\alpha = 240^\circ$ or speeds such as 3,000, 1,000, 600 or 1,500, 750, 500, etc., with $\alpha = 120^\circ$.

By using star-delta switching, phase-shift angles of $\pm 30^\circ$ and $\pm 90^\circ$ can also be used, making (at the expense of very complicated switching) a 9-speed motor. voltages?

These machines are economically sound in sizes down to fractional horsepower, since the short-stator effect is no longer involved.

4. DEVELOPMENT OF THE BASIC SYSTEM

So far the arrangements described involve the use of energized arcs of 240° or 120°. If the latter system is adopted, a balanced arrangement may be designed using two active arcs, α, separated by 60° of inactive arc, as shown in Figure 5(b). Figure 5(c) shows a machine with 2θ, θ, mains, $-\theta$ and -2θ sections in one active arc. One feature of the double-block systems of Figures 5(b) and (c) which should be noted is that the centre slot of each block is fed from the mains supply only, so that the two centre slots always have the same phase. This allows only certain pole configurations to exist, as shown in Figure 6. Figure 6(a) shows a developed diagram of a 2-block system wound to give a 6-pole machine in the condition of zero phase shift. The flux wave joins up as shown in Figure 6(b). At the instant shown, the centre A of one block is of one polarity while the centre B of the other is of opposite polarity. When switched to give four poles, equations (1) - (5) are obeyed, but the machine does not join up, since a 4-pole arrangement demands that A and B be instantaneously of the

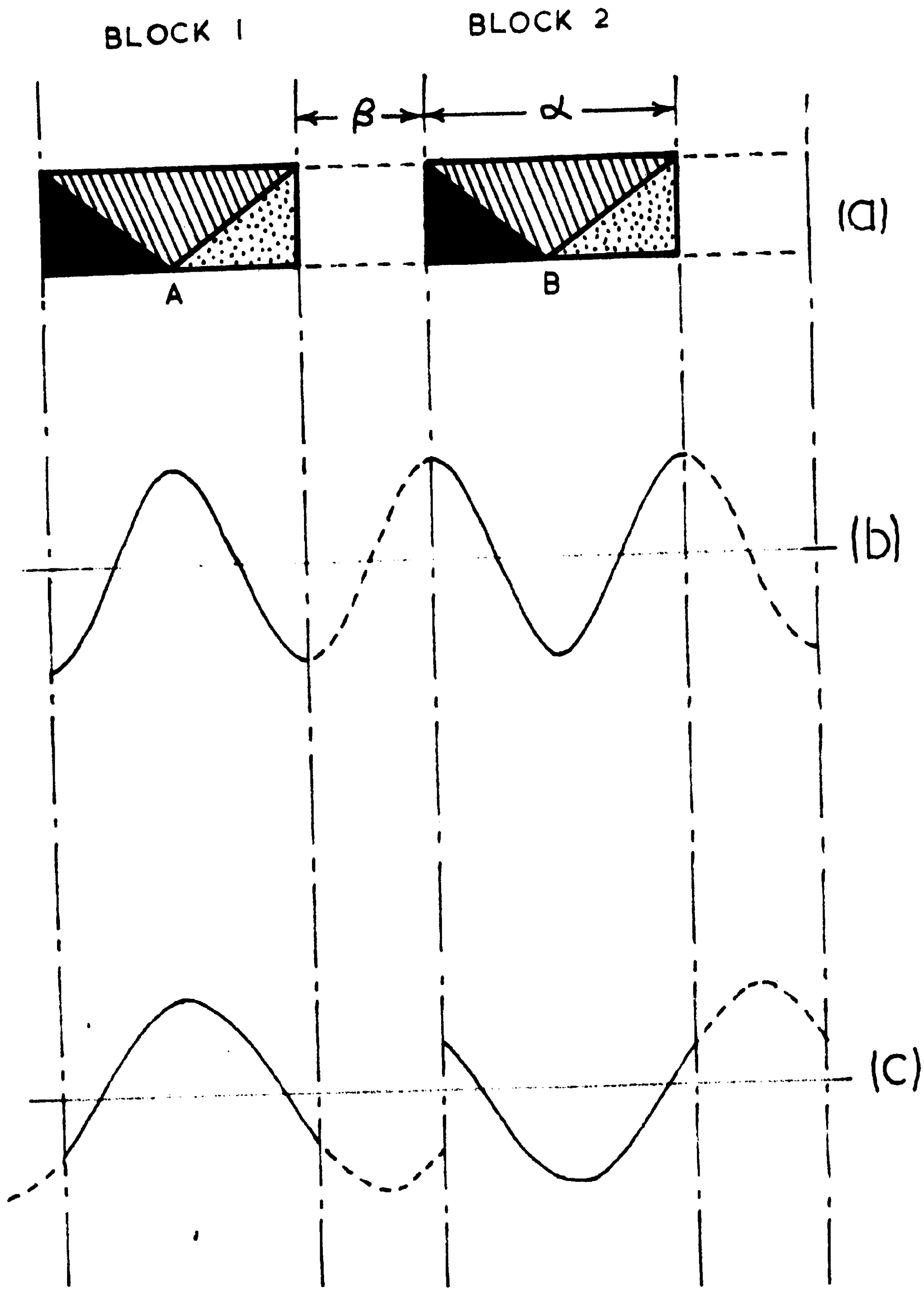


Figure 6. Waveform inversion in double-block systems.

same polarity. Figure 6(c) shows the relationship of the theoretical flux waves, and it is clear that phase inversion of block 2 will effect the desired result. It follows, therefore, that for 2-block systems the switching arrangements must be such that phase inversion of one block is included for the set of pole numbers 2, 6, 10, ... etc., compared with pole numbers 4, 8, 12, ... etc. The obvious disadvantage of all of the arrangements shown in Figure 5 is that only two-thirds of the winding space is usefully employed. A second point to be noted is that the block diagrams indicating the idealized current contributions give no indication as to how the machine is to be wound. If Gramme-ring type windings are to be eliminated, it is not possible to design windings which will produce the sudden termination of the active zones shown in Figure 5. An examination of possible winding systems is therefore necessary before the basis of a practical machine is developed from the above principles.

4.1. <u>Winding Arrangements</u>

Without the provision of an infinite number of slots it is impossible to produce perfectly smooth current distributions. Conventional windings are, in fact, only good approximations to the ideal. The triangular distribution of turns which approaches the ideal in phase-mixer machines, from the point of view of providing the ideal phase of total current per slot, can be obtained very nearly by using sets of coils as illustrated in Figure 7. Such sets of coils can be used to form machines of the partially energized type, and a practical winding diagram for such a machine using full-pitched coils is shown in Figure 8. The chief difficulty occurs in the sections β of the periphery which are supposed to be unenergized. If the phase-shifted

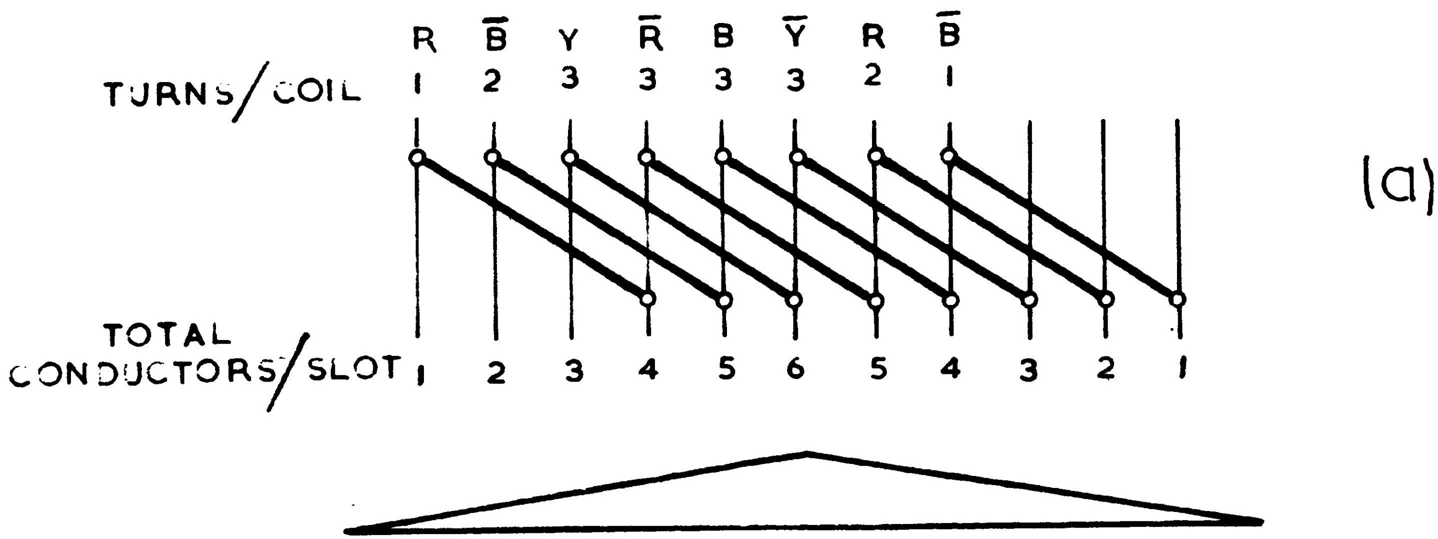


Figure 7. Winding arrangements to produce triangular current distribution.

(a) 1 slot/pole/phase, full-pitch.

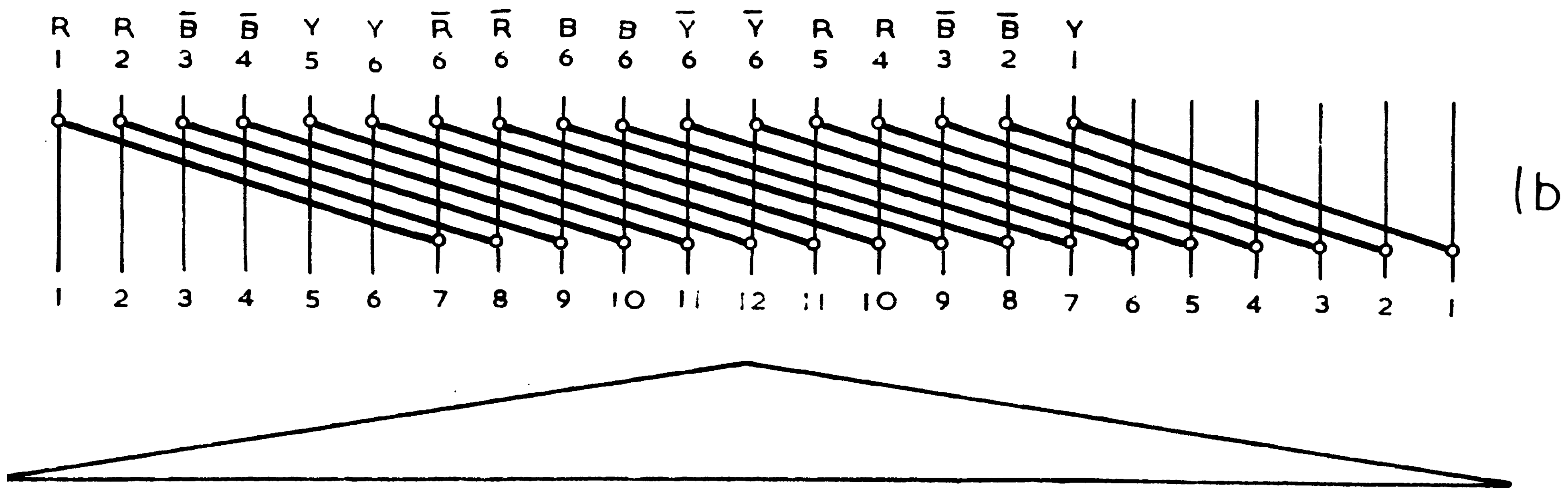
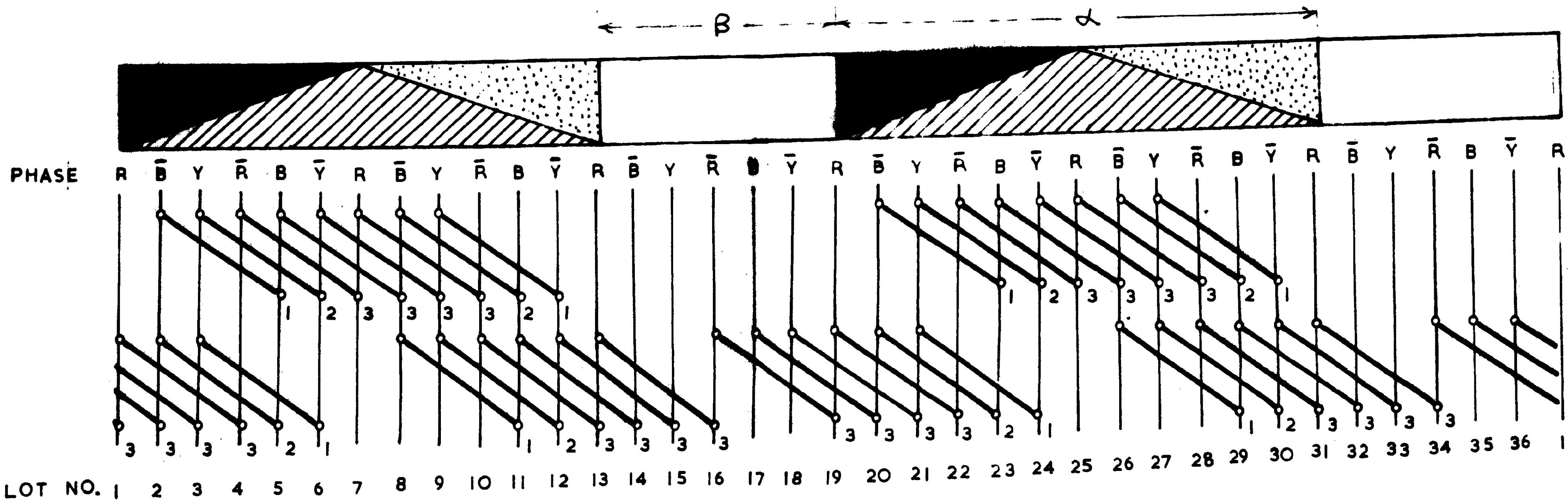


Figure 7. Winding arrangements to produce triangular current distribution.
(b) 2 slot/pole/phase, full-pitch.

Figure 8. Complete winding diagram for a double block machine -0, M, 0.

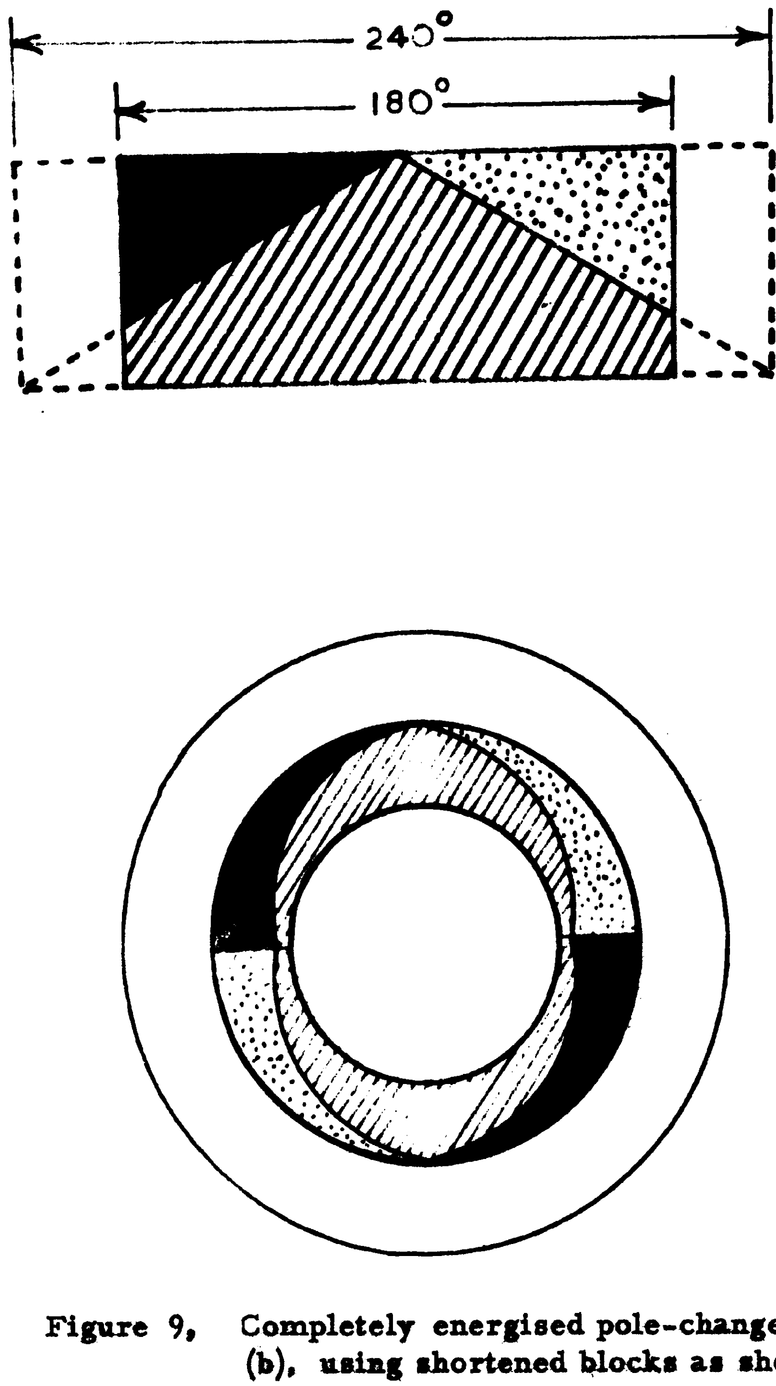


sections of the winding be limited to the turns required to complete the active arcs, there remain, inevitably, coil sides to be disposed in the arc β. An examination of Figure 8 shows that the numbers of turns and phases of current are not such that complete cancellation could be obtained by repositioning the conductors in the arc β. In any case, such repositioning would involve the use of coils of non-standard pitch, and an arrangement which would give reasonable cancellation at one switch position would not necessarily do so at another. The conclusion is that the use of a winding system with 'unenergized' arcs inevitably leads to sections with inappropriate slot currents. Quite apart from the foregoing considerations, the utilization of the stator slot space is limited to two-thirds of the total available space, with consequent penalties in power/weight ratio.

4.2. <u>Machines with No Inactive Arcs</u>

The windings described earlier in this Section and shown diagrammatically in Figures 5(a), (b) and (c) provide the correct phase of current in each slot over the active arc. The machine will therefore operate in a satisfactory manner if some of the coil sides at each end of the energized portions are omitted. In particular, if the total arc energized is reduced to 180° in a symmetrical fashion, a favourable arrangement can be formed by winding two such 180° arcs on a machine to fill all the slots. Such a configuration is illustrated in Figure 9.

These arrangements enable full utilization of the stator periphery to be made, but the problem of accommodating the return conductors which are left over after completing the 180° arcs remains; in fact, the problem



(a)

(b)

Figure 9, **Completely energised pole-change machine, (b), using shortened blocks as shown at (a).**

is rather worse than before, in that no slots at all remain, and an examination of Figure 7 shows that the return conductors carry currents which do not equate to zero.

5. THE 3k SYSTEM

So far, the systems described have been based upon the block winding arrangements of the phase-mixer motor [1]. The terms θ, 2θ, mains, $-\theta$, -2θ are somewhat inappropriate in the case of switched systems, since all the coils are supplied from the mains. Figure 10 illustrates the use of a new terminology in which blocks which are to be phase switched relative to each other are labelled 0, 1, 2 ...... (m - 1), 0 being the reference block. In such a system it is to be understood that the phase difference between the first block and the second block is the same as that between the third block and the second block, etc. With a switching system such as that shown in Figure 4, this phase difference must be 60° or 120°. Thus the winding arrangement described with reference to Figure 2 as shown in Figure 10(a) is now represented as a 3 block system as shown in Figure 10(b), and a winding as described in the preceding Section cannot be made such that the two ends of Figure 10(b) will join up.

Examination of Figure 10(c), however, shows that, if an extra block be added, the amount of phase shift in such a block will be $\pm 360^{\circ}$ for a shift per block of $\pm 120^{\circ}$. Thus this added block may be labelled 0 and the winding joined up completely.

Figure 11(a) shows an equivalent block diagram of a three block

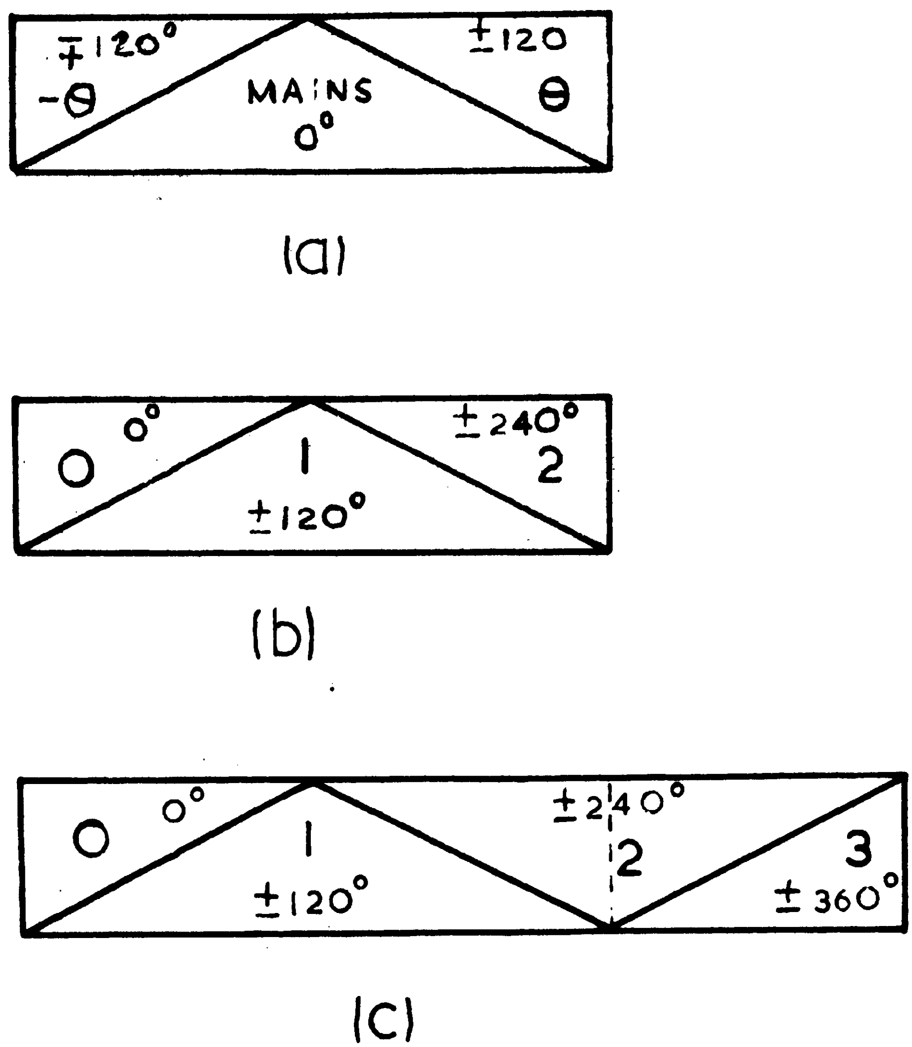
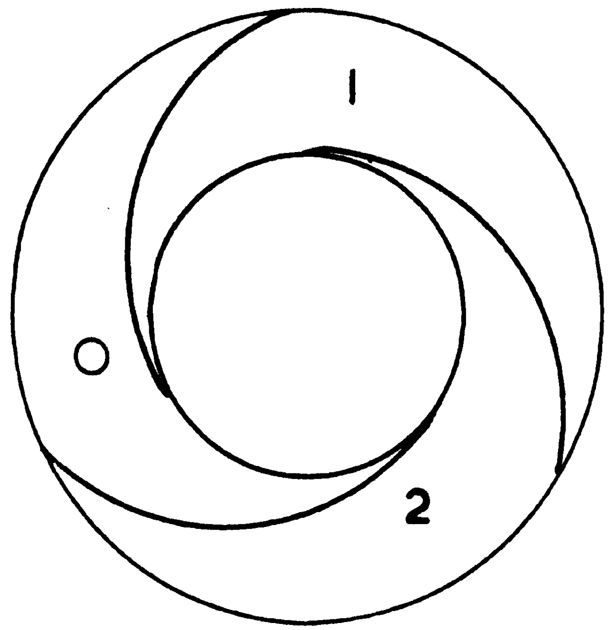
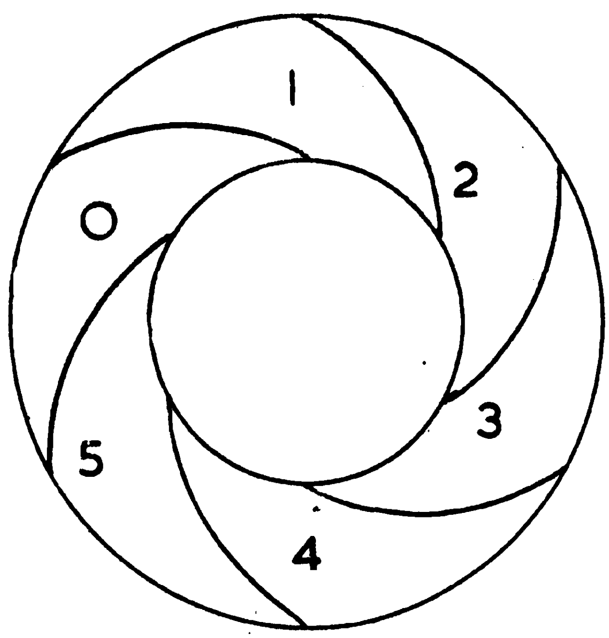


Figure 10. Re-arrangement of blocks for complete continuity.
(a) $-\theta$, M, θ system.
(b) Re-labelled system 0, 1, 2.
(c) Addition of extra block for continuity.



(a)



(b)

Figure 11. Schematic diagrams illustrating the 3k block system.

(a) k = 1
(b) k = 2

machine and Figure 12 shows the winding arrangement using fully pitched coils. Figure 11(b) shows a block diagram for a six block machine in which the phase shift per block may be both $\pm 60^{\circ}$ or $\pm 120^{\circ}$. Thus the number of poles which are added to the basic number in a three block machine for a shift of $\pm 120^{\circ}$ is ± 2 poles. Similarly, a shift of $\pm 60^{\circ}$ in a six block machine adds ± 2 poles whilst a shift of $\pm 120^{\circ}$ adds ± 4 poles.

In general, if there are 3k blocks, shifts of $\pm 60^{\circ}$ and $\pm 120^{\circ}$ produce changes in pole number of $\pm$ k and $\pm$ 2k poles respectively.

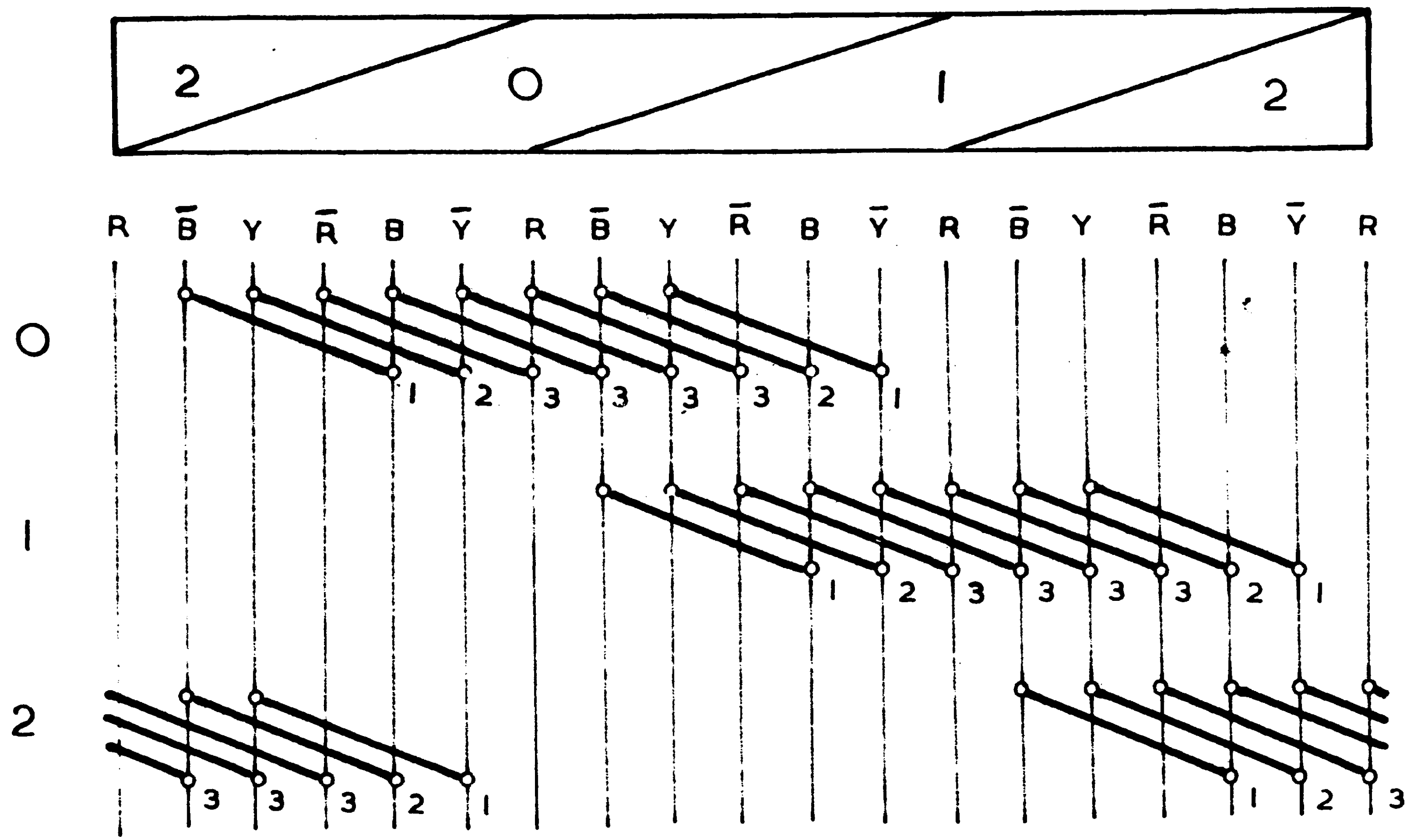
5.1. Winding Arrangements in a 3k block System.

Figure 7 shows winding arrangements which are as suitable for the 3k system as they were for the earlier system. Examination of both Figures 7(a) and (b) shows that the base of the triangle is 4 coil pitches long. Thus, if full-pitched coils are used, the base of the triangle is 4 pole pitches long, and Figure 10(c) shows that for $k = 1$ the total periphery will contain six poles in the condition of zero phase shift. Shifts of $\pm 120^{\circ}$ will therefore produce 4-pole and 8-pole speeds, and a 3-speed motor having 4, 6 or 8 poles results. Similarly a full-pitched machine with $k = 2$ would have a middle speed equivalent to 12 poles and be capable of being switched $\pm 60^{\circ}$ or $\pm 120^{\circ}$ to yield pole-numbers of 8 - 10 - 14 - 16 in addition to the 12-pole centre speed.

5.2. Chorded Coils

Whatever the pitch of the coil, the base of the triangle is 4 coil pitches. If chorded coils are used having a coil-pitch/pole-pitch ratio γ, then the number of pole pitches in a group is clearly 4γ for zero phase shift.

Figure 12. Complete winding arrangement of a 3 block system.



Thus the number of poles around the machine in this condition is $(3k/2)4\gamma = 6k\gamma$.

Now k can be any integer; hence, $6k\gamma$ must be an even whole number and $3k\gamma$ an integer.

A phase shift of $\pm 60^\circ$ gives a pole-number change of $\pm$ k poles, while a phase shift of $\pm 120^\circ$ gives a pole-number change of $\pm$ 2k poles. When k is odd the 60° positions are redundant since odd pole numbers do not give practical machines. If k is unity, that is, using the three block system, 3γ must be a whole number and γ can have the values 1/3, 2/3, 1, 4/3, 5/3. Table 1 tabulates the pole-number ranges available using the chorded system with $k = 1$.

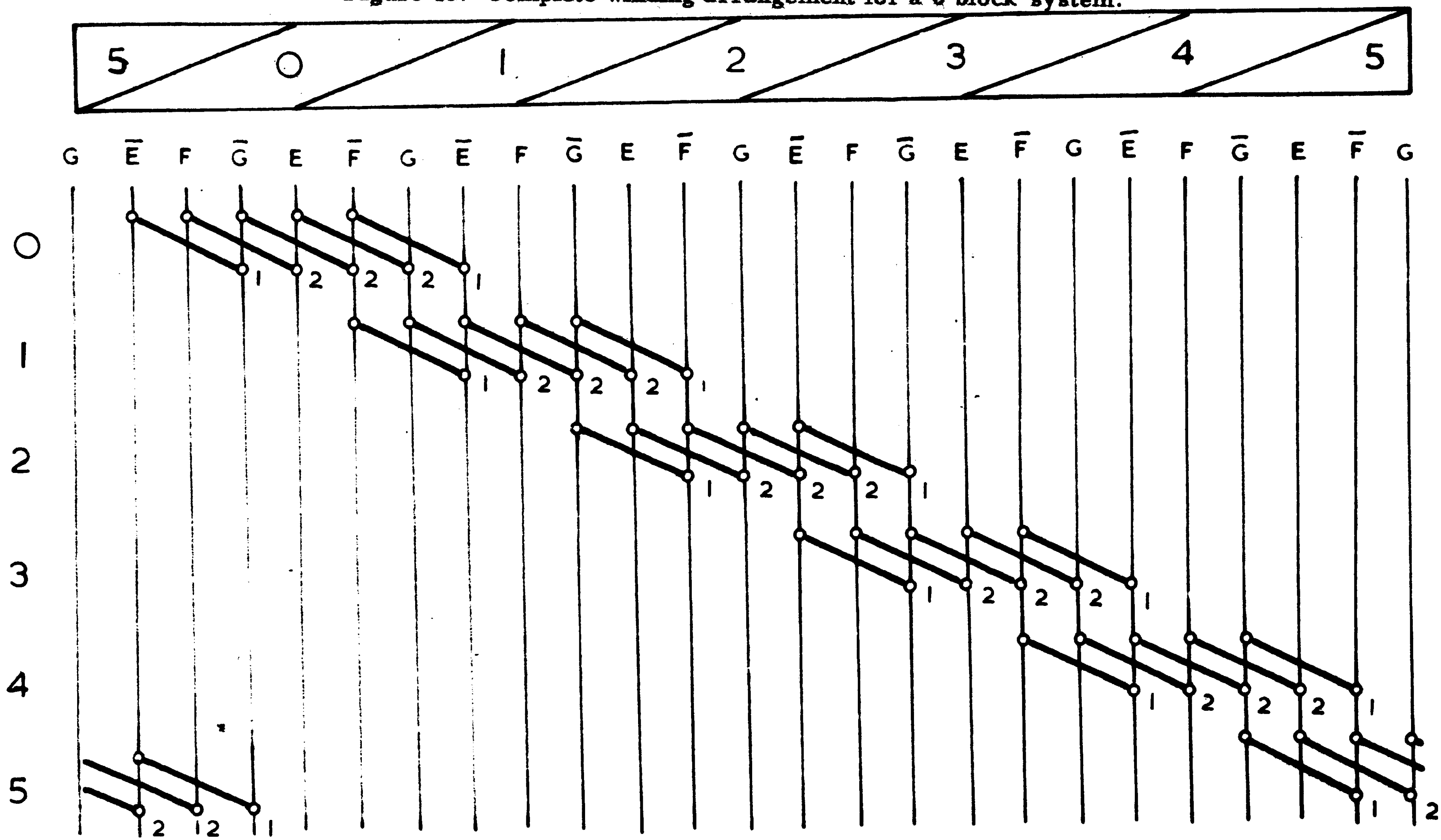
If $k = 2$, that is using the six block system, then 6γ must be a whole number; the pole numbers obtained are tabulated in Table 2.

Similar Tables can be calculated for all values of k. Figure 13 shows an example of the winding obtained using six blocks with $\gamma = \frac{2}{3}$ for 8 poles at mid-speed.

5.3. A Second Basic Coil Arrangement

The cases for the second basic coil arrangement, that is the one illustrated in Figure 14, can be obtained as follows. Using chorded coils, the number of pole pitches in the block in the condition of zero phase shift is 8γ. The number of poles around the machine is $(3k/2)8\gamma = 12k\gamma$. This must be an even whole number. Using this as a condition, the pole numbers obtainable can be calculated. The chorded winding system can be extended considerably by making the shape of the blocks non-triangular. In particular,

Figure 13. Complete winding arrangement for a 6 block system.



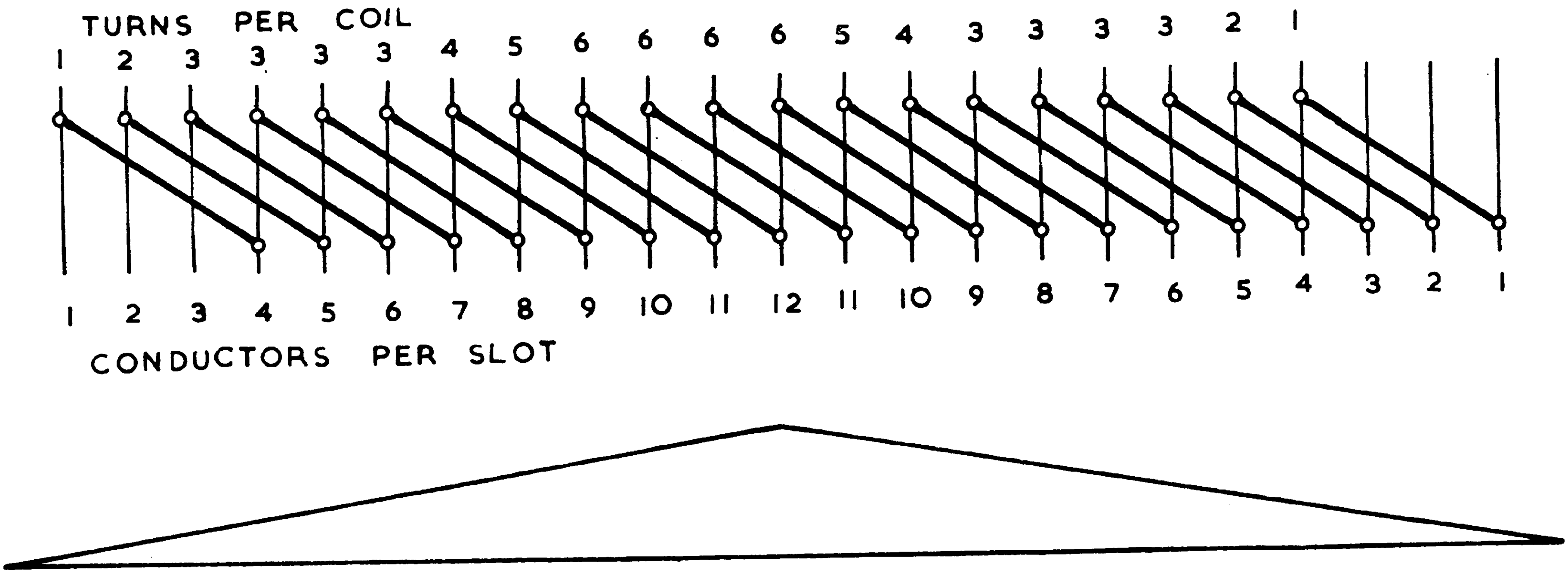


Figure 14. Double sequence winding arrangement.

TABLE 1.

POLE NUMBERS FOR k = 1

Phase shift	Functions of k, γ	Calculated values of functions				
-	γ	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$
-	$k\gamma$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$
-120^{o}	$6k\gamma + 2k$	4	6	8	10	12
0^{o}	$6k\gamma$	2	4	6	8	10
$+120^{o}$	$6k\gamma - 2k$	0	2	4	6	8

TABLE 2.

POLE NUMBERS FOR k = 2

Phase shift	Functions of k, γ	Calculated values of functions								
-	γ	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1	$\frac{7}{6}$	$\frac{8}{6}$	$\frac{8}{6}$	$\frac{10}{6}$
-	$k\gamma$	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{8}{6}$	$\frac{10}{6}$	$\frac{12}{6}$	$\frac{14}{6}$	$\frac{16}{6}$	$\frac{18}{6}$	$\frac{20}{6}$
-120°	$6k\gamma - 2k$	0	2	4	6	8	10	12	14	16
-60°	$6k\gamma - k$	2	4	6	8	10	12	14	16	18
0°	$6k\gamma$	4	6	8	10	12	14	16	18	20
$+60^{\circ}$	$6k\gamma + k$	6	8	10	12	14	16	18	20	22
$+120^{\circ}$	$6k\gamma + 2k$	8	10	12	14	16	18	20	22	24

the triangles can be truncated. Figure 7(a) shows a coil arrangement which produces the triangular distribution; Figure 15 shows one which produces a truncated triangular distribution. The top flat section was obtained by adding two extra coils of three turns in the centre. Figure 15 shows that the length of the base is now $4\frac{2}{3}$ coil pitches, compared with 4 in the case of Figure 7(a). This arrangement would enable a machine to be built using six blocks having a mid-speed (zero phase shift) of 14 poles. The number of extra coils of three turns can be chosen at will, so that by adjusting the length of the flat portion of a truncated triangular distribution it is possible to obtain different pole numbers at the mid-speed, with the same chording factor.

6. AN EXPERIMENTAL MACHINE

A machine was built using the three block system with 72 slots and full-pitched coils to produce a 3-speed motor with pole numbers 4 - 6 - 8 by switching $\pm 120^{\circ}$. The winding diagram of one block is shown in Figure 16. In order to run the machine on a reasonably low voltage, 12 conductors per slot were needed. If the procedure outlined previously had been adopted for the production of the triangular block shape, 24 conductors per slot would have been needed. The conductor numbers were therefore duplicated as shown, giving an approximation to a triangular distribution which contains rather bigger steps than would have been the case otherwise. If any other number of turns had been required to produce a different voltage, a different approximation to the triangle could have been produced. The harmonic analysis of the experimental machine was performed using the method described

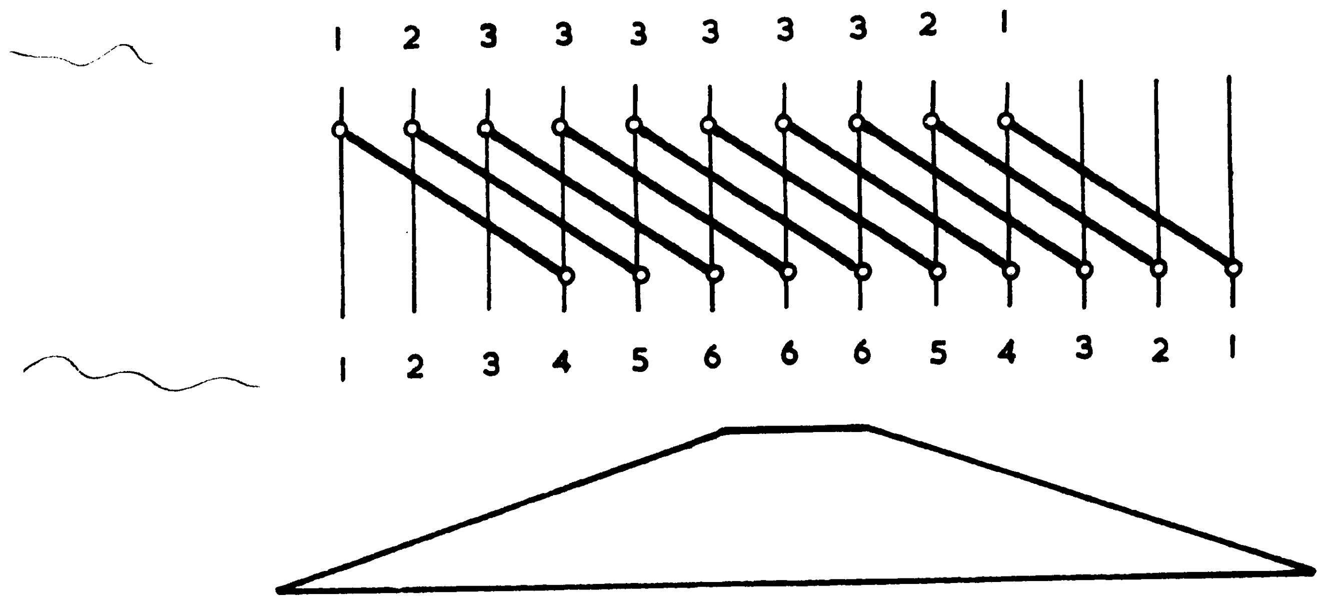


Figure 15. Winding arrangement for truncated triangular distribution.

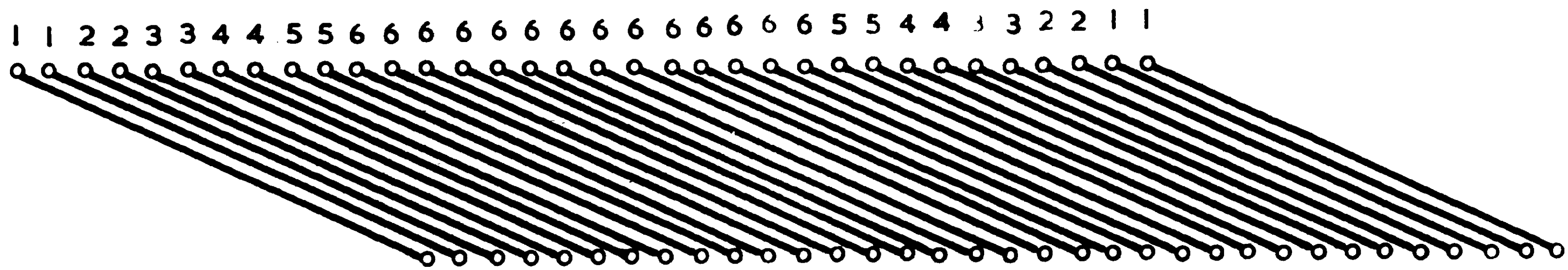


Figure 16. Winding arrangement of one block of the first experimental machine.

in Reference 4 but with the results expressed in relative current density. The results of the calculations for the 4- and 8-pole settings are shown in Table 3: the results for the 6-pole connection are not shown since the machine is completely conventional in this condition.

The winding factors in the 4-, 6- and 8-pole settings are 0.66, 0.96 and 0.65, yielding relative air-gap flux densities of 0.97, 1.0 and 1.96 respectively, when the machine is connected in series star at each setting. Clearly, if the machine is used in this manner, the output on the 6- and 4-pole settings will be severely curtailed. However, if the windings are connected in series delta for the 8-pole speed and series star for the other two speeds, the relative air-gap densities become more nearly uniform at 0.97, 1.0 and 1.13 for the 4-, 6- and 8-pole settings respectively.

In order to assess the effect of the harmonic content, the torque/speed curves of the machine were measured at low voltage using the method described in Reference 6. These curves are shown in Figure 17. The high 10-pole m.m.f. on the 4-pole setting is reflected in a very large dip in the speed/torque curve. However, if the control gear is suitable designed, so that the machine must be started in the low-speed connection, it will accelerate to each of the synchronous speeds in turn. The 8-pole speed/torque curve is shown both for the series-star and series-delta connections. If the series-star 8-pole connection is used in conjunction with the 4- and 6-pole series-delta connections, the machine exhibits approximately constant torque for the three settings. The output from the 6-pole setting is the same as that normally obtained from that frame size using a conventional 6-pole winding, since the flux density is sensibly constant from speed to speed.

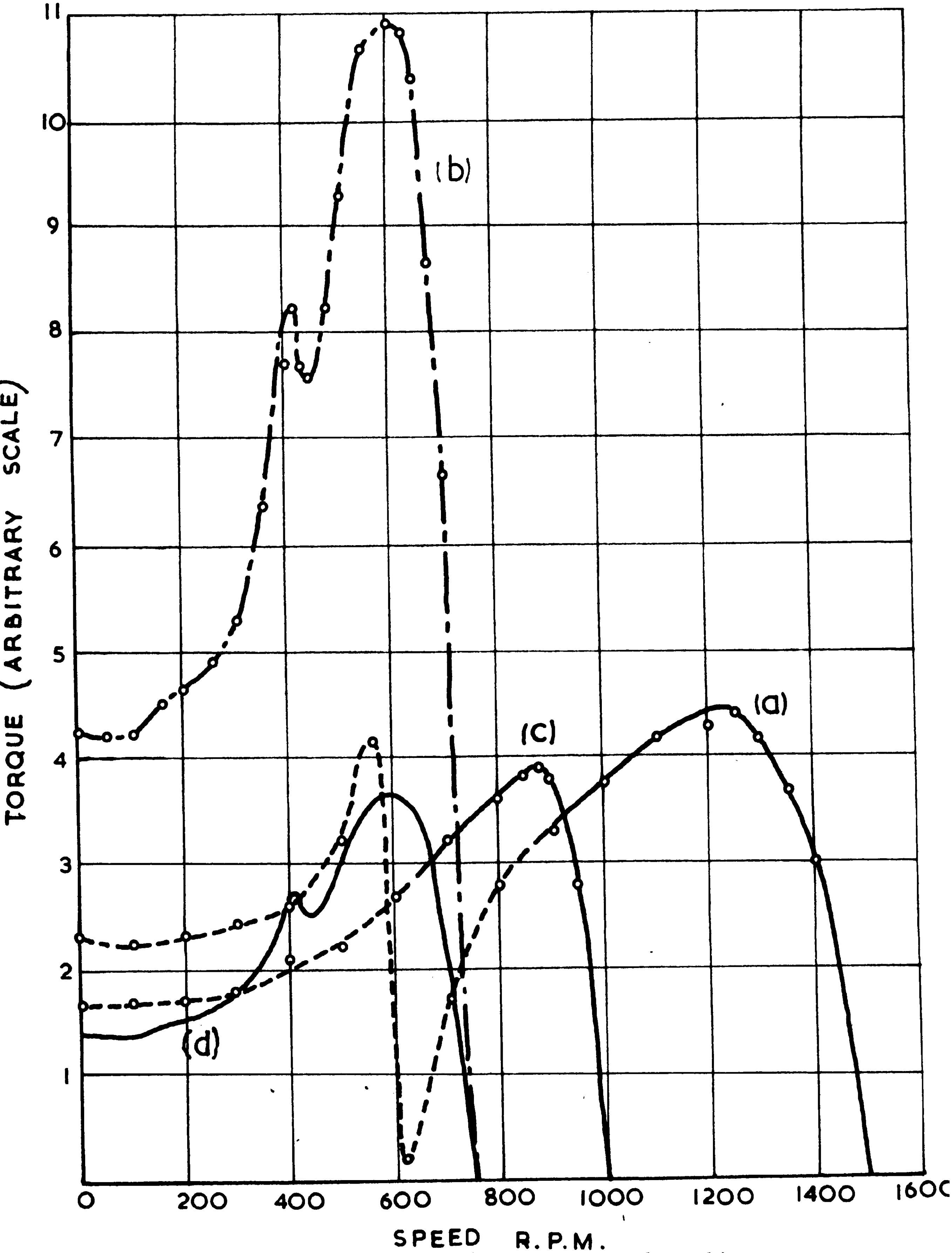


Figure 17. Speed-torque curves for experimental machine.
(a) 4 pole series star. (b) 6 pole series star.
(c) 8 pole series star. (d) 8 pole series delta.

TABLE 3.

4-pole connection			8-pole connection		
Pole No.	Rotation	Relative current density	Pole No.	Rotation	Relative current density
4	+	100%	2	+	25%
10	+	24%	8	+	100%
16	+	4.4%	14	+	8.7%
24	+	5.4%	20	+	4.7%
			26	+	1.3%

All figures in the Table are referred to a 100% fundamental.

7. A GENERAL SCHEME FOR ANY BLOCK SHAPE OR POLE NUMBER

Two basic difficulties are still present in chorded windings. First, the harmonic content of the current densities produced is extremely high and can produce serious dips in the torque/speed characteristics. Secondly, the chording factor γ to some extent fixes the pole number of the machine at the centre speed. This means that the flux density at the various settings is not in any way a design variable.

Figure 18 shows a schematic winding arrangement which consists of two windings of the 3k block type superimposed. By displacement of one winding relative to the other, any specified harmonic may be eliminated. In such a scheme both sides of any coil form part of the same system. Much greater flexibility, however, is obtained by returning the coil sides from one system to the other. Figure 19 illustrates how the scheme is derived. Figure 19(a) shows how the numbers of conductors per slot would be made up from the coil groups of a winding in 12 slots with $k = 1$. If the winding is to be, say, 4 poles in the condition of zero phase shift, there is one slot per pole and phase, and the currents in the windings are as shown. If the first group only is considered, it is apparent that these currents alone cannot be formed using coils, since, in order to be able to do this, the number of currents from each phase must sum to zero; for example, the E phase has + 1, - 4 and + 1 in slots 2, 5 and 8, respectively, which sums to - 2. If, however, a second set of conductors, consisting of three groups, is arranged as in Figure 19(b), it is evident, when considering, say, the first group currents, that the current in slot 5 of winding 2 can be carried in one side

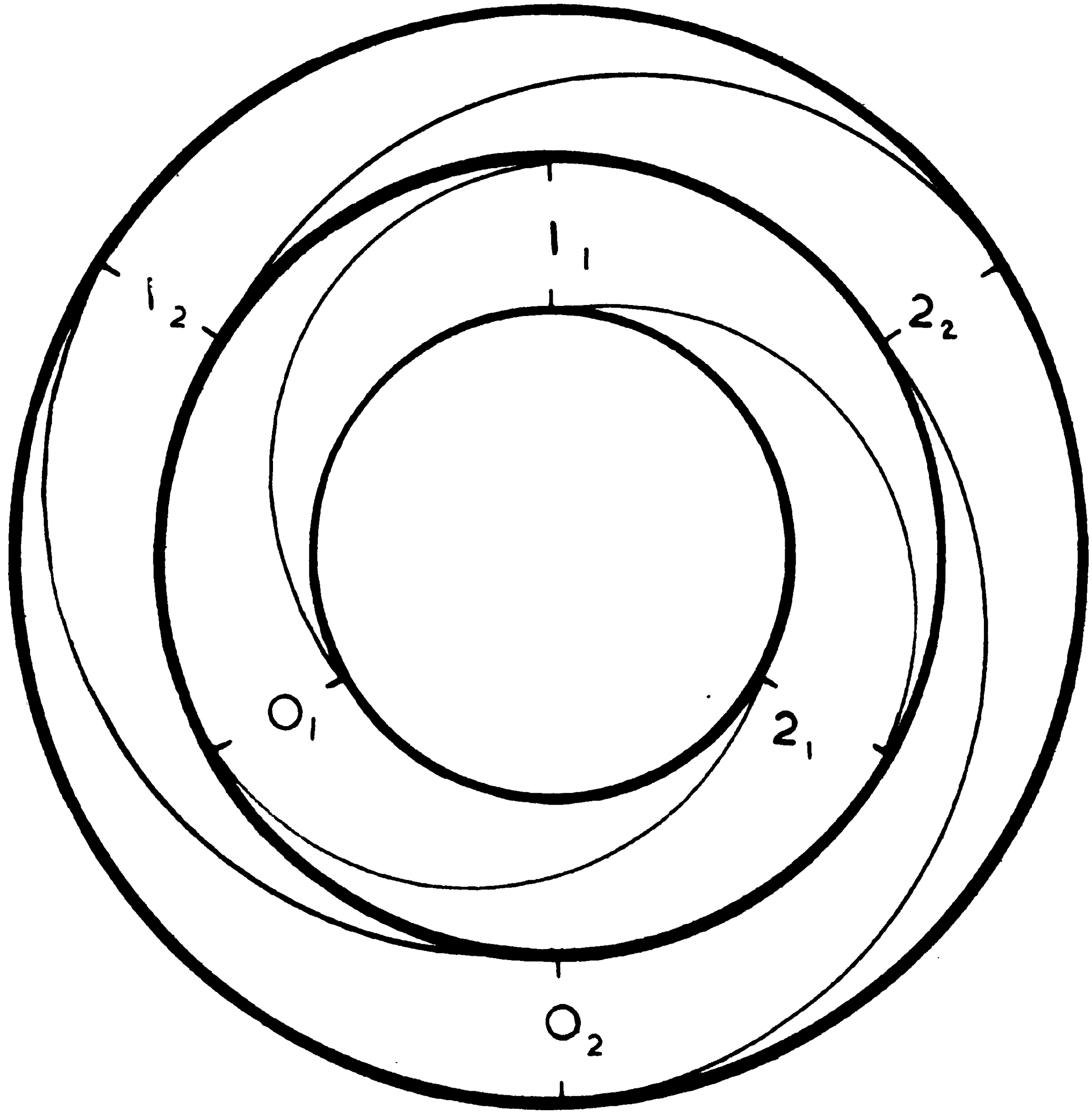


Figure 18. Schematic arrangement of a double layer 3 block system.

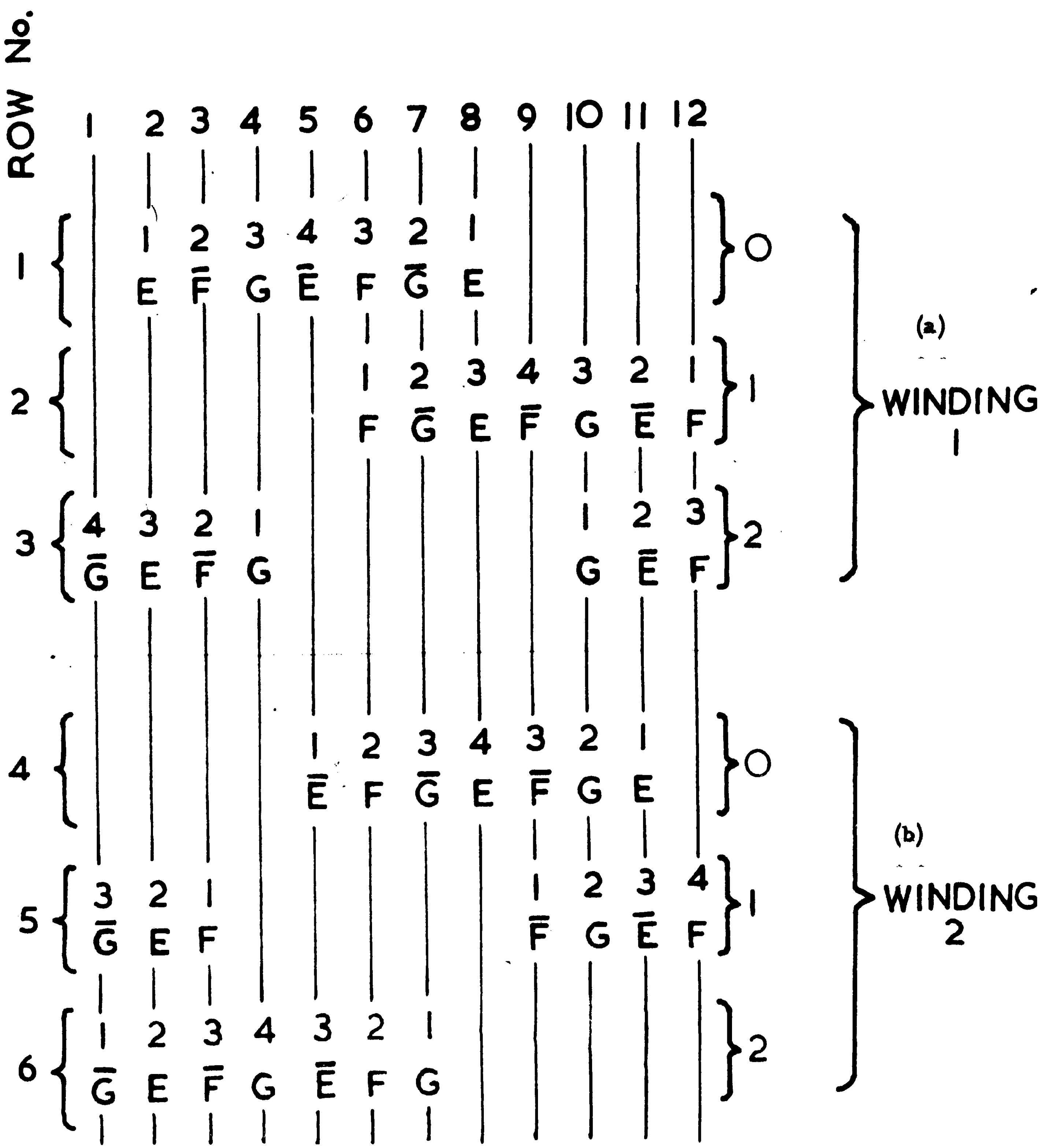


Figure 19. Conductor arrangements in a double layer system.

of a coil which has its other side in slot 2 of winding 1. Thus it can be seen that windings 1 and 2 can be combined as a single winding, as illustrated in Figure 20. The coils in this winding span three slots and are therefore full-pitched, but it will be appreciated that any chording factor can be obtained by sliding winding 2 with respect to winding 1. Hence, using this system, any pole number at any chording factor can be accommodated.

It is now apparent that, with this technique, the block can be adjusted to any desired symmetrical shape and, in fact, it may be possible to produce shapes which are of much simpler form and give satisfactory performance.

8. SOME TWO-SPEED SWITCHING ARRANGEMENTS

If only two speeds are required from a switched phase-mixer machine, then the switching arrangement is simplified. In particular, some preferred arrangements result. In these machines some of the coil groups are merely reversed with respect to the rest, in order to change the pole number. An example of this can be developed from a 6 block machine. Figure 21(a) shows such a machine with the coils marked with the phase of current which must flow for the $+60^\circ$ condition, when two poles are added to the basic pole number. Similarly, Figure 21(b) shows the coils marked with the phase of current necessary for the -120° condition, when four poles are subtracted from the basic pole number. It can be seen from these diagrams that the currents flowing in the 0, 2 and 4 groups are unchanged as the machine is switched from $+60^\circ$ to -120°, while the currents in the 1, 3 and 5 ones are merely reversed. Hence, the 0, 2 and 4 groups can be permanently connected together, as can the 1, 3 and 5 groups (see Figure 21(c)). The

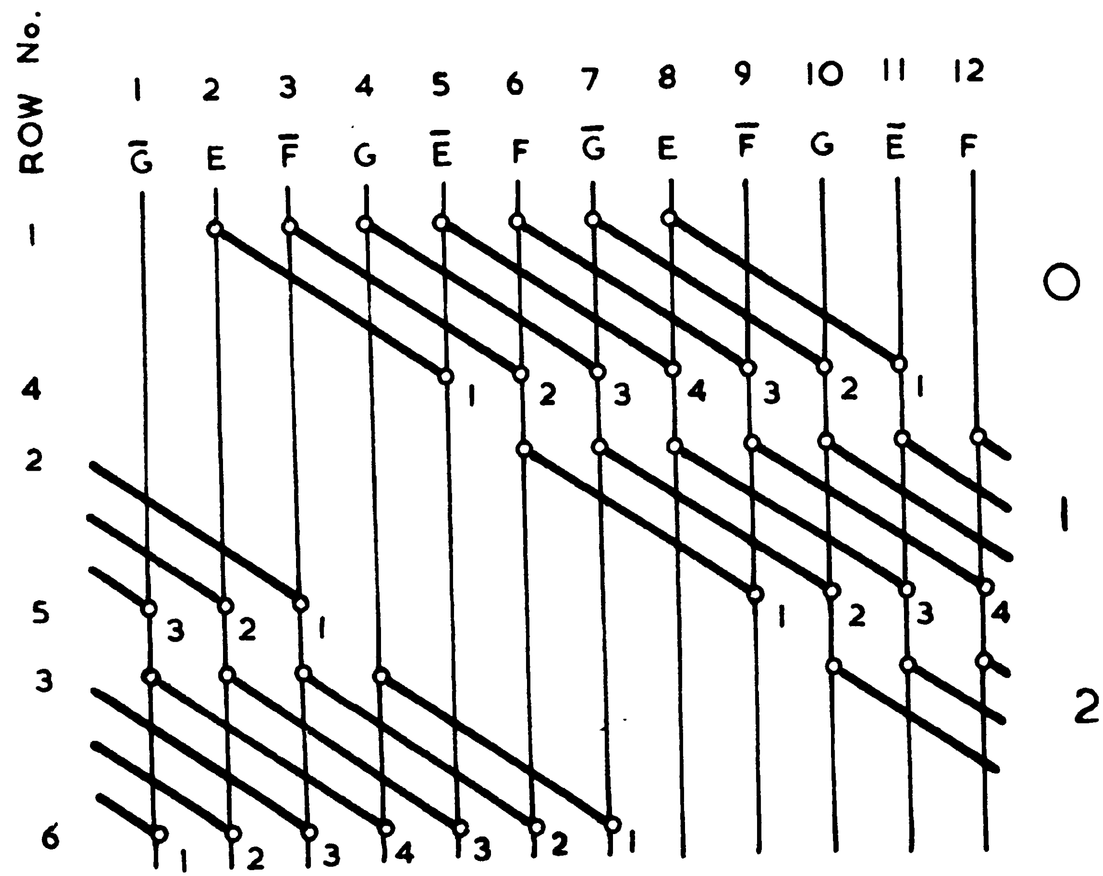
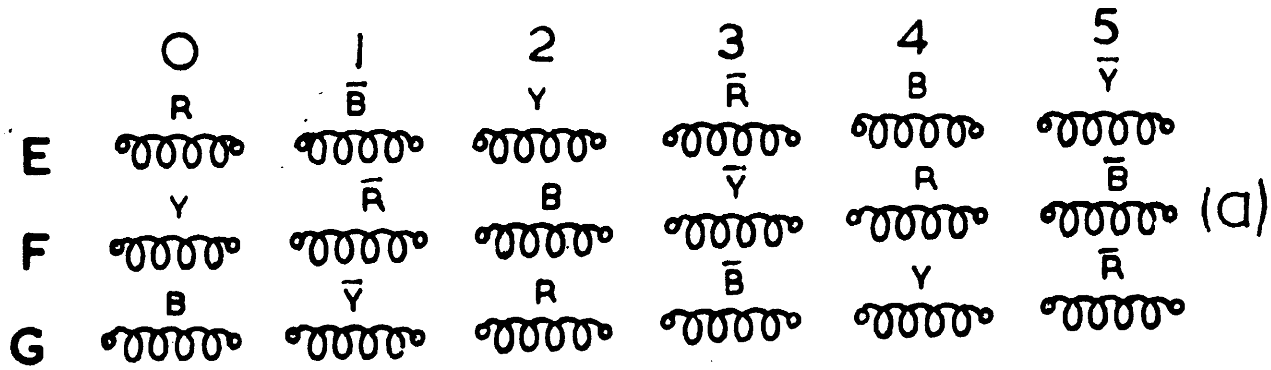
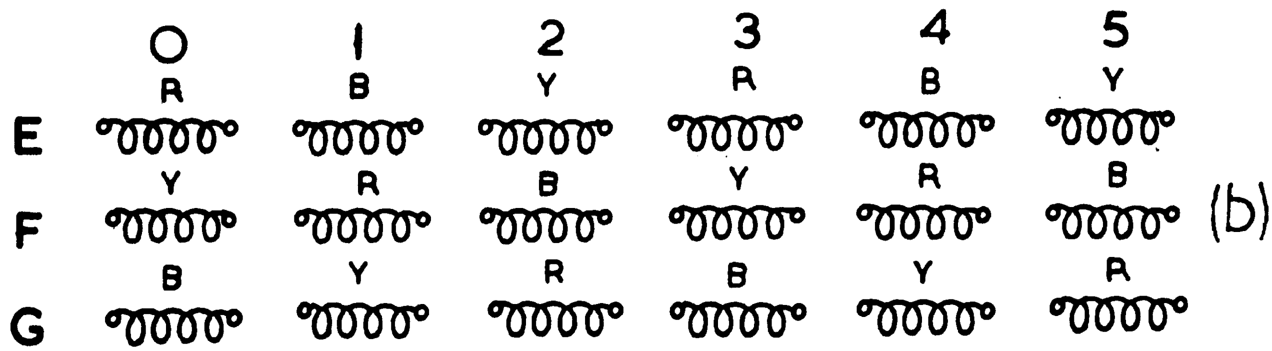


Figure 20. Coil arrangements in a double layer system.

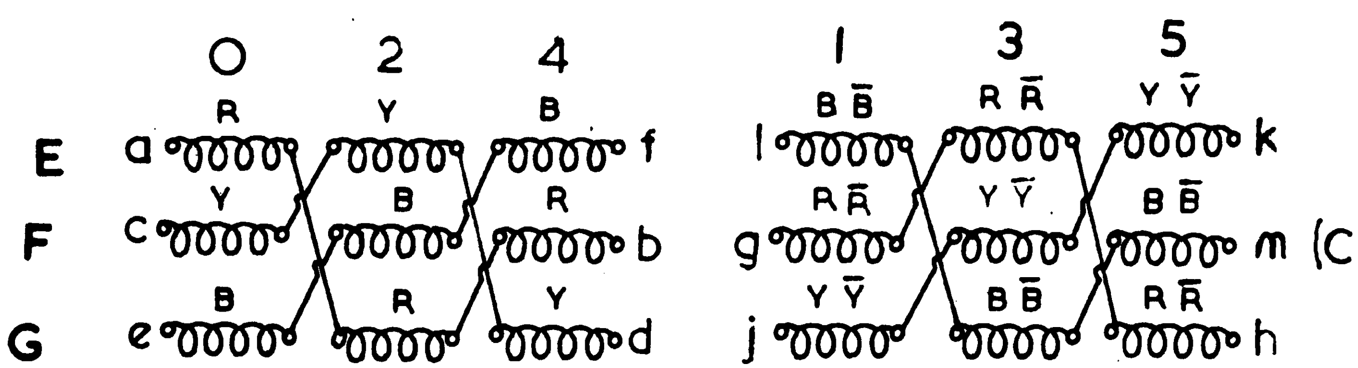
Figure 21. Connections for a 2 speed machine.



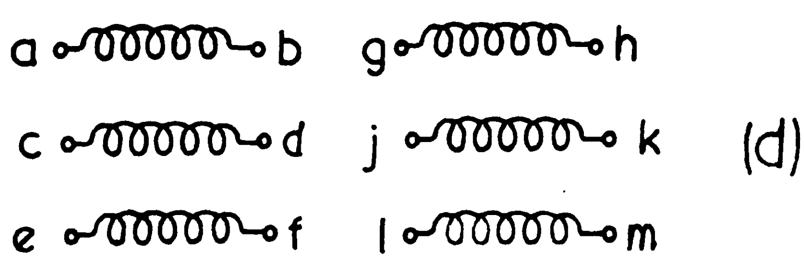
(a) Currents required for + 60°



(b) Currents required for - 120°



(c) Permanent connections between coils.



(d) Coil groups to be switched.

machine has now two sets of coils connected in each phase, and the wiring diagram can be further simplified to that of Figure 21(d). Switching from one speed to the other now involves the same technique as that used for the conventional 2:1 speed-change machines, and similar arrangements can be used. These are shown in Figure 22.

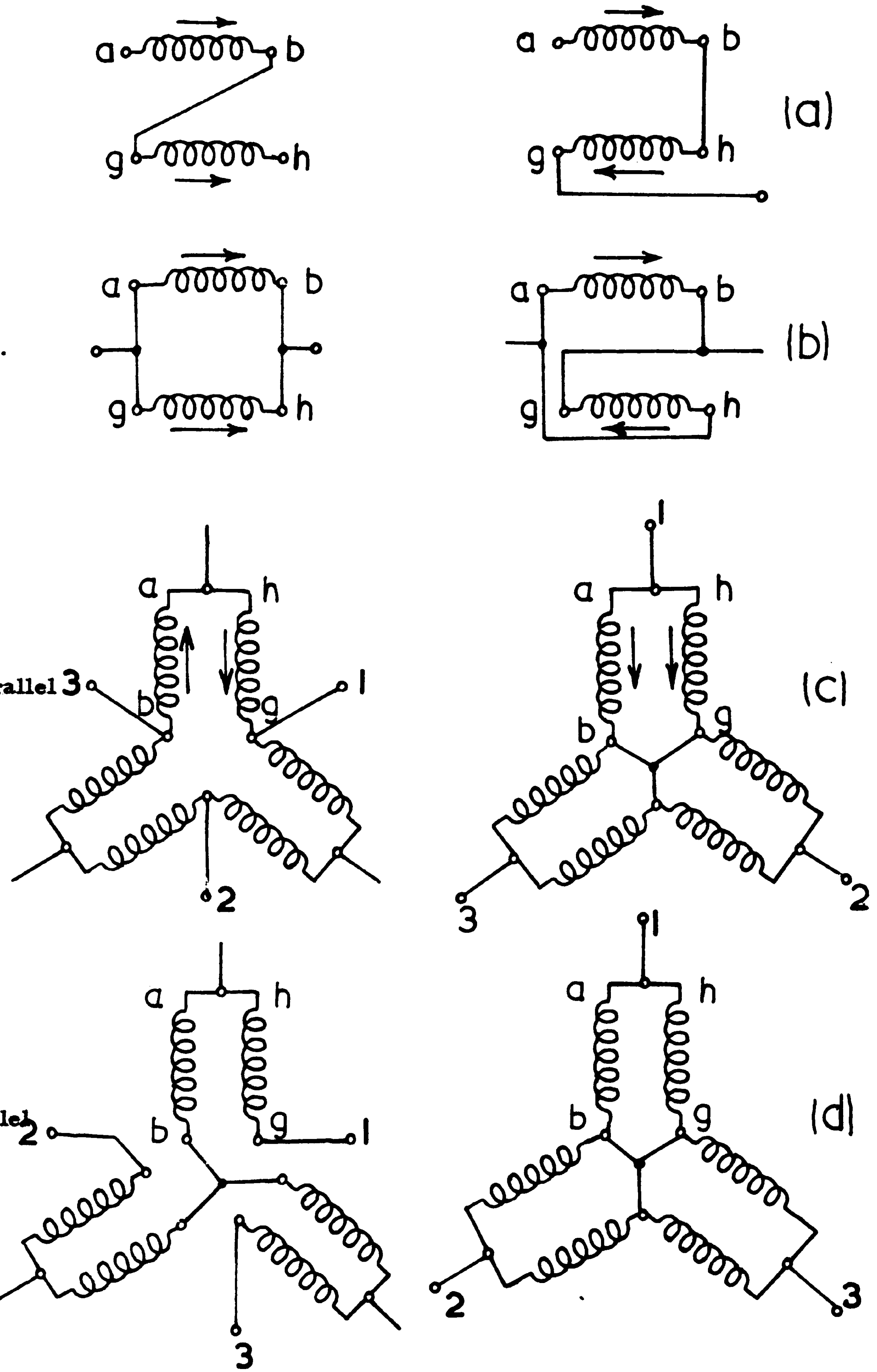
Figure 22 illustrates the basic arrangements for one phase group of coils. Figure 22(a) shows an arrangement in which the two groups are in series for both of the connections, whilst Figure 22(b) shows the corresponding parallel configuration. Figure 22(c) shows a series-delta/parallel-star switch. The flux-density relationship between the two speeds is related to the voltage applied to a group of coils, and is also affected by the pole number; hence it is possible to compensate for the difference in flux density produced by the difference in pole number by suitable choice of the switching arrangement.

It will be noted that only six leads are required from the machine, using the arrangements of Figure 22. However, it is possible to keep all the coils in series in both connections, and a suitable switch system is shown in Figure 23.

It will be appreciated that a similar arrangement could be constructed using the basic positions of + 120° and - 60°, i.e. adding four poles and subtracting two, respectively, from the basic pole number.

When only two speeds are required, the number of blocks is not necessarily a multiple of three. This is illustrated by Figure 24. If we regard the centre block Y of Figure 24(a) as the fixed reference, then in

Figure 22. Group connections for 2 speed machines.



series-series.

parallel-parallel.

series delta-parallel star.

series star-parallel star

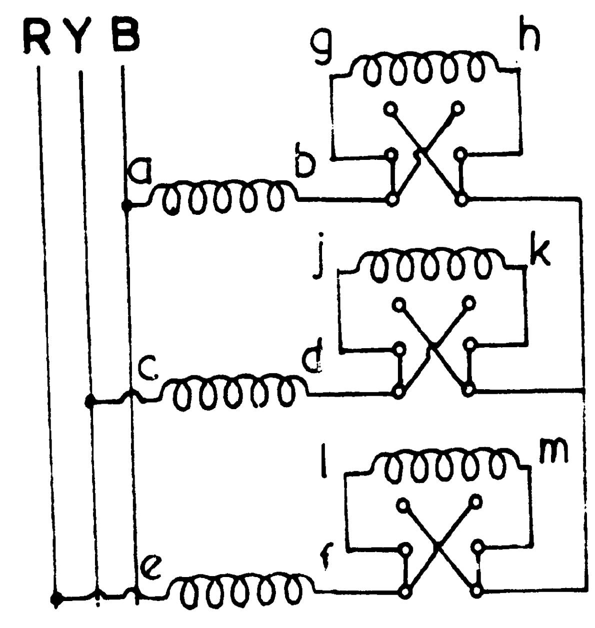


Figure 23. Switch connections for series-series arrangement.

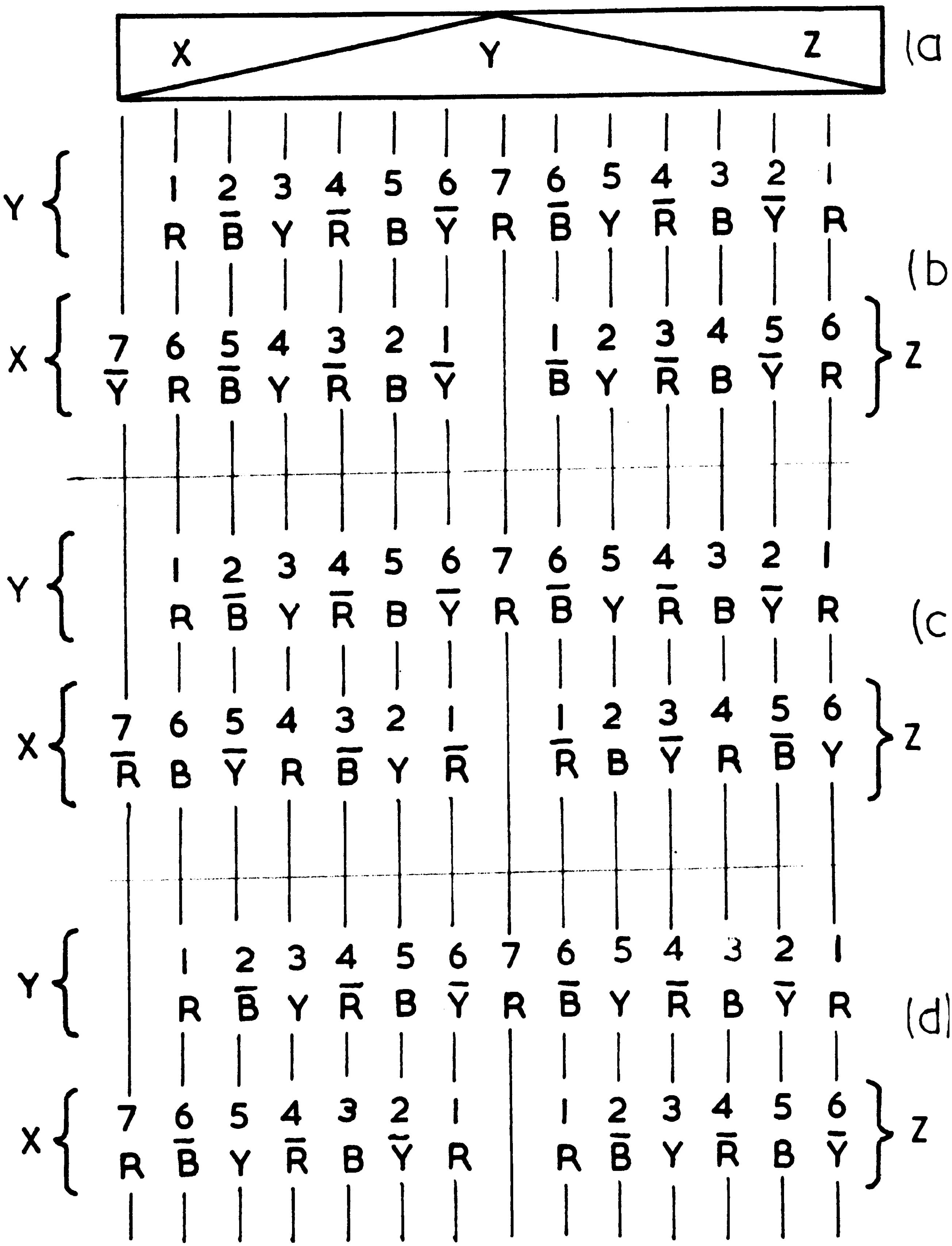


Figure 24. Conductor arrangements in a two block machine for 4 - 6 poles.

order to effect a change in pole number, X must be advanced in phase by θ and Z must be retarded in phase by θ. Figure 24(b) shows the system of Figure 24(a) with a winding which gives $4\frac{2}{3}$ poles in the condition of zero phase shift. The top layer only is shown, it being assumed that the bottom layer of the winding is formed from the return conductors of the top layer. Figure 24(c) shows the conditions when Z is advanced in phase, and X retarded, by 120°. This is defined as the $+ 120^{\circ}$ condition in order to maintain consistency with figures in previous Sections. In the $+ 120^{\circ}$ condition $1\frac{1}{3}$ poles are added to the original pole number, so that the machine has six poles. Figure 24(d) shows the currents when Z is retarded in phase, and X advanced, by 60°. This condition is then the $- 60^{\circ}$ position, and $\frac{2}{3}$ pole is subtracted from the original pole number. In the $- 60^{\circ}$ condition the machine has four poles. Examination of Figures 24(c) and (d) shows that simple reversal of the X and Z sections with respect to the Y section results in the required change in pole number. It will also be observed that sections Z and Y form a continuous winding in both of the required switch positions and can therefore be regarded as one winding. This is then a two block system formed by two similar blocks.

If a winding of $5\frac{1}{3}$ poles had been drawn in place of that of Figure 24(b), then phase shifts of $+ 60^{\circ}$ and $- 120^{\circ}$ would have resulted in adding $\frac{2}{3}$ pole, and subtracting $1\frac{1}{3}$ poles from $5\frac{1}{3}$ poles, and again a 4 - 6 pole-changing machine would be obtained. In general, the pole numbers are either

(a) $n - \frac{2}{3}$ and $n + 1\frac{1}{3}$

or (b) $n - 1\frac{1}{3}$ and $n + \frac{2}{3}$

where n is the number of poles in the condition of zero phase shift. Table 4 gives some of the possible 2-speed machines which can be constructed in this fashion.

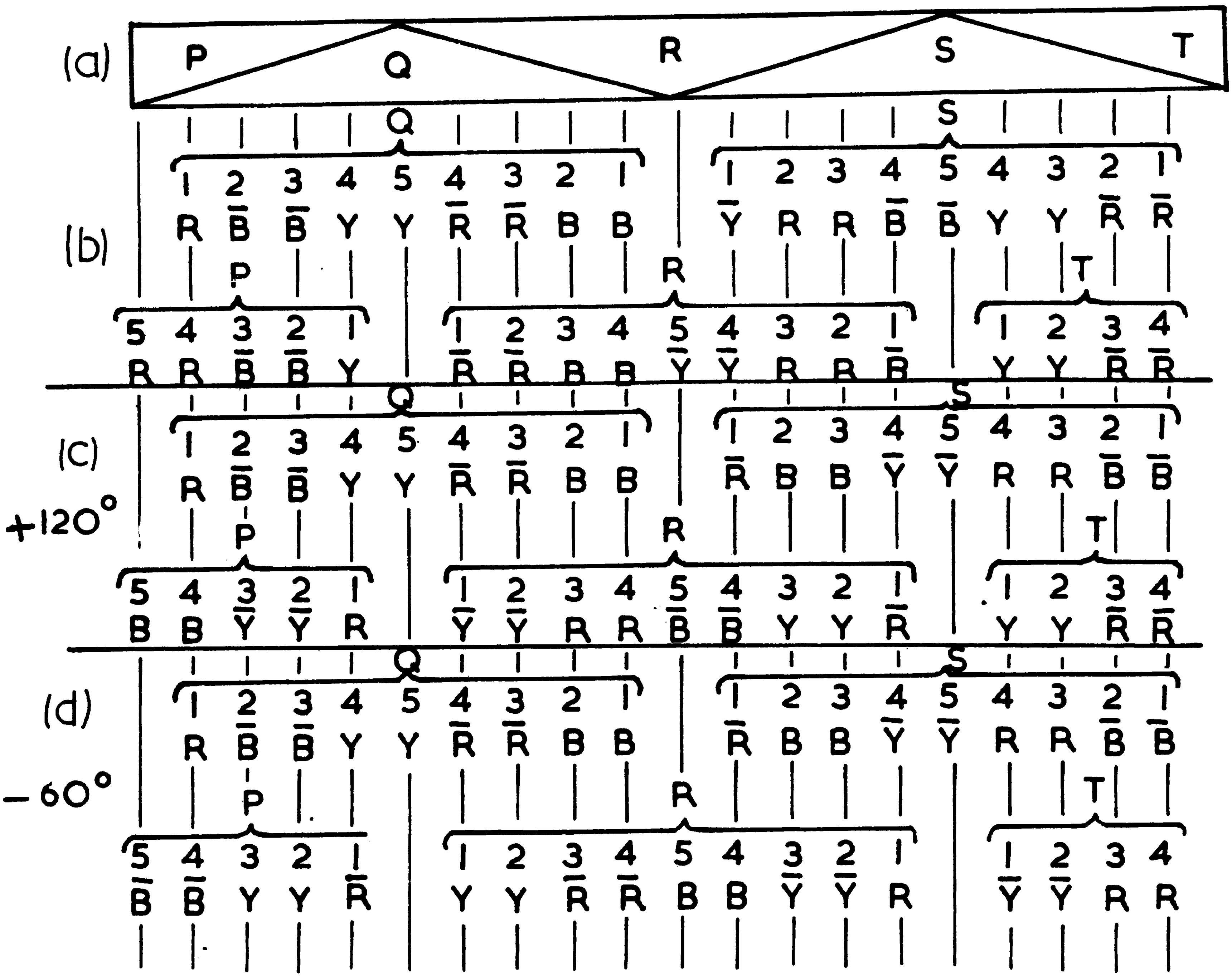
TABLE 4.

(a)			(b)		
$n - 2/3$	n	$n + 4/3$	$n - 4/3$	n	$n + 2/3$
2	$2\frac{2}{3}$	4	2	$3\frac{1}{3}$	4
4	$4\frac{2}{3}$	6	4	$5\frac{1}{3}$	6
6	$6\frac{2}{3}$	8	6	$7\frac{1}{3}$	8
8	$8\frac{2}{3}$	10	8	$9\frac{1}{3}$	10
10	$10\frac{2}{3}$	12	10	$11\frac{1}{3}$	12

If a three block machine is considered, a phase shift of 60° produces a change in pole number of unity, while a phase shift of 120° produces a change of two poles. Hence, if only simple reversal is allowed, i.e. from -60° to $+120^{\circ}$, or -120° to $+60^{\circ}$, a difference in pole number of 3 is produced, and if one position gives an even number of poles, the other gives an odd number, which is not permissible.

Figure 25 shows how the blocks of another 2-speed machine might be formed. Figure 25(b) shows the top layer of a winding to give $3\frac{1}{3}$ poles. The reference block is taken to be Q, and Figure 25(c) shows the configuration for a phase shift of $+120^{\circ}$. In this system, a phase shift of $+120^{\circ}$ adds $2\frac{2}{3}$ poles to the original pole number, and hence the machine has 6 poles in

Figure 25. Conductor arrangements in a four block machine for 2 - 6 poles.



this condition. Similarly, Figure 25(d) is drawn for a phase shift of -60°, when the pole number is 2. Again, T and P form a continuous winding in both the required conditions and can be considered as one block. The machine is then a four block machine. In order to change from one pole number to the other, P, R and T are reversed with respect to Q and S.

A similar 2 - 6 pole machine could be constructed by designing the winding to have $4\frac{2}{3}$ poles in the zero phase-shift configuration, when phase shifts of -120° and $+60^{\circ}$ would produce 2 and 6 poles, respectively. In general, for a four block machine there is a choice of

(i) $n - 4/3$ and $n + 8/3$

or (ii) $n - 8/3$ and $n + 4/3$

Some of the possible machines are shown in Table 5.

TABLE 5.

	(i)			(ii)	
$n - 4/3$	n	$n + 8/3$	$n - 8/3$	n	$n + 4/3$
2	$3\frac{1}{3}$	6	2	$4\frac{2}{3}$	6
4	$5\frac{1}{3}$	8	4	$6\frac{2}{3}$	8
6	$7\frac{1}{3}$	10	6	$8\frac{2}{3}$	10
8	$9\frac{1}{3}$	12	8	$10\frac{2}{3}$	12
10	$11\frac{1}{3}$	14	10	$12\frac{2}{3}$	14

The general pattern of these machines is now evident. The number of blocks must be even, since the difference in the two pole numbers is the same as the number of blocks. If there are 2a blocks, where a is any integer, for phase shifts of 60° and 120°, pole differences of $2a/3$ and $4a/3$, respectively, are produced. In general there is a choice of either

$$n - 2a/3 \text{ and } n + 4a/3$$

or

$$n - 4a/3 \text{ and } n + 2a/3$$

By the technique described so far, different effective current densities are manufactured from the winding in the two switched positions, since in one case currents in some of the slots are adding vectorially at 120°, whilst in the other switched position they add at 60°. Hence, by this technique, one excellent setting and one mediocre setting are produced. A better mean result can be obtained by virtual phase shifts of $\pm 90^\circ$; for example, in the arrangement shown in Figure 26, which is the top layer of a section of winding between two blocks, if the currents have the phases shown in Figure 26(b) in the zero phase-shift condition, a virtual phase shift of 90° for the second block can be obtained by winding it with its phase groups displaced by half a pole pitch, which in this case is three slots. This is illustrated in Figure 26(c).

Reversal of this winding will produce a virtual phase shift of -90°, as indicated by Figure 26(d).

On a two block machine, $+90^\circ$ produces a pole number of $n + 1$, where n is the pole number in the middle-speed position, while a phase-shift of -90° produces a pole number of $n - 1$, and in general, on a 2a machine, the pole difference between the two speeds is 2a, where a is any integer. Thus again, any two pole numbers can be produced. An example of this

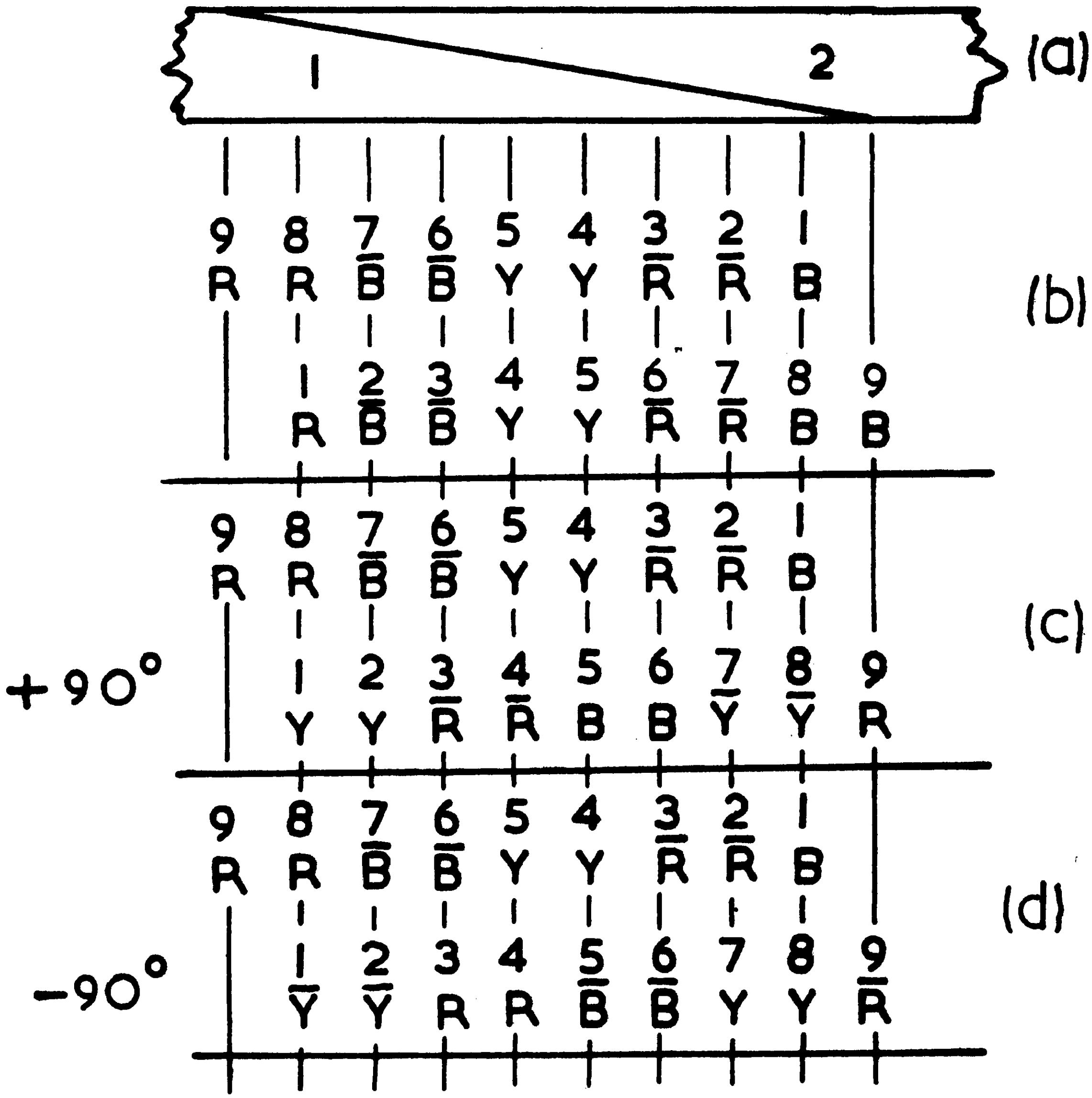


Figure 26. Conductor arrangements using $\pm$ 90° switching

system is illustrated by Figure 27, which shows the top layer of an 8 block machine, based on six poles, producing 2 - 10 poles by reversal of the blocks labelled 0, 2, 4 and 6 with respect to those labelled 1, 3, 5 and 7.

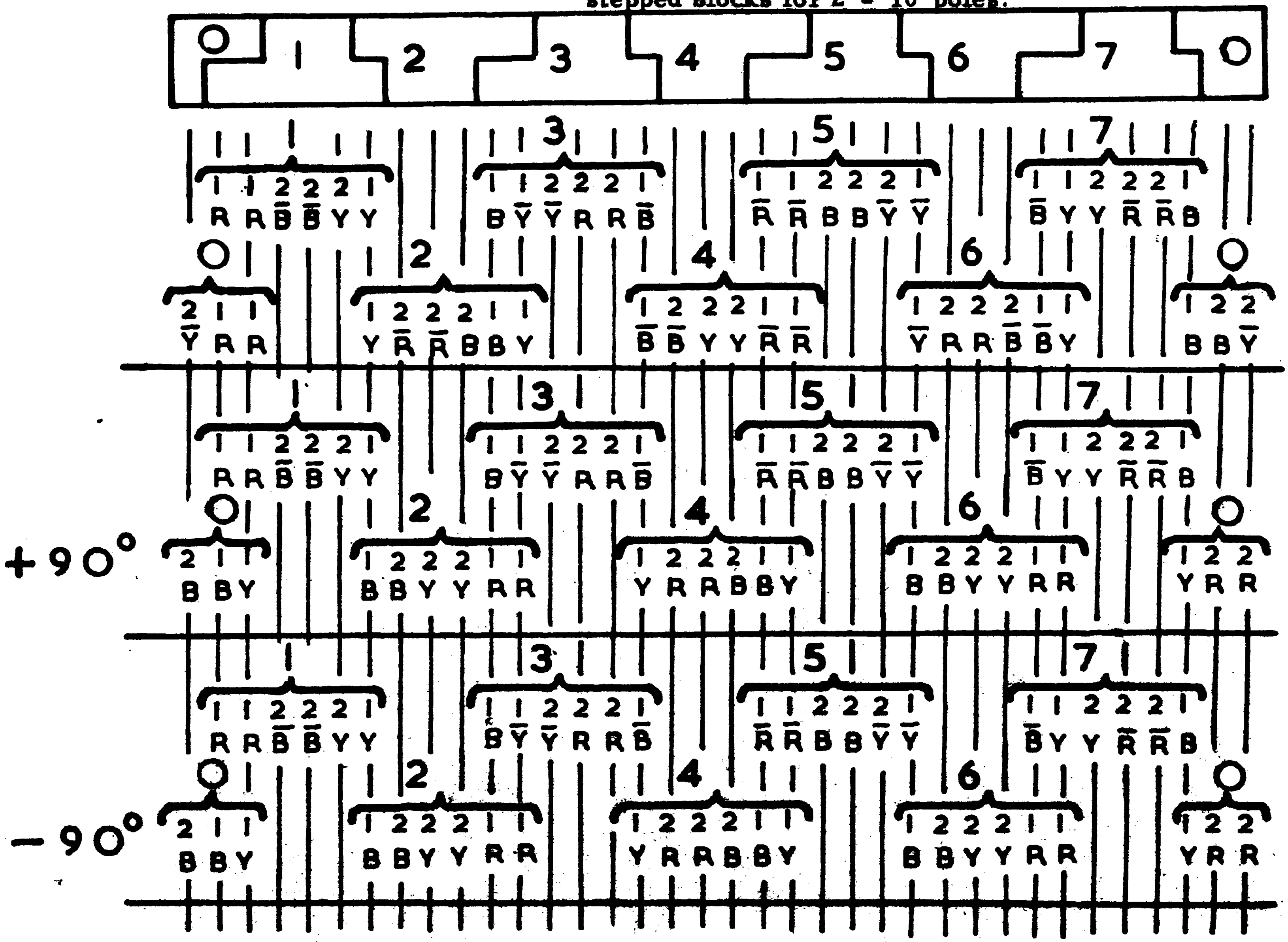
The block shape is of stepped form in Figure 27 to emphasize further that any block shape is possible.

9. THE SECOND EXPERIMENTAL MACHINE

The second experimental machine employed a 48-slot stator and six groups producing eight poles in the centre connection. One such group is shown in Figure 28. The machine can be used as either a 2, 3 or 5 speed machine. If only two speeds are required, the switching and connections are as described in Section 8. Three speeds can be obtained by using the system illustrated by Figure 4(a) and replacing winding A in this diagram by the 0 and 3 groups in series; similarly winding B is replaced by the 1 and 4 groups in series, and winding C by the groups labelled 2 and 5 in series. The 5-speed connection results if the windings are connected as for three speeds but with provision for the reversal of the odd-numbered groups with respect to the even numbered groups. Thus, if the switch on Figure 4 is in the second position, yielding + 120° virtual phase shift, - 60° virtual phase shift can be obtained by reversal of the odd-numbered groups. Similarly the + 60° condition can be obtained from the - 120° connection.

The machine was tested as a 2-speed machine having a pole combination of 4/10. The harmonic analysis is shown in Table 6, harmonics

Figure 27. Conductor arrangements in an eight block machine using stepped blocks for 2 - 10 poles.



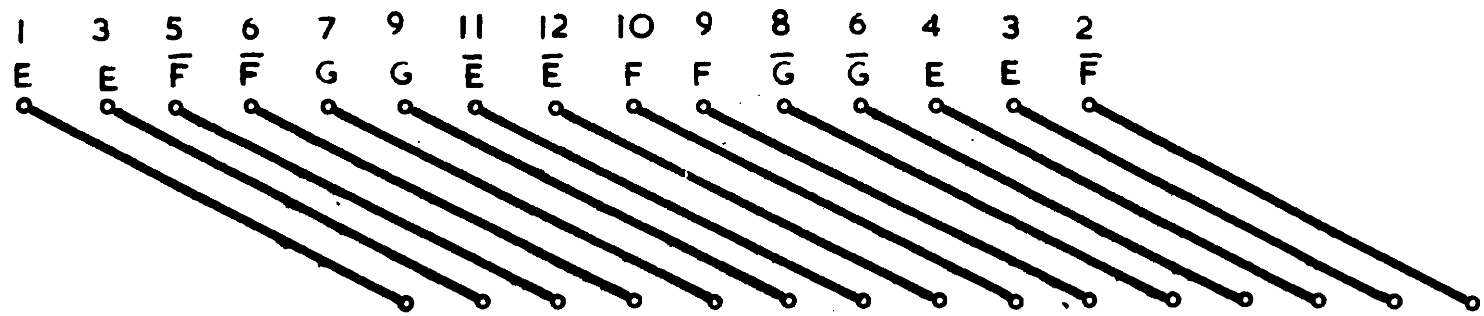


Figure 28. Winding arrangement of one block of the second experimental machine.

greater than 1% being listed, up to 28 poles.

TABLE 6.

4-pole connection			10-pole connection		
Pole number	Direction	Current-density amplitude relative to 100% fundamental	Pole number	Direction	Current-density amplitude relative to 100% fundamental
4	+	100	2	-	1.33
8	-	10.9	10	+	100
14	-	6.44	28	-	1.97
16	+	20.9			
28	+	7.6			

The winding factors for the 4- and 10-pole connections are 0.405 and 0.88, respectively, and if series-delta/parallel-star switching is used as shown in Figure 22(c) the ratio of flux densities B_4/B_{10} is 1.003. The ratio of magnetizing currents I_4/I_{10} is 0.98. Figure 29 illustrates the relationship between the quadrature component of current against voltage with the machine running light for the two settings, the ratio of magnetizing currents agreeing tolerably well with the expected values. Inspection of Table 6 indicates a possibility of a torque dip at the 16-pole speed on the 4-pole setting; accordingly the torque/speed curve in this condition was obtained experimentally and is plotted in Figure 30. The torque dip produced is not serious in this particular case, since the rotor employed was of double-cage construction with a high starting-torque characteristic. It cannot, however, be assumed that a harmonic of this size is necessarily

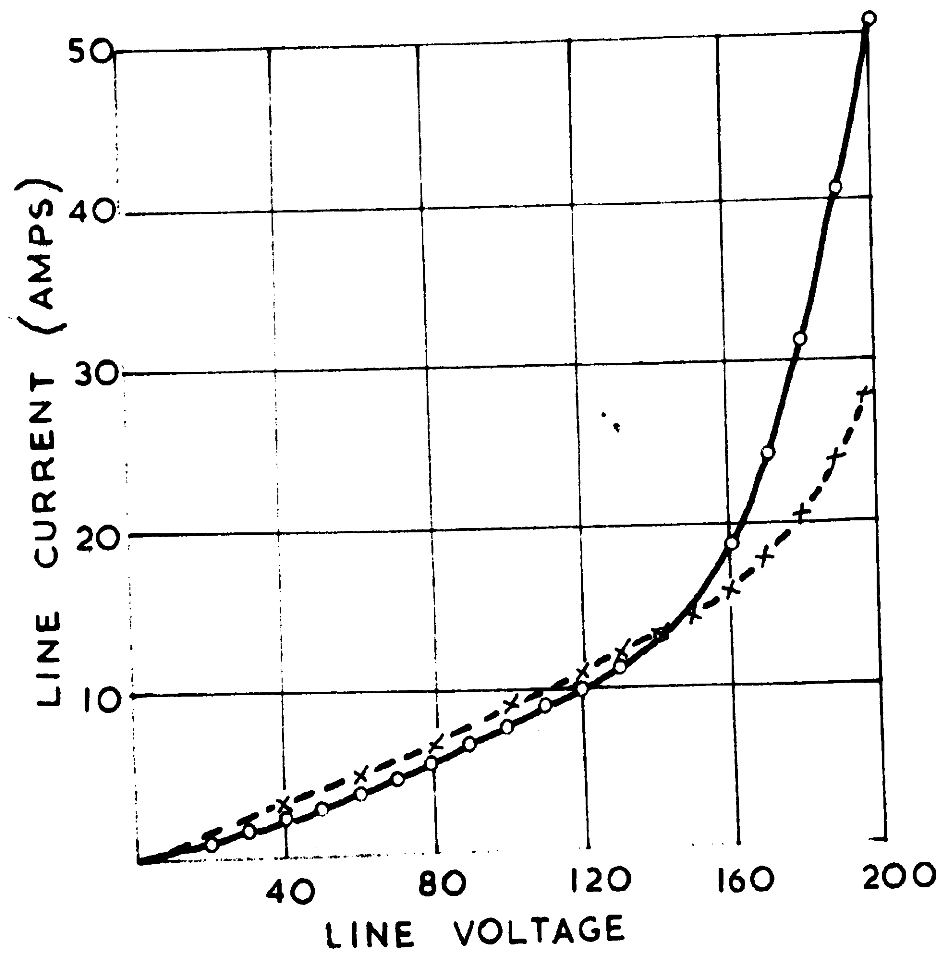


Figure 29. Comparison of magnetising currents for a 4-10 pole machine.

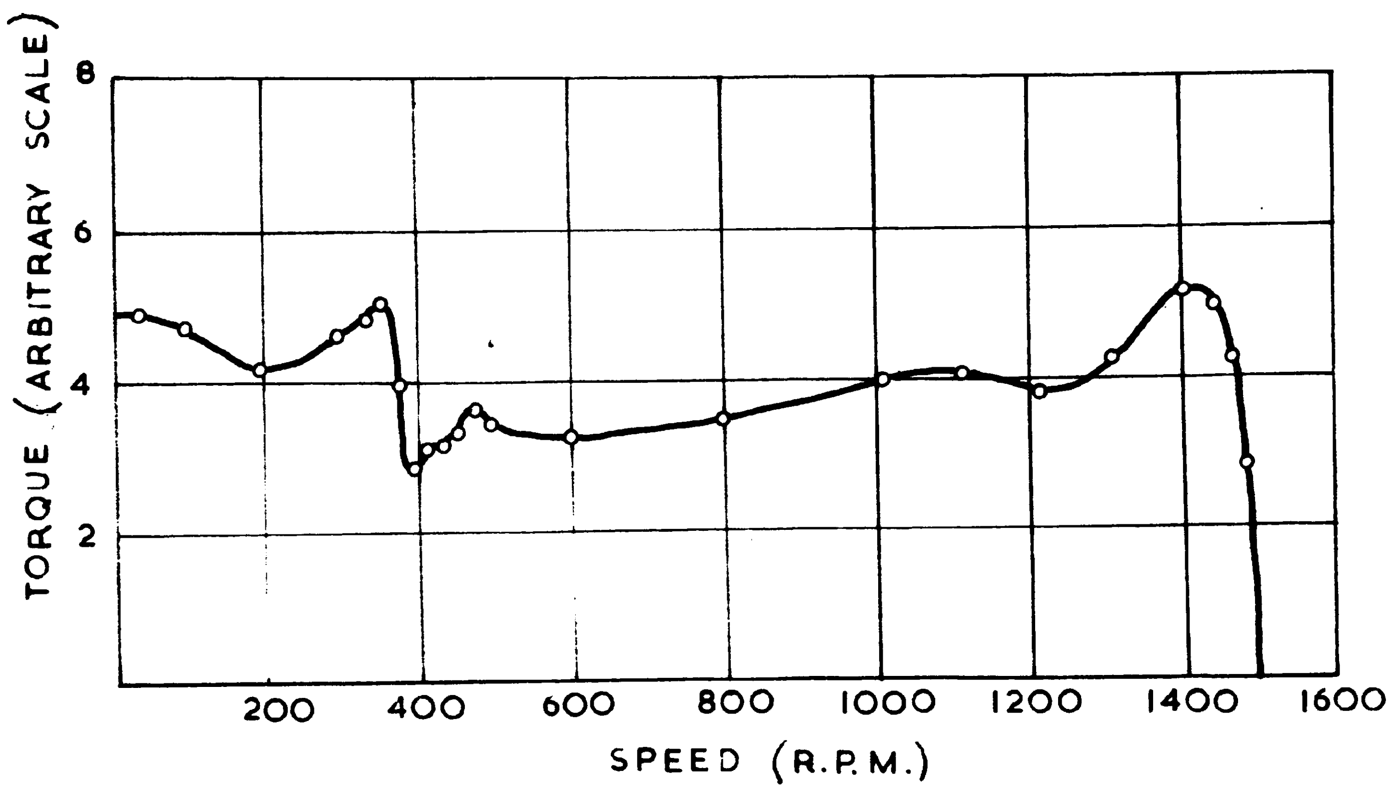


Figure 30. Speed-torque curve for a 4-10 pole machine on the 4-pole setting.

innocuous. The harmonic torque dip produced clearly depends on the characteristic of the rotor bar employed and the pole pitch of the machine. The machine exhibits constant-torque characteristics. The fan on the machine was intended for 1,500 r.p.m. and was not entirely suitable for the 10-pole setting, but it can be taken that the output in this connection was sensibly the same as would have been obtained from a conventional winding of this pole number, since the flux-density ratio is unity, and the winding factor approaches very closely that for a conventional machine.

The test results indicated an output of 5.5 hp. in the 10-pole setting and 14 hp. in the 4-pole condition, the machine frame being rated at 20 hp. at 1,500 r.p.m. Full-load efficiencies in the two cases were 81% and 87% on the 10- and 4-pole settings, respectively.

10. CONCLUSIONS

The principles of a whole range of new pole-changing machines have been outlined. The development of these machines has been carried out by a process of proceeding from the general to the particular, starting with the continuously variable brushless phase-mixer and ending with 2-speed 6-terminal machines. It is interesting to note how development of 2-speed machines has proceeded along two separate channels. The work of Professor Rawcliffe and his associates[6, 7, 8] has produced 2-speed machines which were developed from the basic idea of pole-amplitude modulation. The 2-speed machines in this Chapter were developed from the idea of phase mixing and phase shifting and have resulted in completely different winding arrangements. This is hardly surprising, since with

S slots there are 6^S ways of filling them with a 3-phase single-layer winding and 15^S ways with a double-layer winding, a large number of which, of course, are not apt. However, if a general comparison of the techniques of the two sets of machines is to be made, the 2-speed machines described in this Chapter might be said to operate on the principle of phase modulation as opposed to amplitude modulation.

Obviously a great deal of work is still to be done in connection with harmonic suppression and increase of copper utilization. This must take the form of finding the most advantageous block shapes and chording factors, and is attempted in the subsequent chapters.

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CHAPTER II

WINDING DESIGN FOR POLE-CHANGE MACHINES USING PHASE-SHIFT TECHNIQUES.

1. INTRODUCTION

The previous chapter describes methods of obtaining multi-speed operation from a single induction machine winding. The analysis used to find the harmonic content of the windings is essentially a coil by coil summation and as such is of great utility in assessing the performance of a winding which is specified. However, it cannot be adapted easily to design purposes. It is necessary, therefore, to develop a generalised analysis of the action of the new windings.

In section 7 of Chapter I, a winding technique is described which permits any block shape for the groups. It is, of course, the object of the designer to simplify the windings as far as possible, in particular if a simple rectangular block could be used the winding would be of conventional double layer construction. The development of this simple block shape is shown in Figure 1 which illustrates the front or leading coil sides. Figure 1(a) shows the triangular block shape which is used in much of the work described in the previous chapter, similarly Figure 1(b) illustrates the stepped form of block which can be constructed using two sizes of coil, whilst Figure 1(c) shows the simple rectangular form. The block shape of Figure 1(b) can be synthesized from two simple rectangles and therefore the analysis begins by considering a machine formed from 'm' rectangular segments spaced around the periphery, each of which carries a travelling wave of current. This is illustrated in Figure 2.

In order to produce a reasonably compact theory, it is necessary to specify that the blocks are all of equal length and that their centres are

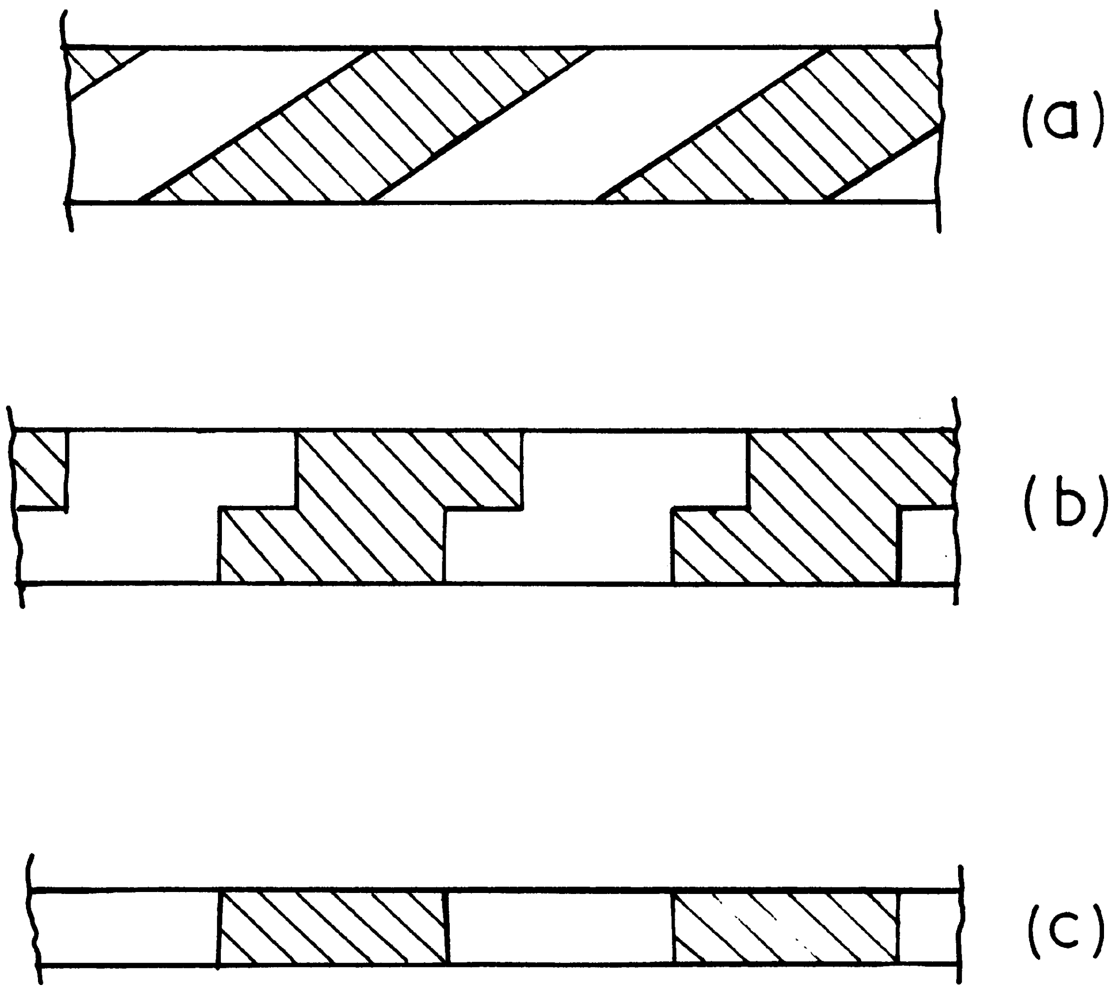


Figure 1. Block Shapes in Pole-change Machines.

(a) Triangular
(b) Stepped
(c) Rectangular

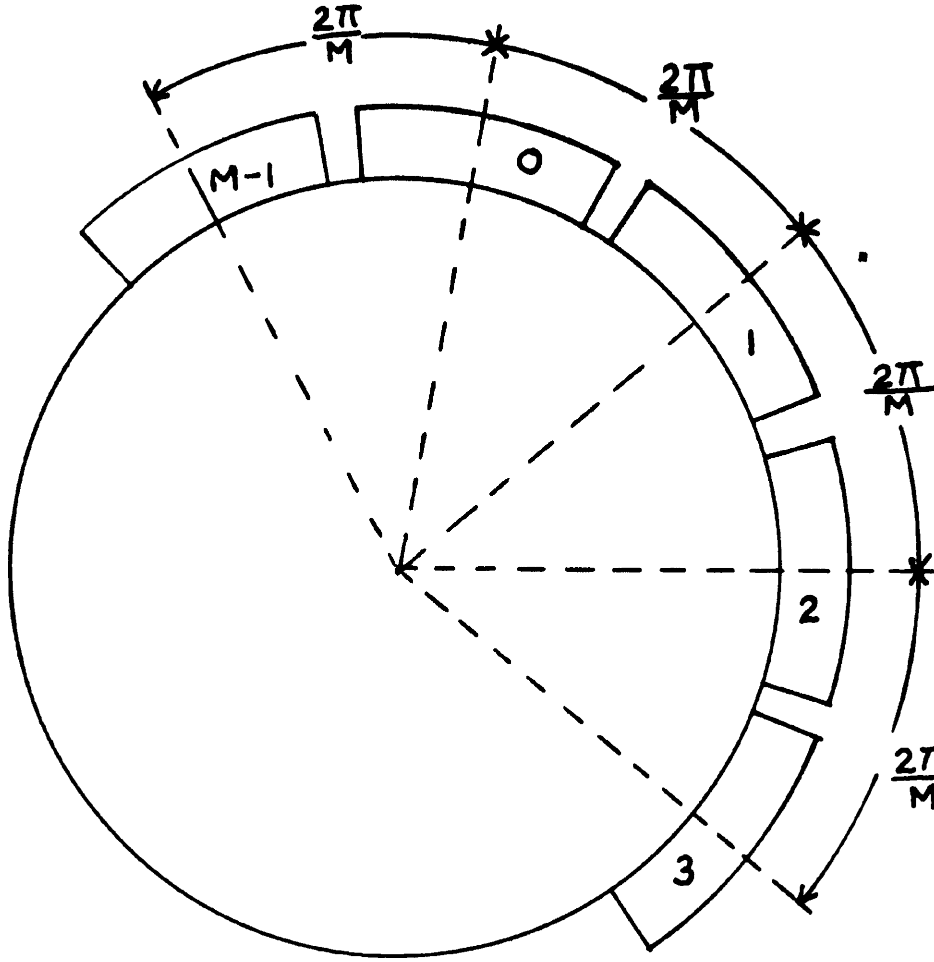


Figure 2. The basic machine to be analysed.

equally spaced.

2. THE DATUM MACHINE

It is convenient to define a basic or datum condition, in which each segment is energised with an identical travelling wave of current of angular pole pitch, y_ω, given by:-

$$J_s \, e^{j(\omega t - \phi_o)} \, e^{-j\pi\left(\frac{y}{y_\omega} - x_o\right)} \qquad \ldots\ldots\ldots \qquad (1)$$

where y is measured relative to the centre of a particular segment and ϕ_o and x_o are arbitrary constants. This datum machine is shown in Figure 3.

It is apparent from equation (1) that the progressively increasing phase-shift proportional to segment number, which is required to change the machine from its datum condition, can be achieved in two ways. It can be done electrically by changing the phase of the excitation supplied to the segments, that is in the ℓ^{th} segment ϕ_o is replaced by $\phi_o + \ell\phi$, or it can be done mechanically by moving the exciting wires (which in any practical machine will make up the segment) to new positions. This involves replacing x_o by $(x_o + \ell x)$ where x is measured in pole pitches.

Only particular values of electrical phase shift are conveniently possible, whereas the refinement of shift which is possible using mechanical methods is, in practice, limited only by the angular separation of the slots provided. This practical difference between mechanical and electrical shift is illustrated by Figure 4. Figure 4(a) shows a section of the top layer of a winding in its datum condition whilst Figure 4(b) illustrates a 'mechanical' shift of block B with respect to block A. Block B is

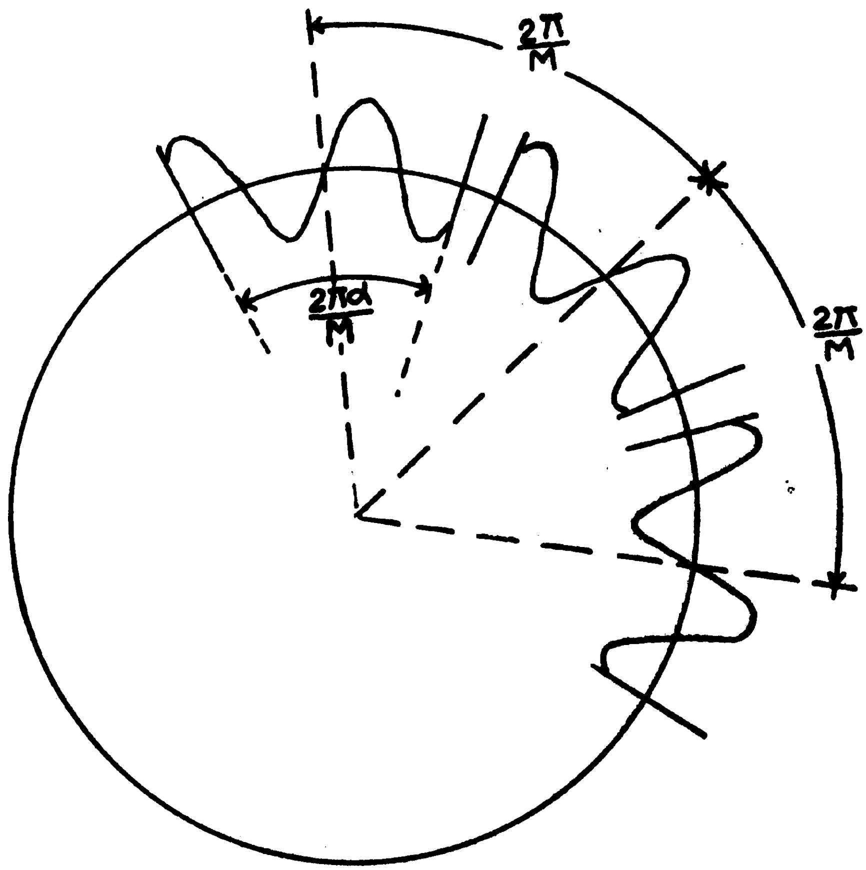


Figure 3. The Datum Condition.

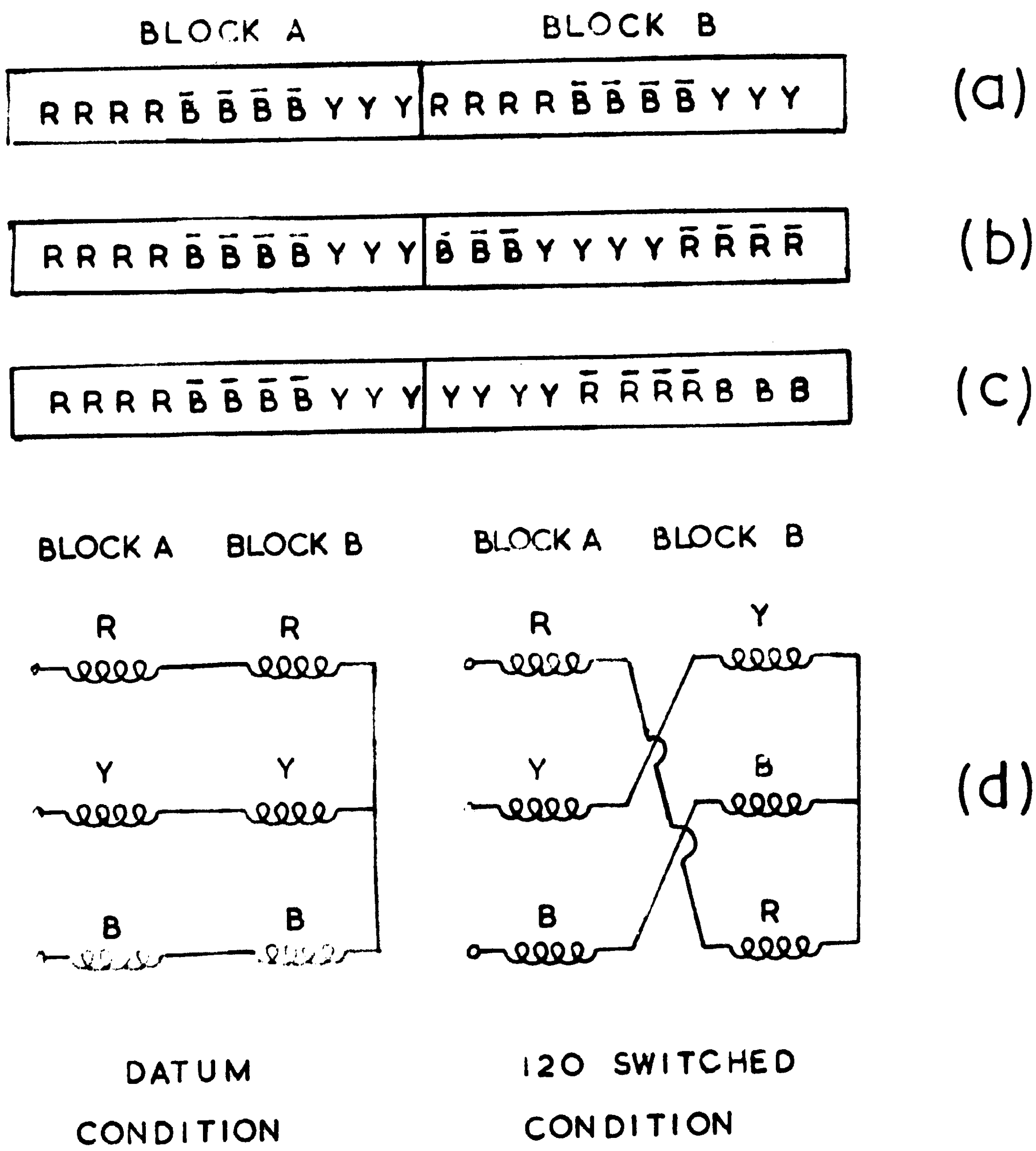


Figure 4. Illustrating the difference between mechanical and electrical shift.

(a) The datum condition
(b) Mechanical shift
(c) Electrical shift
(d) Connections for the electrical shift of (c).

phase shifted 5 slots or 75° on the scale of the excitation. Similarly Figure 4(c) shows an electrical phase shift of 120°. Clearly the configuration of Figure 4(c) can be obtained by switching the phase connections as Figure 4(d) illustrates, but the configuration of Figure 4(b) cannot be produced by simple switching from that of Figure 4(a) since it involves reconnection of the phase-bands. Thus the mechanical shift must be regarded as a displacement which occurs at the design stage; all subsequent displacements occur during use and must therefore be available electrically. Appendix (1) shows that the amplitude of the 2p pole harmonic produced by the rectangular segment is given by:-

$$\frac{aJ_s}{m} \left| \frac{\sin \frac{a\pi}{m}\left(p - \frac{\pi}{y_\omega}\right)}{\frac{a\pi}{m}\left(p - \frac{\pi}{y_\omega}\right)} \right| \quad \ldots\ldots\ldots \quad (2)$$

where a is the fraction of the stator periphery occupied by all the segments. This equation is illustrated in Figure 5. The envelope is a '$\left|\frac{\sin\gamma}{\gamma}\right|$' curve centred on $\frac{2\pi}{y_\omega}$ poles, which need not be an even pole number or even integral, but ordinates exist only at the even pole numbers. The 'band width' between the two zeros adjacent to $2\pi/y_\omega$ is $\frac{4m}{a}$, and subsequent zeros occur every $\frac{2m}{a}$. Returning now to the complete machine set up in the datum condition, appendix (2) shows that the harmonic contributions of the segments add up exactly in phase for all pole numbers that are multiples of 2m poles and that all other segment harmonic pole numbers cancel out completely. Thus, the envelope curve defining the amplitude of these harmonics is the same as the envelope curve for one

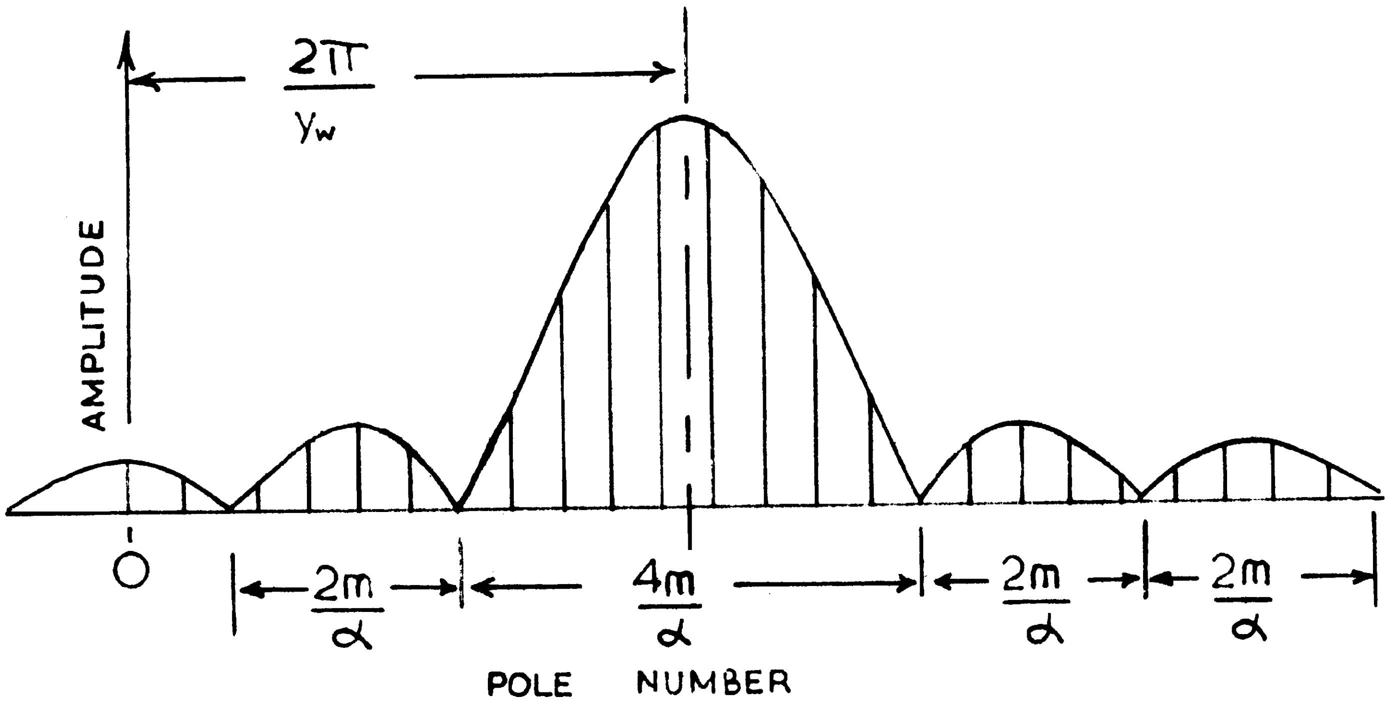


Figure 5. The segment harmonic envelope.

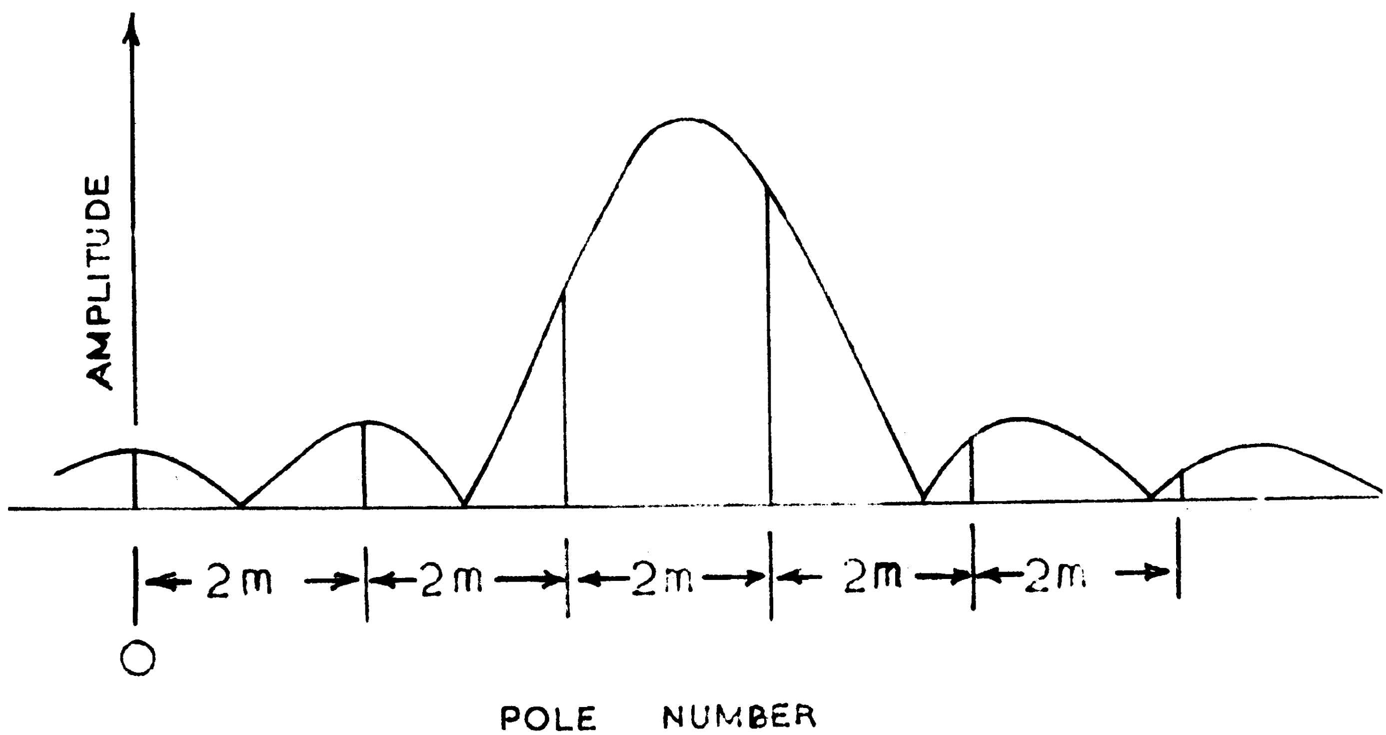


Figure 6. The harmonics due to the assembly of segments.

segment as Figure 6 illustrates. Appendix (2) further shows that if now 'mechanical shift' of amount $x\ell = 2r\frac{\ell}{m}$ is applied to segment ℓ where ℓ ranges from 0 to (m - 1) and r is an integer, every ordinate associated with the datum condition will be moved to the right under a fixed envelope by 2r poles; in particular there will be an ordinate at 2r poles and in general 2r will be caused to be near $\frac{2\pi}{y_\omega}$ so as to maximise its amplitude.

With the machine in this condition, electrical phase shift is applied to the segments. Using 3-phase supplies the possible values of ϕ are limited to 120° obtained by switching round the phases, 60°, obtained by switching round the phases combined with reversal of the windings, and 180° obtained by winding reversal alone. The segment ℓ has the phase of its current delayed relative to the 0^{th} segment by $\ell\phi$ where $m\phi = 2\pi q$ and q is an integer. Appendix (2) shows that when this is done all the ordinates will be moved further to the right by 2q poles under the fixed envelope. The machine now has a dominant pole number of (2r + 2q) and will run at the corresponding speed.

3. MAGNITUDE OF HARMONIC POLE NUMBERS

Examination of the spectra under the various conditions indicates the possibility of undesirable harmonics and it is necessary to study their magnitudes. The machines can be divided broadly into two main groups, the 3k block machines described in Section 5 of Chapter 1 and the two-speed windings discussed in Section 8 of Chapter 1. The 3k block machines use electrical phase shifts of $\pm 60^\circ$ and $\pm 120^\circ$ whilst the electrical phase shift produced by the switching in the two speed case is 180°.

3.1. <u>3 and 5-Speed Machines.</u>

As an example, Figure 7(a) shows a 6 block machine in the datum condition. The winding is designed for 5-speed operation, the excitation has a pole number of 8 and the blocks occupy all the periphery, that is $\alpha = 1$. Figure 7(b) illustrates the spectra produced by a single block, whilst Figure 7(c) shows the spectra produced by the assembly of blocks when the only harmonics present are those which are multiples of 12. Mechanical shift is now applied to the blocks so that the ordinate at 0 poles moves to the peak of the envelope at 8 poles. In order to do this, 2r must be 8 poles so that $x\ell = \frac{8\ell}{6}$ and $x = 1^1/3$ poles. When this shift is applied to the configuration of Figure 7(a) the condition shown in Figure 7(d) results, an ideal 8 pole wave - this is as it should be; when the 8 pole shift is applied to the spectra of Figure 7(c) all the harmonics except the one at 8 poles disappear into the zeros of the envelope, as Figure 7(e) indicates.

With the machine in this condition electrical phase shift ϕ can be applied to the segments.

Now $m\phi = 2\pi q$ where q is an integer if the phase discontinuity is to be the same at each block boundary. Thus, to move the ordinate at 8 poles 2 poles to the right, i.e., $q = 1$, a phase shift of $\phi = \frac{\pi}{3}$ is required. When this phase shift is applied, all the ordinates in Figure 7(e) move 2 poles to the right as shown on Figure 7(f), thus reducing the height of the

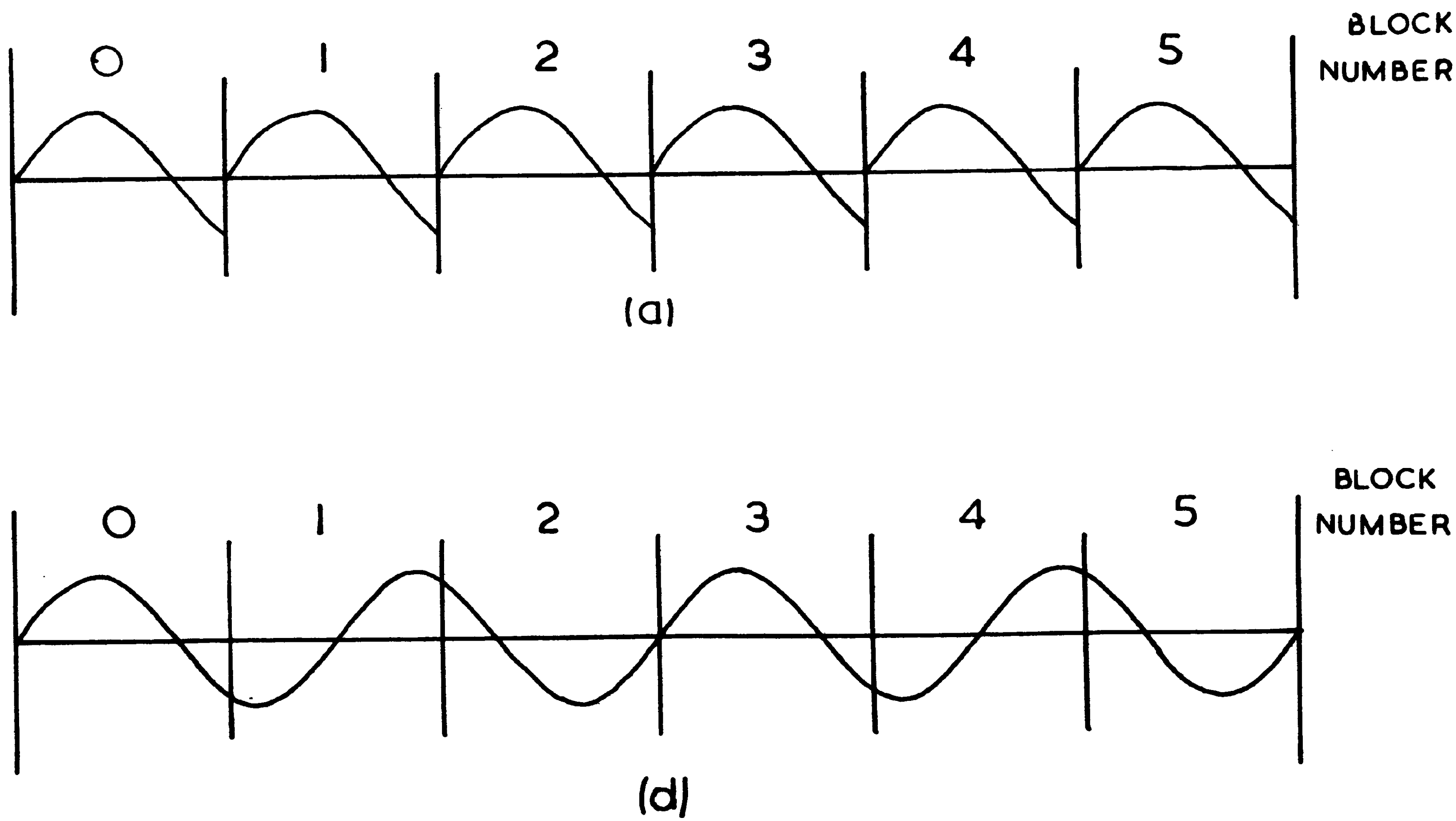
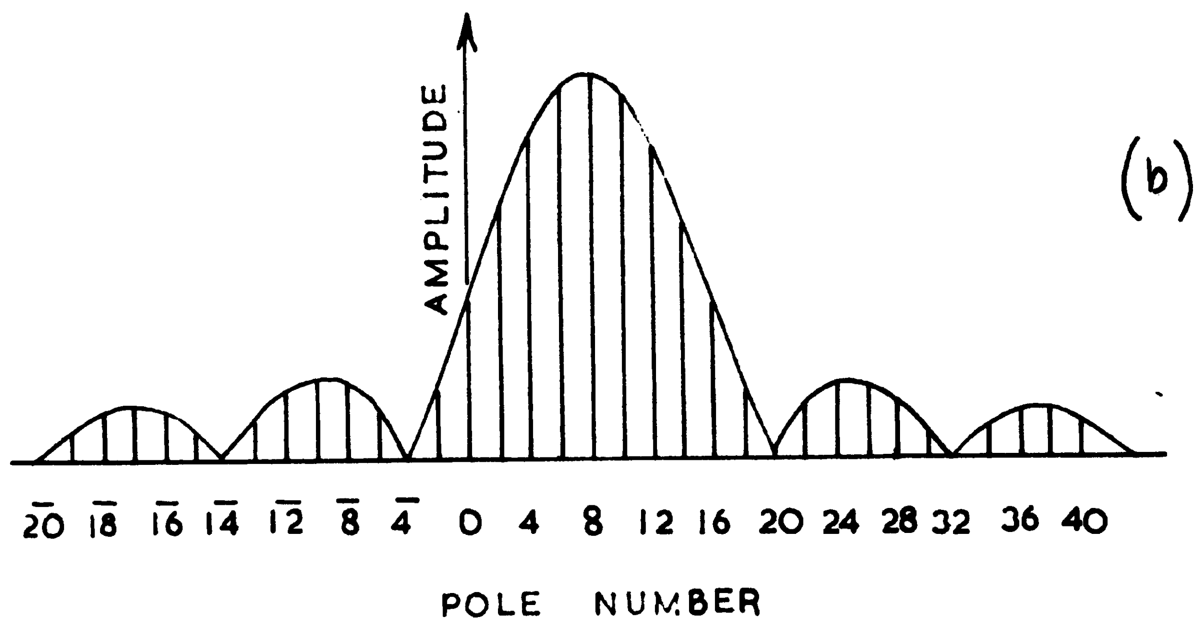


Figure 7. <u>5 speed winding example.</u>

(a) Segmental current sheets in the datum condition.
(d) Segmental current sheets after mechanical shifting.



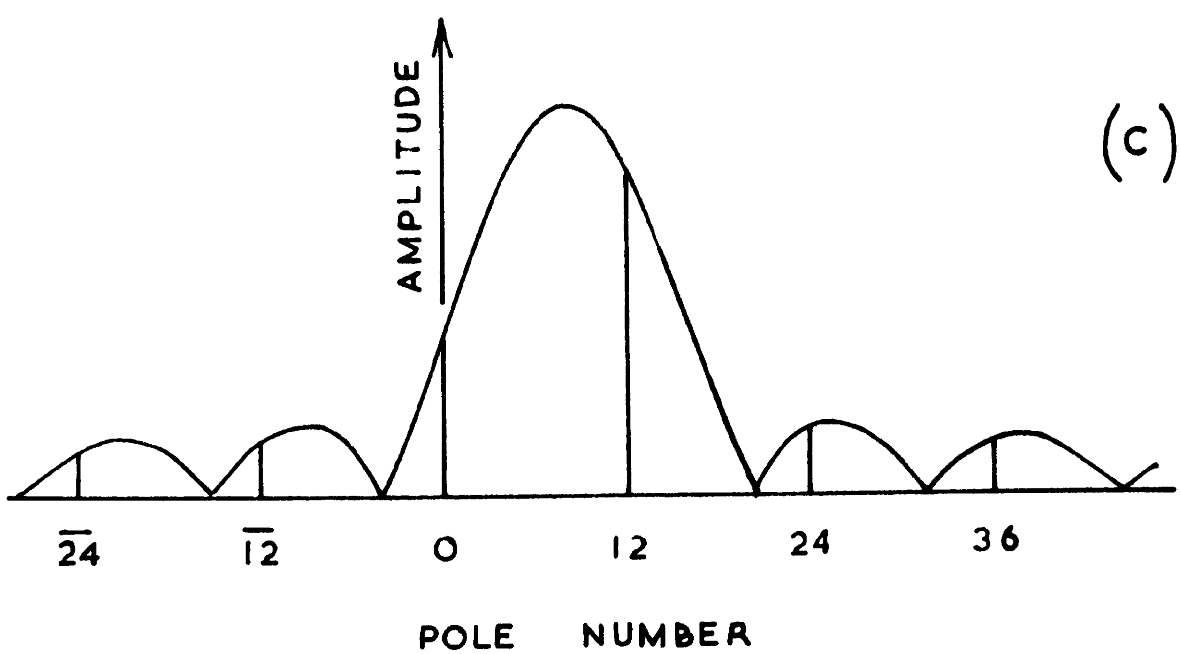
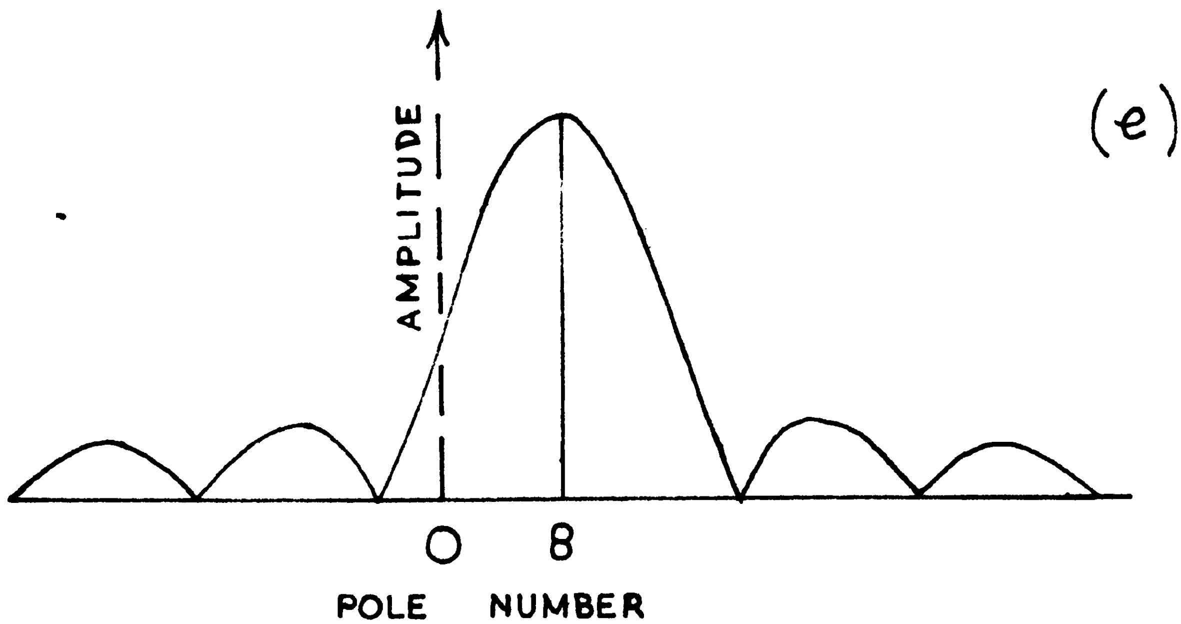


Figure 7. 5 speed winding example.

(b) Harmonic ordinates produced by a single segment.
(c) Harmonics produced by the assembly of segments in the datum condition.



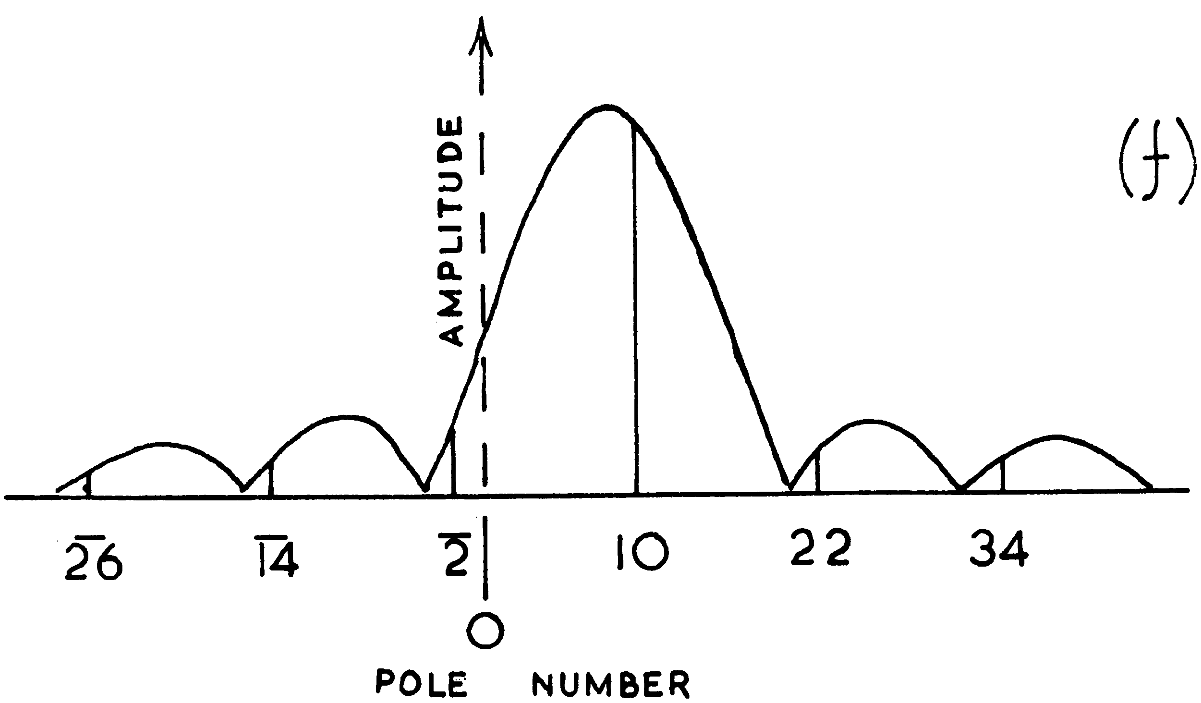


Figure 7. <u>5 speed winding example</u>.

(e) Harmonic ordinates after mechanical shifting.
(f) Harmonic ordinates after electrical shifting.

maximum ordinate and bringing the harmonic pole number ordinates out of the zeros. Similarly, a shift of $\phi = \frac{2\pi}{3}$ will move the ordinates of Figure 7(e) 4 poles to the right, and shifts of $-\frac{\pi}{3}$ and $-\frac{2\pi}{3}$ produce dominant pole numbers of 6 and 4 respectively. In the general case the maximum ordinate at any setting is given by

$$U = \frac{\sin \frac{\pi(2q)}{2m}}{\frac{\pi(2q)}{2m}}$$

The value of this expression may be called the 'utilization factor' and measures the fraction of the stator current which is usefully employed at the wanted pole number; clearly this should approach unity which requires that $\frac{q}{m}$ be as small as possible.

The harmonic amplitudes to the right of the wanted ordinate, numbered progressively, in s are given by

$$U_s = \frac{\sin \pi(\frac{q}{m} + s)}{\pi(\frac{q}{m} + s)}$$

whilst those to the left are given by

$$U_s' = \frac{\sin \pi(s - \frac{q}{m})}{\pi(s - \frac{q}{m})}$$

numbered progressively in s'.

Hence $$\frac{U_s}{U} = \frac{(-1)^s}{\frac{ms}{q} - 1} \qquad \ldots\ldots\ldots \qquad (3)$$

and $$\frac{U_s'}{U} = \frac{-(-1)^s}{\frac{ms}{q} + 1} \qquad \ldots\ldots\ldots \qquad (4)$$

Once again, if these ratios are to be kept small, $\frac{m}{q}$ must be large, in particular if $\frac{U_s}{U} < 10\%$ were required then $\frac{m}{q} > 11$. In the case of the 5 speed machine $\frac{m}{q} = 6$ for the 60° switched case and $\frac{m}{q} = 3$ for the 120° case. Thus even for 60° switching the harmonic amplitudes are undesirably large.

3.2. Two-speed machines

In the general two-speed system, using 180° switching, $\frac{m}{q} = 2$. Figure 8(a) illustrates the ordinates produced by a machine in which mechanical shift has set $2r = \frac{2\pi}{y_\omega}$. That is, the machine is an ideal 2r pole machine in one of the settings with a utilization factor of unity and no harmonics. However, an electrical phase shift of 180°, which implies the reversal of alternate segments, changes the ordinates under the envelope to the new positions shown in Figure 8(b). There are two ordinates of equal heights and an extremely high harmonic content. Such a machine is not of general utility. A much more useful machine can be devised by setting $\frac{2\pi}{y_\omega}$ midway between 2r and (2r + 2q). The spectra for the two speeds are illustrated by Figures 9(a) and (b).

The displacement from the maximum of the envelope is now $\frac{2q}{2}$ instead of 2q, and the amplitude of the harmonics is given by equations (3) and (4) using $\frac{m}{q} = 4$ and it is again apparent that the harmonic amplitudes are undesirably large.

This arrangement, in which $\frac{2\pi}{y_\omega}$ is chosen midway between 2r and (2r + 2q), corresponds with the $\pm 90^\circ$ switched winding with the 0° condition suppressed, which is described in Section 8 of Chapter 1. The basic pole number $\frac{2\pi}{y_\omega}$ need not be set up midway between 2r and

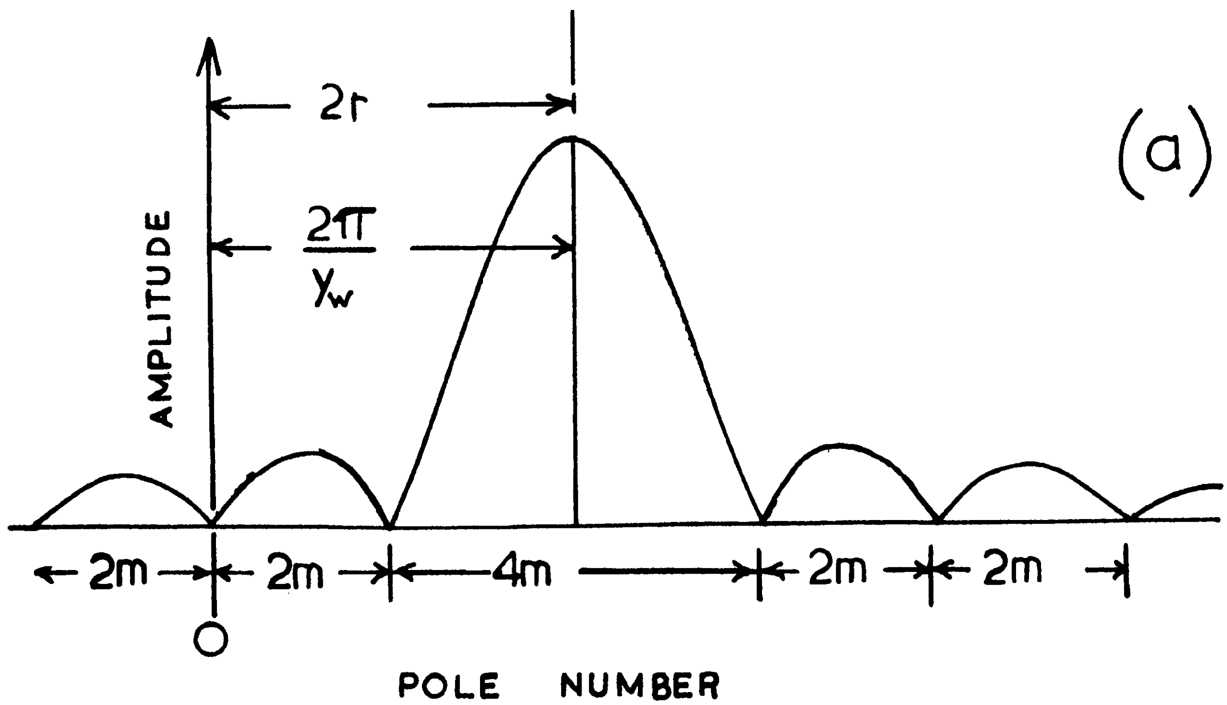


Figure 8(a) The ordinates produced by an ideal 2r pole machine.

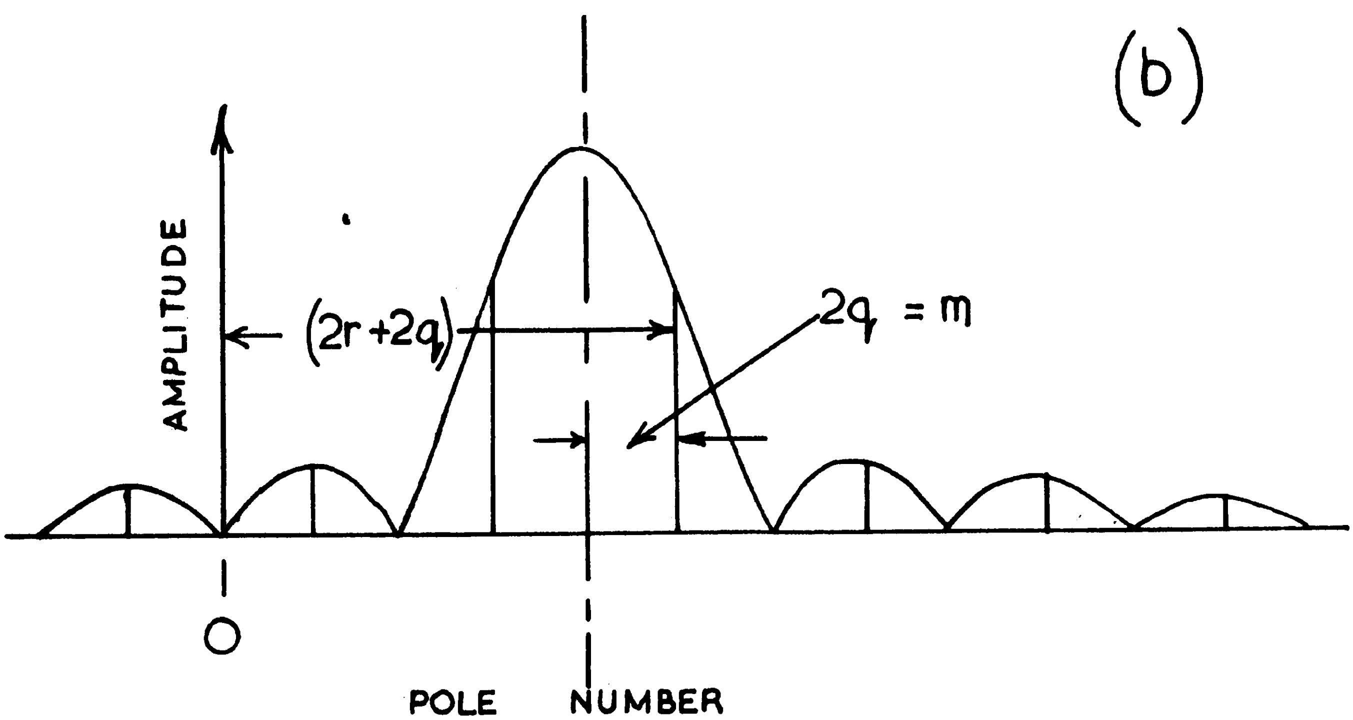


Figure 8(b). 180° electrical shift applied to the machine of Figure 8(a).

$(2r + 2q)$; if it is set one third of the way from $2r$ to $(2r + 2q)$ the setting up of $2r$ by mechanical phasing corresponds with a system switched-60° away from $\frac{2\pi}{y_{\omega}}$. 180° switching from this condition yields $2r + 2q$ which is effectively $+ 120^{\circ}$ switching from $\frac{2\pi}{y_{\omega}}$. In general, the relevant values of $\frac{2\pi}{y_{\omega}}$ will not be integers for example for $2r = 8$, $2r + 2q = 10$, $\frac{2\pi}{y_{\omega}} = 8\frac{2}{3}$ poles. The discussion on 60° and 120° switching relates unchanged to these cases.

In the general two-speed switching system, one of the speeds must imply a virtual phase shift of at least 90° and thus in all the machines the harmonic amplitudes produced by using simple rectangular segments are undesirably large for at least one of the speeds.

3.3. Application of Chording to Harmonic Reduction

In conventional machines unwanted harmonics are frequently diminished by choosing an appropriate pitch for the coils. The equivalent procedure with the segmental current sheet would be to duplicate it in a reverse sense spaced y' away where y' is the angular coil pitch, as Figure 10 illustrates.

Appendix 3 shows that the effect is to multiply any harmonic amplitude U_p of pole number $2p$ by a chording factor

$$C = \left| \cos \frac{1}{2} y' (\delta p) \right|$$

where $2\delta p$ is the departure of the pole number of the harmonic from the pole number, not necessarily an integer, for which y' would represent full pitch. The envelope of this expression is shown in Figure 11 together with a typical segment harmonic envelope with its wanted and unwanted

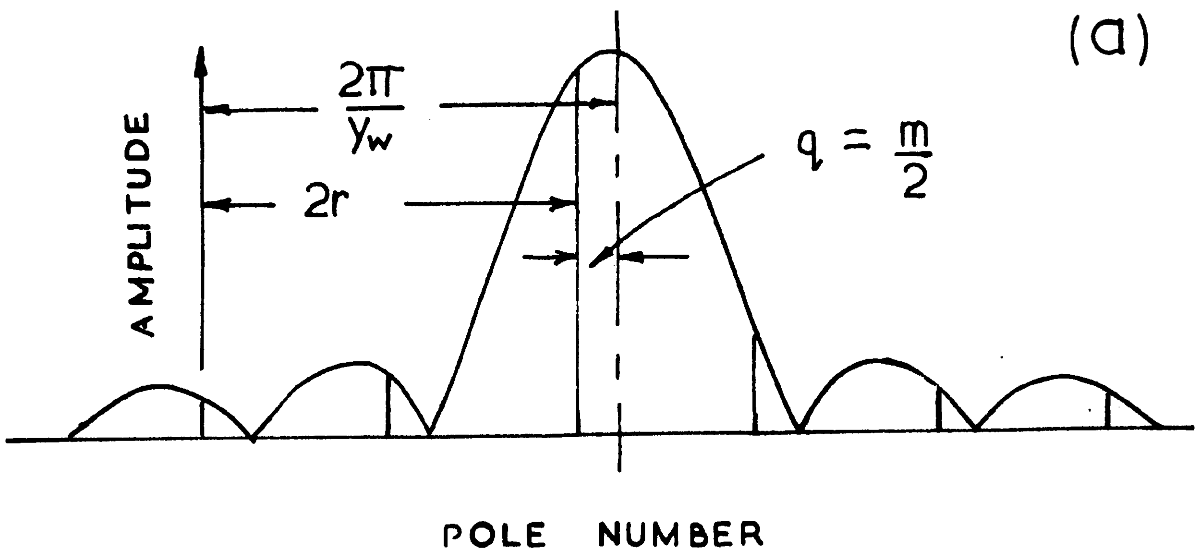


Figure 9(a) The ordinates produced by a machine with $\frac{2\pi}{y_\omega}$ set in between $2r$ and $2r + 2q$.

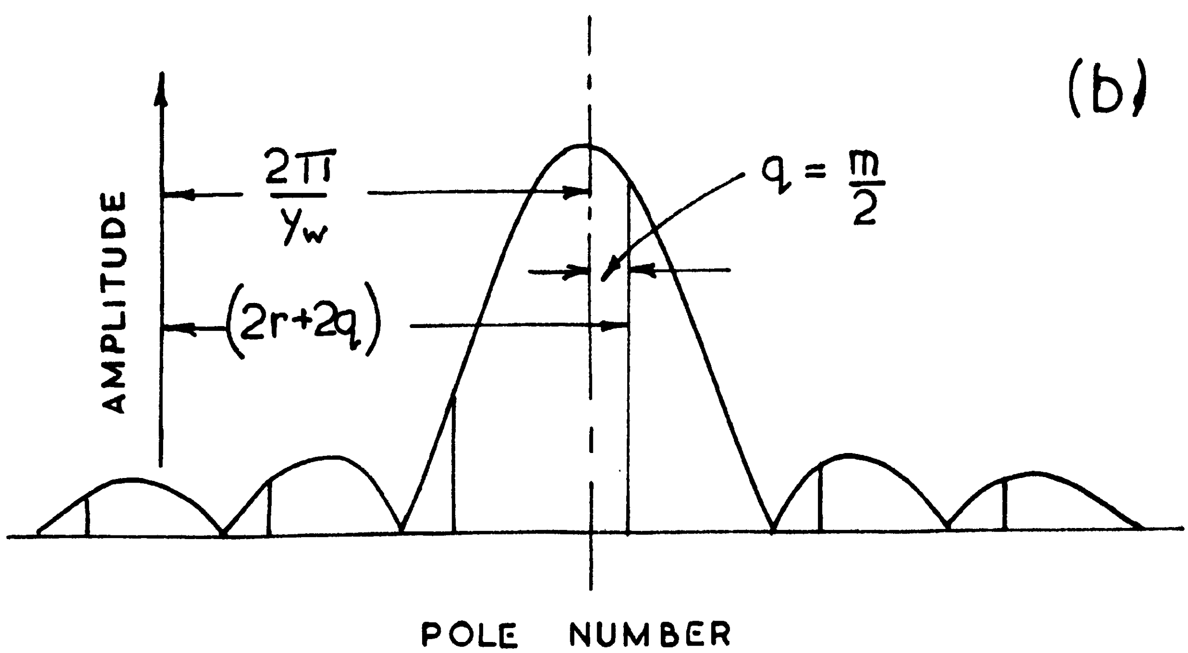


Figure 9(b). 180° electrical shift applied to the machine of Figure 9(a)

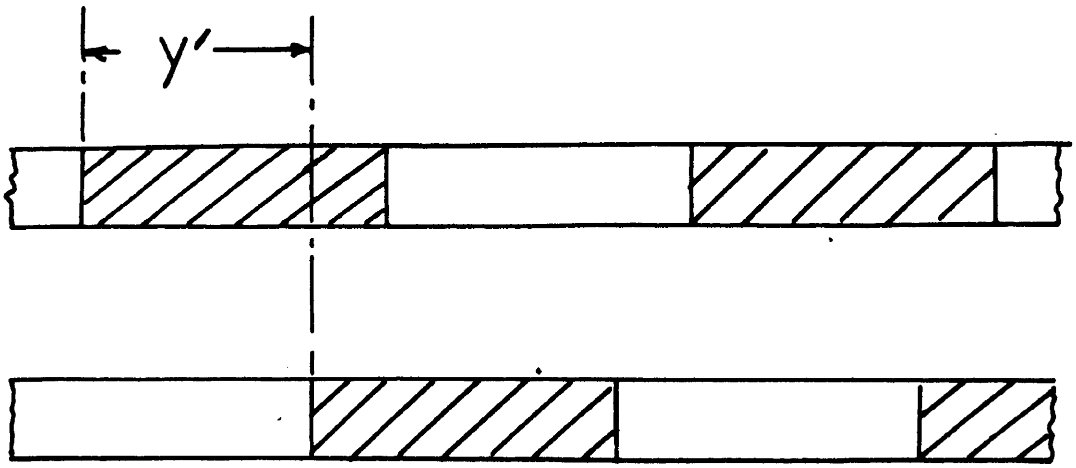


Figure 10. Segmental current sheets produced by chording.

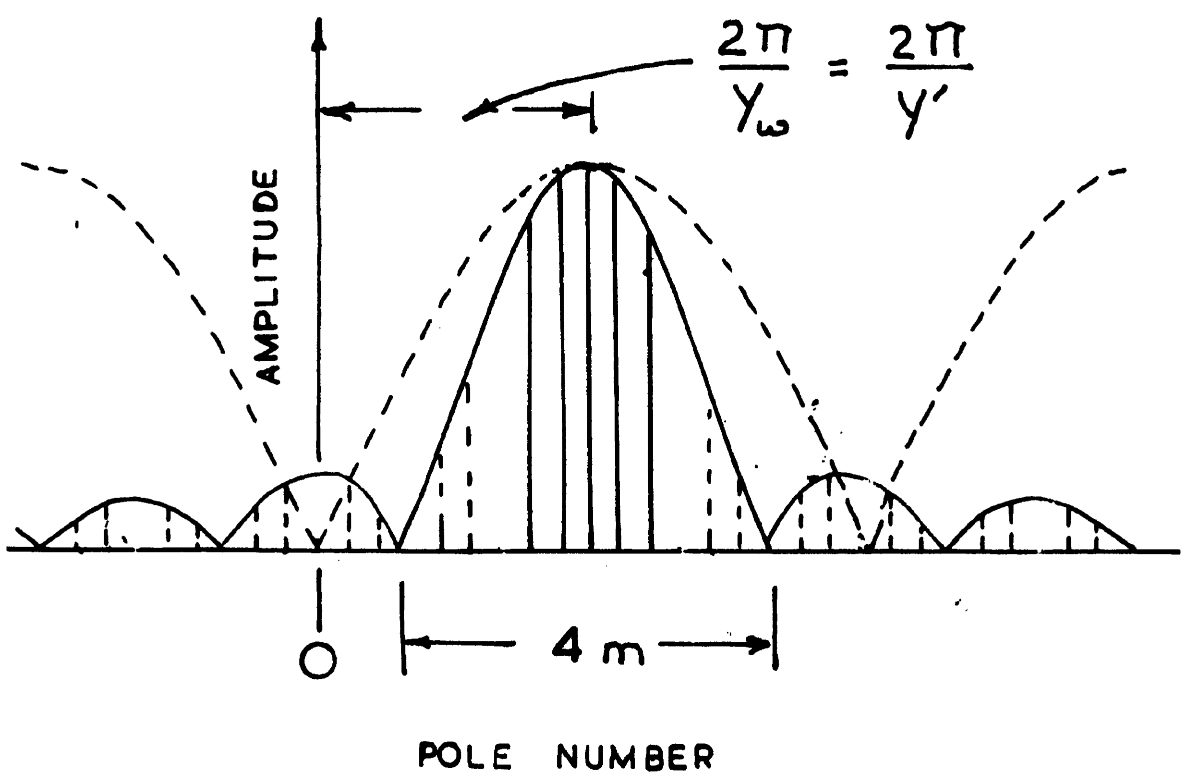


Figure 11. Illustrating the effect of chording on the segment envelope.

ordinates. Comparison of these curves shows that $\frac{2\pi}{y'}$ must be near $\frac{2\pi}{y_\omega}$ if the utilization factor is not to be adversely affected and that the chording factor C is unlikely to be useful as a method of eliminating the large 'close in' harmonics U_1 and U_1' though it may well be of service in reducing more remote harmonics. The exception occurs when

$$\frac{2\pi}{y_\omega} \approx \frac{2\pi}{y'} \approx 2m$$

i.e. each segment contains about two poles of the basic excitation. This situation may arise but it is not of general utility, and some other means of harmonic reduction must be sought. Chording will always be incorporated but since its effect is represented by C, attention will be concentrated on the unchorded 'front edges' of the winding.

Considering the first segments only, that is the current sheets corresponding to the leading edges of the coils, it is possible for harmonic ordinates of finite height to fall on the 'zero poles' point on the abscissa. 'Zero poles' corresponds with infinite pole pitch, that is with net current flowing from one end of the stator bore to the other. When the blocks identified with the pitched returns are included this condition is automatically excluded, and it can be said that any coil size 'pitches out' zero poles.

4. 'TWO LAYER' SEGMENTS

Figure 12 shows the developed diagram of part of a two layer machine. Each layer consists of a set of m identical segments with $\alpha = 1$, but one set is displaced $\phi_m = 2\frac{\beta}{m}\pi$ radians mechanical relative to the other and is energised with an electrical phase difference ϕ_e. This produces block shapes of the general form of Figure 27 of Chapter 1.

If each simple segment produces a harmonic of amplitude U_h at pole number $2p_h$ the combination of this segment with a displaced segment will produce a resultant at $2p_h$ poles given by

$$2U_h \left| \operatorname{Cos} \frac{1}{2} \left[(2p_h) \frac{1}{2} \frac{2\pi\beta}{m} + \phi_e \right] \right|$$

$$= 2U_h C_p$$

$$\text{where } C_p = \left| \operatorname{Cos} \frac{1}{2} \left[(2p_h) \frac{\pi\beta}{m} + \phi_e \right] \right|$$

The resulting harmonic amplitudes are then found by multiplying the relevant ordinates of the harmonic envelope by the relevant ordinates of the envelope of C_p; the 2 may be omitted since its sole significance is that now there are two layers, each of strength J_s.

The envelope of C_p is found by treating $2p_h$ as a running variable, and is the modulus of a cosine curve exhibiting a positive maximum at $\frac{2\pi}{y_t}$ poles where

$$\frac{2\pi}{y_t} \cdot \frac{\pi\beta}{m} = -\phi_e$$

and C_p can be re-expressed as

$$C_p = \left| \operatorname{Cos} \frac{1}{2} \left(2\delta p\right) \frac{\pi\beta}{m} \right| \qquad \ldots\ldots\ldots\ldots\ldots \qquad (6)$$

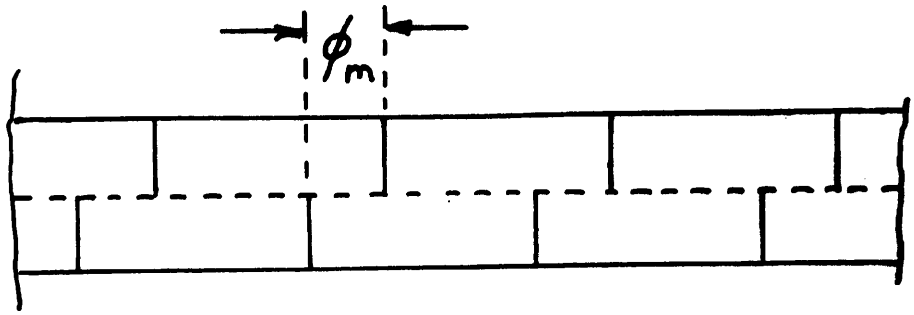


Figure 12. Two layer segmental current sheets.

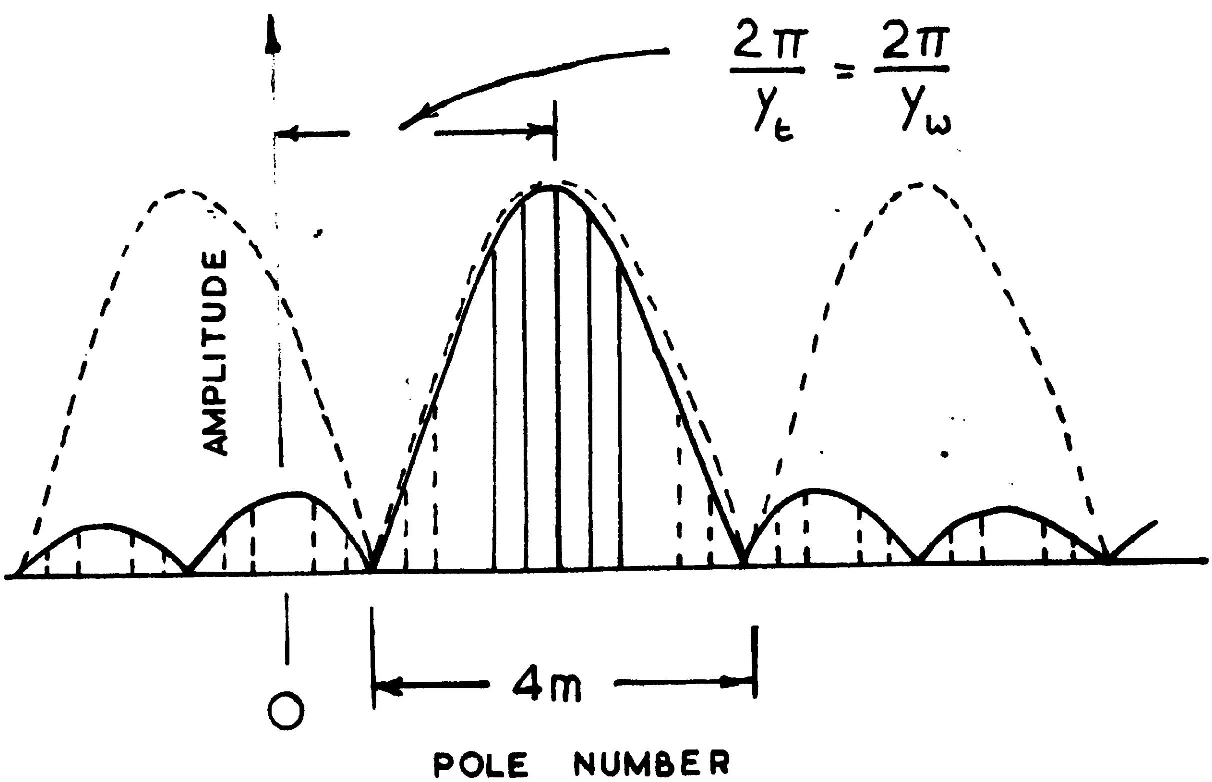


Figure 13. Illustrating the effect of the second layer on the segment envelope.

where $2\delta p$ is the departure of the pole number considered from $\frac{2\pi}{y_t}$.

Since minimum diminution of harmonics at pole numbers near $\frac{2\pi}{y_\omega}$ will be required it is often convenient to centre C_p on $\frac{2\pi}{y_\omega}$ poles, by making

$$\frac{2\pi}{y_t} = \frac{2\pi}{y_\omega} = -\phi_e \frac{m}{\pi\beta}$$

that is,

$$-\phi_e = \frac{2\pi\beta}{m} \cdot \frac{\pi}{y_\omega}$$

This means that the overlapping portions of the composite segment are in phase.

The first pair of zeros symmetrically dispersed about $\frac{2\pi}{y_t}$ occur when

$$\frac{1}{2}(2\delta p)\frac{\pi\beta}{m} = \pm\frac{\pi}{2}$$

i.e.
$$(2\delta p) = \pm\frac{m}{\beta}$$

and the "bandwidth" can be made equal to that of the segment harmonic envelope by making

$$\frac{2m}{\beta} = 4m$$

i.e.
$$\beta = 0.5$$

Figure 13 shows a typical segment envelope together with the cosine curve corresponding to $\beta = 0.5$, whilst the new composite segment envelope for these conditions is shown by Figure 14. With these arrangements the values of utilization factor and harmonic amplitudes relevant to $\pm 60^\circ$, $\pm 90^\circ$ and $\pm 120^\circ$ switching become:-

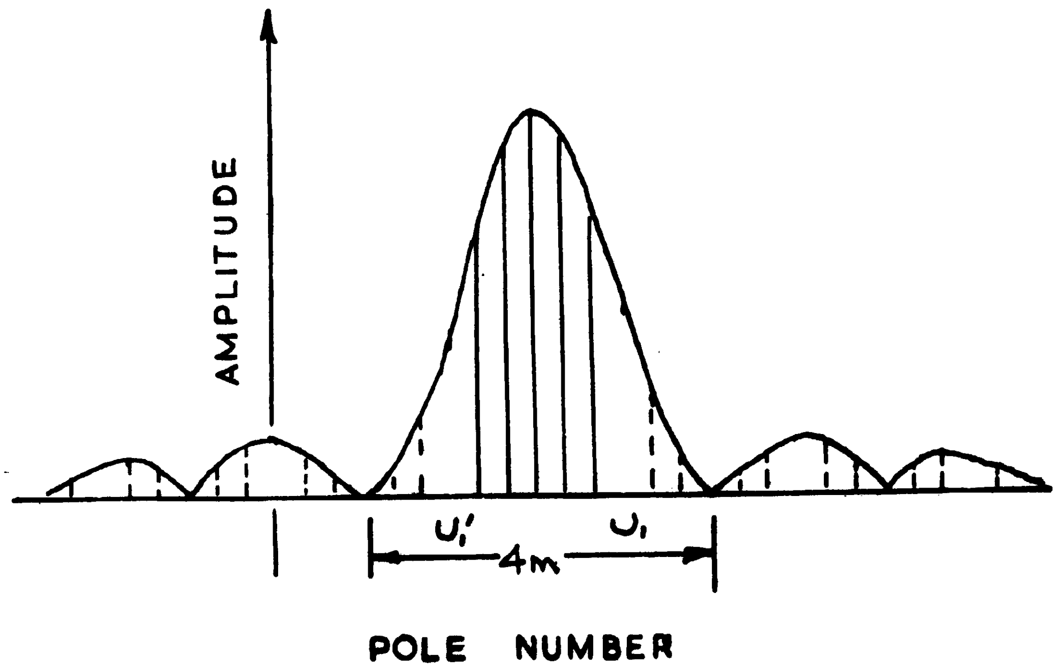


Figure 14. Two layer segment envelope.

TABLE 1.

Electrical Phase-Shift.	Amplitude of Harmonic Ordinates						
$+60^{\circ}$	.015	.084	.049	.92	.035	.071	.013
-60°	.013	.071	.035	.92	.049	.084	.015
90°	.031	.119	.115	.832	.069	.092	.027
-90°	.027	.092	.069	.832	.115	.119	.031
120°	.052	.143	.207	.716	.103	.102	.041
-120°	.041	.102	.103	.716	.207	.143	.052

Comparison of Figure 13 and equation (6) of this section with Figure 11 and equation (5) of section 3 dealing with chording shows a close parallelism; in each case the basic layer of length $\frac{2\pi}{m}$ is accompanied by a second displaced layer, and in each case the effect is represented by a multiplicative cosine wave. The difference lies in the fact that in the new arrangement the added segment cannot be joined to the first to form coils. Because of the similarities and because of the difference, the process of introducing a displaced additional segment is locally referred to as Phantom chording.

It is apparent from the table that the technique permits the design of multi-speed motors using $\pm 60^{\circ}$ switching that have acceptably low harmonic content.

The harmonic content in the $\pm 90^{\circ}$ switched case is less satisfactory, but the use of a composite block coupled with ordinary chording techniques produces good designs in most cases. It may well be worth while in some instances to increase β slightly from 0.5 to reduce $\frac{U_1}{U}$ at the expense of increasing $\frac{U_1'}{U}$ or to reduce $\frac{U_2'}{U}$ at the expense of increasing $\frac{U_1}{U}$.

Satisfactory designs using two layer blocks switched $\pm 120^{\circ}$ are difficult to arrange. In particular cases normal chording may prove sufficiently helpful, in others it may be necessary to obtain a further set of zeros by introducing a second stage of phantom chording. This is done by repeating the composite segment with a displacement $V\frac{2\pi}{m}$. Since the design adopted will depend considerably on the particular pole numbers required, and the duty to be performed, i.e. fan drive, constant torque, constant power, etc. it is of little value to pursue the matter here in general terms. The techniques available for obtaining the desired results are:

Choice of y_{ω}

Choice of y'

Choice of β

and if necessary

Choice of V.

Some examples are given at the end of the Chapter.

5. THE EFFECT OF USING PRACTICAL WINDINGS.

The basic segment of any machine will in general consist of a short section of a conventional winding, such as for example a two slot/pole/phase winding. When such a winding is used for a conventional machine it can be analysed into fundamental and harmonic pole numbers. The original winding can then be synthesized exactly from fundamental and harmonic current sheets of appropriate strength and phase. A suitable section taken out of this synthesized machine with the harmonic current sheets summed over the segment length would exactly reproduce the current sheet due to the segment winding, and it follows that the segment winding can itself be treated as if made up of a fundamental wave of pole pitch y_ω together with harmonics of appropriate strength of pole pitch $\frac{y_\omega}{N}$ where N = -5 +7 -11 +13 etc., the negative sign indicating reversed rotation.

5.1. <u>The Effect of mechanical shift on winding harmonics.</u>

In the datum condition all the segments are identical, and the N^{th} harmonic pole number is no different from the fundamental considered so far except that its pole pitch is $\frac{y_\omega}{N}$ and it may be travelling backwards. It will also produce a harmonic envelope centred on $\frac{2N\pi}{y_\omega}$ poles, as Figure 15 shows. However, when a mechanical shift of $x\ell = \frac{2r\ell}{m}$ poles measured on the fundamental scale is applied to the segments the shift becomes $\ell x = \frac{2Nr\ell}{m}$ measured on the N^{th} harmonic scale, and the machine, in so far as this harmonic pole number is concerned, will become a 2Nr pole machine. This is illustrated in Figure 15 for positive and negative values of N.

In this figure only the defining ordinates at 2r and 2Nr are shown, but of course each is accompanied by its attendant string spaced at intervals of 2m poles. The 2Nr ordinate is shown inside the related segment envelope, but it need not be, and in general will not be for the higher values of N. That the forward going harmonic ordinate should appear at 2Nr when the fundamental appears at 2r is easily understood, but it might be thought that backward going harmonics might be treated differently since

Figure15. Segment envelopes due to winding harmonics.

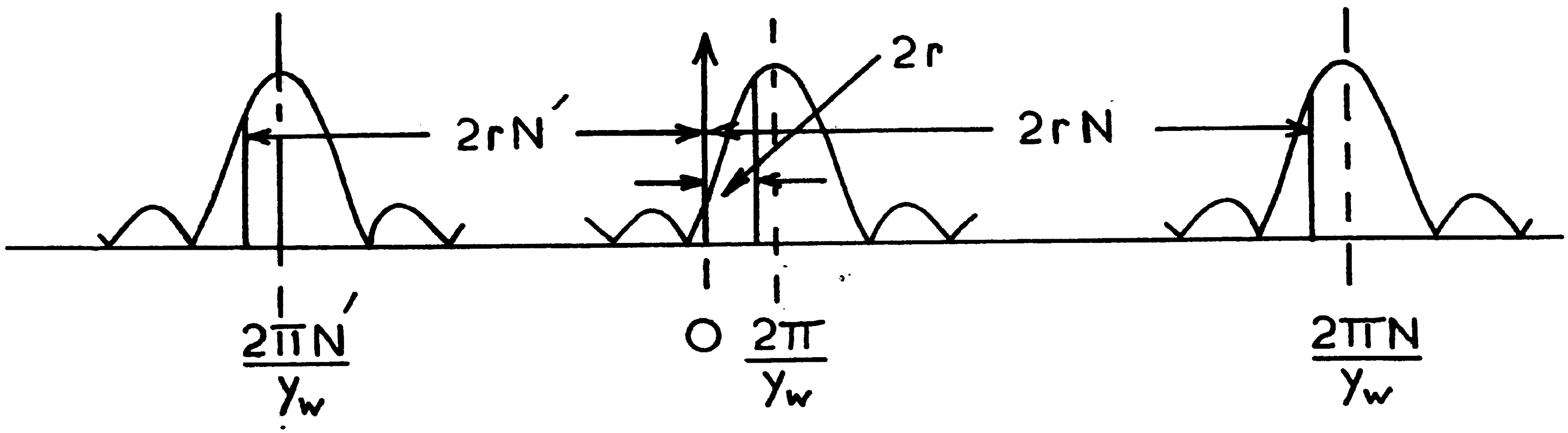
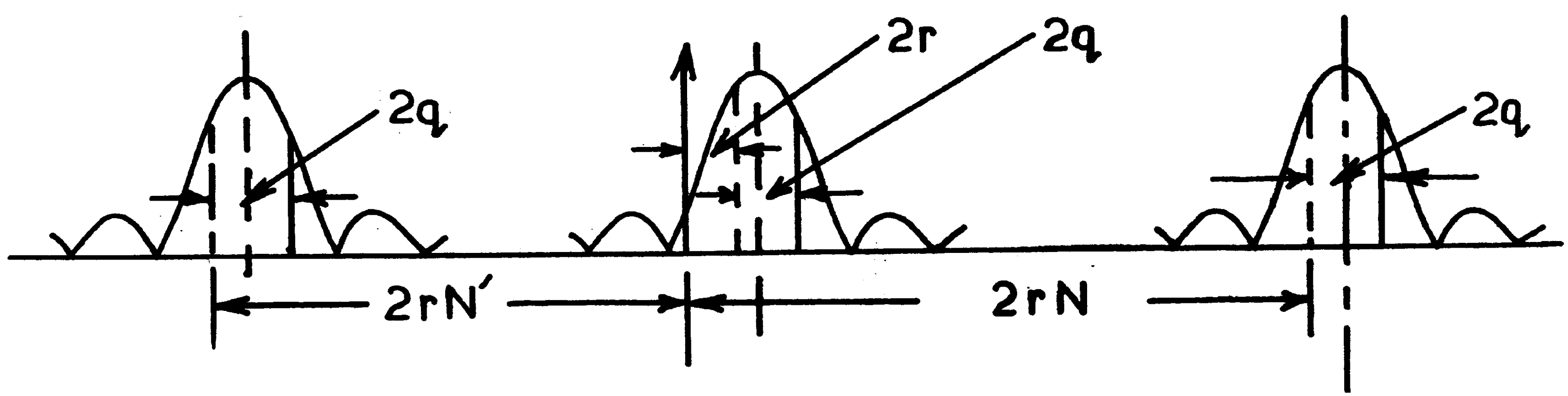


Figure 16. Electrical shift applied to the ordinates of Figure 15.



their direction of motion is different relative to the direction of the mechanical displacement x. Appendix (2) shows that this is not so, 'mechanical phasing' by an amount x where the sense of x is such as to move the waves in adjacent segments <u>towards</u> each other produces an increase in the modulus of pole number irrespective of whether the wave is moving forwards, backwards, or is merely pulsating.

These additional 'winding harmonic' excitations of the segments introduced by practical considerations will contribute additional harmonic pole numbers to the machine. With many designs the harmonic pole numbers deriving from the segment 'winding harmonic' pole numbers will be no more obtrusive than are winding harmonic pole numbers in conventional machines. In terms of Figure 14, 2Nr itself will be too far away from the working range near 2r poles to be troublesome, and its attendant string in the region of 2r poles will be so far from 2Nr as to be negligible. Certain harmonics may, however, contribute either positively or negatively to the utilization factor at 2r. For this to happen the requirement is that

$$2Nr - 2r = 2m\,n \qquad \text{where n is an integer}$$

i.e.
$$\frac{\left(\frac{N-1}{2}\right)2r}{m} = n$$

With due attention paid to the sign of N and with $N = -5\ +7\ -11\ +13$ etc., $\frac{N-1}{2}$ is always a multiple of 3, and it follows that if m is a multiple of 3 or will divide into 2r, the utilization at 2r will be modified either upwards or downwards by contributions from the winding harmonics. Any of the conventional methods of reducing 'winding harmonics' will reduce these effects.

It is necessary now to consider how subsequent electrical phase shift will influence these findings.

5.2. Effect of Electrical Phase Shift on Winding Harmonics.

Appendix (2) shows that electrical phase shift of amount $\phi = \frac{2q}{m}$, shifts the defining pole number of the harmonic strings 2q poles to the right irrespective of the winding harmonic number or of the sense of rotation of that harmonic number. Thus after such phase shifting Figure 15 becomes Figure 16. The effect on harmonic excitations is therefore different according as N is negative or positive and the 'symmetry' about zero poles which was present with mechanical phase shift has been lost.

However, the defining ordinates at $2Nr + 2q$ with N having values $1 \ -5 \ 7 \ -11 \ +13$ etc. each have attendant harmonic strings spaced every 2m away from $2Nr + 2q$ and it may be that one of these will fall on $N(2r + 2q)$ and so reproduce the condition that would have arisen had the shift of 2q been achieved mechanically instead of electrically. For this to be so,

$$N(2r + 2q) - (2Nr + 2q) = 2m\,n$$

$$(N - 1)\,2q = 2m\,n$$

or

$$\frac{2q}{m} = \frac{2n}{N-1}$$

i.e.

$$\phi = \frac{2q\pi}{m} = \frac{2n}{N-1}\,\pi$$

Since $N - 1$ is always a multiple of 6 this equation is satisfied for all values of N provided.

$$\phi = \frac{\pi}{3}, \ \frac{2\pi}{3} \ \text{or} \ \pi$$

Subject to this limitation, therefore, the findings of the previous section apply equally to electrical switching. Since these values of ϕ are the only ones readily available the limitation is not serious.

6. TERMINAL VOLTAGE

In practical designs the current sheets so far discussed are replaced by individual conductors each carrying currents obtained in general from an 'n' phase supply, but most likely from a three phase supply. In designing windings it is convenient to assume that the currents form a pure three phase set free from zero and reverse phase sequence components. The supply, however, will be a three phase voltage supply and even if this is assumed free from zero and reverse phase sequence components the currents drawn from it will not be balanced unless the terminal voltage of the machine, assumed fed with pure three phase current, exhibits a balanced terminal voltage. It is therefore important to evaluate the terminal voltages corresponding with the various proposed configurations of current sheets.

6.1. Reverse Phase Sequence Voltages.

It is shown in Appendix (4) that for any configuration of conductors the ratio of the reverse phase sequence voltage to the forward phase sequence voltage at any particular pole number is the same as the ratio of the reverse rotating current harmonic to the forward rotating current harmonic. If the dominant forward rotating pole number is 2r then the distance separating this from the equal pole number reverse rotating harmonic is 4r. Associated with 2r there is a harmonic string with ordinates every 2mn, the condition that one of these shall lie on -2r is

$$4r = 2m\,n$$

or
$$\frac{2r}{m} = n$$

That is, the number of poles must be an integral multiple of the number of segments. This will be inevitable for m = 2, and increasingly less probable as m gets larger. The magnitude of the reverse phase sequence voltage will be set by the ordinate at -2r of the segment harmonic envelope centred on $\frac{2\pi}{y_\omega}$. Clearly $\frac{2\pi}{y_\omega}$ could be chosen to set a zero on -2r, but in general the machine will be required to be switched from 2r to some other pole number, 2r + 2q, and $\frac{2\pi}{y_\omega}$ will commonly be placed midway between. For minimum ordinates on both reverse pole numbers it is best to set a zero of the harmonic envelope on $\frac{-2\pi}{y_\omega}$.

The harmonic envelope has zeros at $\frac{2\pi}{y_\omega} \pm 2mn$ due to the $\left|\frac{\sin\gamma}{\gamma}\right|$ component and at $\frac{2\pi}{y_\omega} \pm \frac{m}{\beta}(2n+1)$ due to the "phantom cord" component. Since the difficult cases are those when m is a multiple of 2, the zeros on the $\left|\frac{\sin\gamma}{\gamma}\right|$ curve are useful only when $\frac{2\pi}{y_\omega}$ is a multiple of m. The zeros of C_p are more flexible, the required condition, given in section 4 being

$$\frac{2\pi}{y_\omega} = \frac{m}{\beta}(2n+1)$$

i.e.
$$\frac{\beta\pi}{m} = \phi_m = y_\omega \cdot \frac{2n+1}{2}$$

This requires that the second layer in Figure 12 should start an odd number of half pole pitches of the segment winding away from the start. This can frequently be arranged without departing far from the recommendation of section 4 that calls for $\beta = .5$.

Appendix (5) shows that this condition is also helpful in minimising negative phase sequence components due to some 'winding' harmonics. This appendix also discusses the effect of dividing the winding into a number of parallel paths, the analysis above assumes that all the conductors of a particular phase are connected in series.

6.2. <u>Zero Phase Sequence Components.</u>

Appendix (4) shows that the zero phase sequence component of voltage can be evaluated by assuming that the 3 phase windings forming the segments are all energised with single phase current of standard amplitude and then estimating the resulting pulsating harmonic at 2r poles.

Starting with the machine in its datum condition, each segment consists of a part of a travelling wave of fundamental pole number $\frac{2\pi}{y_\omega}$. This travelling wave is formed by three phase currents suitably arranged in phase bands. In general, when such a winding is excited by connecting all its phases to the same single phase supply the predominant result is a pulsating excitation of $\frac{6\pi}{y_\omega}$ poles of amplitude (relative to 3 phase excitation amplitude) depending on the length of the phase bands. This pulsating excitation can be split into a $\frac{6\pi}{y_\omega}$ pole wave travelling forwards and a $\frac{\overline{6\pi}}{y_\omega}$ pole wave travelling backwards, each of half amplitude.

Each of these waves can be treated independently as a travelling wave. Thus when '2r' poles is set up by "mechanical shift" the defining ordinate of the forward travelling wave will appear at + 6r poles, as Figure 17 shows.

Figure 17. Harmonic envelopes produced by single phase excitation of all phases.

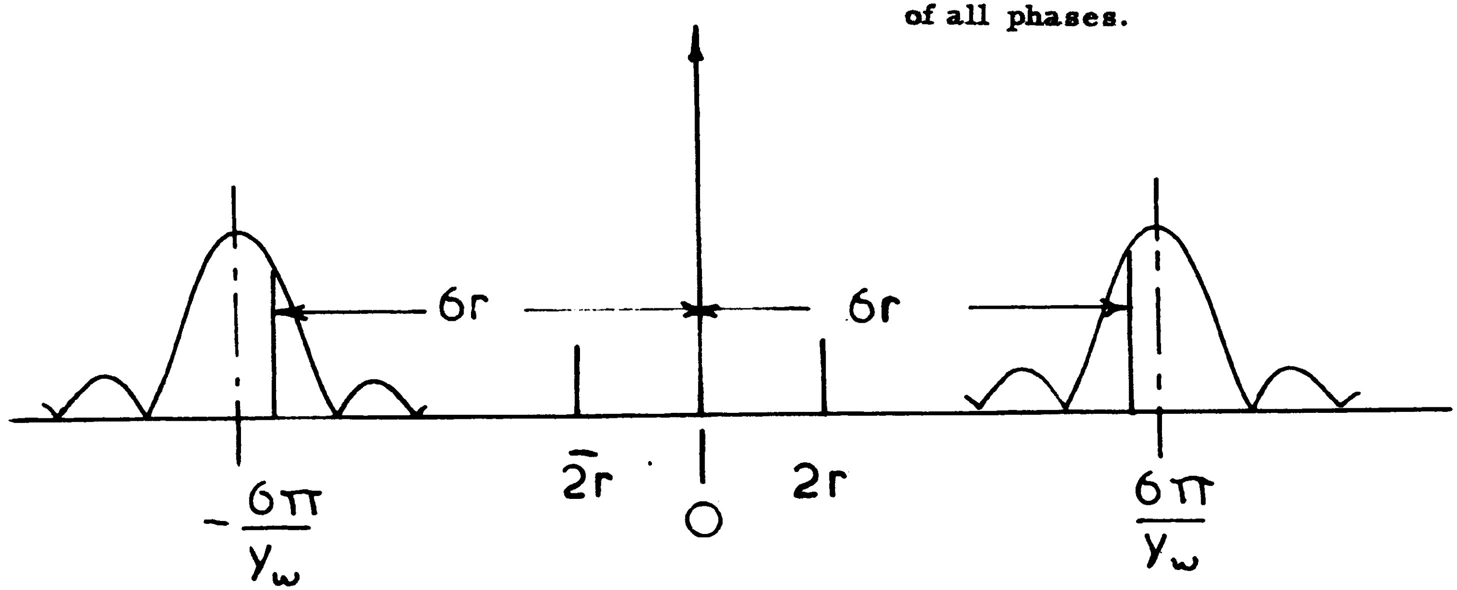
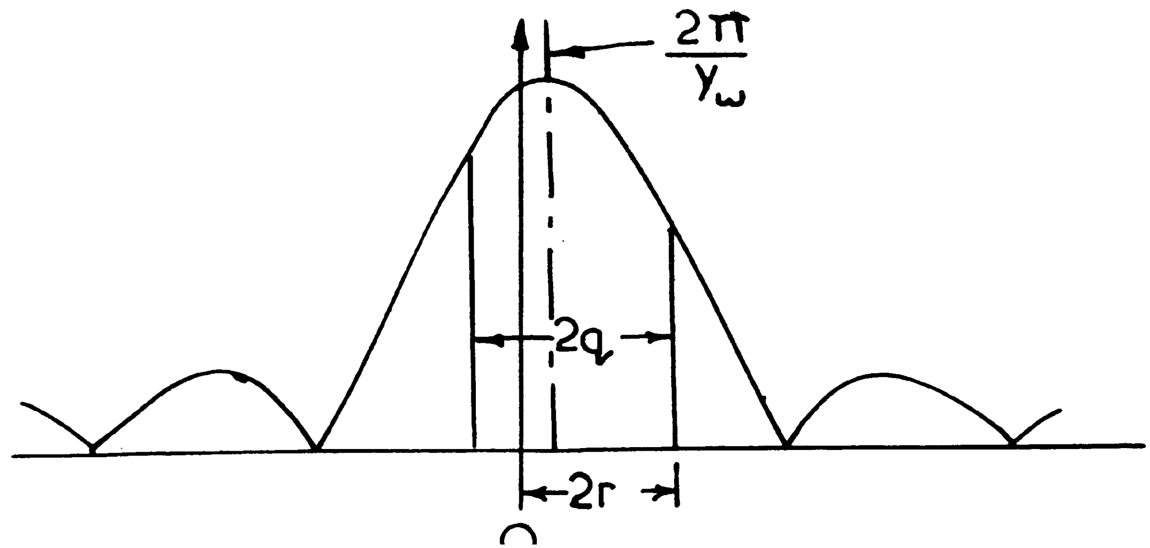


Figure 18. Ordinates produced by a machine in which 2q > 2r.



The amplitudes of the non zero ordinates will be defined by the segment harmonic envelope based on $\frac{6\pi}{y_\omega}$ poles. Similar treatment will be accorded to the backward travelling component by 'mechanical' shift.

The 6r excitation produces a harmonic at 2r if

$$6r - 2r = 2m\,n, \text{ i.e. } \frac{2r}{m} = n$$

and at $\overline{2}r$ if

$$6r + 2r = 2m\,n, \text{ i.e. } \frac{2r}{m} = \frac{1}{2}n$$

The $\overline{6}r$ excitation produces similar harmonics at $\overline{2}r$ and 2r.

The first condition is the same as that for the existence of negative phase sequence voltages, the second is more difficult to avoid.

Examination of Figure 17 suggests that the most important ordinate to minimise is the ordinate due to the $\frac{6\pi}{y_\omega}$ excitation which falls on 2r. This ordinate is likely to be the largest as it is the nearest to the maximum value of its associated envelope.

Once again it is uxeful to set a zero of the phantom chording factor C_p which will now have a maximum value at $\frac{6\pi}{y_\omega}$ on $\frac{2\pi}{y_\omega}$ by making

$$\frac{6\pi}{y_\omega} - \frac{2\pi}{y_\omega} = \frac{m}{\beta}\frac{(2n+1)}{2} \qquad \ldots\ldots\ldots\ldots \qquad (10)$$

This yields the same values of β which were shown to be appropriate for negative phase sequence reduction in section 6.1.

A zero of C_p will fall on $\frac{\overline{2}\pi}{y_\omega}$ if

$$\frac{6\pi}{y_\omega} + \frac{2\pi}{y_\omega} = \frac{m}{\beta}\frac{(2n+1)}{2} \qquad \ldots\ldots\ldots\ldots \qquad (11)$$

a condition which can never be satisfied simultaneously with that of equation (10)

Equations (7), (8) and (9) show that a zero phase sequence component due to the $-\frac{6\pi}{y_\omega}$ excitation may occur in the absence of either zero phase sequence due to the $+\frac{6\pi}{y_\omega}$ excitation or negative phase sequence components. In such cases it may be useful to satisfy equation (11) by making

$$\frac{\beta\pi}{m} = \phi_m = \frac{(2n+1)}{4} y_\omega$$

i.e. $$\phi_m = \frac{y_\omega}{4}, \frac{5y_\omega}{4}, \frac{7y_\omega}{4} \text{ etc.}$$

and choosing that value which is nearest to $\beta = .5$, in the interests of harmonic reduction.

These findings relate to 'mechanically' set up pole numbers, but section 5.2 showed that similar rules relate to electrical phase shift provided

$$\phi = \frac{2n}{N-1}\pi$$

the relevant value of N being ± 3.

Since the only possible value of ϕ with single phase excitation is π, this condition is always met. Because the very existence of a zero phase sequence component depends on the existence of a $\frac{6\pi}{y_\omega}$ excitation of the basic winding, when single phase connected, chording of that winding against 3rd harmonic, i.e. $\frac{2}{3}$ chording, will eliminate the effect entirely.

Appendix 6 extends the above analysis to cover winding harmonics 3, 9, 15 etc. times $\frac{2\pi}{y_\omega}$.

6.3. Magnitude of the Supply Voltage.

The terminal voltage, V, of a 2r pole machine when running close to its synchronous speed is given by

$$V = K\frac{B\,K_\omega}{2r}$$

where K is a constant, B is the air gap density and K_ω the winding factor appropriate to the pole number considered. K_ω will be maintained as large as possible and hence in general if the terminal voltage remains constant the gap flux will be high on high pole numbers and low on low pole numbers . If the speed ratios are close, this may not entail severe loss of output, but if the ratio is wide the loss will be intolerable. It follows that some adjustment of terminal voltage between speeds will in general be necessary, for instance by switching from parallel star to series star or from parallel star to delta. These arrangements are best considered in relation to particular cases since the required action depends vitally on the speed ratio.

It is unlikely that exact flux equality on the different speeds will be obtained by these switching methods, and in general one speed will yield a high flux value. It may sometimes be possible to compensate for this by designing for a high value of K_ω at this speed so as to reduce B, but this will inevitably be at the expense of the winding factors at other speeds, and the desirability or otherwise of such action will depend on the specification to be met.

7. POLE CHANGE COMBINED WITH REVERSAL.

So far it has been assumed that all the pole numbers set up by switching have been positive. Figure 18 shows a machine which is mechanically set up to 2r poles and electrically switched to $2r - 2q$ poles with $2q > 2r$.

There is of course no reason why the machine should then in fact run backwards, since reversal of the applied phase sequence can be incorporated with the pole change switching which produced it.

The analysis of the preceding section applies unchanged to these cases, but with differences in emphasis. For example, if the two pole numbers are numerically close together the appropriate value of $\frac{2\pi}{y_\omega}$ will lie near to 0, as Figure 18 illustrates. Since 2q is now the sum of the moduli of the pole numbers instead of the difference, m will be large and the 'band width' of the $\left|\frac{\sin \chi}{\chi}\right|$ curve correspondingly greater. The harmonics U_s and U_s', though they will retain the same amplitudes relative to U, will occur at pole numbers more remote from 2r and 2q - 2r. This is advantageous since they are then more open to attack by conventional as distinct from 'phantom' chording. The favourable cases, from the reverse and zero phase sequence point of view, where m is a multiple of 3, represent different speed ratios here than when $2q < 2r$, and so, particularly in 2 speed machines, a choice between methods may well depend on this consideration alone.

So far, most of the comments have suggested advantages; there are, however, also disadvantages.

Since $\frac{2\pi}{y_\omega}$ is small, the winding harmonics of $\frac{2\pi}{y_\omega}$ may well lie near to 2r or 2q - 2r, and can introduce significant harmonic components which are no longer 'well down the skirts' of their envelope before they approach the working range.

The condition for negative phase sequence voltages (that the pole number be divisible by m) can never occur, since m is greater than 2r; on the other hand, winding harmonics can contribute extensively to the negative phase sequence component since they are multiples of $\frac{2\pi}{y_\omega}$, which is small, and have very wide response envelopes. They can no longer be conveniently eliminated by choice of β because the number of segments is large and the length of each is short, probably much less than y_ω. In any particular case, however, the winding harmonics supplying the negative phase sequence voltage can be identified by using the methods of Appendix 5, and these harmonics can then be chorded out of the basic winding, inevitably however at some loss of winding factor.

Similar remarks apply to zero phase sequence voltages, and once again the procedure is to attack the basic winding harmonic responsible.

Because in the main it is harmonics of the basic winding that cause trouble, the design problem is more complex because there are more factors to take into account and the matter will not be pursued here.

8. WINDING EXAMPLES

4/6/8 Change Pole Machine; $+60^\circ$ 0° -60° Switching.

Although there are special features of this machine which permit a simplified approach, the general design procedure will be followed. The initial pole number to be set up by 'mechanical' shift is $2r = 6$; the 'side' pole numbers are obtained by 60° switching. The number of segments will therefore be $m = 6q = 6\,\frac{(6-4)}{2} = 6$. If there is no reason to favour either of the side pole numbers at the expense of the other, the proper choice of $\frac{2\pi}{y_\omega}$ is 6, i.e. $y_\omega = \frac{2\pi}{6}$. Since $m = 6$ the length of a basic segment is $\frac{2\pi}{m} = \frac{2\pi}{6}$ so that each segment contains exactly one pole of the basic wave.

It is now necessary to choose a suitable number of slots per segment. This must be such as to permit easy winding of 1 pole, and will preferably be 3n where n is the number of slots/pole/phase; six slots per segment is a suitable choice. The 'datum' condition can now be set up, each segment being as shown in line 2 of Table 2.

The next step is to convert this machine into a 6 pole machine by mechanical phasing. This is done by shifting successive segments, by "2θ" where $\theta = \frac{6\pi}{m} = \pi$. The generalised method of doing this is first to draw up a list showing the contents of successive slots of a winding of pitch y_{ω} (now counted in slots); this is shown in Table 2. The angle π is now represented by 6 slots. Had the 'mechanical shift' not been a whole number of slots, the slots/pole/phase would have required adjustment.

The 6 pole machine is now set up by copying segment 0, replacing segment 1 by what appears in the list starting from slot $0 + (1 \times 6) = 6$, replacing segment 2 by what appears in the list starting from slot $0 + (2 \times 6) = 12$ and so on. This is shown below the datum state in Table 2. It may be seen that in this particular case a conventional 6 pole machine has resulted. Clearly there will be no difficulties with harmonics or unbalanced voltage in this condition. The two electrically switched states of the machine can now be set down by leaving segment 0 unchanged, moving round 1 on the $R\bar{B}Y\bar{R}B\bar{Y}$ clock for segment 2, moving round 2 for segment 3 and so on, as lines 4 and 5 of Table 2 illustrate.

In the electrically switched condition harmonics will enter, but the negative and reverse phase sequence voltage components will remain zero since neither 4 poles nor 8 poles is divisible by m ($=6$) or $\frac{m}{2} = 3$, in accordance with section 6 and Appendices 5 and 6.

TABLE 2.

4/6/8 Change Pole Machine

Switching:	-60° 0° 60°
Bias:	Symmetrical
Basic winding pitch:	$y_{\omega} = \frac{2\pi}{6}$
Segment No:	$m = 6q = 6 \times \left(\frac{4-2}{2}\right) = 6$
Segment Arc:	$\frac{2\pi}{m} = \frac{2\pi}{6}$
Poles per segment:	$\frac{2\pi}{m} \times \frac{1}{y_{\omega}} = 1$
Slots per segment:	6 (say)
Total slot number:	$6 \times m = 36$

List for obtaining mechanical shift.

24 25					
12 13	14 15	16 17	18 19	20 21	22 23
0 1	2 3	4 5	6 7	8 9	10 11
R	$\overline{B}'$	Y	$\overline{R}$	B	$\overline{Y}$

Segment No.	0	1	2	3	4	5	(1)
	0	6	12	18	24	30	
Datum Condition	$RR\overline{BB}YY$	$RR\overline{BB}YY$	$RR\overline{BB}YY$	$RR\overline{BB}YY$	$RR\overline{BB}YY$	$RR\overline{BB}YY$	(2) ?
Mech. Shift.	$RR\overline{BB}YY$	$\overline{RR}BB\overline{YY}$	$RR\overline{BB}YY$	$\overline{RR}BB\overline{YY}$	$RR\overline{BB}YY$	$RR\overline{BB}YY$	(3)
-60° Elect. Phase Shift.	$RR\overline{BB}YY$	$YY\overline{RR}BB$	$BB\overline{YY}RR$	$RR\overline{BB}YY$	$YY\overline{RR}BB$	$BB\overline{YY}RR$	(4)
$+60^{\circ}$ Elect. Phase Shift.	$RR\overline{BB}YY$	$BB\overline{YY}RR$	$YY\overline{RR}BB$	$RR\overline{BB}YY$	$BB\overline{YY}RR$	$YY\overline{RR}BB$	(5)

Reduction of harmonics can be achieved by phantom chording using $\beta = .5$.

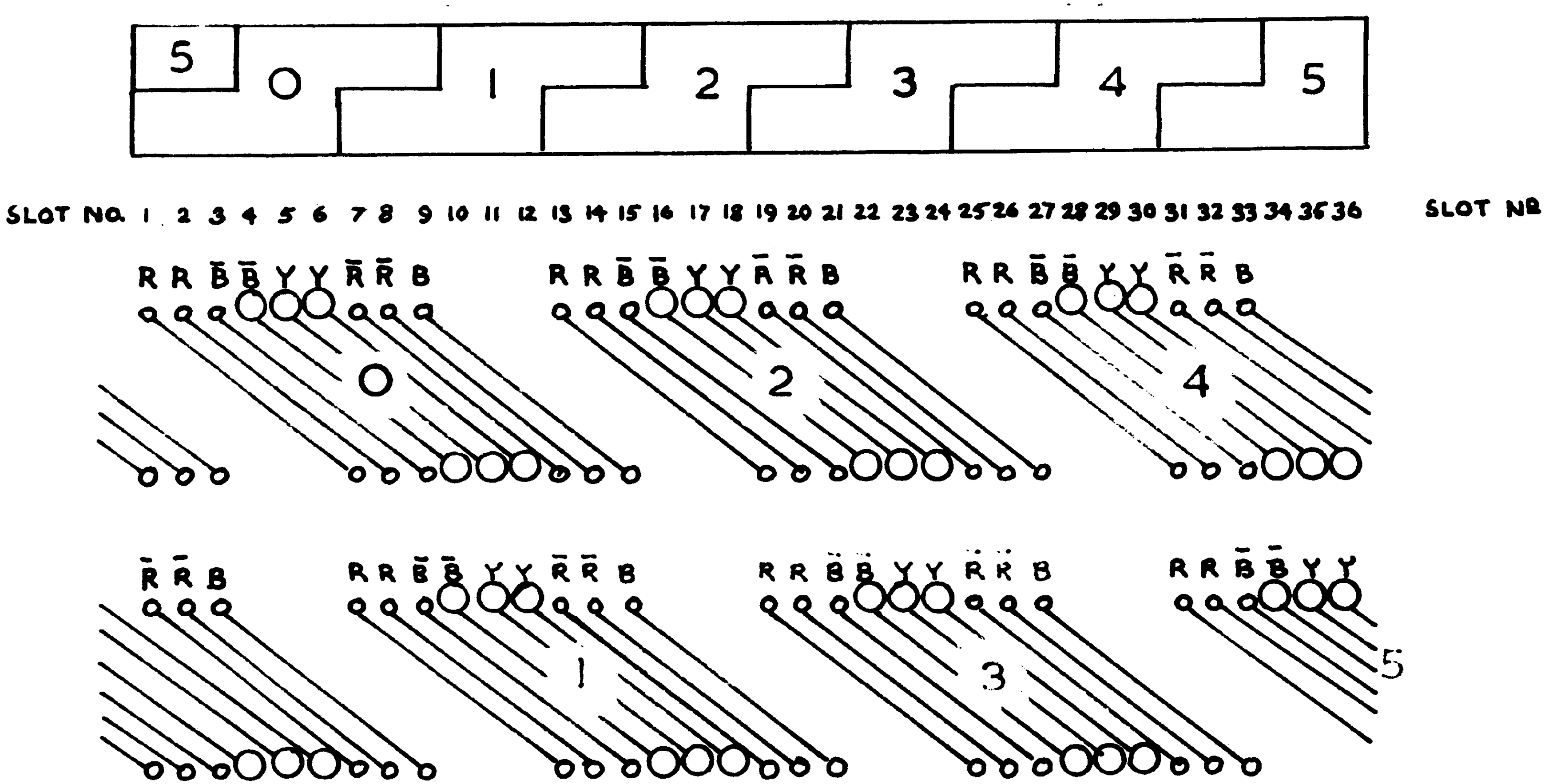
It remains now only to choose a suitable coil pitch for the winding. For symmetrical treatment of 4 and 8 poles, the appropriate value is 6 slots. The resulting winding is shown in Figure 19. The utilization factors and harmonic amplitudes for the 4 and 8 poles conditions derived from the true current sheet analysis are listed in Table 3, and the results of direct slot by slot computation by digital computer of the actual windings of Table 2 used to replace these current sheets are shown alongside for comparison.

TABLE 3.

	4 poles			8 poles	
Pole No.	Computer Analysis	Current Sheet Analysis.	Pole No.	Computer Analysis	Current Sheet Analysis.
-20	.0753	.0615	-16	.0826	.0615
-8	.0374	.0306	-4	.0472	.0428
4	.772	.796	8	.772	.796
16	.0472	.0428	20	.0374	.0306

The analysis of windings using computer techniques is discussed in Appendix 4. The differences between the two sets of results are a measure of the contribution made by the harmonic content of the basic winding. Suitable switching arrangements are described in Section 3 of the first Chapter. The computer results indicated a complete absence of zero and negative phase sequence components, thus confirming the work of Section 6 and Appendices 5 and 6.

Figure 19. Winding arrangements for a 4/6/8 pole machine.



A 4/6 Pole Change Machine.

TABLE 4

Pole numbers	6 4
Switching	0° 6 poles:star
	180° 4 poles:star two parallels.
Service:	Fan drive.
Bias:	prefer 4 poles.
Segment number:	$m = 2q = 2$
Symmetrical pole number:	$\frac{2\pi}{y_{\omega}} = 5$
Poles/segment for this number:	2.5

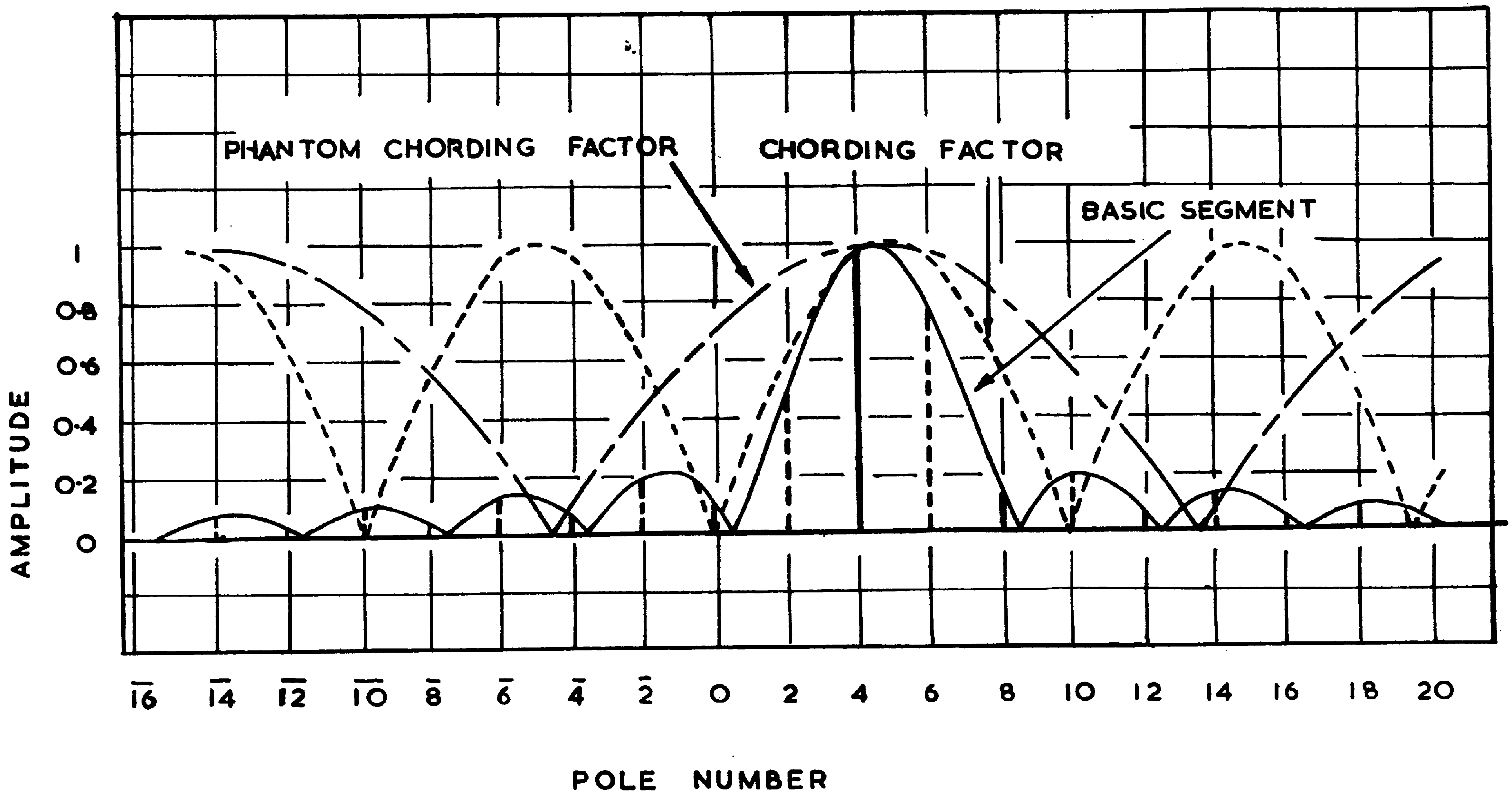
Segment arrangement for 2.5 poles/segment on 4 slots/pole/phase.

$$\overset{1}{\mathrm{RRRR\overline{YYYY}BBBB}} \quad \overset{1}{\mathrm{\overline{RRRR}YYYY\overline{BBBB}}} \quad \overset{.5}{\mathrm{RRRRYY}}$$

Bias towards 4 poles is achieved by shortening the segment. Omitting the last 6 slots puts $\frac{2\pi}{y_{\omega}} = 4$. Sufficient bias may be obtained by omitting 3 slots, then the segment length becomes 27, the stator has 54 slots, y_{ω} is 12 slots and $\frac{2\pi}{y_{\omega}} = 4.5$ poles. The relevant harmonic envelope curve for simple segments is shown in Figure 20. This is centred sufficiently close to 4 poles to ensure that all harmonics on this setting (shown by the full lines ordinates) will be small. The important harmonics (shown dotted) on 6 poles setting are at $\bar{6}$, $\bar{2}$, 2, 10, 14 poles and are of considerable magnitude. A coil-pitch near $\frac{2\pi}{5}$ ($=$ 11 slots) will reduce the 10 poles ordinate greatly, and 2 and $\bar{2}$ poles somewhat, as illustrated by the chording envelope shown on the Figure.

The ordinate at -6 poles shows that considerable negative phase

Figure 20. Illustrating the design of a 4/6 pole machine.



sequence voltage will be present on the 6 pole setting. This can be reduced by 'phantom chording' such that $\frac{2\pi\beta}{m} = \frac{1}{2}$ pole or $1\frac{1}{2}$ poles. The envelope due to phantom chording on $\frac{1}{2}$ pole is illustrated on the Figure, and it may be seen that this also greatly attenuates the 14 pole harmonic. The harmonic amplitudes calculated from Figure 20 are shown in Table 5.

TABLE 5.

Pole No.	4 pole setting. Current sheet analysis	Digital analysis.
$\overline{8}$	0.0122	0.0084
$\overline{4}$	0.00483	0.0172
4	0.928	0.893
8	0.062	0.062
12	0.011	0.016
16	0.0165	0.0085

Pole No.	6 pole setting. Current sheet analysis	Digital analysis.
$\overline{6}$	0.0273	0.0274
$\overline{2}$	0.0467	0.0448
2	0.261	0.25
6	0.712	0.682
10	0.0648	0.061
14	0.0106	0.018

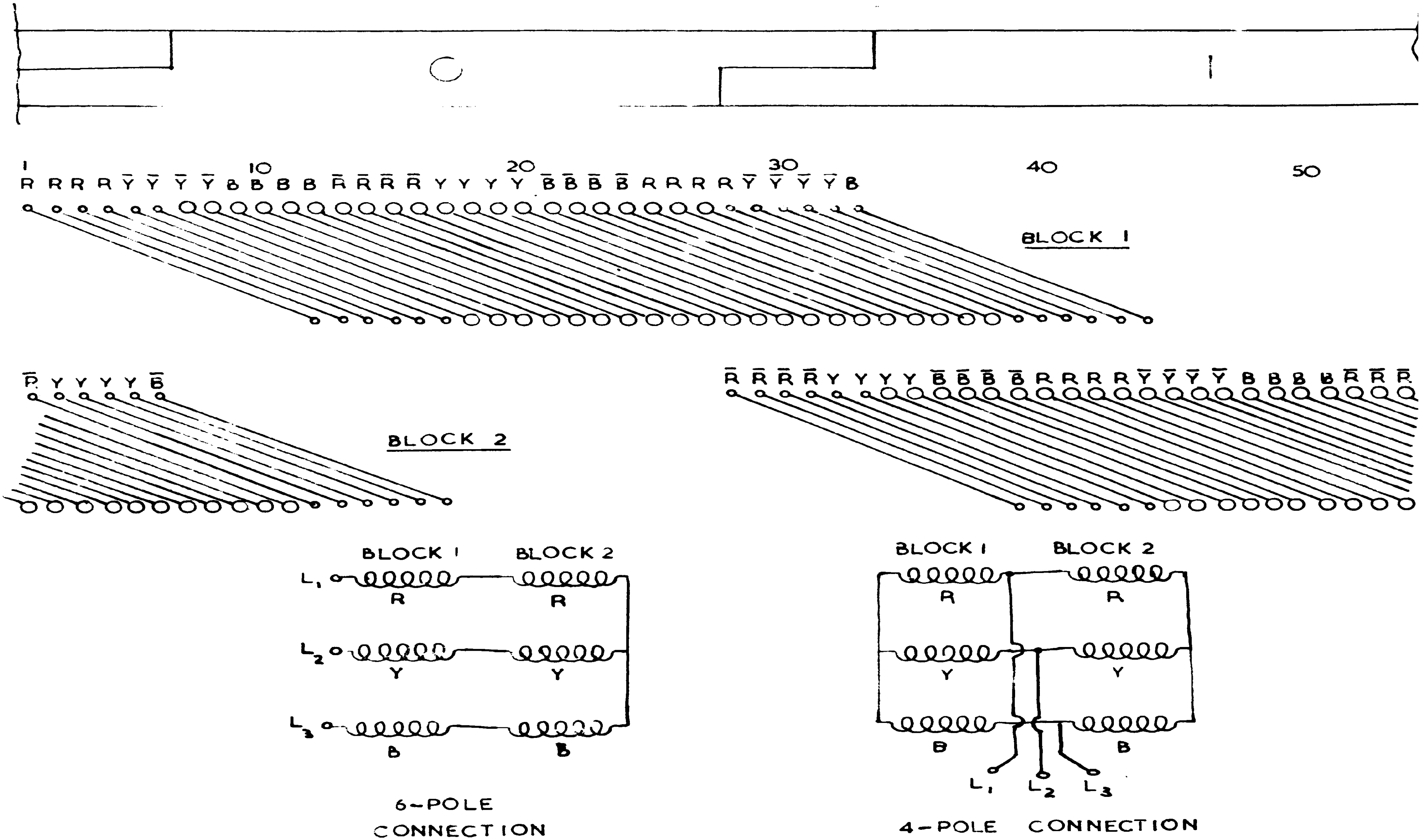
The full winding is shown in Figure 21. This winding has been analysed by digital computation and the results are also shown in Table 5; these results differ from those given by the current sheet analysis slightly, due to the effect of winding harmonics. The only harmonic in excess of 10% is the forward going 2 pole harmonic on the 6 pole setting. It is inconceivable that this harmonic could ever drive the fan; at its synchronous speed it would require 8 times full load power. Thus it is significant only to the extent that it will produce additional loss. At 6 poles the rating of the machine is 29.5% of that on 4 poles, so this additional loss should easily be accommodated. The air-gap flux density ratio using the method of Section 6.3 is 1.02, thus since the machine has a four pole winding factor which is almost equal to that of a conventional winding the machine should produce sensibly the same output on the 4 pole setting from a particular frame as may be normally obtained.

9. AN EXPERIMENTAL MACHINE.

A machine was constructed to the specification of the second winding example. The frame employed was intended for use as a 20 H.P. 1000 r.p.m. machine using a conventional double layer winding. Figure 22 shows the torque-speed curves produced by the machine which are suitable for use with a fan load. The tests were performed at half rated voltage in order to prevent excessive rotor heating at the high slip values.

Table 6 gives the full voltage performance results. The full load temperature rises were each lower than might be expected if the full load torque is assumed to occur at half peak torque. This may be because on

Figure 21. Winding arrangement for a 4/6 pole machine.



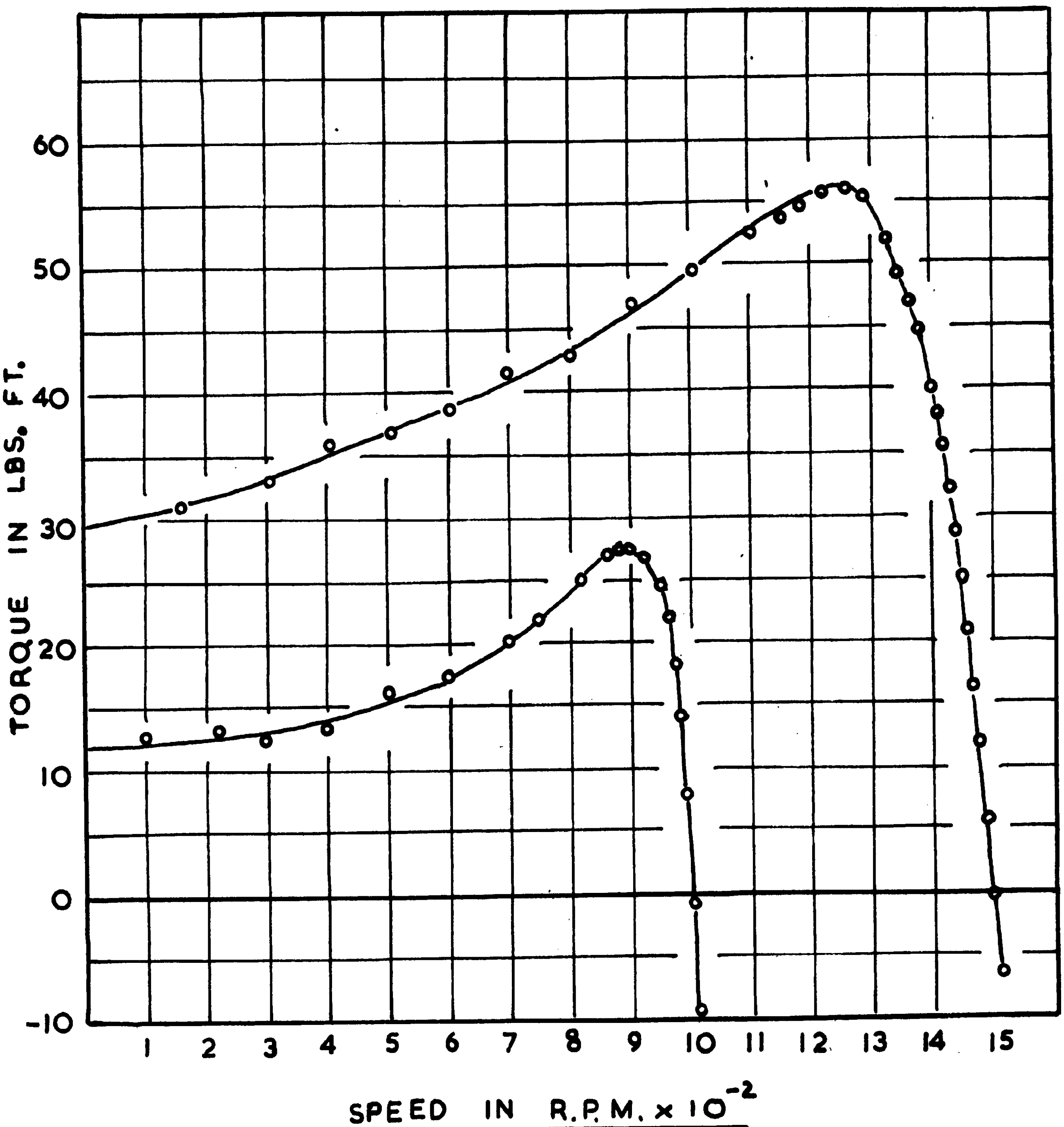


Figure 22. Torque speed curves for the 4/6 pole machine.

TABLE 6.

4 poles

Output Power:	29 H.P.
Full load efficiency:	87.3%
Full load power factor:	.91
Pull out torque:	2 x full load torque.
Starting torque:	1.3 x full load torque.
Starting current: (mean)	4.35 x full load current.
Winding temperature rise:	34°C

6 poles

Output Power:	9.8 H.P.
Full load efficiency:	86%
Full load power factor:	.84
Pull out torque:	2 x full load torque.
Starting torque:	.91 x full load torque.
Starting current: (mean)	4.1 x full load current.
Winding temperature rise:	32°C

the 6 pole speed the power output is less than is normally obtained from the frame, whilst in the 4 pole case the machine fan, which is intended for 6 pole operation, provides an excess of cooling air.

The effect of the winding unbalance can be assessed by considering the relative line currents drawn from a balanced source at starting, full load, and no load speeds; these are shown in Table 7.

TABLE 7.

	4 poles			6 poles		
	I_1	I_2	I_3	I_1	I_2	I_3
Starting	1.0	.926	.926	1.0	.852	.882
Full load	1.0	.83	.962	1.0	.73	.94
No load	1.0	.84	.866	1.0	.7	.81

In this table the currents are scaled to the greatest value.

In accordance with general practice these results were obtained with the neutral point of the winding disconnected, and therefore only the negative phase sequence voltage can affect the current balance. The current balance is better in the 4 pole case than in the 6 pole, in agreement with the factors in Table 5. Quantitative agreement between the measured currents and the calculated phase sequence components is difficult mainly because any winding irregularities can also cause unbalance.

9.1. An Industrial Machine.

In order to assess the commercial possibilities of the new windings, a design study was undertaken in collaboration with Messrs. Bruce Peebles

Limited. A 4/6 machine using two separate windings had been built previously for 70/25 horse-power, and it was found possible to provide a tapped winding machine having the same performance in a smaller frame. The design study indicated a saving of 18% in D^2L factor, and when the machine was constructed and tested, its performance was found to agree with the predictions within the normal tolerances.

10. CONCLUSIONS

The principles of winding design for pole-change machines using phase-shift techniques have been established. The normal procedure in attempting a particular design is to obtain the required winding factors, harmonic content and voltage balance using the simple current sheet analysis and then to confirm these results using the digital approach which takes into account all the harmonics produced by the excitation.

APPENDIX 1.

The excitation of the ℓ^{th} block is given by

$$J_{\ell} = J_s \, e^{j(\omega t - \phi_o - \phi \ell)} \, e^{-j\pi\left(\frac{y}{y_\omega} - x_o - x\ell\right)} \quad \ldots\ldots\ldots \quad (1)$$

in the region $-\frac{a\pi}{m} < y < \frac{a\pi}{m}$ and is zero elsewhere.

The complex amplitude of the travelling harmonic corresponding to 2p poles is:-

$$\frac{J_s \, e^{-j(\phi_o + \phi_\ell - \pi x_o - \pi x\ell)}}{2\pi} \int_{-\frac{a\pi}{m}}^{\frac{a\pi}{m}} e^{\left(-j\frac{\pi y}{y_\omega} + jpy\right)} dy$$

$$= J_s \, e^{-j(\phi_o + \phi_\ell - \pi x_o - \pi x\ell)} \left[\frac{\sin \frac{a\pi}{m}\left(p - \frac{\pi}{y_\omega}\right)}{\pi\left(p - \frac{\pi}{y_\omega}\right)} \right]$$

The magnitude of this harmonic component is:-

$$\frac{a\left|J_s\right|}{m} \left| \frac{\sin \frac{a\pi}{m}\left(p - \frac{\pi}{y_\omega}\right)}{\frac{a\pi}{m}\left(p - \frac{\pi}{y_\omega}\right)} \right| \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

APPENDIX 2.

If $J_{\ell} = \sum_{-\infty}^{\infty} a_p \, e^{-jpy}$ for one segment then a_p is given by equation (2) in Appendix 1. The effective current density of the assembly of m segments is obtained by summing the separate Fourier expansions

$$J = \sum_{\ell = 0}^{\ell = m-1} \sum_{p} a_p \, e^{-jp(y - \frac{2\pi \ell}{m})}$$

When there is no relative mechanical shift or electrical phase difference between the elements the complex amplitude of the travelling harmonic corresponding to 2p poles is

$$a_p \sum_{\ell = 0}^{\ell = m-1} e^{j\frac{2\pi \ell p}{m}} = a_p \left[\frac{1 - e^{j2\pi p}}{1 - e^{j\frac{2\pi p}{m}}} \right]$$

The numerator is always zero, but the denominator is also zero when $\frac{p}{m}$ is an integer; then the expression tends to m a_p, and for other values of p it is zero. Thus ordinates appear at $p_1 = ms$ where s is integral.

If now the phase of the ℓ^{th} segment is delayed relative to the 0^{th} segment by $\ell \phi$ then

$$J = \sum_{\ell = 0}^{\ell = m-1} \sum_{p} a_p \, e^{-j(py - \frac{2\pi p \ell}{m} + \ell \phi)}$$

and the complex amplitude of the 2p pole component becomes

$$a_p \sum_{\ell = 0}^{\ell = m-1} e^{j(\frac{2\pi p}{m} - \phi)\ell} = a_p \left[\frac{1 - e^{j(2\pi p - m\phi)}}{1 - e^{j(\frac{2\pi p}{m} - \phi)}} \right]$$

$$= a_p \, e^{j(\frac{\pi p}{m} - \frac{\phi}{2})(m - 1)} \left[\frac{\sin (\pi p - \frac{m\phi}{2})}{\sin (\frac{\pi p}{m} - \frac{\phi}{2})} \right]$$

The magnitude of this component is thus

$$\frac{a\left|J_s\right|}{m}\left|\frac{\sin\frac{a\pi}{m}(p - {}^{\pi}/y_{\omega})}{\frac{a\pi}{m}(p - {}^{\pi}/y_{\omega})}\right|\left|\frac{\sin\left(\pi p - \frac{m\phi}{2}\right)}{\sin\left(\frac{\pi p}{m} - \frac{\phi}{2}\right)}\right|$$

The phase discontinuities at the segment boundaries must all be equal. Thus $m\phi = 2q\pi$, where q is an integer, and the magnitude of the 2p pole component now becomes

$$\left|a_p\right|\left|\frac{\sin \pi(p - q)}{\sin\frac{\pi}{m}(p - q)}\right|$$

This is zero unless $\pi(p - q) = ms\pi$, s being an integer; its value is then $m\left|a_p\right|$. The ordinates appear at $p_2 = p_1 + q$ and thus the harmonic which existed at 2p poles has been shifted to $2(p + q)$ poles. If the excitation were such as to imply a negative going wave in the segments this shift to the right must still apply.

Since the wave of equation (1) of Appendix 1 can be written

$$J_e = J_s\, e^{j(\omega t - {}^{\pi y}/y_{\omega})}\, e^{-j\pi\left(\frac{\phi\ell}{\pi} + \frac{\phi_o}{\pi} - x_o - x\ell\right)}$$

It follows that an electrical phase lag ϕ has exactly the same effect as mechanical phase lead $\frac{\phi}{\pi}$, and the spectrum will be shifted by 2r poles, if the "mechanical phase lead" is made $\frac{2r}{m}$ pole-pitches per segment. However, if the excitation is negative going then a mechanical shift of $\frac{\phi}{\pi}$ in the same direction, i.e. a direction such as to move the waves in adjacent segments towards each other, is equivalent to an electrical phase advance rather than a delay. Thus harmonics due to negative going excitation will be shifted to the left rather than the right and therefore mechanical shift increases the modulus of the pole number whether the excitation be positive or negative going or merely pulsating.

APPENDIX 3.

If the 2p pole harmonic produced by the top layer is given by

$$J_T = R_p \, e^{-jpy}$$

then the return sides of the coils produce

$$J_B = - R_p \, e^{-jp(y - y')}$$

hence the resultant from both layers is:-

$$R_p \left[e^{-jpy} - e^{-jp(y - y')} \right]$$

and its modulus is

$$= 2 \, R_p \left| \cos \left(\frac{\pi - py'}{2} \right) \right|$$

$$\text{Thus } C = \left| \cos \frac{\pi - py'}{2} \right|$$

$$C = \left| \cos \frac{1}{2} y' (\delta p) \right|$$

where $2\delta p$ is the departure of the pole number of the harmonic from the pole number, not necessarily an integer for which y' would represent full pitch.

APPENDIX 4.

A slot current in the stator of a machine may be considered to act over a small distance 2δ. If there are N_1 turns in the slot, each carrying a current I, then the amplitude of the n^{th} harmonic due to this single slot is given by

$$a_n' = \frac{1}{2\pi}\int_{\phi'-\delta}^{\phi'+\delta} \frac{N_1 I}{2} e^{jn\theta}\, d\theta$$

where ϕ' is the displacement of the slot from an arbitrary zero.

$$a_n' = \frac{N_1 I}{2\pi n}\left[\frac{e^{jn\phi'} \sin n\delta}{\delta}\right]$$

If the current is considered to act at a single point then

$$a_n' = \frac{N_1 I \; e^{jn\phi'}}{2\pi}$$

For q slots in the machine the amplitude of the n^{th} harmonic due to one phase of the excitation becomes

$$a_n = \frac{I_p}{2\pi} \sum_{s=1}^{s=q} N_{sp}\, e^{jn\phi_s}$$

where ϕ_s is the displacement of the s^{th} slot, I is the phase current and N_{sp} the number of turns due to the phase p in the s^{th} slot.

If the number of turns in the s^{th} slot for the red, yellow and blue phases are given by N_{SR}, N_{SY} and N_{SB} respectively, then the forward rotating current density on the n^{th} harmonic, a_{nf}, due to all the phases, is given by

$$\frac{2\pi}{I} a_{nf} = \sum_{s=1}^{s=q} N_{SR} e^{jn\phi_s}$$

$$+ e^{\frac{j2\pi}{3}} \sum_{s=1}^{s=q} N_{SY} e^{jn\phi_s}$$

$$+ e^{\frac{-j2\pi}{3}} \sum_{s=1}^{s=q} N_{SB} e^{jn\phi_s} \quad \ldots\ldots\ldots \quad (A)$$

and the backward rotating n^{th} harmonic is given by:-

$$\frac{2\pi}{I} a_{nb} = \sum_{s=1}^{s=q} N_{SR} e^{jn\phi_s}$$

$$+ e^{\frac{-j2\pi}{3}} \sum_{s=1}^{s=q} N_{SY} e^{jn\phi_s}$$

$$+ e^{\frac{j2\pi}{3}} \sum_{s=1}^{s=q} N_{SB} e^{jn\phi_s} \quad \ldots\ldots\ldots \quad (B)$$

If each phase of the winding is connected to a single phase supply then the resulting pulsating amplitude of the n^{th} harmonic is a_{nz} given by:

$$\frac{2\pi}{I} a_{nz} = \sum_{s=1}^{s=q} N_{SR} e^{jn\phi_s}$$

$$+ \sum_{s=1}^{s=q} N_{SY} e^{jn\phi_s}$$

$$+ \sum_{s=1}^{s=q} N_{SB} e^{jn\phi_s} \quad \ldots\ldots\ldots \quad (C)$$

If the flux density is given by $B_n e^{j(\omega t + n\theta)}$ then the p phase voltage is:

$$V_p = \frac{B\omega a \ell}{n} \sum_{s=1}^{s=q} N_{sp} e^{jn\phi_s}$$

where ℓ is the length of the machine and a is the radius.

Thus the positive phase sequence component due to the red, yellow and blue phases is given by V_{nf} where:

$$\frac{nV_{nf}}{B\omega a\ell} = \sum_{s=1}^{s=q} N_{SR}\, e^{jn\phi_s} + e^{\frac{j2\pi}{3}} \sum_{s=1}^{s=q} N_{SY}\, e^{jn\phi_s} + e^{\frac{-j2\pi}{3}} \sum_{s=1}^{s=q} N_{SB}\, e^{jn\phi_s} \quad \ldots\ldots\ldots \quad \text{(D)}$$

and the negative phase sequence is given by:

$$\frac{nV_{nb}}{B\omega a\ell} = \sum_{s=1}^{s=q} N_{SR}\, e^{jn\phi_s} + e^{\frac{-j2\pi}{3}} \sum_{s=1}^{s=q} N_{SY}\, e^{jn\phi_s} + e^{\frac{j2\pi}{3}} \sum_{s=1}^{s=q} N_{SB}\, e^{jn\phi_s} \quad \ldots\ldots\ldots \quad \text{(E)}$$

and similarly the zero phase sequence component is

$$\frac{nV_{nz}}{B\omega a\ell} = \sum_{s=1}^{s=q} N_{SR}\, e^{jn\phi_s} + \sum_{s=1}^{s=q} N_{SY}\, e^{jn\phi_s} + \sum_{s=1}^{s=q} N_{SB}\, e^{jn\phi_s} \quad \ldots\ldots\ldots \quad \text{(F)}$$

Comparison of equations (A), (B), (C) and (E) shows that for any configuration of conductors the ratio of the reverse phase sequence voltage to the forward phase sequence voltage is the same as the ratio of the reverse rotating current harmonic to the forward rotating current harmonic.

Similarly, comparison of equations (A), (C), (D) and (F) shows that the ratio of the pulsating harmonic, caused by feeding the three phases of the winding from a single phase source, to the forward rotating current harmonic is the same as the ratio of forward and zero phase sequence components of voltage.

Either analysis is in a convenient form for use on the digital computer, since the effect of each slot for a particular phase may be summed in a short cycle of programme. It is then easy to form the symmetrical components produced by the phases taken together.

Due to rotor action, the flux density distribution in the machine, when its speed approaches the synchronous speed of the dominant harmonic, is substantially a sinusoidal space distribution of a pole pitch corresponding to the dominant harmonic. Thus to a first approximation, in the absence of information about a particular rotor, it is sufficient to consider only this distribution of flux. Absence of zero or negative components due to a forward travelling wave of flux is therefore defined as "complete voltage balance".

APPENDIX 5.

If the winding has P parallel paths, where each path is made up by connecting every P^{th} segment in series, then each series path constitutes a $\frac{2m}{P}$ block machine. Thus the harmonic ordinates due to the path will appear at multiples of $\frac{2m}{P}$ under the segment envelope when the machine is in the datum condition. Hence the condition for negative phase sequence voltages due to the N^{th} harmonic is:

$$2Nr + 2r = \frac{2mn}{P}$$

i.e. $$\frac{(N+1)}{2} \frac{(2r)}{{}^{m}/P} = n$$

where N takes 1, -5, 7, -11 etc. and n is an integer.

For the fundamental $N = 1$

$$\text{and} \quad \frac{2r}{{}^{m}/P} = n$$

Thus there is the possibility of a negative phase sequence component due to the fundamental, only when 2r is an integral multiple of ${}^{m}/P$, and under these circumstances all the harmonics will also produce negative phase sequence components.

For the possible values of N, N + 1 is never divisible by 3, hence if ${}^{m}/P$ is a multiple of 3 and 2r is not, there can be no negative phase sequence components. When ${}^{m}/P$ is not a multiple of 3 the harmonics producing reverse phase sequence can be identified from the expression

$$N = \frac{2mn}{2rP} - 1$$

The magnitude of the negative phase sequence component due to this harmonic can be found by centering the harmonic envelope of amplitude equal to the N^{th} winding harmonic amplitude on $\frac{2N\pi}{y_\omega}$ and evaluating the ordinate at $-2r$.

These findings relate to "mechanically" set up pole number $2r$, but Section 5.2 shows that they will also relate to the electrically shifted pole number $2r + 2q$ provided $\phi = \frac{\pi}{3}$, $\frac{2\pi}{3}$ or π, the common values.

APPENDIX 6.

Zero Phase Sequence.

The condition for zero phase sequence voltages is

$$2Nr \pm 2r = \frac{2mn}{P} \text{ where } N = 3,\ 9,\ 15 \ldots\ldots$$

i.e. $(N \pm 1)\dfrac{(2r)}{2\,^{m}/P} = n.$

For $N = 3$,

$$\frac{(2r)}{^{m}/P} = n$$

or

$$\frac{2r}{^{m}/2P} = n$$

For these conditions all harmonics give trouble also. Where these conditions are not satisfied the harmonics causing zero phase sequence components are given by

$$N = \frac{2m}{P}\,\frac{n}{(2r)} \pm 1$$

Hence if $^{m}/P$ is a multiple of 3 and $(2r)$ is not, no harmonics will produce zero phase sequence components.

Again the magnitude can be evaluated by finding the ordinates at $2r$ and $-2r$ of a harmonic envelope centred on $2N\dfrac{\pi}{y_\omega}$.

CHAPTER III

COMPLETELY BALANCED TWO-SPEED MACHINES.

1. INTRODUCTION

In general the windings analysed in the second chapter have two defects. Firstly, a compromise must be reached between the requirements of voltage balance and harmonic content, and in any case in most circumstances only approximate balance can be achieved. Secondly, some of the coils contain twice as many turns as the others. This second defect is particularly onerous in high voltage windings since the "4 layer" effect, at the points where the blocks overlap, necessitates extra slot insulation. Two block close ratio windings which are completely balanced whilst using uniform coils are shown to be possible in this chapter.

2. MACHINES USING INTERLEAVED BLOCKS

Machines using two layer blocks can be converted into machines using uniform coils by designing the winding for S slots and then using it in a stator having 2S slots by transferring half the conductors from each of the S slots into an intermediate slot. The general form of the top layer of such a machine is shown in Figure 1.

3. CONDITIONS FOR VOLTAGE BALANCE

Complete voltage balance can be achieved by providing conductors which produce the same pattern of induced voltage vectors for each of the phases, the patterns being separated by 120° electrical. For example, consider the two pole pitches of the winding shown in Figure 2(a). The vector diagram of Figure 2(b) illustrates the induced voltage vectors for the three phases, assuming that the flux density wave producing these voltages has a pole pitch of 6 slots. The dotted lines on Figure 2(b) indicate the

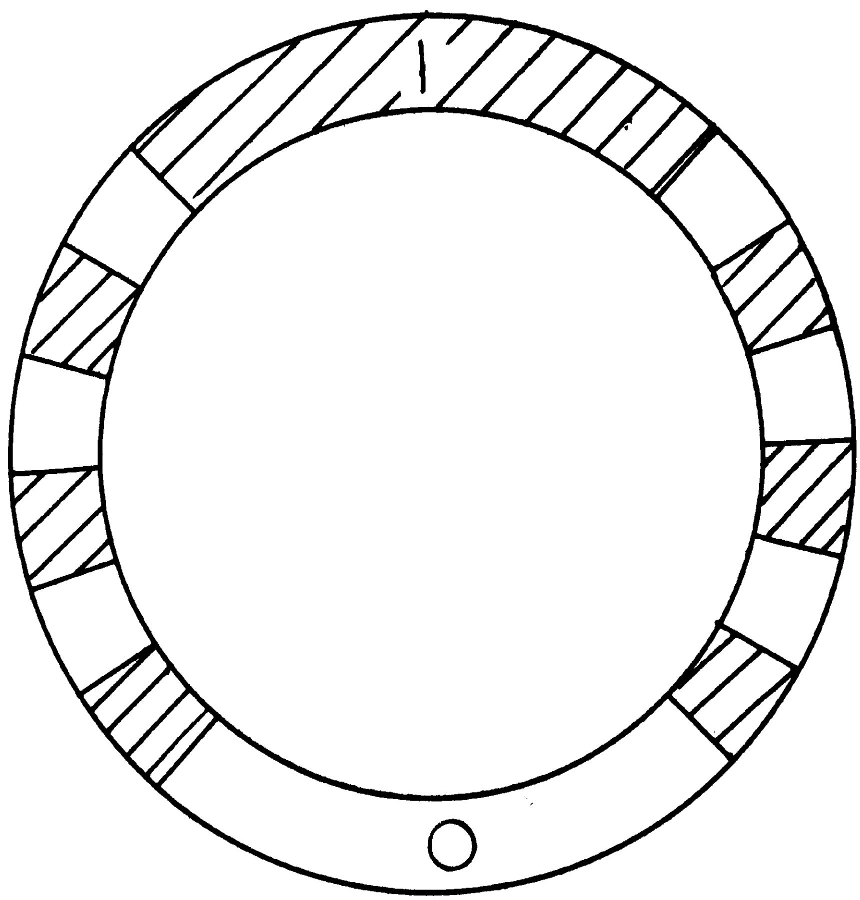


Figure 1. An interleaved machine.

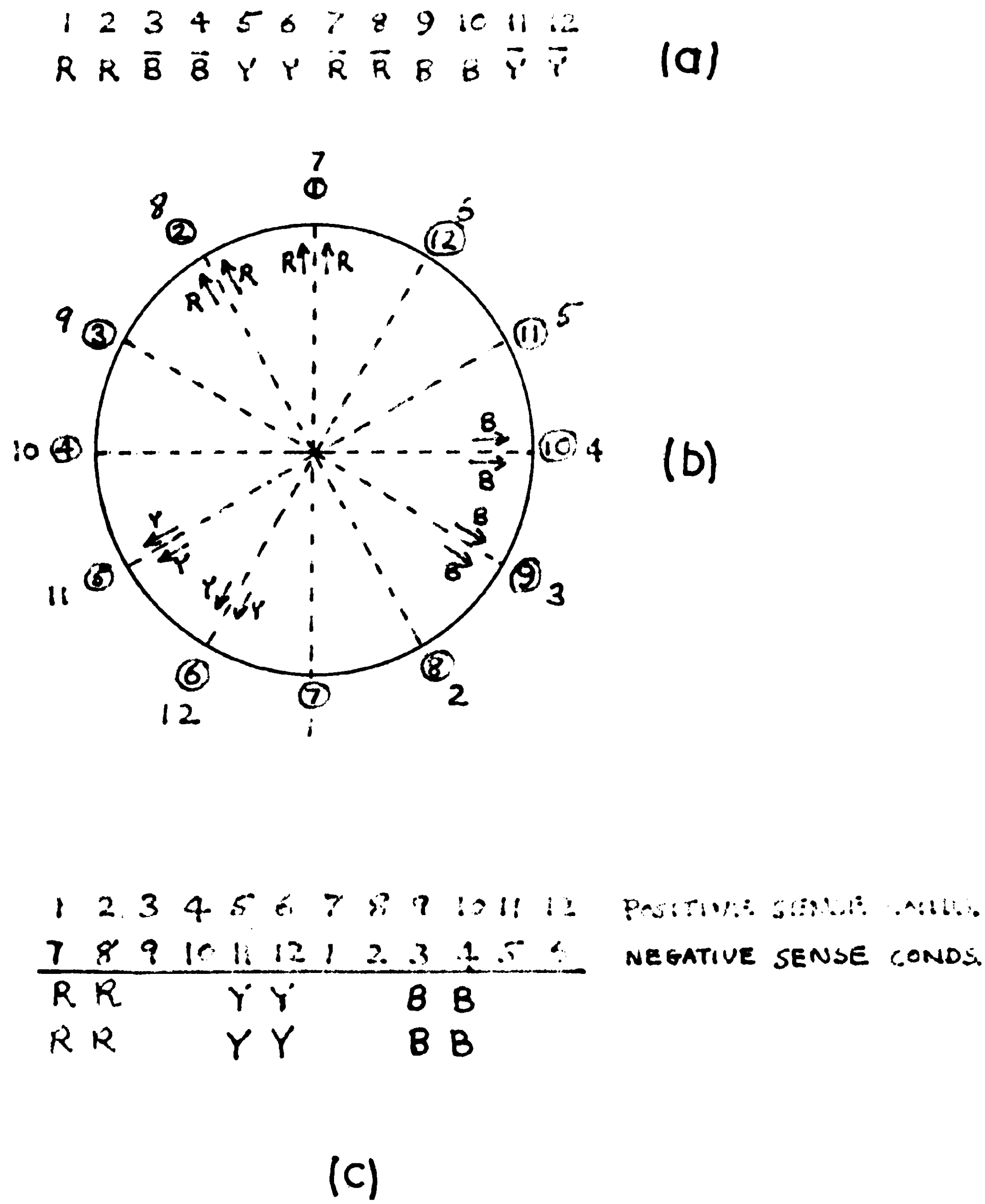


Figure 2. Illustrating balanced conditions.

(a) A winding segment.

(b) The conventional vector diagram corresponding to (a)

(c) A developed diagram corresponding to (b).

phase position of the induced voltage in a conductor placed in the slot number ringed at the end of the dotted line, considering the conductor to be connected in a positive sense. If the conductor is connected in a negative sense then the unringed set of numbers applies. It can be seen that the winding example of Figure 2(a) is balanced since the phase patterns are the same and separated by 120^{o} electrical. In order to simplify the diagram it is possible to consider a linear version; this is shown in Figure 2(c) where again voltage balance is indicated by the same pattern for each phase separated by $^{1}/3$ of the diagram length.

4. BALANCED WINDING SEGMENTS

Suppose for example there are 12 slots in the segment length and a winding having $2^{1}/3$ poles is required. This segment is suitable for use in a 4/6 pole machine. On 4 pole excitation, the condition for which the winding is to be balanced, there are 6 slots per pole of excitation wave. The first step taken in order to achieve balance is shown in Figure 3(a). Conductors giving three poles of excitation are put in the 12 slots. As a second step, R and $\overline{B}$ conductors are placed in slots 11 and 12 as illustrated in Figure 3(b). The conductors displaced from 11 and 12 are replaced in the main body of the winding in some apt position. Thus in Figure 3(c), R and $\overline{B}$ conductors are put in slots 1 and 2 and then the $\overline{B}$ conductor displaced from slot 11 is placed in slot 3. The original winding is then written down in order in slots 4 to 9, the yellow conductor from slot 12 is then inserted, followed by the R and $\overline{B}$ conductors. This winding is then checked on the vector diagram of Figure 3(d), and it can be seen that the

$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

$\mathrm{R}\ \bar{\mathrm{B}}\ \mathrm{Y}\ \mathrm{Y}\ \bar{\mathrm{R}}\ \bar{\mathrm{R}}\ \mathrm{B}\ \bar{\mathrm{Y}}\ \underbrace{\mathrm{R}\ \bar{\mathrm{B}}}\ \bar{\mathrm{B}}\ \mathrm{Y}$ (a)

$\mathrm{R}\ \bar{\mathrm{B}}$ (b)

$\mathrm{R}\ \bar{\mathrm{B}}\ \boxed{\bar{\mathrm{B}}}\ \mathrm{Y}\ \mathrm{Y}\ \bar{\mathrm{R}}\ \bar{\mathrm{R}}\ \mathrm{B}\ \bar{\mathrm{Y}}\ \boxed{\bar{\mathrm{Y}}}\ \mathrm{R}\ \bar{\mathrm{B}}$ (c)

(d)

1	2	3	4	5	6	7	8	9	10	11	12
7	8	9	10	11	12	1	2	3	4	5	6
R			Y	Y	B		B	B			
R		Y	Y				B			R	R

$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12$

$\mathrm{R}\ \bar{\mathrm{B}}\ \bar{\mathrm{B}}\ \mathrm{Y}\ \mathrm{Y}\ \bar{\mathrm{R}}\ \mathrm{B}\ \mathrm{B}\ \bar{\mathrm{Y}}\ \bar{\mathrm{Y}}\ \mathrm{R}\ \mathrm{R}$ (e)

(f)

1	2	3	4	5	6	7	8	9	10	11	12
7	8	9	10	11	12	1	2	3	4	5	6
R		Y	Y	Y		B	B	B		R	R
			Y				B				R

Figure 3. Illustrating the production of a balanced segment.

winding is unbalanced; in order to achieve balance the 3 vector patterns must be the same and mutually displaced by 120°. This can be accomplished in the present case by interchanging the conductors in slots 7 and 12. This is shown in Figure 3(e), with a corresponding vector diagram at Figure 3(f).

4.1. A Machine which is Balanced on one Pole Number.

The sub-block which has been formed so far may now be used together with a similar segment to form an interleaved winding which is approximately balanced on the 6 pole speed. Figure 4 illustrates how two segments may be interleaved. Segment B is the original segment whilst segment A has the same phase band configuration but the order of the phases has been altered to suit the particular overlap chosen. The segments of Figure 4 are interleaved to form one block of the winding of Figure 5 by writing 1, 3, 5, 7 etc. along segment B and 2, 4, 6, 8, 10 etc. along segment A. The vector diagrams of Figures 5(b) and 5(c) show that the block is completely balanced on 4 poles and only approximately balanced on 6 poles.

In a two block machine $x\ell = \dfrac{2r\ell}{m}$ * or $x = r$ and since r is an integer the mechanical shift required to change the machine from its datum condition is always a whole number of poles. All the blocks are the same in the datum condition and hence the second block of a two block machine is always either a copy or a reversed copy of the first block, according to the speed setting. Thus the balanced block of winding can be used for both the first and second blocks of the two block machine. The complete winding is shown in Figure 5.

4.2. A Completely Balanced Machine.

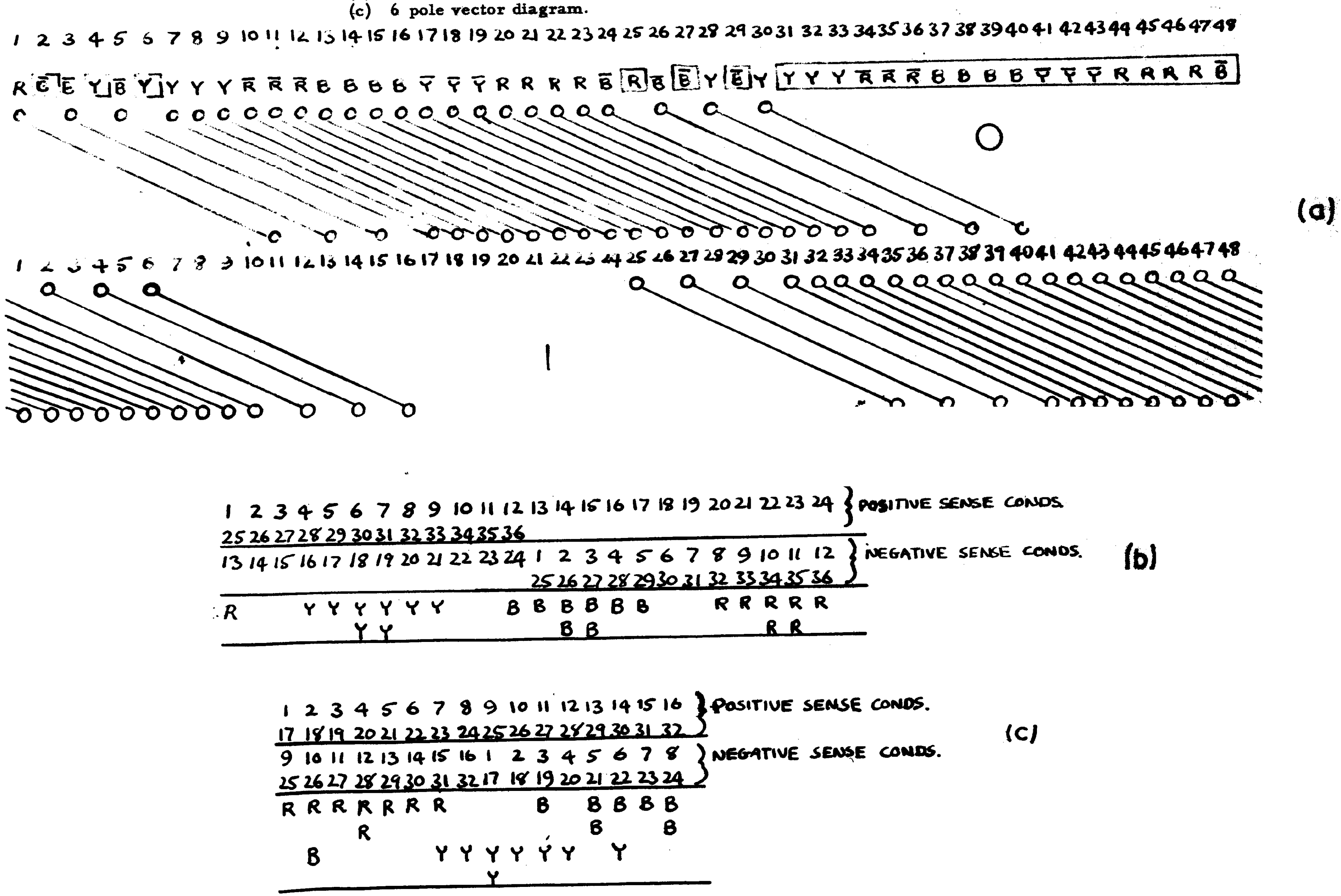
A winding which is completely balanced on the 6 pole setting may be found by interleaving three segments of the form of Figure 3(e). The total

* Section 2 of Chapter II.

2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	
			Y	$\bar{R}$	$\bar{R}$	B	B	$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$	Y	Y	— A
R	B	B	Y	Y	R	B	B	$\bar{Y}$	$\bar{Y}$	R	R				— B
1	3	5	7	9	11	13	15	17	19	21	23				

Figure 4. Two segment interleave.

Figure 5. A 4/6 machine which is completely balanced in the 4 pole connection.
(a) Winding diagram.
(b) 4 pole vector diagram.
(c) 6 pole vector diagram.



number of slots on the machine will then be 72, that is 12 slots per pole in the 6 pole condition. Balance can be achieved for 3 conductors, i.e. one per phase by positioning them so that they occur at a multiple of $^1/3$rd of a pole apart. For example, if we start with a red conductor in slot 1 then positioning the blue conductor in slot 17 and the yellow in slot 9 would achieve balance on the 6 pole condition for the 3 conductors. In general for a red conductor in slot 1, balanced conditions result if any two of the possibilities for the yellow and blue conductor positions shown in the Table below are chosen.

TABLE 1.

1	5	9	13	17	21	S
R	$\bar{B}$	Y	$\bar{R}$	B	$\bar{Y}$	R

The balanced segment may be written in any one of the six ways shown below:

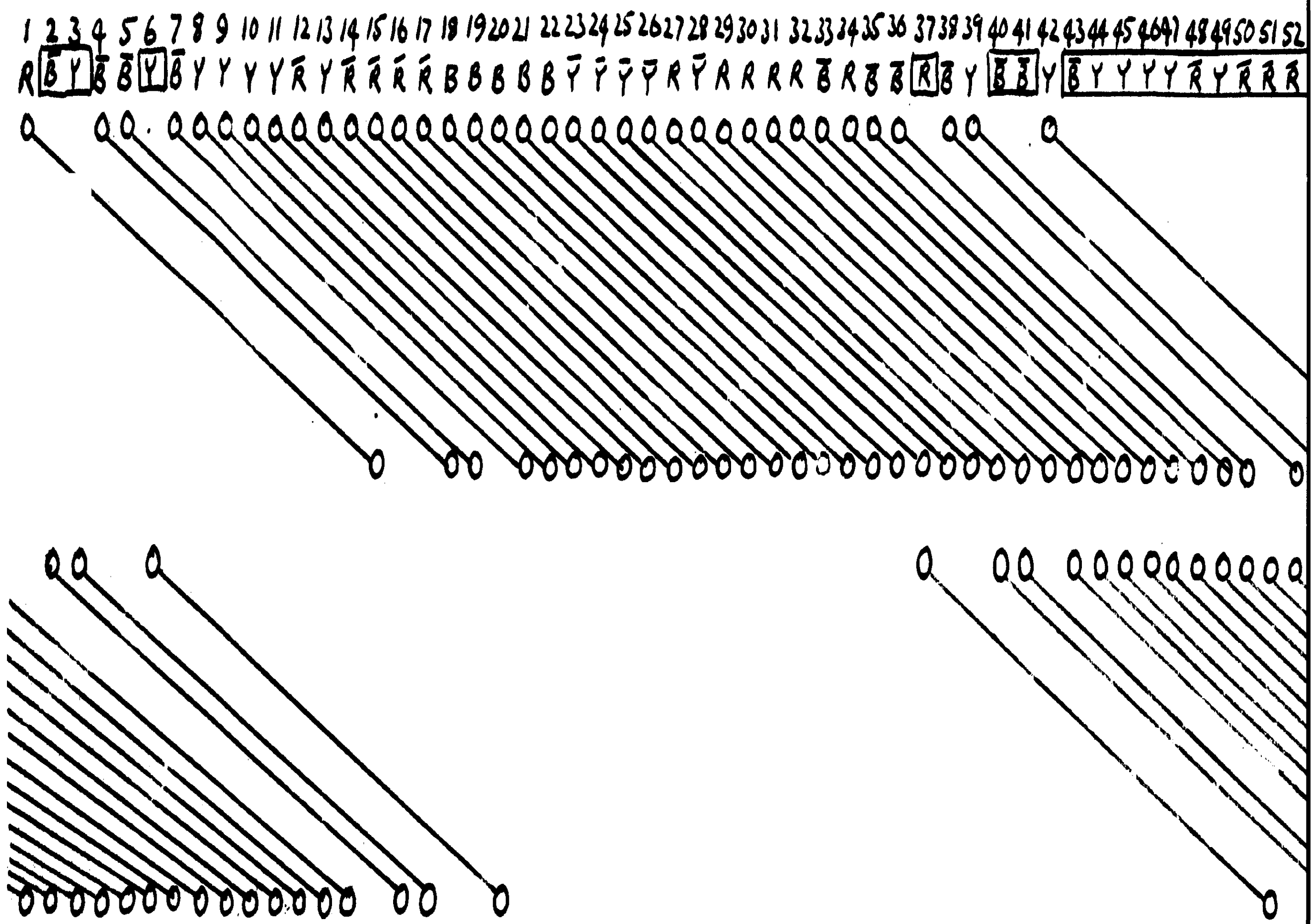
1	2	3	4	5	6	7	8	9	10	11	12	
R	$\bar{B}$	$\bar{B}$	Y	Y	$\bar{R}$	B	B	$\bar{Y}$	$\bar{Y}$	R	R	- E
$\bar{B}$	Y	Y	$\bar{R}$	$\bar{R}$	B	$\bar{Y}$	$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$	- F
Y	$\bar{R}$	$\bar{R}$	B	B	$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$	Y	Y	- G
$\bar{R}$	B	B	$\bar{Y}$	$\bar{Y}$	R	$\bar{B}$	$\bar{B}$	Y	Y	$\bar{R}$	$\bar{R}$	
B	$\bar{Y}$	$\bar{Y}$	R	R	$\bar{B}$	Y	Y	$\bar{R}$	$\bar{R}$	B	B	
$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$	Y	$\bar{R}$	$\bar{R}$	B	B	$\bar{Y}$	$\bar{Y}$	

If, for example, it is decided to use segments E, F and G then clearly examining column 1, we can put the R conductor in slot 1, the $\bar{B}$ conductor in slot 5 and the Y conductor in slot 9, when these 3 conductors produce balanced conditions. The conductors of column 2 may be dealt with in a similar way by placing the $\bar{B}$ Y and $\bar{R}$ conductors in slots 4, 8 and 12 respectively. This process may be repeated until all the conductors have been positioned in the slots. Hence the conductors from the E segment occupy slots 1, 4, 7, 10, 13, etc., i.e. slots 3 slot pitches apart whilst segments F and G occupy slots 5, 8, 11, 14, etc., and 9, 12, 15, 18, 21 etc., respectively. The segments with the appropriate slot number written above the conductors are shown below, forming the winding of Figure 6.

TABLE 2.

1	4	7	10	13	16	19	22	25	28	31	34
R	$\bar{B}$	$\bar{B}$	Y	Y	$\bar{R}$	B	B	$\bar{Y}$	$\bar{Y}$	R	R
5	8	11	14	17	20	23	26	29	32	35	38
$\bar{B}$	Y	Y	$\bar{R}$	$\bar{R}$	B	$\bar{Y}$	$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$
9	12	15	18	21	24	27	30	33	36	39	42
Y	$\bar{R}$	$\bar{R}$	B	B	$\bar{Y}$	R	R	$\bar{B}$	$\bar{B}$	Y	Y

In general, the method is to put the conductors from each segment in slots which are 3 slot pitches apart. The starting number for each sequence is determined by a set of numbers which are $^1/3$rd of a pole pitch apart. (Table 1 in the above example). Since the slots occupied by a segment are evenly spaced each of the segments must remain balanced on 4 poles, and hence the block is balanced in both the 4 and 6 pole conditions. Thus the winding is balanced on a first pole number by adjustment of the phase-band lengths of the segment and on a second pole number by suitably interleaving



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 } POSITIVE SENSE CONDS.
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48

13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 } NEGATIVE SENSE CONDS.
37 38 39 40 41 42 43 44 45 46 47 48 25 26 27 28 29 30 31 32 33 34 35 36

R R R R R R R R R
R R
Y Y Y Y Y Y Y Y Y Y
Y Y B B B B B B B B B
B B

(b)

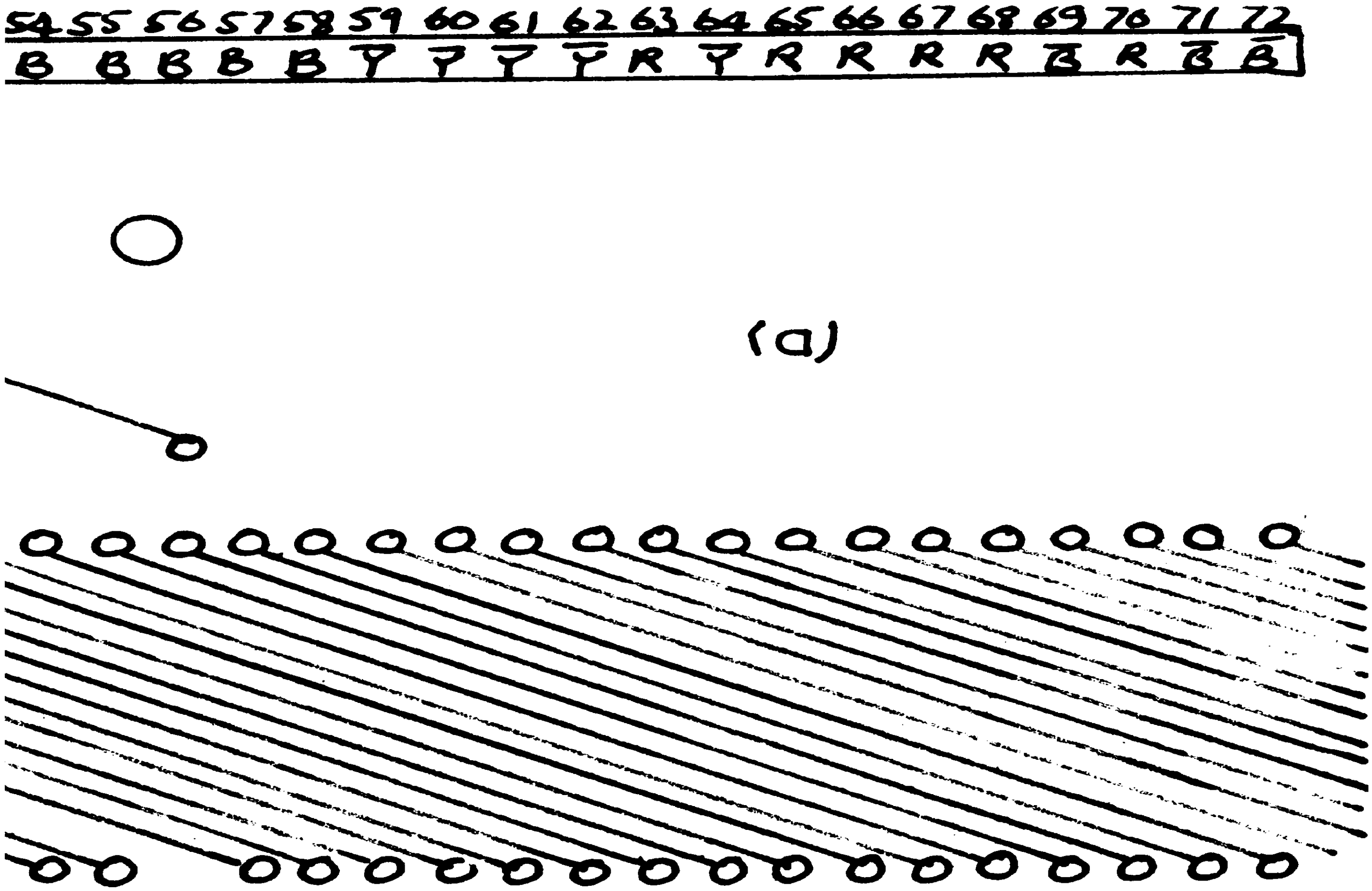
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 } POSITIVE SENSE CONDS.
37 38 39 40 41 42 43 44 45 46

19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 } NEGATIVE SENSE CONDS.
37 38 39 40 41 42 43 44 45 46

R Y Y Y Y Y Y Y Y Y B B B B B B B B B R R R R R R R R
Y Y Y B B B R R R

Figure 6. Comple

(a)
(b)
(c)



(a)

y balanced 4/6 pole machine.

nding diagram.
pole vector diagram.
pole vector diagram.

(c)

three segments.

The method of interleaving three segments to form a block places some restrictions on the stator slot numbers available. To achieve voltage balance the number of slots in a segment must be a multiple of three in order that there shall be an equal number of conductors per phase. Since six segments in all are used to form the machine, the total number of stator slots must therefore be a multiple of 18. When balancing the simple segment it must be regarded as one block of a two-block machine. Thus, if the vector diagram is to be capable of producing 3 patterns which are separated by 120°, twice the number of slots in a segment divided by the number of pole pairs, must be a multiple of three. Whence if the number of slots in a segment is $3k_1$, where k_1 is an integer, then $\frac{12k_1}{r_s}$ must be a multiple of 3, where r_s is the pole number on which balance is to be achieved. Hence for balance to be possible $\frac{4k_1}{r_s}$ must be integral.

Table 3 lists the pole and slot number combinations on which it is possible to balance the segment.

In order to produce balance by interleaving, the number of slots per pole per phase must be an integer which is not a multiple of 3. The exclusion of the multiples of 3 slots per pole per phase is necessary because under these conditions the three segments would occupy the same slots over some portion of the stator. The number of phase bands is $3r_e$ where r_e is the pole number on which balance is to be produced by interleaving. Thus the number of slots per pole per phase is $\frac{18k_2}{3r_e} = \frac{6k_2}{r_e}$. Since k_2 is an integral it is not possible to produce balance by interleaving unless r_e is a multiple of 6.

TABLE 3.

Segment Balance

Slot Number.	Pole Numbers. 4	6	8	10	12
18	P	-	-	-	-
36	P	-	P	-	-
54	P	P	-	-	P
72	P	-	P	-	-
90	P	-	-	P	-
108	P	P	P	-	P
126	P	-	-	-	-
144	P	-	P	-	-

P indicates that segment balance is possible

Hence for 6 poles balance is possible for $k_2 = 1, 2, 4, 5, 7$ etc., i.e. for slot numbers of 18, 36, 72, 90, 126, 144 and so on. Similarly, balance is possible for 12 poles when the slot number is 36, 72, 144 and so on.

In two block machines the two pole numbers produced are separated by 2 poles and hence it is possible to achieve complete balance for 4/6 pole machines using 18, 36, 72, 90, 126 and 144 slots. Similarly it is possible to produce 6/8 pole machines using 36, 72 and 144 slots. Other pole combinations are not possible using a reasonable number of slots.

5. WINDING EXAMPLES

An 8/10 Pole Machine.

It is not possible to balance this machine completely in both connections by using the 3 segment interleave system, and the system outlined in Section 4. 1 must be used.

In order to produce a gap flux density ratio which is sensibly unity, it is most convenient to use delta to parallel star switching. Since the delta connection occurs in the 10 pole condition it is perhaps more important that the machine should be completely balanced in this connection rather than in the 8 pole connection. The machine is to have 120 slots, that is the number of slots per segment is 30, since the total number of segments is 4. A 30 slot segment which is balanced on the 10 pole setting is shown below.

1 10 20 30

$$R\ R\ \overline{B}\ \overline{B}\ Y\ Y\ Y\ \overline{R}\ \overline{R}\ B\ B\ \overline{Y}\ \overline{Y}\ \overline{Y}\ R\ R\ \overline{B}\ \overline{B}\ \overline{B}\ Y\ Y\ \overline{R}\ \overline{R}\ B\ B\ \overline{Y}\ \overline{Y}\ R\ R\ \overline{B}$$

This can be interleaved with a second segment formed by writing Y for R, B for Y and R for B in the above segment, whilst maintaining the positive and negative signs. The interleave must be chosen to minimise the negative phase sequence voltage on the 8 pole setting. Since it is difficult to ascribe an exact pole-pitch to the winding, it is also difficult to say how many slots of overlap are appropriate. However, the recommendations of Chapter II can be followed to the extent of indicating, say, four possible machines. These machines may then be examined using the digital computer technique. In this particular case a good machine is produced by placing the conductors from the first segment in slots 2 4 6 8 10 etc. of the machine, whilst using slots 23, 25, 27 etc. for the second displaced segment, thus forming the first winding block. The second winding block is of course the same as the first but displaced by 180° mechanical or 60 slots. The computer results indicate complete balance on the 10 pole setting with a winding factor of .728. The winding factor on the 8 pole setting is .8199 with a reverse phase sequence winding factor of 0.0021.

The harmonic analysis is shown in Table 4 for a coil pitch of 15 slots.

TABLE 4.

8 pole setting. Pole Number.	Amplitude	10 pole setting. Pole Number.	Amplitude
- 20	.0122	- 18	.002
- 12	.0553	- 14	.0271
- 8	.0021	- 6	.0235
- 4	.0239	- 2	.0411
4	.0043	2	.0169
8	.8199	6	.207
12	.102	10	.728
20	.0455	14	.034
		18	.0291

The harmonic content is satisfactory since the only appreciable harmonic is the 6 pole harmonic on the 10 pole setting. This harmonic cannot produce dips in the speed-torque curve.

A 4/6 Pole Machine

The most difficult windings to form are those using a small number of slots per segment; however, a reasonable number of balanced segments exist for a segment slot number of six. Thus a 36 slot completely balanced machine for 4/6 pole operation can be formed using the segment R $\bar{B}$ Y B $\bar{Y}$ R which is balanced on the 4 pole setting. This segment may be interleaved to produce 6 pole balance using the segments and slot numbers shown below.

1	4	7	10	13	16
R	$\bar{B}$	Y	B	$\bar{Y}$	R
3	6	9	12	15	18
$\bar{B}$	Y	$\bar{R}$	$\bar{Y}$	R	$\bar{B}$
5	8	11	14	17	20
Y	$\bar{R}$	B	R	$\bar{B}$	Y

The complete winding is shown in Figure 7. Computer analysis indicates that this machine is completely balanced and produces the harmonic content for the top layer only which is shown in Table 5.

TABLE 5.

6 pole settings			4 pole settings.		
Pole No.	Top layer.	9 slots coil pitch.	Pole No.	Top layer.	9 slots coil pitch.
- 14	.0253	.0179	- 8	.0296	0
- 10	.0691	.0489	4	.8312	.8312
- 2	.1442	0.1022	8	.3199	0
2	.1989	0.141			
6	.8796	0.622			
10	.0728	0.0315			
14	.1059	0.075			

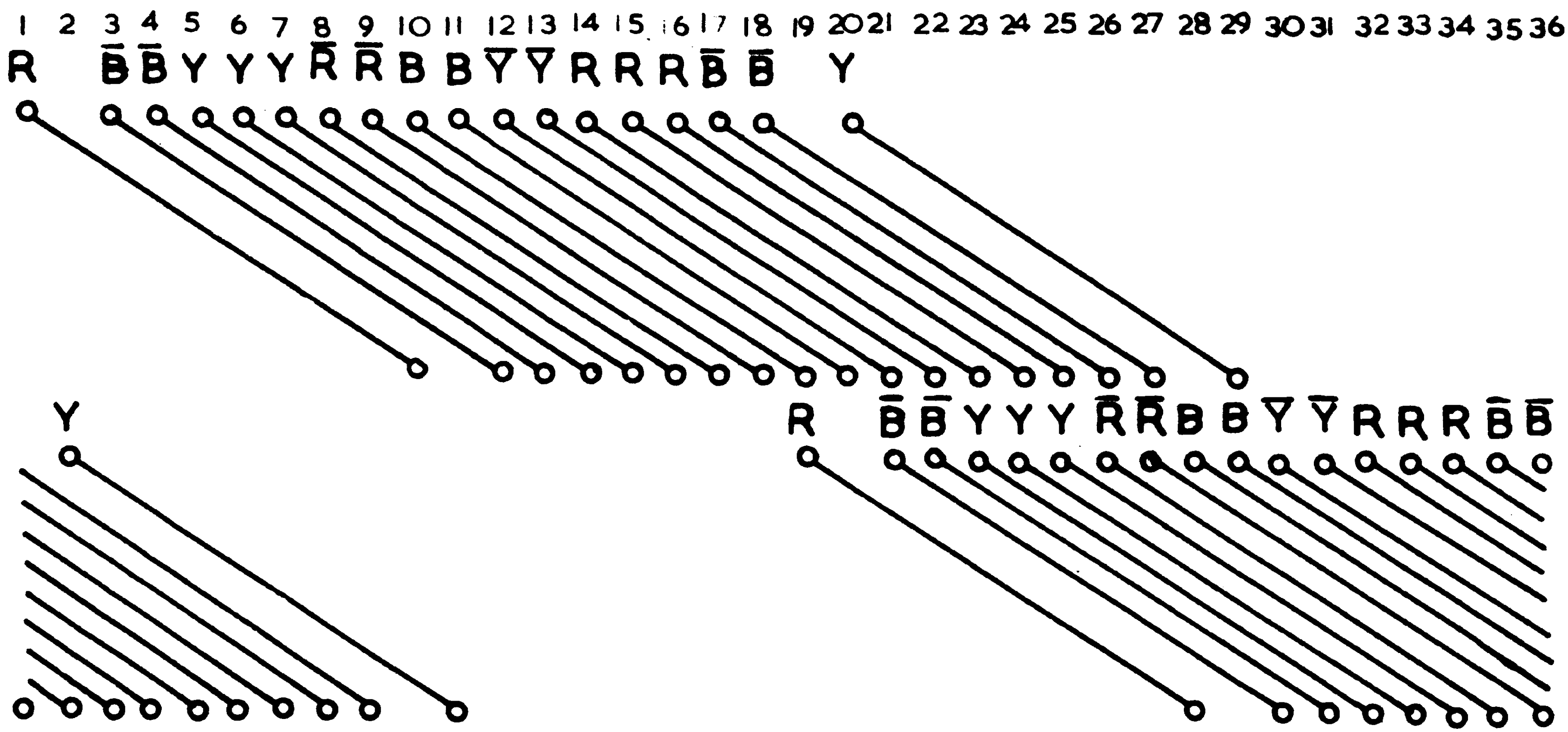


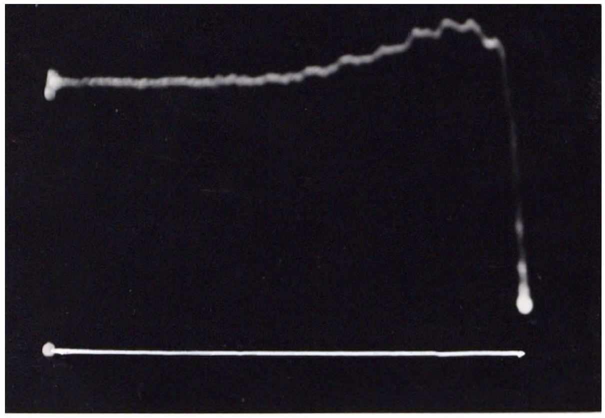
Figure 7. 4/6 Experimental Machine Winding.

Examination of Table 5 shows that a coil pitch of 9 slots is necessary to "pitch out" the high 8 pole content on the 4 pole setting. The analysis under these conditions is again shown in Table 5. The winding factors produce a gap flux density ratio $B_4/B_6 = 1.0$ when used with a star to star two parallels switching arrangement. The machine is suitable for fan drive applications.

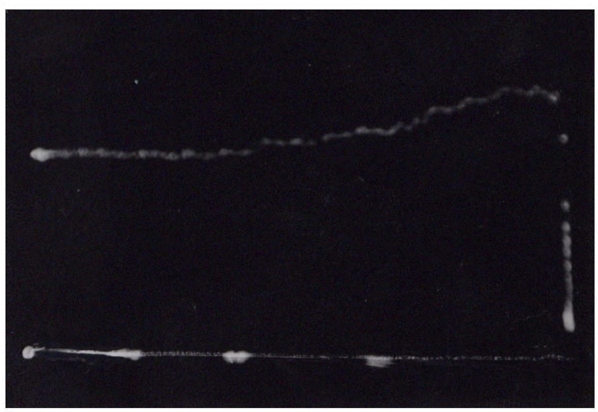
6. AN EXPERIMENTAL MACHINE

A machine was constructed to the specification of the second winding example in a frame normally used for 2 horse-power 4 pole machines. The frame of the machine was supported in bearings which were concentric with the machine shaft. The stator was then constrained with a short steel beam which carried a strain gauge. The strain gauge therefore produced changes in resistance which were proportional to the shaft output torque. The strain gauge was connected in a suitable bridge circuit, and the output from this circuit was amplified and applied to the X-plates of a cathode ray oscilloscope. A signal proportional to speed was then applied to the Y plates of the oscilloscope. Thus if the machine was allowed to accelerate slowly, the trace on the oscilloscope corresponded with the torque-speed curve of the machine. Traces for the 4 and 6 pole settings are shown in Figure 8. In each case the scales are arbitrary but they indicate satisfactory torque performance. Test figures for the machine are shown in Table 6, and since the output is 1.8 horse-power on the 4 pole setting, at the same temperature rise as a single speed 2 horse-power machine, the performance is satisfactory.

The current balance produced by the machine was exceptionally good.



4 pole Torque Speed Curve.



6 pole Torque Speed Curve.

Figure 8. The Torque Speed Curves of the Experimental Machine.

The input currents were always within 1.0% of each other on the 6 pole setting and within 3% of each other in the worst case on the 4 pole setting.

TABLE 6.

Results taken at 410v. line.

	4 poles	6 poles
Peak torque	172 lb-ins.	68 lb-ins.
Starting torque	138 lb-ins.	45 lb-ins.
Full load power	1.8 H.P.	.51 H.P.
Full load speed	1430 r.p.m.	970 r.p.m.
Full load power factor	.82	.54
Winding temperature rise	50°C	48°C
Efficiency	78%	57%
Full load current	2.9 amp.	1.75 amp.
Starting current	16.2 amp.	5.5 amp.

7. CONCLUSIONS

Completely balanced windings for 4/6 and 6/8 pole machines have been shown to be possible. The other close-ratio machines which are commonly constructed include 8/10 and 10/12 pole ratios. These latter machines can be balanced only on one of the pole numbers. However, this is very helpful to the designer, since if a segment is inherently balanced on one pole number a second balanced segment can be positioned so as to almost completely cancel the negative phase sequence component on a second pole number; whereas if the interleave is used to reduce the unbalance on two pole numbers a compromise position for the interleave has to be used.

The analysis of the balanced machines using simple current sheet analysis is difficult since the winding irregularities themselves produce the balanced conditions. However, the work of Chapter II can be used to provide the broad outline of the winding design, and thus reduce the work which is necessary using the digital approach.

The balancing system using three segment interleave is only one way to produce the desired result. There are almost certainly others. For example, if a certain selection of the slots is ascribed to one block of the winding, a trial and error process using the vector diagrams for both pole numbers simultaneously may eventually produce a winding which is balanced on each pole number. It may be possible to programme a computer to do this work by using a random number generator. This will form the basis of future work on close-ratio machines.

The balancing system has been used to produce two block machines only, clearly it could be extended to the other machines, and this again will form the basis of future investigations.

PRINCIPAL REFERENCES.

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THE LOGMOTOR—A CYLINDRICAL BRUSHLESS VARIABLE-SPEED INDUCTION MOTOR

By

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AND

BRUSHLESS VARIABLE-SPEED INDUCTION MOTORS USING PHASE-SHIFT CONTROL

By

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SUMMARY

In a conventional induction motor the synchronous speed at the rotor surface is set by the phase difference between currents in adjacent slots and the spacing of the slots. The synchronous speed may be increased by changing the phase of the current in each slot progressively so as effectively to stretch the poles. The paper describes a transformer arrangement giving a multi-phase output, which can provide such a 'phase-stretching' system to be used to supply the stator of a squirrel-cage induction motor. The transformer is similar to a conventional phase-shifting transformer, except that the pitch of each coil of the primary is proportional to the log of its distance from a fixed point in the system. The primary and secondary of the transformer are analogous to the C and D scales of a slide-rule, and movement of the primary relative to the secondary results in a variation of the phase increment between adjacent secondary coils. The secondary is connected to the stator winding of the motor in such a way as to provide a uniform-velocity field in the latter, the velocity being varied by adjusting the position of the primary of the transformer. The method of construction of an experimental machine is described and test results over a speed range of 4 : 1 are included.

(1) INTRODUCTION

In an earlier paper,[1] the concept of the linear induction motor was described by supposing that a conventional cylindrical stator could be cut open and rolled out flat. The synchronous rotational angular velocity $2f/n$, where n is the number of poles around the periphery and f is the supply frequency, is then replaced by a synchronous linear velocity $u_s = 2pf$, where p is the pole pitch. The operation was demonstrated by using a model of a stator made of rubber. The model was split axially and held together by a zip-fastener. When the latter was opened, the stator could be rolled out flat. In this condition the model was also capable of being stretched. If the model were stretched in the direction of the rotor velocity u_r so that the length of the model was increased to k times the initial length, the new pole pitch would be kp and yield a new synchronous velocity $u_s = 2kpf$. Thus, variations in k would produce variable speed.

Various attempts[2,3] have been made to obtain variable speed by using this 'pole-stretching' technique. Usually the windings of the various poles have been mounted on separate iron structures movable relative to one another. This difficulty was avoided in the authors' variable-speed motors[4,5,6] by adjusting the angle between the direction of motion of the field and the direction of motion of the rotor, so as to give the effect of variable pole pitch as seen from a point on the rotor during rotation. All of these methods depend on changing the actual or apparent spacing of conductors carrying currents of fixed relative phase.

Alternatively, if a fixed structure is used, with conductors in fixed positions relative to one another, 'pole stretching' can be achieved by varying the phase displacement between the currents in adjacent conductors. Thus, referring to Fig. 1, if ϕ_n is the phase of the current in slot n, $x_n - x_{n-1} = \delta x$ is the slot pitch, and $\phi_n - \phi_{n-1} = \delta\phi$, the synchronous velocity is

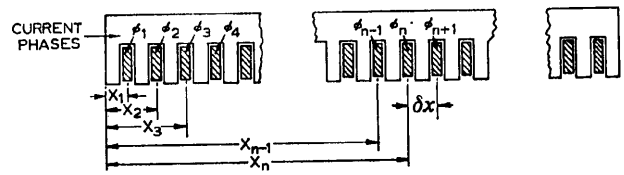

Fig. 1.—The problem of 'phase stretching'.

$$u_s = 2\pi f \frac{\delta x}{\delta\phi}$$

If u_s is to be constant along the length of the block, $\delta\phi$ must be everywhere the same; but if $\delta\phi$ can be varied simultaneously along the whole length of the block, then u_s can be varied. The pole pitch of the winding is $\pi\delta x/\delta\phi$, and is 'stretched' by changing $\delta\phi$. This method might be called 'phase stretching' to distinguish it from 'pole stretching'. The paper is concerned with the development of a 'phase-stretching transformer' designed to supply currents of appropriate phase to a stator block with the object of providing a velocity variation over a wide range.

(2) A PHASE-STRETCHING TRANSFORMER

A conventional wound-rotor induction motor in which the rotor is held at standstill constitutes a conventional rotary phase-shifting transformer, provided that the angular position of the rotor can be adjusted. Such machines usually carry a 3-phase wound rotor. If a squirrel-cage machine were to have one end-ring removed, the free ends of the m rotor bars would provide an m-phase system of a.c. supply when the stator was energized to produce a rotating magnetic field in the air-gap. With the rotor fixed in position the phase of the induced e.m.f. in one rotor bar would be fixed in relation to the phase of the stator supply. A change in the angular position of the rotor would change the phase of the induced e.m.f. in each rotor bar by the same amount, the phase difference between the e.m.f.'s in adjacent bars remaining the same.

The structure just described is characterized by having uniform slotting of both rotor and stator and by having a stator winding of constant pole pitch. An alternative arrangement is shown in the developed diagram of Fig. 2(*a*) and (*b*). Here, both rotor and stator blocks have 80 slots, numbered 10–89 for convenience, but the slots, unlike those of the conventional phase-shifter, are arranged in a non-uniform manner such that the displacement of a slot from the left-hand edge is proportional to log $(x/10)$, where x is the slot number. For the sake of clarity, not all the slots are drawn in this Figure; the first 20 are shown

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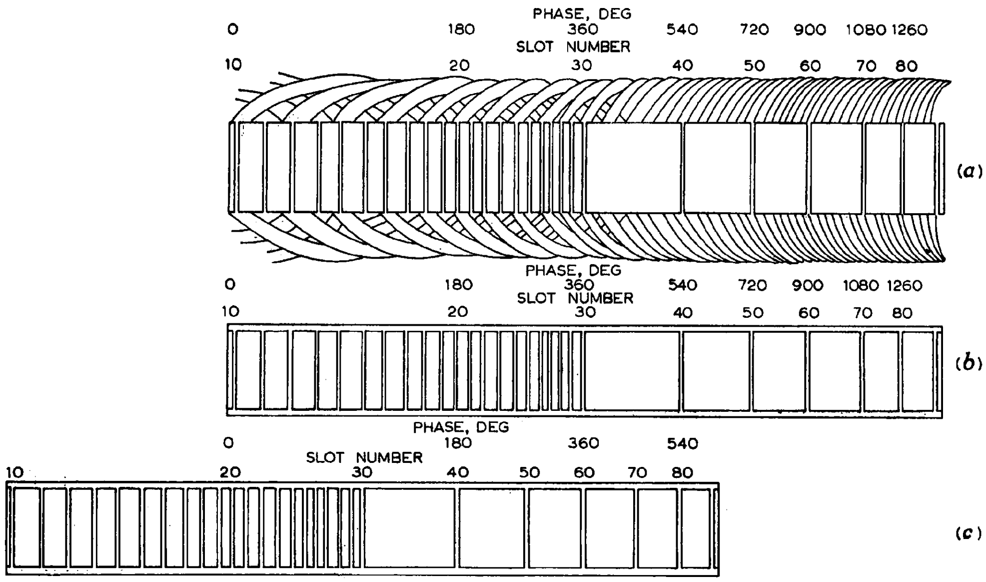


Fig. 2.—The principle of 'phase stretching'.
(*a*) Primary. (*b*) Secondary. (*c*) Secondary after displacement.

in full, thereafter every tenth slot is shown. The stator or primary winding is an exact copy of a conventional stator winding with 80 slots, like-numbered slots carrying corresponding coil sides. The only difference lies in the spacing of the slots. The stator shown in Fig. 2 carries 8 pole pitches, so that the phase increment per slot of the currents in the stator is constant at 18° along the block, but the pole pitch decreases from left to right. With good coupling the rotor or secondary bars will carry equal and opposite currents, and will therefore also exhibit a phase increment of 18° per bar. If one end-ring is removed, as before, secondary-bar e.m.f.'s will exhibit an 18° increment per bar.

Although Fig. 2 is introduced as a developed version of a cylindrical structure, it could equally well relate to a linear structure, and it is convenient to consider it as such for the moment. With this proviso, Fig. 2(*c*) shows the result of displacing the secondary relative to the primary, so that bar 20 of the rotor is now opposite slot 10 on the stator. Because of the logarithmic spacing of the slots, which corresponds to the relative spacings of the incremental tenths on a slide-rule, this displacement brings bar 40 on the secondary opposite slot 20 in the primary, bar 60 opposite slot 30, and so on. Thus, discounting overlap, any 20 bars on the secondary now span 10 slots on the primary, and the incremental phase change per bar on the secondary is therefore 9° instead of 18°. Similarly, if the rotor is displaced from the centre position through an equal distance in the opposite direction, the phase increment per bar on the secondary becomes 36°. Intermediate, or greater, displacements yield corresponding results, as may be checked by using the C and D scales of a slide-rule as a model, or by reference to the analysis of Section 12.1.

Thus, in principle, a winding designed to produce a travelling field with pole pitch decreasing according to a logarithmic scale, mated with a movable secondary block carrying conducting bars arranged on a similar logarithmic scale, provides a multi-phase output from those bars, and movement of the secondary relative to the primary yields continuous variation of the phase increment between adjacent output points. This process has been called 'phase stretching' for want of a better name. The multi-phase output is unusual in two respects. First, owing to variable overlap, the number of energized output points, or phases, is variable, and secondly, in proceeding from the first energized point progressively through the phases to the last, the total phase rotation in general will not be a multiple of 360°. These peculiarities may limit general application of the device but do not hinder its use in variable-speed motors.

(3) APPLICATION TO VARIABLE-SPEED MOTORS

Fig. 3 shows in developed form at (*a*) and (*b*) a phase-stretching transformer as described above, used to supply the stator (*c*) of an induction motor with a squirrel-cage rotor (*d*). The output points of the transformer section are cross-connected to uniformly spaced bars in the motor stator slots. This stator therefore carries currents which show a uniform phase increment per slot. The transformer is capable of movement relative to the secondary and is shown in Fig. 3 in two positions, A and B. The stator is drawn only in part in position A for the sake of clarity. With the transformer primary in position A, the phase increment in the motor stator can be seen to be 18°. When the transformer primary is moved to the position B, the phase increment is seen to be 9°. Variation of phase increment produces a corresponding change in the synchronous speed of the stator field, and hence of rotor velocity. In the linear form, 'overlap' will not cause any particular difficulty; it will merely result in part of the stator being un-energized and part of the primary unused.

The diagram can, however, equally be related to a cylindrical form by first folding the primary on to the secondary, face to face; similarly folding the motor rotor on to the motor stator; and then rolling both sections into cylinders, the transformer primary and motor rotor being inner members, and the transformer secondary and stator being outer members. The general form of the machine is shown in Fig. 4. In the cylindrical machine, overlap of primary and secondary can be harmful because the small-pole-pitch end of one will be mated with the large-pole-pitch end of the other. This difficulty can be overcome in a number of ways, for example, by omitting part of the primary winding so that the 'overlap' is always un-energized. Speed adjustment is obtained by rotating the inner member of the right-hand section, which is the wound primary of the phase-stretching transformer.

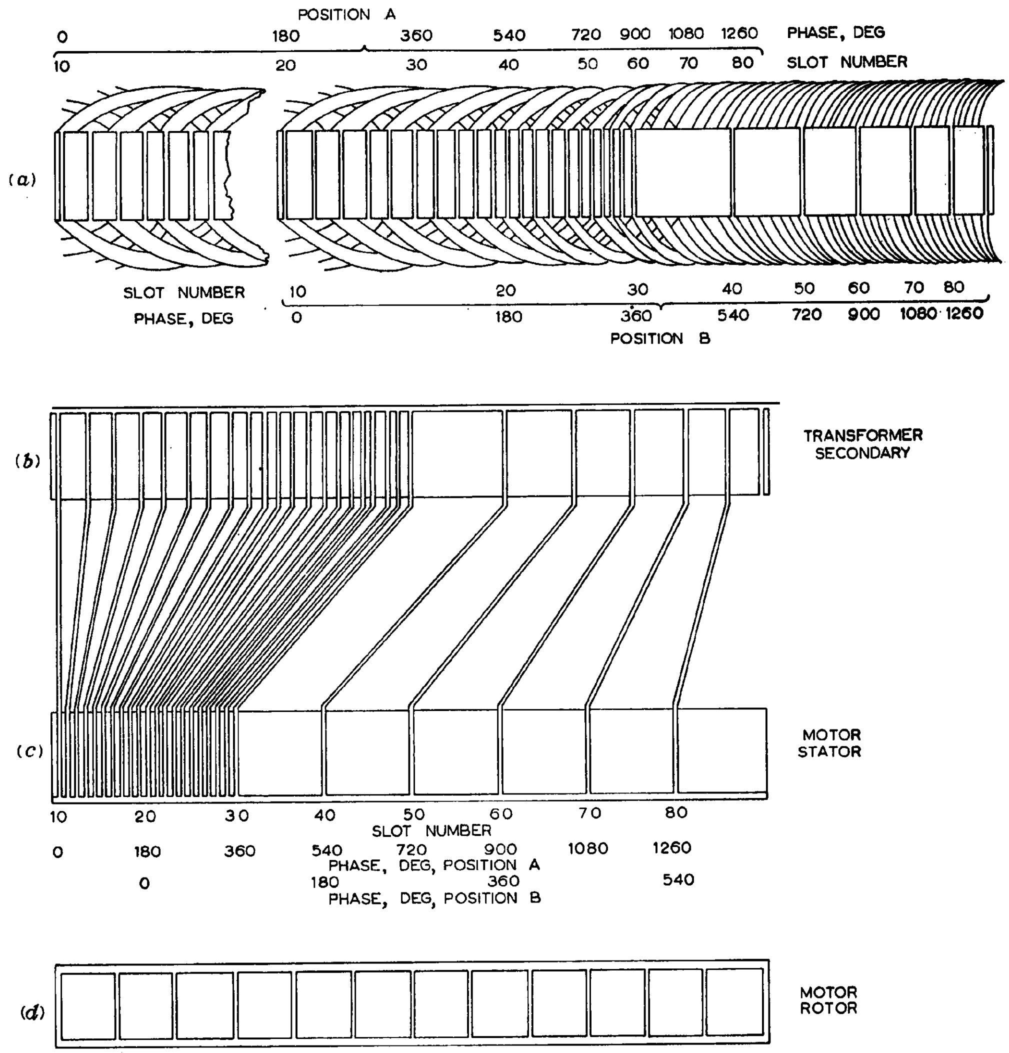


Fig. 3.—Principle of the log motor.

(*a*) Transformer primary in two positions.
(*b*) Transformer secondary.
(*c*) Motor stator.
(*d*) Squirrel-cage rotor.

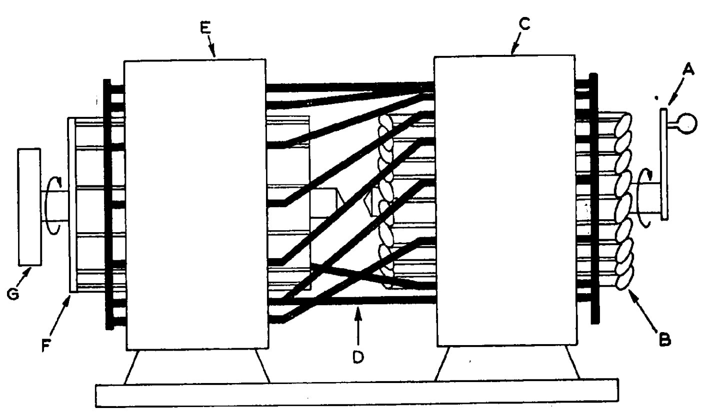


Fig. 4.—Form of construction of transformer and motor as a unit.

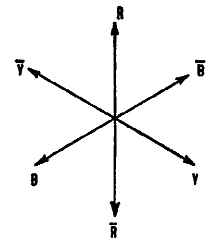
A Speed-adjusting handle.
B Transformer primary.
C Transformer secondary.
D Interconnectors.
E Motor stator.
F Squirrel-cage rotor.
G Output shaft.

(4) EXPERIMENTAL MACHINE

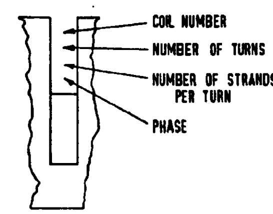
The winding plan of the first experimental machine is shown in developed form in Fig. 5. It will be seen that all the iron members are uniformly slotted but that the windings are distributed in the slots in a non-uniform manner.

(4.1) Primary Winding

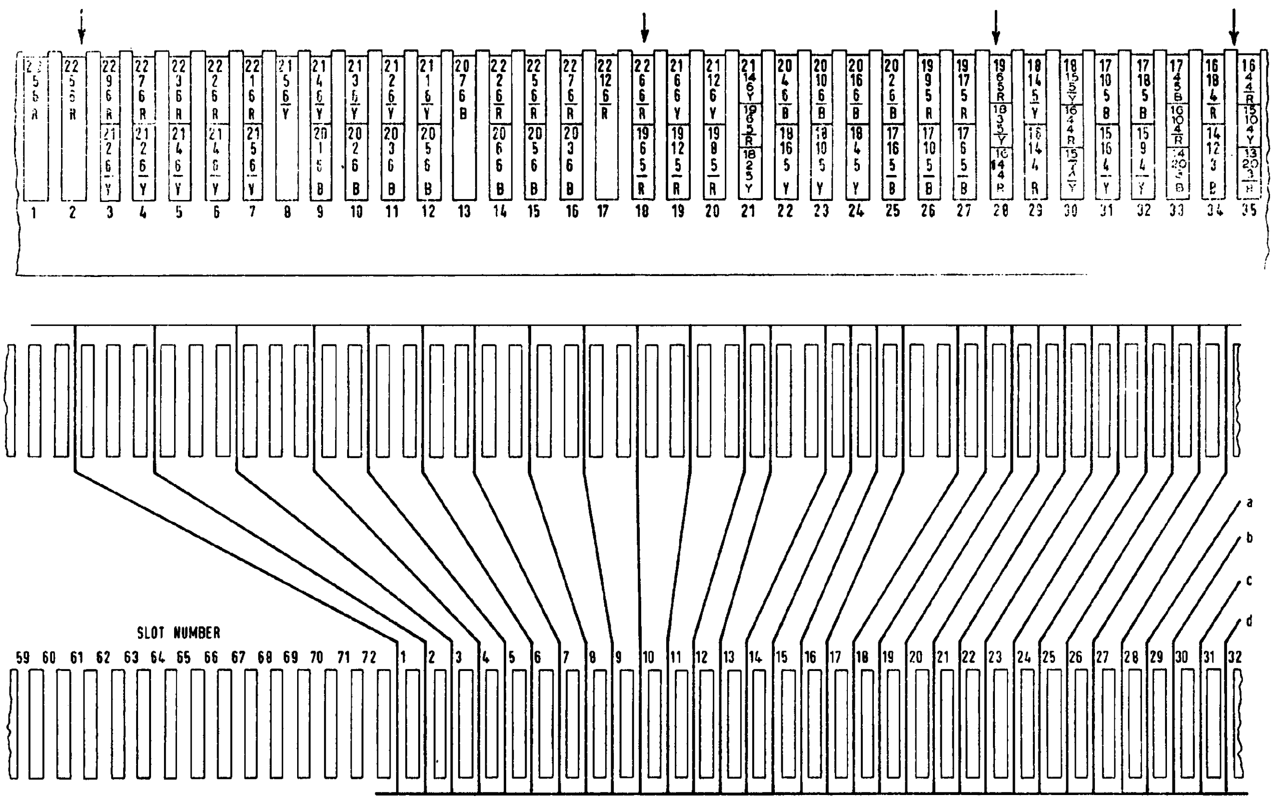
The transformer primary is wound in 54 slots to provide 8 poles, so that the total phase change along it is 1 440°. The logarithmic distribution required is obtained by mixing con-



PHASE CHART



KEY TO PRIMARY DIAGRAM



(*a*)

Fig. 5.—Winding plan of a logarithmic motor.

ductors of different phases in the various slots. If the net current in slot n is arranged to have phase

$$\phi_n = \frac{1\,600^\circ}{10} \text{antilog}_{10} \frac{n}{54}$$

with n ranging from 0 to 54,* then ϕ_n ranges from 160 to 1 600, i.e. a range of 1 440° equal to 8 poles. The positions on the winding corresponding with pure 'red-phase' currents, either positive or negative, are shown on the diagram. At the right-hand end the winding is almost exactly a 'one slot per pole per phase' winding, but the degree of 'spread' increases greatly towards the left, until at the left-hand edge it is equivalent to 10 slots per pole per phase. The winding used is a double layer winding and each coil has the same number of turns. In order to make maximum use of the slot depth, the number of strands per conductor is increased from right to left. Because the 'spread' increases from right to left, the current per slot increases from left to right. Section 12.2 shows that the

* In a cylindrical machine with 54 slots, slot 0 and slot 54 are one and the same.

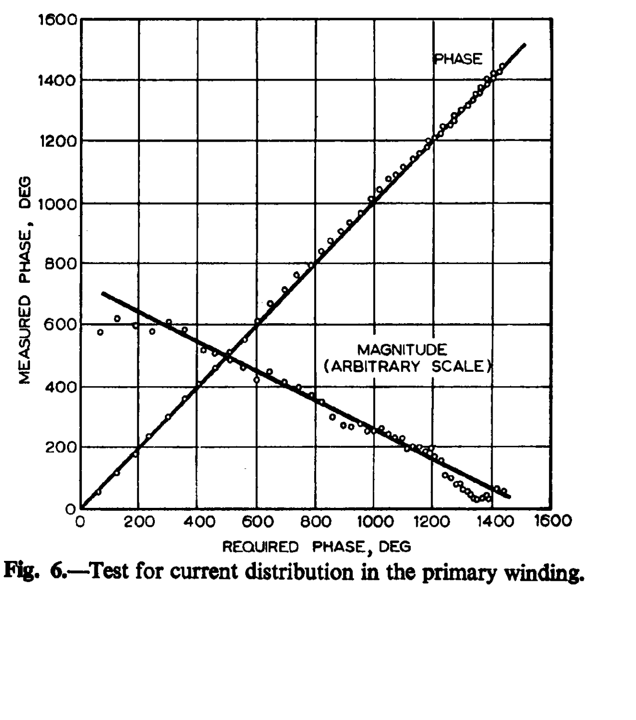


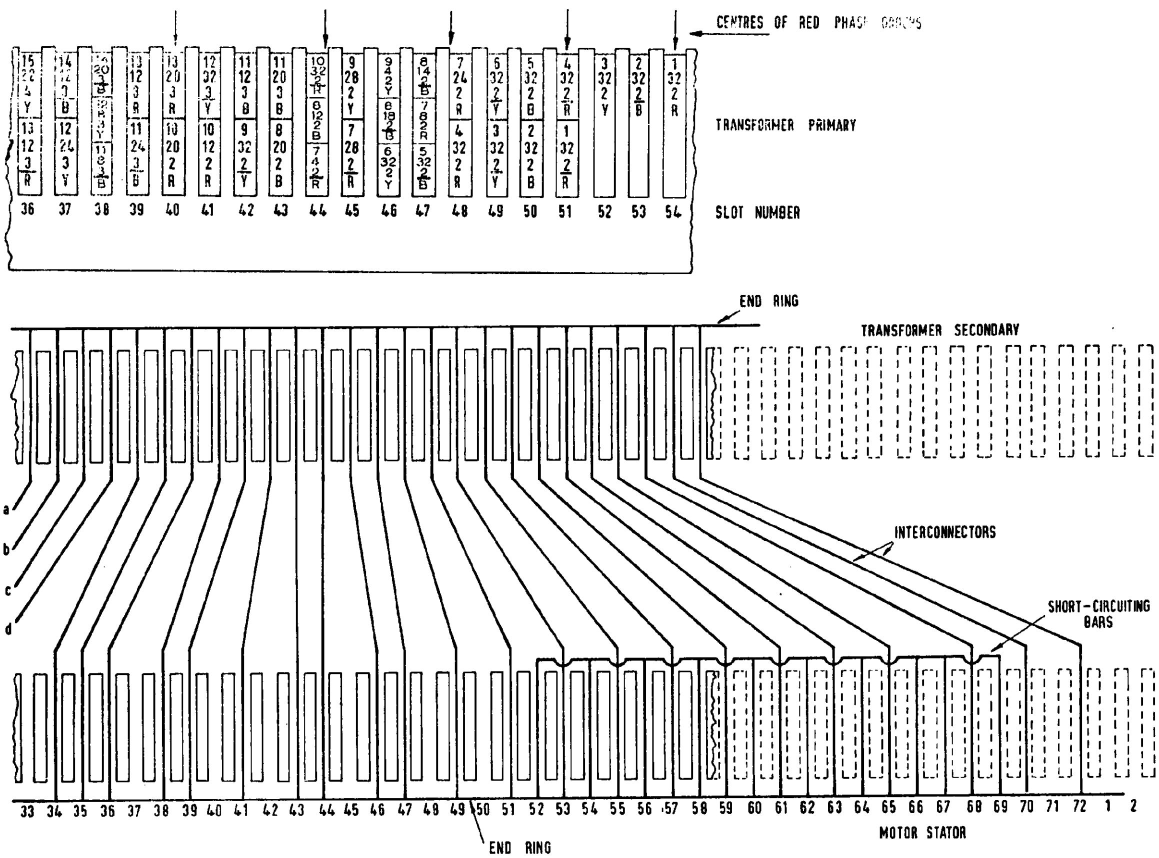
Fig. 6.—Test for current distribution in the primary winding.

appropriate formula for the current density, J_T, at position y from the start of the winding is $J_T = J_S \alpha \lambda \varepsilon^{-\lambda y}$.

Fig. 6 shows the measured phase and amplitude of the currents in the slots plotted against calculated values of ϕ_n. The phase error nowhere exceeds 30°. The amplitude shows the right kind of variation except for the beginning and end, where current amplitudes are low owing to the underlap and overlap of the two layers of the winding. Tapping points were provided in the vicinity of each of the 'red-phase' markers to enable less than 8 poles to be energized if desired.

when this occurs the pairing process is best performed by taking successive values of s and finding the corresponding value of r. The resulting set of interconnections is shown in Fig. 5.

Each interconnecting bar was made up of 20 insulated conductors of 0·116 in diameter. They were arranged accurately in each slot in ten layers of two; each bar was partially transposed in passage from one block to the other, so that the conductors at the top of the slot in one block were at the bottom of the slot in the other. End-connections were made by soldering to stout copper rings.



(*b*)

Fig. 5 (*continued*).

(4.2) Transformer Secondary and Motor Stator Interconnections

The secondary and stator blocks each have 72 uniformly spaced slots, but the interconnections between them are required to transform a logarithmic distribution of phase into a linear one. Starting at the left hand of the motor stator, slot 1 can be assigned a phase of 160° to correspond to slot 1 of the transformer primary. The phase increment per slot along this block is everywhere 1 440°/72 = 20°, and hence the phase in slot m is $\phi_r = 20r + 160r$.

The phase ϕ_s in slot s of the transformer secondary is

$$\phi_s = 160° \text{ antilog}_{10} \frac{s}{72}$$

By listing ϕ_r and ϕ_s for slots 1 to 72 and starting at the left-hand end, it is easy to choose the value of s which corresponds most nearly in phase to each value of r. At first, each value of r will find an individual partner in s, with empty slots intervening between successive values of s, but, as the process proceeds, successive values of r will correspond with the same value of s;

*

Fig. 7 shows the result of a test of the transformer and motor stator, with the motor rotor removed. The amplitude and phase of the currents in the stator bars are shown as a function of the stator slot number. For this test the starting-points of the primary and secondary windings were aligned. The phase is seen to be a linear function of slot number apart from small errors not exceeding 25°. The amplitude is reasonably constant, except at the right-hand end, where it shows peaks at fairly regular intervals. These peaks are due to variation of slot alignment between the primary and secondary of the transformer. The ratio of slot numbers is 54 : 72, which results in slot alignment occurring every fourth stator slot. The effect is greatest at the right-hand end, because here the pole pitch is short and the slotting relatively coarse. The effect could have been reduced by skewing the primary slots, but a better solution, and one to be used in later models, is to use equal numbers of slots.

Fig. 8 shows curves similar to those obtained in Fig. 3 with the primary displaced to the left of the mid-point through an angle corresponding to log 2. It may be seen that there is a considerable range over which the phase is linear with slot number, and of about the expected slope, i.e. 40° per slot. At the right-hand end, however, the phase curve changes slope and falls to about 4° per slot. This is due to the left-hand end of the primary in Fig. 3 overlapping the right-hand end of the secondary. Reference to the amplitude curve, however, shows that the

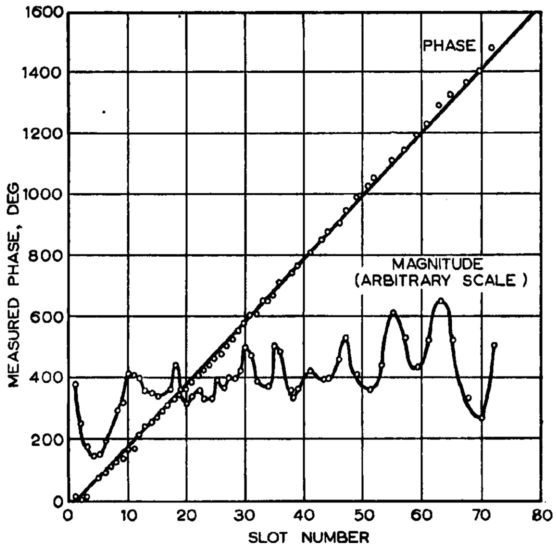


Fig. 7.—Test for current distribution in the motor stator.
Primary and secondary aligned.

current density is reduced by a factor of ten over this range; this part of the curve can therefore be ignored.

Fig. 9 shows the effect of displacement by the same amount in the opposite direction. Now there is a portion at the right-hand end which shows the expected phase slope, 10° per slot, but this is preceded by a portion showing 100° per slot, over which the current density is very high. Again this is due to overlapping, but this time the right-hand end of the primary, where current density is high, is overlapping the left-hand end of the secondary, where conductors are more widely spaced than on the motor stator. The high current density is therefore further increased. The early, incorrect part will therefore be dominant, and a low speed will result. If the high speed corresponding to the right-hand part of the curves is required, it will be necessary to disconnect that part of the transformer primary which overlaps the left-hand end of the secondary in Fig. 3.

These results show that the 'phase-stretching' transformer works as anticipated, and also that it can provide a configuration of current in a motor stator such as will produce an adjustable speed.

(5) MOTOR ROTOR

The rotor had 54 slots and each slot contained four insulated wires each 0·116 in diameter. Rotor resistance could be adjusted over a 4 : 1 range by connecting the appropriate number of rotor conductors to the end-rings.

(6) EXPERIMENTAL RESULTS

For the first test the transformer primary and secondary were aligned as in Fig. 3 (position A) to produce 8 poles on the motor stator. Torque/speed and other characteristics are shown in Fig. 10 with the intake current held constant. These curves show the characteristic shape associated with induction motors; maximum torque occurs at 7½% slip, which indicates a rather

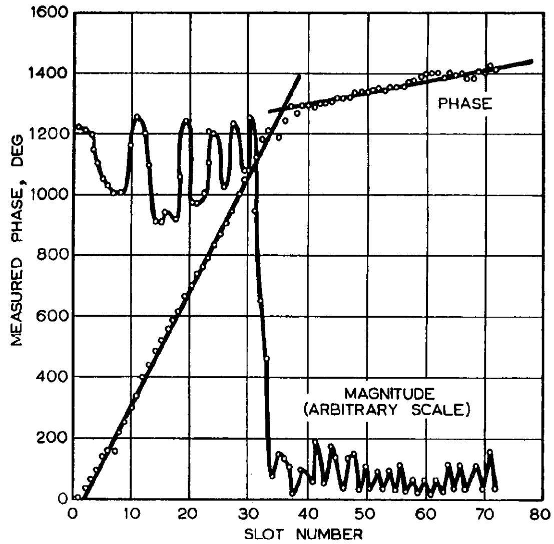


Fig. 8.—Test for current distribution in the motor stator.
Primary displaced to the left by an angle corresponding to log 2.

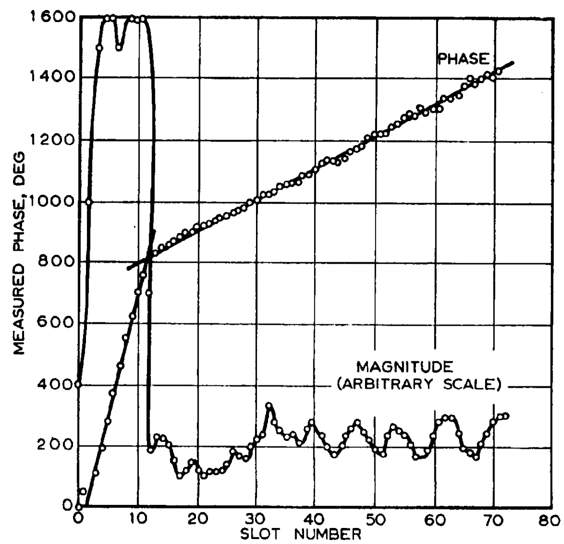


Fig. 9.—Test for current distribution in the motor stator.
Primary displaced to the right by an angle corresponding to log 2.

high rotor resistance. The peak efficiency is rather low, namely 63%. Curve (*b*) shows intake power, P_t, less 'synchronous power', $T\omega_s$, where T is the torque and ω_s the synchronous angular velocity. In a conventional machine this curve represents the sum of stator copper loss, iron loss and stray load loss. By measuring the intake power with the rotor removed from the motor stator, it is possible to obtain an estimate of copper loss per ampere squared; this corresponds to the short-circuit test on a transformer.

By measuring intake power with the rotor inserted, but with all rotor bars disconnected, it is possible to obtain an estimate of iron loss; this corresponds to the open-circuit test on a transformer. Both experiments have been done and used to estimate iron and copper losses in Fig. 10. These are shown at (*d*) and (*c*). Curve (*f*) is the sum of (*c*) and (*d*) and may be seen to approximate closely to (*b*). Thus the poor efficiency can largely be ascribed to conventional causes rather than to any unexplained property of the unconventional winding.

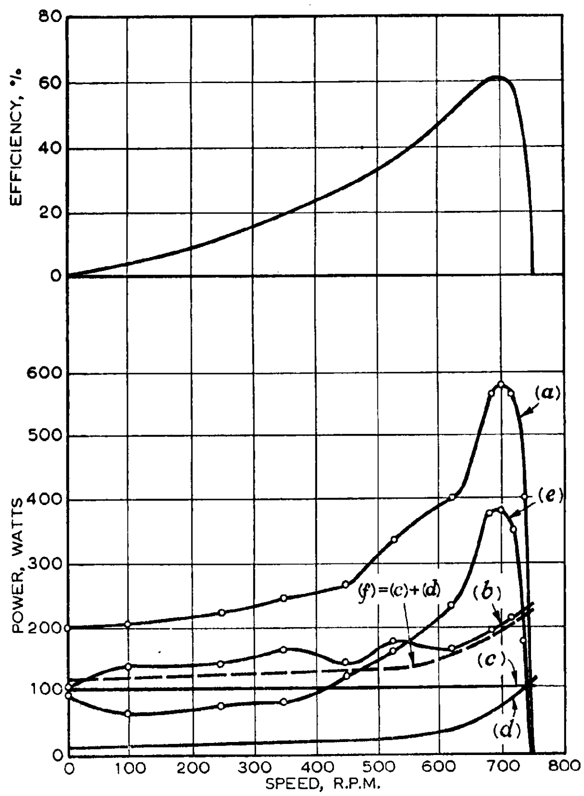


Fig. 10.—Brake test at constant current with primary and secondary aligned.

(*a*) Intake.
(*b*) Intake synchronous power.
(*c*) Estimated copper loss.
(*d*) Estimated iron loss.
(*e*) Synchronous power.

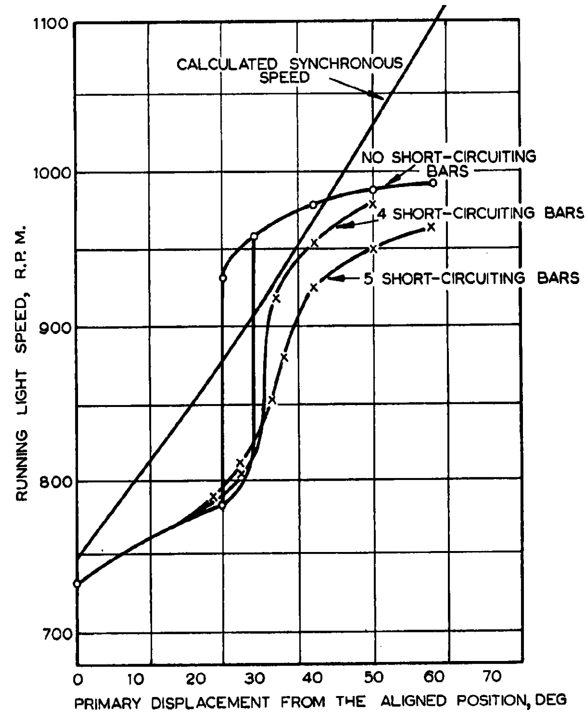


Fig. 11.—Speed variation with change in primary position.

(7) VARIABLE-SPEED WORKING

Fig. 11 shows the results of an attempt to obtain variable-speed working. For this test only the first four poles of the winding were connected, for the reason given in Section 3. As a result only part of the motor stator block was energized. The initial setting was for 750 r.p.m. as in the previous experiment. As the transformer primary was turned there was at first very little speed increase, but at about $+30°$ the speed suddenly increased to near 1 000 r.p.m. Further turning gave relatively little speed increase. The primary was next turned back towards zero, and there was at first very little speed reduction, until, at about 25°, the speed returned abruptly to about 750 r.p.m., thus forming a 'speed/angle' hysteresis loop. As might be expected, the machine showed a marked preference for the speeds corresponding to even numbers of poles. This was because, as the rotor rotated, it carried flux round from the end of the energized part of the stator block back to the beginning of the block. The phase of this 'carried-over' flux, relative to the current at the entry edge of the energized part of the stator block, depends on speed, and either additional positive or negative torque may result.[6]

The effect of a carried-over flux can be eliminated by arranging to destroy the rotor flux between exit from the trailing edge and entry to the leading edge of the energized part of the stator block. For this purpose addition bars were introduced into the motor stator. These are shown in slots 52 to 69 in Fig. 5, and are connected to the end-ring at the top end only. If the free ends of these are short-circuited to each other, then, provided that they are of low resistance, flux cannot readily penetrate the stator block between the extremities of the short-circuited section. The flux in a rotor element passing under this short-circuiting grid is therefore severely attenuated, and sensibly 'dead' rotor is fed continuously to the leading edge of the energized part.

Fig. 11 shows the result of short-circuiting four of these bars. It may be seen that the hysteresis effect has disappeared, though there remains some obvious distortion of the speed/angle relationship. Short-circuiting a further bar reduces the distortion, but at the expense of additional losses, since rather heavy currents flow in the short-circuited bars.

With these five bars connected the next experiment was designed to explore the extent of variable-speed working available. For this purpose the first six poles of the winding were used. The test was made with a fixed load-torque of 3·3 lb-ft, and at each speed setting the voltage supplied was adjusted to yield the maximum efficiency.

The results are shown in Fig. 12. It may be seen that speed variation can be obtained over a range of 4 : 1. Curve (*a*) of Fig. 12 shows the voltage required for the machine to run at maximum efficiency at constant torque. The variation is not excessive and the machine exhibits a close approximation to a 'constant torque at constant voltage' characteristic over this range, i.e. power proportional to speed. From 175 to 350 r.p.m. the efficiency falls as the speed is reduced.

The efficiency is about 50% over the range of speeds between 350 and 700 r.p.m. This rather low value can be ascribed to a considerable extent to excessive copper and iron losses, and it can confidently be expected that a redesigned machine in which these losses are reduced will yield much better efficiency. The efficiency falls off severely at the low-speed end, owing to at least two causes. First, the stator has 72 slots, so that at a synchronous speed of 175 r.p.m. there is less than 1 slot per pole per phase. Secondly, at the angle setting of $-200°$ only $3\frac{3}{4}$ poles of the original 6 are still operative, the remainder having 'disappeared' by overlapping the wrong end of the transformer secondary.

The speeds plotted lie rather randomly above and below the mean line. This is due to the fact that the efficiency/speed curve has a rather flat maximum, and the methods used did not permit of its accurate determination.

In these curves only certain speeds are represented by plotted points, but it should be recorded that speed variation is in fact continuous, in that any desired speed can be obtained at any load within the capability of the machine by appropriate adjustment of primary angle.

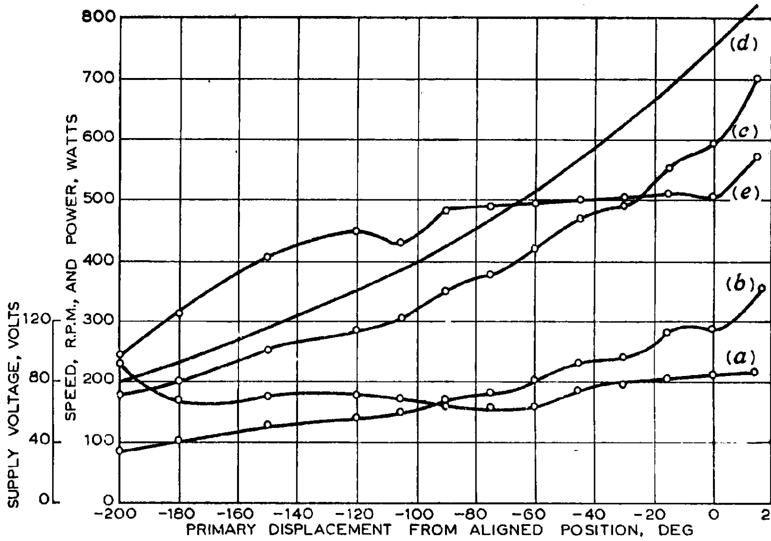


Fig. 12.—Load tests with variable speed (6-pole primary).

(a) Supply voltage.
(b) Output power.
(c) Speed for maximum efficiency.
(d) Calculated synchronous speed.
(e) Efficiency.

(8) FURTHER SPEED/TORQUE CURVES

Fig. 13 shows the results of a test taken at the aligned setting of the transformer primary with four poles connected. The results are plotted for constant supply voltage. The peak efficiency of about 50% occurs at approximately 20% slip. The torque characteristic shows that the motor is free from crawling torques or other vices. Curve (c) shows the intake minus $T\omega_s$, and curves (d) and (e) show estimated stator copper and iron losses of the motor stator and transformer. Curve (f) shows the difference between curve (c) and the sum of (d) and (e). This curve presumably represents additional loss due to the unorthodox nature of the machine. Various sources of additional loss have been described in earlier papers[5,6] and the magnitude of curve (f) is consistent with expectation. They will not be discussed further here, since the purpose of the paper, which is to introduce this new type of machine, has already been served by this first experimental model. In a later machine iron and copper losses will be reduced, and the residue will then be more accurately determinable.

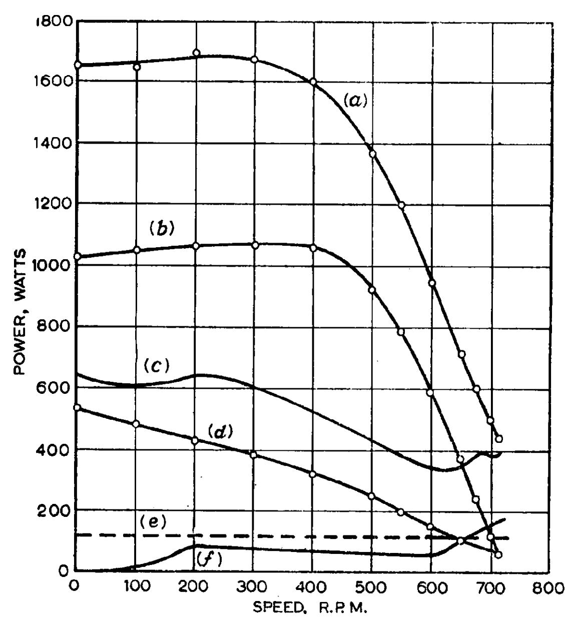


Fig. 13.—Brake test with primary and secondary aligned.

Short-circuited bars producing short-stator effect.
Constant voltage, 100 volts.

(a) Intake.
(b) Synchronous power.
(c) = (a)–(b).
(d) Estimated copper loss.
(e) Estimated iron loss.
(f) Unaccounted loss = (c)–(d)–(e).

(9) CONCLUSIONS

The first experimental machine has shown that the principle of phase stretching may be applied successfully to control the speed of an induction motor, provided that the latter has a break in its stator winding which allows the flux through any rotor tooth largely to be destroyed once per revolution. The induction-motor part of the machine is thus subject to the limitations imposed by 'short stator' working as outlined in Reference 5, which shows that for high efficiencies the number of stator poles must exceed four. In spherical motors, to which Reference 5 specifically relates, the stator is essentially divided into two blocks, so that at least four poles per block are required at the top speed, imposing a top speed limit of about 400 r.p.m.

In the logarithmic motor almost the whole of the stator periphery is available for use as a single block, and the potential top speed is doubled.

The first experimental machine had a poor efficiency, largely because the potential speed range chosen was high. This resulted in small pole pitches in the primary winding, with the attendant high value of magnetizing current.

Development is continuing with a view to formulating design criteria for this type of machine.

(10) ACKNOWLEDGMENTS

The authors' thanks are due to A.E.I (Manchester) Ltd., Motor and Control Gear Division, for supplying the necessary stampings in three days, and to Mr. A. Gledson for constructing the machine in two months.

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(12) APPENDICES

(12.1) Theory of the Logmotor

In Fig. 14, distances along the transformer primary winding are represented along the line AA′. Distances y along the

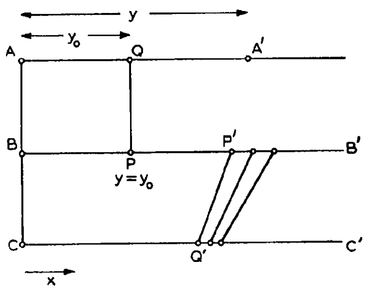

Fig. 14.—Geometrical representation of a logmotor.

secondary are measured along BB′, and distances x along the motor stator along CC′. The line ABC is taken as the origin for distances measured along BB′ and CC′. The origin of the primary is at O and is movable relative to B and C.

When O is opposite a point P on the secondary, where BP $= y_0$, the primary phase at A is $f(y - y_0) = \phi$, the function depending on the winding arrangement of the primary. Points such as P′ on the transformer secondary are connected to points such as Q′ on the stator, according to the relationship

$$x_{Q'} = g(y_{P'})$$

The rate of change of phase which occurs on the stator is then

$$\frac{d\phi}{dx} = \frac{d\phi}{dy}\frac{dy}{dx} = \frac{f'(y - y_0)}{g'(y)}$$

which may be written $d\phi/dx = a(y_0)$, since the phase rate must depend only on the displacement y_0 and be constant for any given value of y_0. Thus

$$f'(y - y_0) = a(y_0)g'(y) \quad . \quad . \quad . \quad . \quad (1)$$

Integrating $\qquad f(y - y_0) = a(y_0)g(y) + h(y_0)$

where $h(y_0)$ is the constant of integration.

Since

$$\frac{\partial}{\partial y_0}f(y - y_0) = -\frac{\partial}{\partial y}f(y - y_0)$$

$$a(y_0)g'(y) = -a'(y_0)g(y) - h'(y_0)$$

or

$$g'(y) + \frac{a'(y_0)}{a(y_0)}g(y) = \frac{h'(y_0)}{a(y_0)}$$

Integrating

$$g(y) = \exp - y\frac{a'(y_0)}{a(y_0)}\left[\int -\frac{h'(y_0)}{a(y_0)}\exp\frac{a'(y_0)}{a(y_0)}y dy + k(y_0)\right]$$

$$= \frac{h'(y_0)}{a'(y_0)} + k(y_0)\exp - \frac{a'(y_0)}{a(y_0)}y$$

$k(y_0)$ being the constant of integration.

$g(y)$ must not be a function of y_0 since the interconnectors remain fixed as the primary is moved.

$$h'(y_0) = -x_0a'(y_0), \text{ where } x_0 \text{ is a constant.}$$
$$a'(y_0) = \lambda a(y_0), \text{ where } \lambda \text{ is a constant.}$$

Therefore $a(y_0) = A\varepsilon^{\lambda y_0}$, where A is a constant.

$$k(y_0) = -\alpha, \text{ where } \alpha \text{ is a constant.}$$

Then $\qquad g(y) = x = x_0 - \alpha\varepsilon^{-\lambda y}$

and $\qquad h(y_0) = -x_0A\varepsilon^{\lambda y_0} + \Phi_0$, where Φ_0 is a constant.

Thus
$$\begin{aligned}\phi &= f(y - y_0) = a(y_0)g(y) + h(y_0)\\ &= A\varepsilon^{\lambda y_0}(x_0 - \alpha\varepsilon^{-\lambda y}) - x_0A\varepsilon^{\lambda y} + \Phi_0\\ &= \Phi\varepsilon^{-\lambda(y-y_0)} + \Phi_0, \text{ say}\end{aligned}$$

Now
$$\varepsilon^{-\lambda y} = \frac{x_0 - x}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and therefore

$$\phi = \Phi\frac{(x_0 - x)}{\alpha}\varepsilon^{\lambda y_0} + \Phi_0$$

If ϕ is a phase lead, then all quantities vary as $\varepsilon^{j(\omega t+\phi)}$. Constant phase along CC′ appears to an observer, who moves a distance δx in time δt, such that $\omega\delta t + \frac{d\phi}{dx}\delta x = 0$

or
$$\frac{dx}{dt} = -\frac{\omega}{d\phi/dx} = u_x, \text{ say}$$

and
$$u_x = \frac{\omega\alpha}{\Phi}\varepsilon^{-\lambda y_0}$$

Similarly
$$u_y = -\frac{\omega}{\partial\phi/\partial y} = \frac{\omega}{\lambda\Phi}\varepsilon^{\lambda(y-y_0)}$$

(12.2) Current Density

It is considered desirable to have a constant surface current density J_s in the stator of the motor. The current contained within a length δx on the motor stator, $J_s\delta x$, is distributed in length δy in the transformer secondary.

Fig. 15 illustrates this by imagining the extremities of the lengths δx and δy to be joined by two of the interconnecting bars. Thus the current density necessary in the transformer

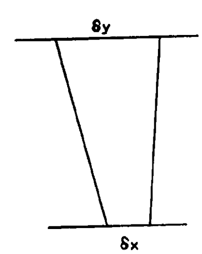

Fig. 15.—Current transference by interconnector.

secondary is $J_s\delta x/\delta y$, where δx and δy are related by eqn. (2) of Section 12.1. Thus $x_0 - x = \alpha\varepsilon^{-\lambda y}$ and $dx/dy = \alpha\lambda\varepsilon^{-\lambda y}$ whence the transformer current density, J_T, is $J_s\alpha\lambda\varepsilon^{-\lambda y}$.

[The discussion on the above paper will be found on page 108.]

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May 1960

BRUSHLESS VARIABLE-SPEED INDUCTION MOTORS USING PHASE-SHIFT CONTROL

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SUMMARY

The paper describes a method of 'pole-stretching' for induction machines in which part of the stator windings are fed directly from the mains supply and part from phase-shifting transformers. Variation of the angle of phase-shift enables continuous speed control to be effected. An experimental machine is described, the test results from which demonstrate that speed control with constant efficiency can be obtained over a speed range of 1·5 : 1. The limitations on the range of such machines imposed by the necessary condition that the stator be discontinuous are discussed, and a method of extending the speed range is then described. Machines of this type may be designed to run with a number of discrete synchronous speeds, in which case no phase-shifting transformers are necessary and speed change is effected by external switches only. The historical link between this type of machine and the spherical motor is outlined.

(1) INTRODUCTION

A new type of brushless variable-speed induction motor, called the 'logmotor', is described in a separate paper.[1] This paper outlines a method of pole-stretching whereby the phase increment per slot of the stator current may be changed in order to change the synchronous speed of the travelling field. The current in each slot of the motor stator is supplied along a separate lead from a special type of transformer. A cylindrical machine such as the logmotor, whose synchronous speed is capable of continuous variation, must necessarily have an arc of the stator which is not energized (see Section 7 of Reference 1). The present paper is concerned with an alternative method of changing the phase increment per slot, which leads to the development of a type of cylindrical brushless variable-speed induction motor quite different from the logmotor.

(2) THE PRINCIPLE OF PHASE MIXING

Fig. 1 shows a section through the stator of the new induction motor; a sector has been removed for the reason just discussed. The stator slots contain three separate 3-phase windings A, B and C, the connections to which are shown in Fig. 2. Windings A and B are supplied from 1 : 1 phase-shifting transformers whose primary windings are in series with winding C, so that the current flowing in A and B is substantially of the same magnitude as that in C. The phase of the currents in A may be advanced or retarded by means of phase-shifter 1. Phase-shifter 2 is mechanically coupled to phase-shifter 1 in such a way that, when the latter is set to advance the phase of the current in A by an angle θ, phase-shifter 2 retards the current in B by the angle θ and vice versa.

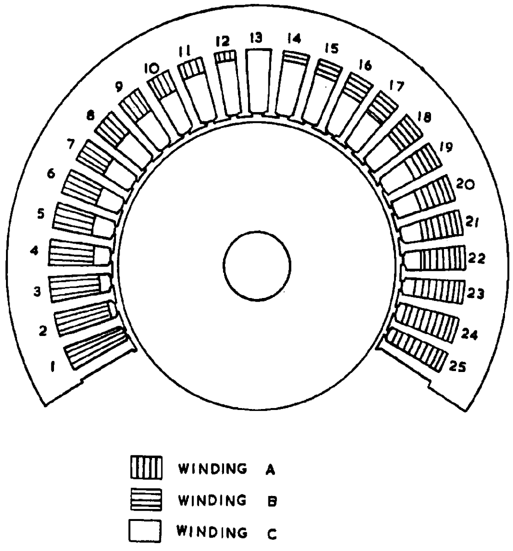

Fig. 1.—Arrangement of stator windings in a pole-stretching motor.

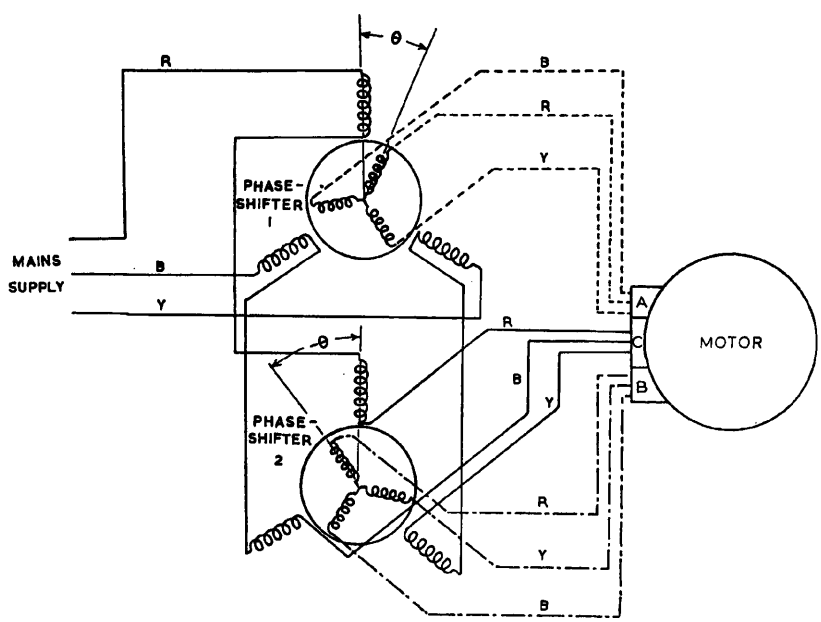

Fig. 2.—Connections between windings in a pole-stretching system.

Each of the windings A, B and C is arranged in the manner of a conventional 3-phase winding so as to produce phase progression around the periphery. The number of conductors of a particular winding in each slot is graded as indicated by the shading, so that the magnitude of the stator current loading produced by any one winding varies around the periphery. The windings are so arranged that when both phase-shifters are set

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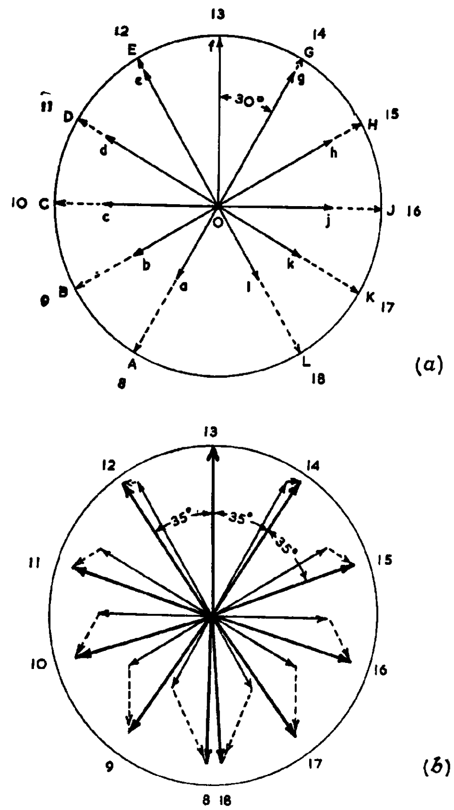

Fig. 3.—Vector diagrams for effective current per stator slot.
(a) $\theta = 0$.
(b) $\theta = 45°$.

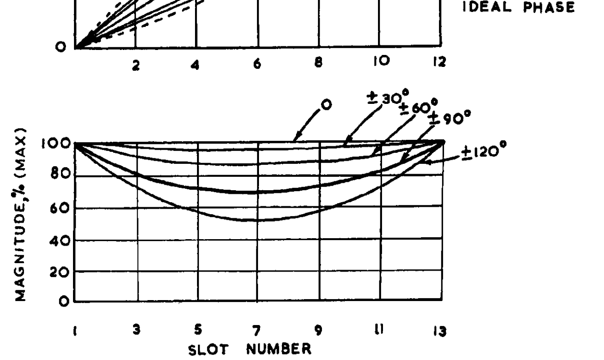

Fig. 4.—Phase and magnitude of effective current per slot obtained by phase-mixing.
Full lines indicate ideal phase.

to $\theta = 0$ the effective stator-current distribution is identical with that of a sector of a conventional induction motor. Accordingly, the vectors which represent the effective slot currents are uniformly phase-spaced by the appropriate amount. For example,

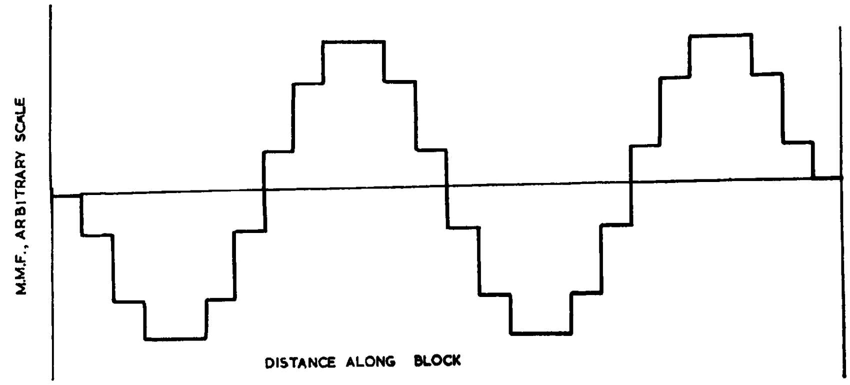

Fig. 5.—M.M.F. diagram for $\theta = 0$.

if the windings shown in Fig. 1 are arranged to have two slots per pole per phase, the vector diagram for the condition $\theta = 0$ is shown in Fig. 3(*a*); for the sake of clarity in the Figure, only slots 8–18 are included. The contributions from winding C are represented by the full lines Oa, Ob, Oc, . . ., etc., those from winding A by the broken lines aA, bB, . . ., eE, and those from winding B by the broken lines gG, . . ., lL. The phase increment per slot is 30°. If now the phase-shifters are set to make $\theta = 60°$, the vector diagram is modified to that shown in Fig. 3(*b*). If the relative numbers of conductors in windings A, B and C have been suitably chosen, the phase increment per slot is increased uniformly. Since slot 25 in Fig. 1 contains only conductors energized from phase-shifter 2, the phase angle of the current in slot 25 has been advanced 60° and therefore the phase increment per slot has been increased by 60/12 = 5°, there being 12 slots containing winding B. Similarly, the phase in slot 1 has been retarded by 60° so that the total phase angle between slots 1 and 25 has been increased from 720° to 840°, with the corresponding reduction in the angular velocity of the field. Similarly, if the phase-shifters are moved through the same angle in the opposite direction the total phase change will be reduced from 720° to 600°. Thus variation in θ from $-60°$ to $+60°$ gives a speed range of 840/600 = 1·4.

If all the slots contain the same total number of conductors, so that both the magnitude of the current per slot and the phase increment are uniform for $\theta = 0$, and if for one value of θ (in this case 60°) the phase increment is again uniform, the magnitude is seen from Fig. 3 to be non-uniform. Furthermore, for values of θ other than 0 and 60°, both the magnitude and the phase increment will be non-uniform.

A machine may therefore be designed to have correct current phase in all slots at one particular setting only. The greatest discrepancy in magnitude occurs where the currents in two windings are mixed in equal proportions, and a calculation of the phase discrepancy is completed in Section 12.2; as an example of the size of the errors introduced in a particular system, the phase and magnitude values of the current per slot are computed for the motor shown in Fig. 1. In this calculation, the conductor ratios are chosen according to Section 12.1, so that the phase increment is uniform for $\theta = 90°$. The calculated values of magnitude and phase are compared with the ideal values in Fig. 4. The phase errors for values of θ between 0 and 90° are nowhere greater than 1·7°, and the maximum error for $\theta = \pm 120°$ is less than 6°. Since a conventional 2 slot/pole/phase winding, 2/3 chorded, has a phase error of 30° in alternate slots, no problem is presented from purely phase considerations. The magnitude errors are more serious. With the arrangement described it is probably uneconomic in terms of stator copper loss to increase θ beyond 90°, although some benefit might be obtained by choosing conductor ratios which give uniform magnitude at $\theta = 60°$ rather than at $\theta = 0$. Furthermore, some magnitude error is clearly tolerable, for a conventional winding with 2 slots/pole/phase, 5/6 chorded, has a 14% difference in magnitude between currents in adjacent slots.

Fig. 5 shows the m.m.f. pattern obtained from a motor wound in accordance with Fig. 1 and having 2 slots/pole/phase and 5/6 chording, at one particular instant in time. In this Figure the phase-shifters are set to the zero position, and since this corresponds to a conventional winding in the same slots, only the resultant m.m.f. is shown. Fig. 6 shows how the resultant m.m.f. waveform [curve (*f*)] is built up from the components supplied by the three mains windings [curves (*a*), (*b*) and (*c*)] and the two sets of phase-shifter-fed windings. The instant of time chosen is when the red phase of the mains winding is at a maximum and the diagram refers to a phase shift of 90°. The centre slot contains the red phase only. It will thus be noted that the contribution from the red phase of either phase-shifter is zero, and this simplifies the illustration. It will be appreciated that the m.m.f. for other instants of time could also be synthesized. The increase in pole pitch between Fig. 6(*f*) and Fig. 5 is clearly indicated.

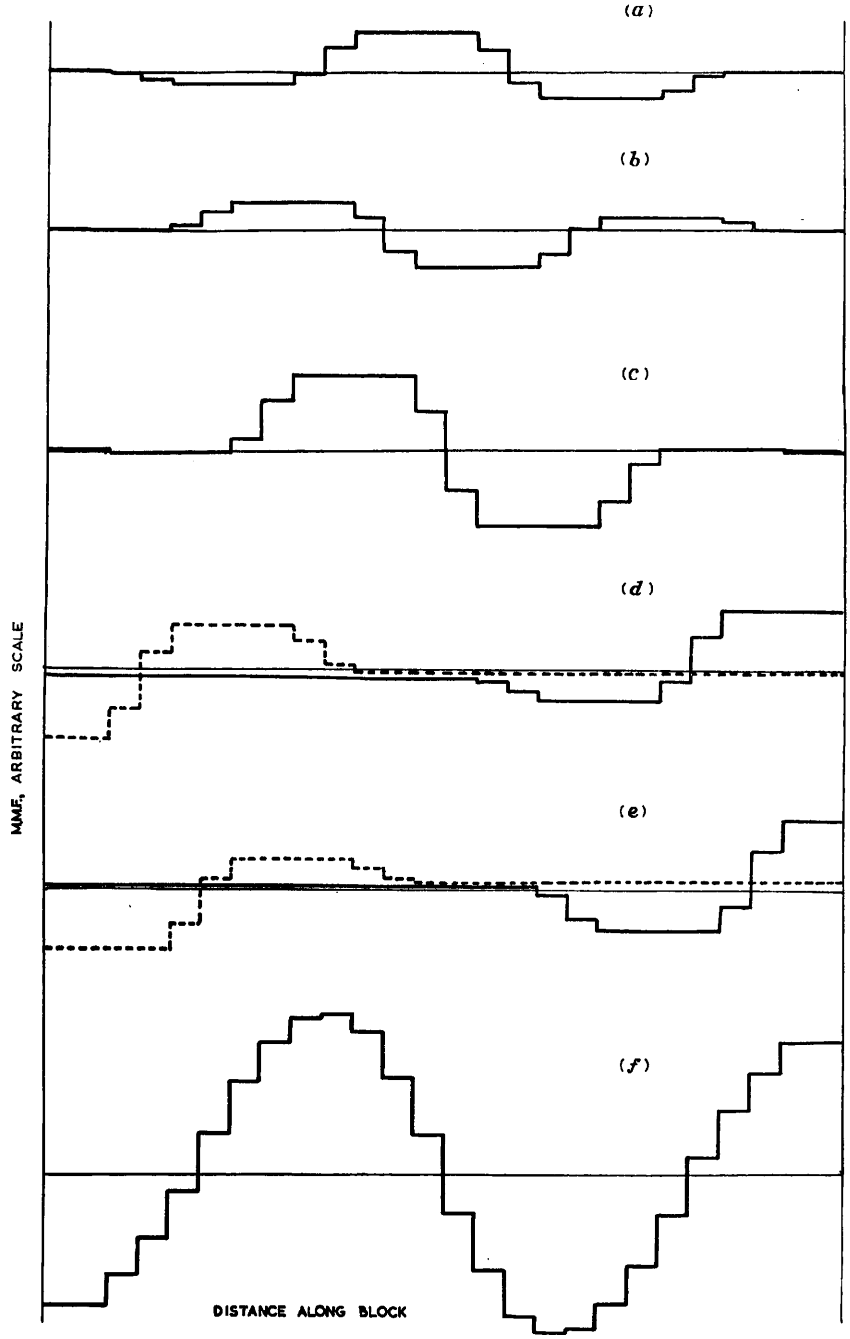

Fig. 6.—M.M.F. diagram for $\theta = 90°$.

(*a*) Contribution from mains winding (blue phase).
(*b*) Contribution from mains winding (yellow phase).
(*c*) Contribution from mains winding (red phase).
(*d*) ——— Contribution from phase-shifter 2 (blue phase).
– – – – Contribution from phase-shifter 1 (blue phase).
(*e*) ——— Contribution from phase-shifter 2 (yellow phase).
– – – – Contribution from phase-shifter 1 (yellow phase).
(*f*) Sum of the components (*a*)–(*e*).

(3) SHORT-STATOR EFFECT

The phase-shift-controlled motor, like the logmotor[1] and the spherical motor,[2] is a short-stator machine which manages to achieve synchronous speeds corresponding to non-integral pole numbers by providing a section or zone of the periphery over which the rotor flux can decay to a low value before re-entering the active zone of the stator. The behaviour of short-stator machines has been investigated in some detail.[2,3] Their efficiency cannot exceed $n/(n+1)$, where n is the number of effective poles around the active zone. The output cannot exceed n times the rotor copper loss. Accordingly, it is usually uneconomic to design machines in which n falls below 4 at any setting. Earlier work[3] has shown that an inactive zone of about 90° is necessary to avoid undesirable 'carry over' of flux, so that the top speed of both the phase-mixing motor and the logmotor for 50 c/s operation is limited to about 1 000 r.p.m. for reasonable efficiency and power/weight ratio. For variation of θ between $-120°$ and $+120°$ the total phase change from end to end of the active zone is $\pm 240°$, which means that $1\frac{1}{3}$ poles can be added to or subtracted from the number of poles on the winding C (Fig. 1). For n to remain greater than 4 at top speed, winding C should contain $5\frac{1}{3}$ poles, so that the speed range is $6\frac{2}{3}/4 = 1\frac{2}{3}$.

(4) EXPERIMENTAL PHASE-MIXING MOTOR

The first machine built was designed to have six poles for zero phase-shift on the grounds of rather better theoretical efficiency. To ensure that no carry-over of flux made analysis of the results difficult, a 120° arc was cut away from the stator. The remaining 240° had 45 slots. The rotor was 13 in in diameter and had 54 slots and the core length was 4 in. Initial tests were made to establish the nature of the flux distribution in the air-gap on open-circuit for the 5-, 6- and 7-pole settings. The flux distribution at particular instants of time may be plotted by feeding direct currents of the appropriate values into the three windings to correspond with one instant of the cycle and measuring the flux across the block with a search coil and fluxmeter. For this test the rotor was replaced by a laminated iron cylinder containing no slots. Shturman[4] measured flux distributions in short-stator machines by using this technique on a tubular structure with no associated iron. When iron is present the technique is difficult to apply, because of the effect of residual magnetism. The method was extended by feeding alternating currents into the 3-phase windings, the relative values of the three currents being appropriate to one instant of the cycle. Flux measurements can then be made with a search coil and voltmeter. The experimental results are shown in Fig. 7. The areas above and below the axis are seen to be equal in each case at all instants of time, indicating that there is no shaft flux and that an appropriate pulsating flux occurs in the case of odd numbers of poles. The results demonstrate clearly that the technique of phase mixing enables pole-stretching to be effected.

The next experiment consisted of load tests at constant current with both even and odd numbers of poles, and the results are shown in Figs. 8 and 9. The difference between the intake-less-stator copper loss and the synchronous power is shown in curve (*d*) in each case. Curve (*e*) on each Figure shows the iron loss. These iron-loss curves were computed by measuring the flux density at many points along the gap from which the density in the yoke was estimated and calculating the iron loss from known data for the type of stampings used. The crosses on Figs. 8 and 9 are obtained by adding exit edge loss to the iron losses. The latter were calculated according to the method outlined in Reference 3, estimating a value of rotor time-constant under the block from the experimental characteristics and measuring the time-constant outside the block by observing the decay of flux using a search coil. Clearly there are very few unaccountable losses.

Fig. 10 shows constant-voltage torque/speed and efficiency curves for various angles of phase shift from $-120°$ to $+120°$. These curves indicate that some undesirable effects occur for $\theta = \pm 120°$. The efficiency is lower at both settings, while the speed/torque curve has a double peak at the top speed setting. These effects could probably be reduced by designing the winding to be correct at $\theta = 60°$ rather than at $\theta = 0$. The results also show that a working speed range from 430 to 630 r.p.m. is possible with the system described. The figures quoted refer to the motor only. Overall efficiencies are not shown, since the phase-shifters used were not matched to the experimental machine and the phase-shifters were working at low load and therefore low efficiency. When well-matched, the estimated phase-shifter efficiency is 90%, and since about one-half of the total power is fed directly to the motor, a reasonable estimate of the overall efficiency is 62%.

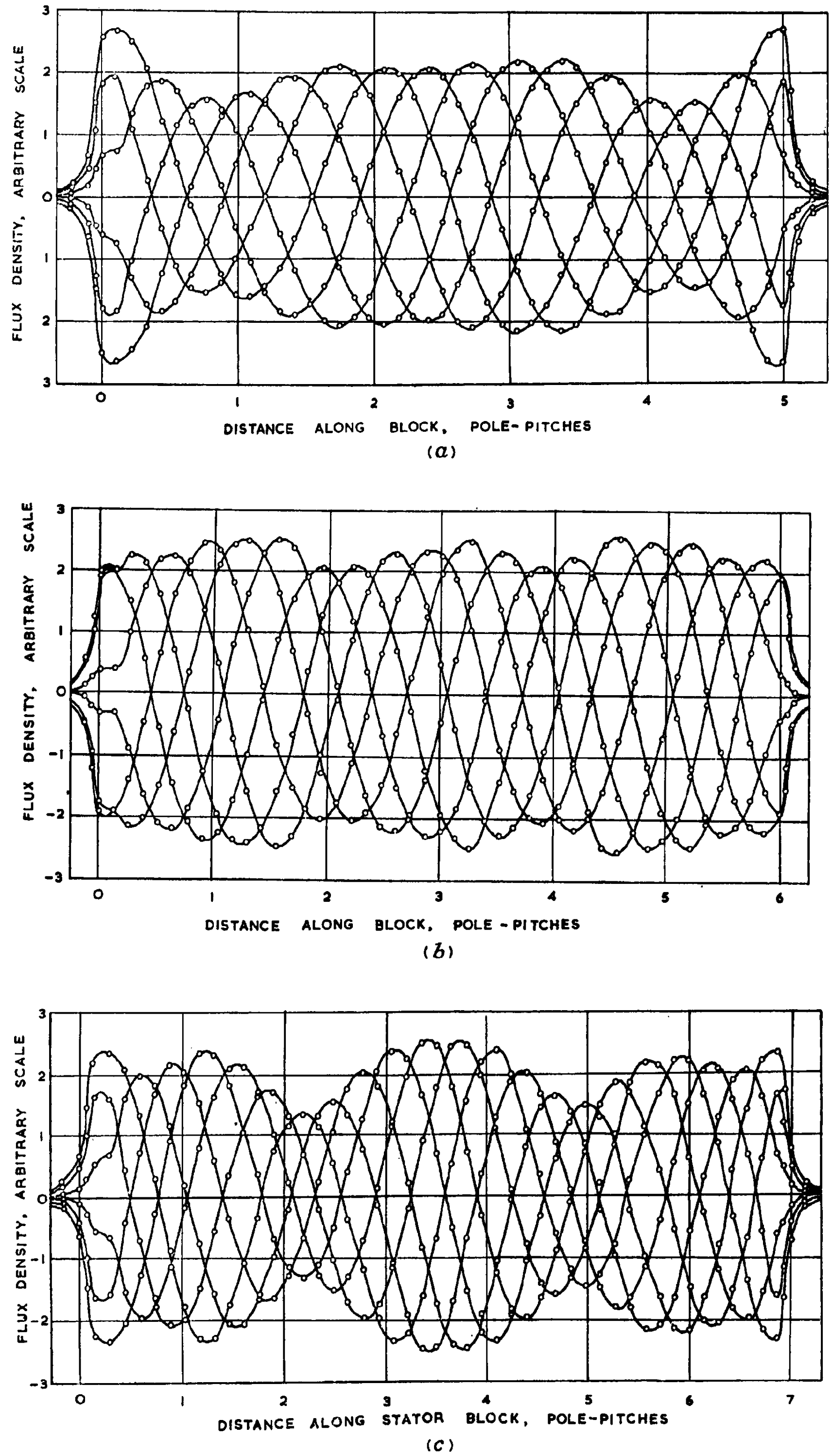

Fig. 7.—Open-circuit flux distribution under a multi-pole stator block.
(*a*) 5-pole. (*b*) 6-pole. (*c*) 7-pole.

(5) EXTENSION OF THE SPEED RANGE

The speed range may theoretically be extended indefinitely without reducing the minimum number of poles on the stator block and without mixing currents which are more than 90° apart by the use of the technique illustrated in Fig. 11. The stator winding of the machine described in Section 4 is represented in this Figure by the rectangle BB′b′b. The proportion

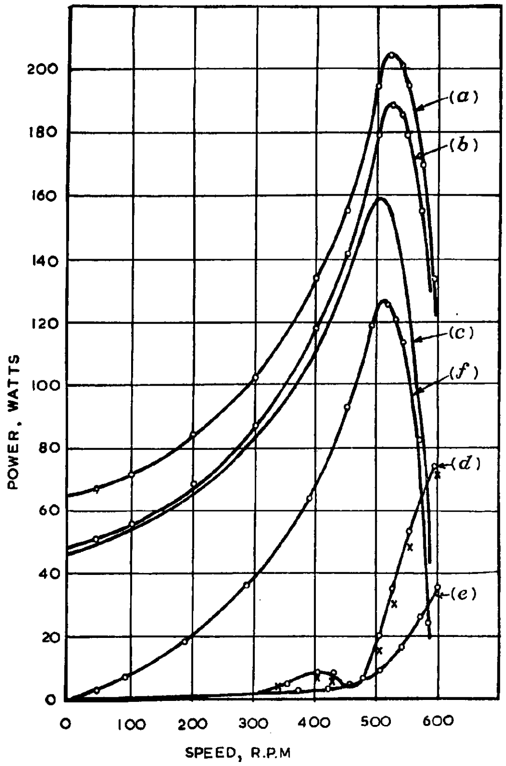

Fig. 8.—Brake test results for a 6-pole stator block (θ = 0).

(*a*) Intake.
(*b*) Intake less stator I^2R loss.
(*c*) Synchronous power.
(*d*) (*b*)–(*c*).
(*e*) Calculated iron loss.
(*f*) Output.

of mains-fed coils in any slot in a section such as ABB′A′ does not decrease linearly with the distance of the slot from AA′ (see Section 12), although it is a fairly close approximation to a linear relationship and it may conveniently be so represented in a schematic such as Fig. 11. If two additional phase-shifters are employed they may be connected respectively to the first pair of phase-shifters through 2 : 1 gearing, such that when the phase-shifter supplying slots in section ABB′A′ is set to give a phase shift of θ, the phase-shifter which is geared to it gives a phase shift of 2θ. When four phase-shifters are used the whole of the stator periphery is arranged to correspond with section CC′c′c of Fig. 11. Examination of the Figure will show that there are slots which mix currents of zero phase-shift (mains fed) and phase-shift θ (section ABB′A) and there are also slots mixing currents of phase-shift θ with currents of phase-shift 2θ (section CC′B′B), but no slots carrying currents of both zero and 2θ phase-shift. A similar situation obtains to the left of AA′ with $-\theta$ and -2θ phase-shift. The maximum value of θ without undue loss of effective m.m.f. may still be 90°. However, the extreme slots at CC′ and cc′ contain 100% of conductors with phase-shifts of 180° and −180° respectively, which means that two poles can be added to the block by a phase-shift in one direction only. Reversal of the phase-shift will therefore sub-

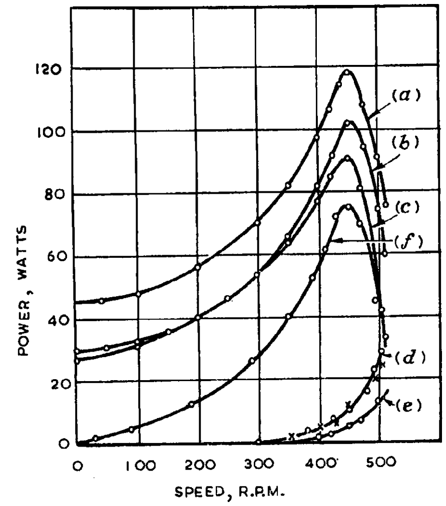

Fig. 9.—Brake test results for a 7-pole stator block (θ = 90°).

(*a*) Intake.
(*b*) Intake less stator I^2R loss.
(*c*) Synchronous power.
(*d*) (*b*)–(*c*).
(*e*) Calculated iron loss.
(*f*) Output.

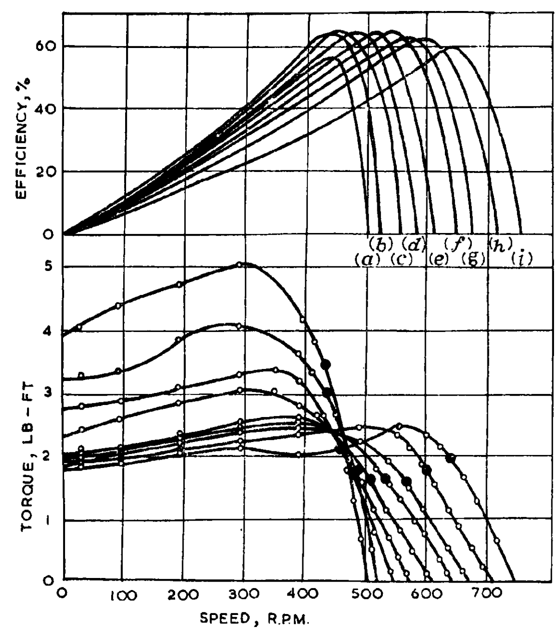

Fig. 10.—Torque/speed and efficiency curves for various angles of phase-shift.

(*a*) θ = 120°.
(*b*) θ = 90°.
(*c*) θ = 60°.
(*d*) θ = 30°.
(*e*) θ = 0°.
(*f*) θ = −30°.
(*g*) θ = −60°.
(*h*) θ = −90°.
(*i*) θ = −120°.

● = Full-load points.

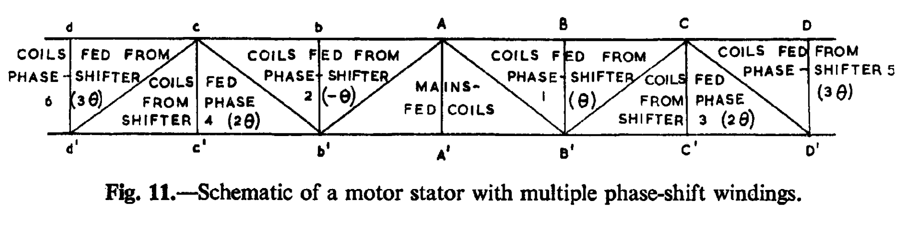

Fig. 11.—Schematic of a motor stator with multiple phase-shift windings.

tract two poles. Hence a stator can be wound to give a 6-pole configuration with zero phase-shift, so that it becomes a 4-pole and an 8-pole device in the extreme positions. Thus a 2 : 1 speed range is obtained with a minimum pole number of four. Without the additional phase-shifters this speed range could be obtained only by reducing the number of poles to two at the top speed setting.

The system shown in Fig. 11 is clearly extendible indefinitely in both directions by the use of more phase-shifters. The use of many phase-shifters is not attractive economically, for two reasons. First, as the number of phase-shifters increases, the fraction of the motor which each supplies becomes smaller and the rating of the phase-shifter is lower for a given size of motor; small phase-shifters, like any other a.c. machine, become less efficient and have a lower power/weight ratio. Secondly, the greater the speed range, the greater is the number of poles at bottom speed, for the top speed is limited to, say, a 4-pole stator block; as the number of poles increases, the magnetizing current may become excessive. The first of these limitations is not so serious as the second, since the phase-shifters may be 2-pole machines.

It was considered well worth while, however, to develop a system using four phase-shifters and the same top speed as in the first machine, since 2 : 1 is a much more useful speed range than the 1·5 : 1 obtainable with only two phase-shifters. To this end a scheme was developed for producing currents of phase-shift angles θ and 2θ from one machine, and such a system will now be described.

(5.1) The θ–2θ Phase-Shifter

A developed diagram of one phase of the stator and rotor windings is shown in Fig. 12. Both stator and rotor carry two windings such that the pitch of one is twice that of the other. There is no coupling between the two windings on either member, between the large-pitch stator winding S_1 and the small-pitch rotor winding R_2, and between S_2 and R_1. The only windings which are mutually coupled are S_1 and R_1 and S_2 and R_2. Windings S_1 and S_2 are in series. If the rotor be displaced from the position shown in Fig. 12 so as to produce current of phase-

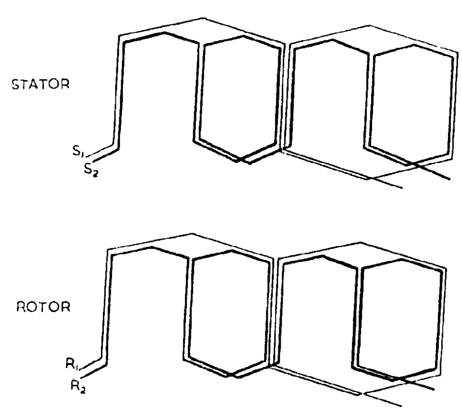

Fig. 12.—Developed diagram of stator and rotor windings of a double phase-shifter.

shift θ in R_1, then the current in R_2 must be shifted 2θ and R_1 and R_2 can be used to feed sections AB′C and B′CC′ respectively (Fig. 11). A second similar machine can be used to supply currents of phase-shift $-\theta$ and -2θ. The primaries of the two phase-shifting machines and the mains-fed coils are all in series.

A linear version of the θ–2θ phase-shifter was constructed to measure the effective coupling between the four windings when all four were chorded. The large-pitch coils constituted a 2 slot/pole/phase system, so that the small-pitch coils provided a 1 slot/pole/phase winding. Each winding was short-chorded by one-third of a pole pitch. The device was found to operate satisfactorily, cross-coupling being less than 1%. A new experimental machine using this type of phase-shifter is now in process of construction.

(6) PROPERTIES OF PHASE-MIXING MACHINES

The pole-stretching motor described here has several features in common with those of the logmotor described in Reference 1, both being examples of the more generalized type of pole-stretching arrangement shown in Fig. 13. Machine A is a device

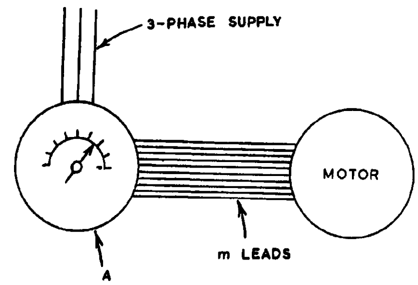

Fig. 13.—The generalized pole-stretching machine.

which takes in power from a 3-phase supply and redistributes it into m leads which feed the motor. The arrangement within the machine A and the motor are such that, when the position of the control handle on A is changed, all the pole pitches of the stator-current distribution on the motor are changed by the same amount, yielding a uniform-velocity field of a different value.

In the arrangement described here, phase mixing is performed in the motor itself as well as in machine A, which in this case consists of the two phase-shifters. If these are of the θ–2θ type there are 15 leads between A and the motor. If only single phase-shift control is used, there are nine interconnecting leads. In the logmotor with uniform slotting, phase mixing is performed in machine A and there are almost as many leads between A and the motor as there are stator slots. This difference between the principles of the types of machine results in a difference in the method of manufacture. To minimize the copper losses in the interconnecting leads the logmotor is constructed as a unit so that the interconnectors are as short as possible. The phase-mixer motor is designed to be controlled from a remote point. In this case the power/weight ratio of the motor alone is high, being of the order of 25 watts/lb. Each of the phase-shifters in the 10 h.p. experimental machine is called upon to handle a maximum of 3 kW. In the case of the logmotor the whole of the motor power is handled by the control machine. A phase-mixing motor fed from a pair of single phase-shifters obtains approximately half of its power directly from the mains; if it is fed from double phase-shifters about three-quarters of the total power is handled by the control. The power/weight ratio of the phase-mixing motor and its control is therefore theoretically higher than that of the logmotor, but the logmotor clearly has a superior speed range. Extension of the speed range of a phase mixer beyond 2 : 1 would be uneconomical, because of the complication of the extra phase-shifters, whereas the logmotor range is extendible merely by the rearrangement of the primary winding and interconnectors. Furthermore, when the logmotor is used as a 'vary arc' machine, the torque increases with the speed, whereas the phase-mixing machine is virtually a constant-power device when run at maximum efficiency.

(7) A NEW FORM OF POLE-CHANGING MOTOR

An interesting by-product of the continuously-variable-speed phase-mixing motor is a machine which provides a limited number of discrete speeds, and for which the control system A of Fig. 13 consists only of a set of switches. An example of the connections for this type of machine is shown in Fig. 14. The motor is wound according to the arrangement shown as BB′b′b of Fig. 11. The three sets of coils normally fed from the mains and the phase-shifters 1 and 2 are designated C, A and B respectively. The arrangement shown in Fig. 14(*a*) gives the

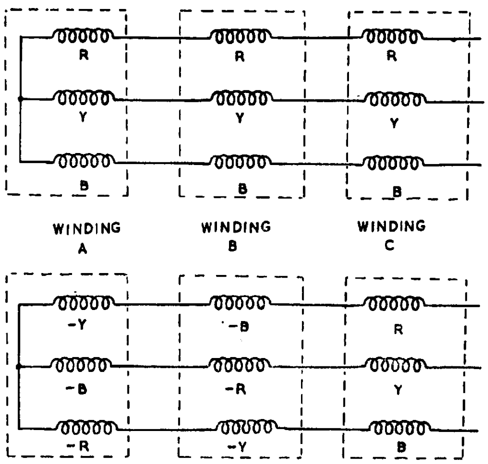

Fig. 14.—Winding connections for pole-changing machine.
(*a*) Zero phase-shift.
(*b*) Effective phase-shift of 60°.

effect of zero phase-shift. Reconnection as in Fig. 14(*b*) produces the same effect as a phase-shift of 60°. If the roles of A and B are interchanged, an effective phase shift of $-60°$ is obtained. The method is applicable to systems having effectively any number of phase-shifters.

Effective phase-shifts of $\pm 90°$ and $\pm 30°$ can be produced at the expense of a standard 3-phase transformer which enables star-delta switching to be used.

(8) THE LINK BETWEEN ANGLED-FIELD MOTORS AND POLE-STRETCHING MACHINES

The principle on which the spherical motor is based[2,3] is that a rotor is arranged so that points on its surface are constrained to move at an angle to the direction of motion of the travelling field set up by the stator. Variation of speed is achieved by mechanically changing the angle between the lines of action of the field and the rotor. This necessitates the use of a rotor and stator whose surfaces are parts of concentric spheres, so that a uniform air-gap can be maintained as the one is moved relative to the other. It is mentioned at the end of the first paper on this machine that a cylindrical version of the angled-field machine can be made in which the necessary adjustment in angle is simulated by using a fixed mechanical structure and phase-shifting transformers.

The principle of such a machine is shown in Fig. 15. The stator of the motor is divided into strips as shown, so that in the first instance relative movement between adjacent strips is possible. Figs. 15(*a*) and (*b*) show two possible configurations.

The resulting field from a stator arrangement such as that in Fig. 15(*a*) travels in the direction θ_1 to the strips at velocity $u_1 = 2p_1 f$. A rotor constrained to move in the direction AB has an effective synchronous speed $u_1/\cos\theta_1 = 2p_1 f/\cos\theta_1$. If the stator strips are reset to the position shown in Fig. 15(*b*),

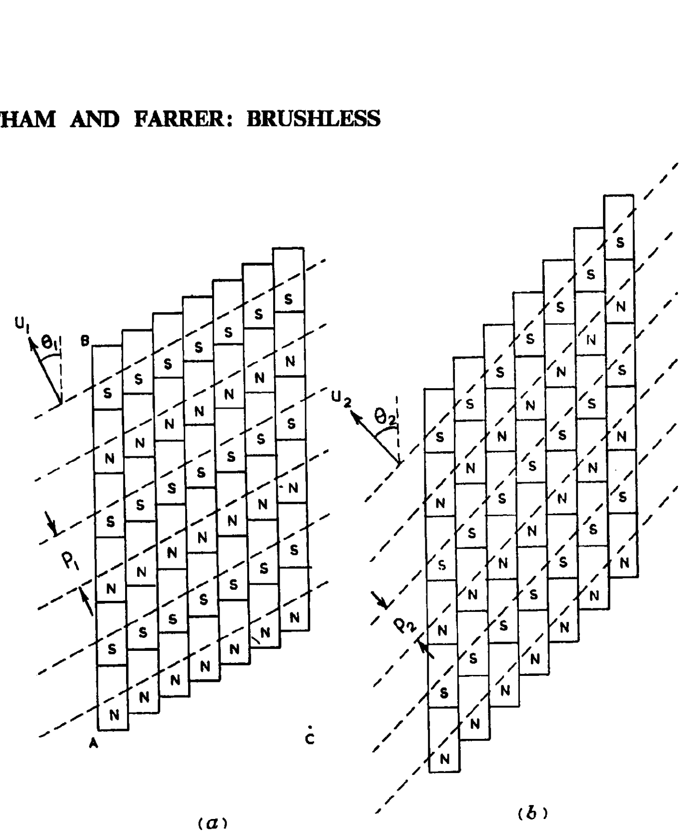

Fig. 15.—The angled-field principle.
(*a*) Pole pitch p_1, angle θ_1.
(*b*) Pole pitch p_2, angle θ_2.

the angle between the directions of the field and the rotor is increased to θ_2, suggesting a higher synchronous speed.

However, reference to Fig. 15 shows that the pole pitch p_2 is smaller than p_1; in fact, $p_2/\cos\theta_2 = p_1/\cos\theta_1$, so that the synchronous speed for the configuration in Fig. 15(*b*) is

$$\frac{u_2}{\cos\theta_2} = \frac{2p_2 f}{\cos\theta_2} = \frac{2p_1 f}{\cos\theta_1}$$

i.e. there is no change in synchronous speed.

This result may also be obtained by observing the rate at which a point on the rotor crosses the poles of the system whatever their configuration, provided it is assumed that the rotor can conduct equally in all directions—which is a requirement for the exploitation of the angled-field principle.

The important aspect of Fig. 15, however, is that a rotor constrained to move parallel to the direction AC has a synchronous speed $u_1/\sin\theta_1 = 2p_1 f/\sin\theta_1$ in the case of Fig. 15(*a*) and of

$$\frac{u_2}{\sin\theta_2} = \frac{2p_2 f}{\sin\theta_2} = \frac{2p_1 f\cos\theta_2}{\cos\theta_1\sin\theta_2}$$

in the case of Fig. 15(*b*), so that variation of speed is achieved.

The next step is to replace the mechanical displacement of the stator strips by an apparent displacement due to a change of phase of the stator current in the different strips, the method being illustrated in Fig. 16. In this example the stator is divided into six strips AA′, BB′, . . ., FF′, each of which is wound with a conventional 2-pole winding so as to produce a travelling field along AA′, etc. The squares on the Figure are used to represent phase groups. An array such as Fig. 16(*a*) produces a field travelling at an angle of 45° to AA′. According to the principle outlined with reference to Fig. 15, the rotor must be constrained to move parallel to AF, and it is apparent from Fig. 16(*a*) that each point on the rotor will traverse two pole pitches as it crosses the block. The supplies to the various strips are now phase-shifted by various amounts as shown in Fig. 16(*b*), yielding the

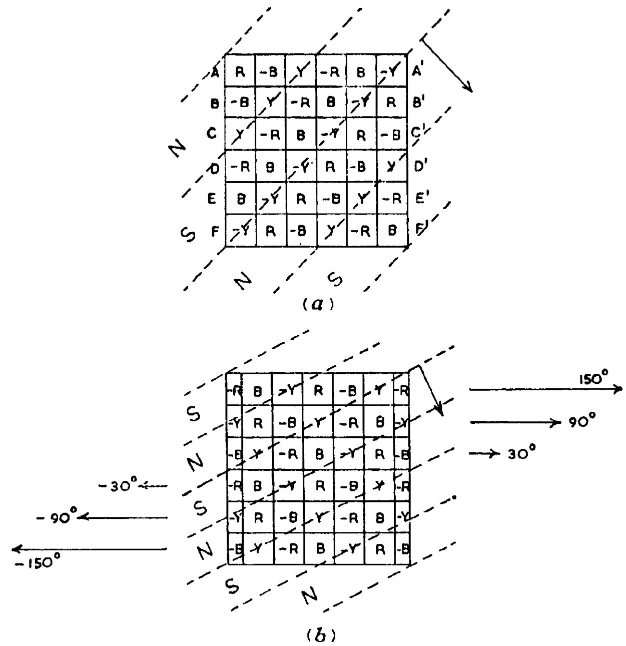

Fig. 16.—An angled-field motor using phase-shift techniques.

(*a*) Condition for zero phase-shift.
(*b*) Phase-shift applied.

phase pattern shown, which produces a field travelling at a greater angle to AA′. The new synchronous speed can easily be determined by noting that a point on the rotor now traverses four pole pitches in crossing the block. The two conditions shown in Figs. 16(*a*) and (*b*) provide two speeds, the second of which is exactly half the first. Clearly any intermediate speed is possible by suitable choice of phase-shift. The boundaries between the strips are effectively the slots of the machine so far as a point on the rotor is concerned, and it is clear that if the number of effective slots per pole is to be reasonable, a large number of strips is required, each one demanding its own phase shift. It was for this reason that the method of phase mixing was first applied to this problem to remove the necessity for a large number of phase-shifters.

An experimental machine was constructed using six blocks arranged around the entire periphery of the rotor as shown in Fig. 17. The slots on each block were pre-skewed at 45° and

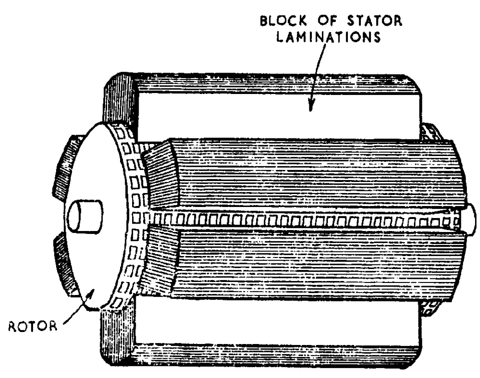

Fig. 17.—Mechanical layout of a helical-field motor.

arranged to form continuous helices, and the windings were arranged so that a point on the rotor crossed three pole pitches in one revolution for $\theta = 0$. A phase-shift of $+90°$ yielded four poles around the periphery, while a shift of $-90°$ produced two poles in 360°. The machine was found to have excellent characteristics at both these settings, but was incapable of stable running at any speed between the two for the reasons given in Section 1.

The departure from the angled-field principle to the pole-stretching principle involves only one fundamental change. With the method of construction shown in Fig. 17, the windings on each strip may be arranged so that an even number of poles may be contained within the axial length. It would appear that any number of poles may be contained around the periphery of the active arc with such an arrangement, since each N-pole has its opposite S-pole alongside it axially. With the arrangement in which conventional stator punchings are used with axial slots and with a segment removed, there is no axial flux and the net flux crossing the air-gap should be zero. The effects of this limitation on the air-gap flux were investigated before the method was accepted. One of the attractive features of the conventional method of construction is that a conventional squirrel-cage rotor may be used, and no additional conducting peripheral rings are required.

(9) CONCLUSIONS

The principle of the phase-mixing motor has been demonstrated and the limitations of the system are fairly clear. The motor itself is essentially a short-stator machine whose output is limited theoretically to n times the rotor copper loss for n poles on the stator. This limitation on the choice of pole numbers determines the speed range of practical machines. The facility of being able to control the motor by means of auxiliary apparatus situated at a distance is valuable for many applications. The characteristics appear suited to constant power, as are those of spherical motors, although the power/weight ratio of the whole system may be lower than that of a spherical motor of the same output. The mechanical construction is conventional, except for the removal of an arc of the motor stator. Experiments with machines in which a short-circuited section of stator is used instead of removing the arc are continuing with a view to making the construction entirely conventional. The winding arrangements on both motor and phase-shifters are conventional, apart from the graded number of turns. The design of this type of machine consists of two parts—first the design of a short-stator machine and secondly the design of conventional phase-shifters to match the impedance of that machine.

(10) ACKNOWLEDGMENTS

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(2) Williams, F. C., Laithwaite, E. R., and Piggott, L. S.: 'Brushless Variable-Speed Induction Motors', *ibid.*, Paper No. 2097 U, June, 1956 (**104** A, p. 102).
(3) Williams, F. C., Laithwaite, E. R., and Eastham, J. F.: 'Development and Design of Spherical Induction Motors', *ibid.*, Paper No. 3036 U, December, 1959 (**106** A, p. 471).
(4) Shturman, G. I.: 'Induction Machines with Open Magnetic Circuits', *Elektrichestvo*, 1946, No. 10, p. 43.

(12) APPENDICES

(12.1) Proportion of Turns used in Phase Mixing

The system is designed to give a uniform phase increment per slot when the phase-shifters are set to 90°. In Fig. 18(a), which

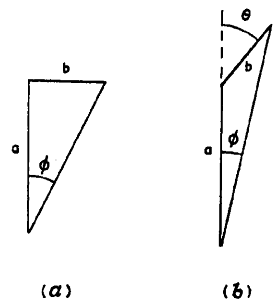

Fig. 18.—Illustrating the problem of phase error.

(a) rth slot of a system for $\theta = 90°$.
(b) Condition with phase-shift angle of θ.

shows the vector diagram for the rth slot of a system having S slots in each half, it is assumed that each slot contains the same total number of conductors, N, and thus

$$a + b = N \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\tan 90°\frac{r}{S} = \frac{b}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$\frac{b}{N - b} = \tan \frac{90°r}{S}$$

or

$$b = \frac{N}{1 + \dfrac{1}{\tan 90°r/S}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(12.2) Phase Errors when $\theta \neq 90°$

The vector diagram for a slot carrying currents differing in phase by an angle θ is shown in Fig. 18(b). Again, $a + b = N$ and

$$\tan \phi = \frac{b \sin \theta}{a + b \cos \theta} = \frac{b \sin \theta}{N - b + b \cos \theta}$$

The desired conditions are that when $\theta = 90°/x$, ϕ should equal ϕ_0/x where $\phi_0 = 90°r/S$. Thus the error is given by

$$\delta = \arctan \frac{b \sin 90°/x}{a + b \cos 90°/x} - \frac{\phi_0}{x}$$

Substituting for a and b from eqns. (1) and (3) gives

$$\delta = \arctan \frac{\tan \phi_0 \sin 90°/x}{1 + \tan \phi_0 \cos 90°/x} - \frac{\phi_0}{x} \quad . \quad . \quad (4)$$

It can be seen that when $\phi_0 = 45°$ ($a = b$) the error is zero for all values of x, i.e. the middle slot of the phase-shift section is always correct. The maximum value of δ for all values of x between $1 \cdot 0$ and ∞ and all values of ϕ_0 between 0 and 90° has been computed and found to be $1 \cdot 67°$. The penalty for exceeding 90° phase-shift so far as slot phase error is concerned has been assessed by computing the maximum value of δ for all values of ϕ_0 from 0 to 90° and $x = \frac{3}{4}$, and this is found to be $5 \cdot 66°$.

DISCUSSION ON THE ABOVE TWO PAPERS BEFORE THE UTILIZATION SECTION, 10TH NOVEMBER, 1960

Mr. C. C. Inglis: Returning to electrical engineering after many years of work in other fields, I asked what had happened in the design of machines, to find that, apart from a very welcome improvement in general detail, technique of installation, finish, etc., nothing very fundamental had occurred. But three years ago our long-cherised belief that the speed of an a.c. machine depended on the number of poles (and that had to be even) was shattered, and now the authors talk about fractional numbers of poles, such as 5·2. The next thing which affects the speed of the motor is the frequency; no longer will the a.c. machine designer be able to set 50 c/s on a slide-rule and work from there—he will have to set any figure between 1 and 100 c/s and design his machine under those conditions.

The future therefore holds tremendous possibilities. Much will be achieved by the use of solid-state devices. A year ago workers on these devices were talking about 10 kW controlled rectifiers, but today they are talking about 100 kW; I am sure there will be a further tenfold increase before long.

One question of great importance to all those who deal with transport is reversal of rotation. I am not quite clear about what happens when one tries to reverse the variable-speed machine. Is the machine reversible, and, if so, does it work equally well in either direction?

The potentialities of a.c. electrification are tremendous, and I therefore welcome the authors' challenging of past concepts through the potentialities of variable-speed a.c. motors, which can be of the greatest importance in railway electrification.

Mr. A. F. M. Ashworth: The antecedent of these machines is the spherical motor, whose output must exceed 50 hp if it is not to have poor performance. Do the authors recommend any limit to the output of their new machines?

The experimental machines which have so far been built have been of comparatively low output and wound for low-voltage supply. This has demanded a large number of turns in each stator slot, which thus permits coils with differing numbers of turns around the periphery of the transformer in the logmotor and the graded mixing of phases in the phase-shift motor. Machines in the range 250–500 hp, which are commonly l.v. devices, require two or fewer effective conductors per slot. It would therefore seem impossible to apply these new machines to such ratings.

Professor G. H. Rawcliffe: Prophecy is a dangerous exercise, but I think the phase-shifting motor is more likely to prove an industrial success than the logmotor. The principles of the two have much in common, but the phase-shifting motor can be constructed in semi-conventional form. There is much less physical and electrical 'hangover' about it. If the authors can devise means of dealing with this 'hangover', whereby the energy which is lost as each rotor element comes out of the stator field is made available as the element goes back into the field again, I think the phase-shifting motor has a chance of industrial success.

Further, the phase-shifting motor is easier to manufacture and has less waste iron than the logmotor: only part of the power has to go through the phase-shifter(s), whereas the whole power has to go through the 'slide-rule' in a logmotor. Both types of motor present the same difficulty in relation to the making and breaking of fields. If this is overcome, the advantages mentioned will, I believe, tell in favour of the phase-shifting motor.

I strongly advise the authors to try to make this machine work over a moderate speed range using one phase-shifter rather than over a greater speed range with two. A little done simply is usually better than a lot done with much complication. I also recommend the authors to publish all future test results with constant applied voltage; and not with constant current flow, whatever experiments may be performed at constant-current. The real test of a machine must always be at constant voltage. I do not think the phase-shifters are 'conventional': they are current transformers and not voltage transformers, and it would be interesting to know what voltage and power they absorb.

In my view, the authors should avoid speaking of the phase error (as they call it) in conventional windings. Machines using these windings are highly successful; and, even if the authors' arguments were correct, I think it is dangerous to seek to justify the shortcomings of a half-developed machine by reference to the alleged defects of something else which is already widely accepted.

Whether or not either of the authors' machines will be used industrially remains to be seen; but they are much to be commended for helping to infuse vigour into electrical machine engineering, which has become very stereotyped.

Mr. J. S. Holmes: I should like to draw attention to work which has been done on short-stator machines by the aircraft electrical industry.

A design was carried out to assess the weight of a 400c/s 40kVA spherical induction generator with a 2 : 1 input speed range and a 3-phase 208-volt output. The weight of this machine, with a heat rejection of 2·5 watts/cm^2, was 300lb.

Fig. A shows the output of a conventional aircraft salient-pole alternator based on a frame size relative to 300lb in weight, and the output is directly proportional to speed. For the short-stator machine the output is theoretically inversely proportional to the speed, as shown by the broken curve. This is not realized in practice, since an increase in the number of poles approaches the limiting pole pitch to which the authors refer and the actual curve falls away in the manner illustrated.

Design exercises were also carried out on the phase-mixer and log machine of similar performance, and the calculated weight for the former was 240lb. This machine had a 2θ phase-change and required 160lb of phase-shifting equipment. A log generator was built and tested, its weight being 480lb.

Mr. W. Hill: We have heard a good deal about the economic aspects of these new machines, and bets of 3 : 1 or 4 : 1 have been mentioned. This leads me to make a suggestion which I am sure the authors will not take amiss. Textbooks never mention economic parameters, which we regard as just as important as others, but I should like to think that engineers, who are trained to deal with complicated subjects, would take the economic side as just another subject in the syllabus. Engineers like to deal with facts, and there are economic facts. Can we not have some guidance on these? Can we not sit down together and elicit the economic facts and determine what a machine will be worth?

Mr. R. D. Ball: These papers are of very great interest, especially since we find that the spherical motor, which is difficult to manufacture and expensive, is suddenly changed into

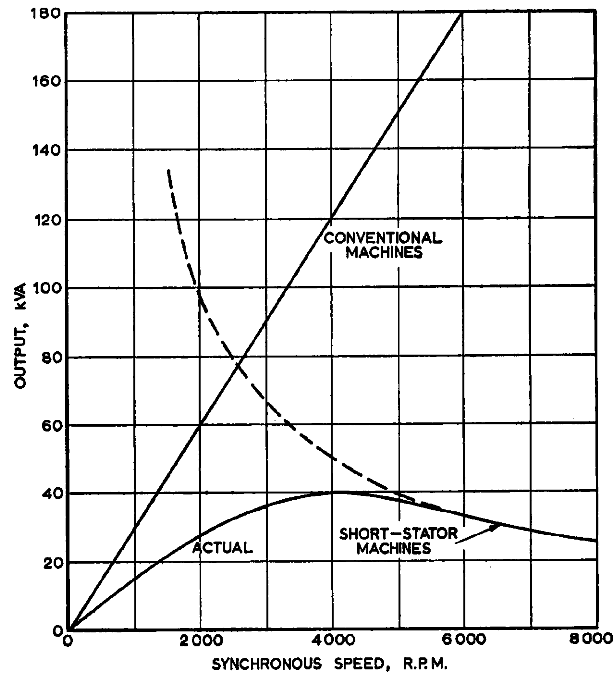

Fig. A.—Approximate output curves for 400c/s machines with a fixed frame size.

a cylindrical-rotor machine, and with the logmotor and the phase-shift pole-stretching motor comes into the practical engineering field. I believe that if we made them large enough and were content with low speeds, they would be extremely efficient.

This advance over the last few years has brought out other possibilities. I wonder whether the authors have considered a hysteresis permanent-magnet-steel type of spherical or cylindrical rotor, which might cut out the slip losses? At Manchester University they have developed linear arrays, and it would seem to be a possible scheme to have mains, $+\theta$ and $-\theta$ pole-stretching windings on these arrays and use this device for driving planers, since these require a linear motion. In such an arrangement the rotor losses would be much reduced by switching from pole-to-pole grouping and feeding back some of the kinetic energy by induction-generator action into the mains.

Dr. W. Fong: Will the authors indicate briefly the winding arrangement for the phase-shifter motor, since it is not given in the paper? It is stated in Section 5 that there are two disadvantages in extending the speed range of this motor: the mains-fed coils become a smaller proportion of the total, and the coils fed through the phase-shifter become greater as the number of phase-shifters increases. Would not this also be a disadvantage in so far as the power flow through the phase-shifters would become larger.

[The authors' reply to the above discussion will be found overleaf.]

SOUTH-WEST SCOTLAND SUB-CENTRE, AT GLASGOW, 23RD MARCH, 1960*

Dr. A. J. Small: This motor is a further attempt to obtain a continuously variable speed control of a cage-type induction motor. Having obtained a speed range of 4 : 1 the authors certainly demonstrate that the principle of phase stretching can be successfully applied.

* Discussion on Paper No. 3149 U only (see page 91).

Since part of the stator of the machine requires to be left unenergized for successful operation, full use is not being made of all the available active material, and consequently it appears that the machine will be more bulky than a conventional one of the same output.

For the experimental motor the efficiency has a peak value of

63%, and only 50% over the limited speed range 750–350 r.p.m., i.e. about 2 : 1, and still lower at lower speeds. It would have been informative if results had been given of brake tests at fixed displacements of the primary from the aligned position, i.e. a demonstration of the variation of speed and efficiency as the load is varied at each displacement setting. Would increased rotor losses at lower speeds cause a marked departure from the approximate constant-speed characteristic of an induction motor?

In the Introduction the units of angular velocity, expressed as $2f/n$, are not stated, but are presumably revolutions per second instead of the usual radians per second. The unconventional use of n and p for the number of poles and the pole pitch respectively is somewhat confusing, particularly when p is almost universally adopted for the number of pole pairs.

I am pleased to hear that development of the design of this type of machine is continuing, since the machine clearly has characteristics which would ensure its application in situations where sliding brush contacts are prohibitive. I trust that as industrial techniques are applied to the construction of the machine the present difficulties as well as the losses will become progressively less.

Mr. H. E. Clapham: It would seem that the discontinuity effects are inseparable from machines employing pole- or phase-stretching techniques in order to obtain variable speed. Fig. 11 demonstrates the effects of discontinuity very clearly. To overcome these undesirable effects the authors had to provide a 'flux killing winding' on the motor stator, which leads to increased and concentrated losses. This situation could be a very great obstacle on a machine of large output. The effects shown in Fig. 11 were obtained on light load; would they be different if the machine were on load?

The logmotor seems easier to construct than the spherical motor, although the provision of bearings between the transformer and the motor presents problems. Access to these bearings must be provided for assembly and maintenance, and this is made difficult by the multiplicity of winding connections. In view of this, is it not possible to use more than one conductor per slot on the stators and so make these connections lighter? The two stator short-circuiting rings can be a potential source of trouble, since fractures are likely to occur where the slot conductors make a T-joint with the ring. Such troubles were once very common on squirrel-cage rotor windings, but today are encountered only on poorly designed machines.

Since two air-gaps are involved one would expect the magnetizing current to be relatively large and the power factor low.

Although the demand for variable-speed drives has been largely met by the a.c. commutator motor, so far as this country is concerned, recent advances in germanium and silicon rectifiers have caused d.c. motors and the Kramer-cascade system to appear as competitors in this field. The logmotor has thus appeared at a time when competition in variable-speed drives has already become more lively.

THE AUTHORS' REPLY TO THE ABOVE DISCUSSIONS

Professor F. C. Williams, Dr. E. R. Laithwaite and Messrs. J. F. Eastham, L. S. Piggott and W. Farrer (*in reply*): Both machines are capable of reversal by interchanging any two phases of the supply, as with a conventional induction motor. In the logmotor, however, the current loading varies around the periphery of the transformer. Short-stator effect produces non-uniform flux distribution around the air-gap, so that the logmotor has a preferred direction of rotation.

This same short-stator effect, which places a lower limit on the size of good machines in the case of the spherical motor, has precisely the same effect on both these machines, and we do not envisage either type being capable of good performance under 50 hp, except for very small speed ranges. It seems probable that the low-voltage machines in the range Mr. Ashworth quotes could not be designed as logmotors. The phase-shift type, however, contains phase-shifters in which it is possible to introduce a turns ratio which would enable the secondary voltage to be stepped up to the value required to allow the necessary distribution of turns. The penalty occurs in that the mains-fed section of the motor ($\frac{1}{3}$ of the whole for two phase-shifters or $\frac{1}{5}$ for four phase-shifters) would need to be fed through an additional static transformer.

In so far as it is possible to judge at this stage, we agree with Prof. Rawcliffe's preference for the phase-shift machine and with his reasons for that preference. We would go further and prophecy that, if a method could be devised to institute a correct entry-edge flux powered by the wasted exit-edge magnetic energy, there would be no doubt whatsoever of the commercial success of these devices, since they would then operate with full conventional machine efficiency and power output. Success in this direction is not, however, at present in sight. We find the constant-current approach much more informative for analytical purposes, though we recognize that where application is in question, constant-voltage results are essential and each of the papers includes sets of constant-voltage characteristics. The term 'conventional' as applied to the phase-shifters refers to their construction rather than their use. A large phase-shift motor is at present under construction and a detailed analysis of phase-shifter losses will be undertaken. Conventional machines do have phase errors and there seems to be no objection to our calling them such. With regard to Prof. Rawcliffe's last paragraph, certainly the most satisfactory practical result so far has been the enthusiasm shown by the present generation of students for electrical-machine research.

With reference to Mr. Holmes's comparison of the conventional and short-stator machines, the output of the latter is inversely proportional to the speed only so long as the output is limited by rotor heating, i.e. to the right of the point on Fig. A where the full and dotted lines join. If this be the case and a more efficient rotor cooling system is devised so that the rotor dissipation is doubled, then the rectangular hyperbola shown in Fig. A will be raised so that each ordinate is doubled, and the output of the short-stator machine will approach that of the conventional machine at 4000 r.p.m. We agree that, of the three machines mentioned by Mr. Holmes, the phase-mixer has the best chance of meeting power/weight requirements.

We agree with Mr. Hill concerning the importance of economic factors, and regard as of great importance the fact that these developments have already fostered closer co-operation between the industrial and academic sides.

Mr. Ball's suggestion of the hysteresis-motor application is interesting, but we can foresee difficulties introduced by the discontinuities in the stator magnetic circuit, although we have not, so far, put the suggestion to practical test. With regard to planing machines, the speed of planers is not sufficiently high to enable a linear induction machine to be designed economically for this application.

Mr. Fong is correct in his comment on Section 5. There are several ways of arranging the windings of phase-mixer motors so as to obtain the triangular (or other shapes) of block as required. Space will permit only one example, which is shown in Fig. B. It will be observed that all coils have the same

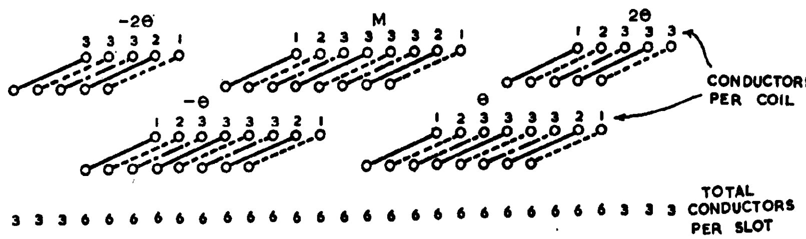

Fig. B.—Arrangement of windings for four phase-shifters.

——— Red phase mains.
– – – – Yellow phase mains.
—●—●— Blue phase mains.

This notation applies to the phase-shift windings only when $\theta = 0$.

pitch and that each slot contains the same total number of turns.

These machines are more bulky than conventional machines of the same speed and output, as Dr. Small suggests. Their efficiency and power factor are likely to be lower. This is the price of continuously-variable speed. A similar price is paid in all other types of variable-speed motor. A.C. commutator motors, Ward Leonard sets, etc., are generally between three and four times as bulky. Even if our machines are still lower in power/weight ratio, they may be worth while in some applications where a brushless machine is essential. The logmotor characteristics at different settings are similar to those of Fig. 10 of the phase-mixer paper, except that the power levels are lower at the lower speeds.

The concentrated losses in a flux-killing winding may not be so serious as Mr. Clapham suggests. If the short-circuited winding is to be effective, its resistivity will be about $\frac{1}{3}$ to $\frac{1}{4}$ that of the rotor. Since the coupling with the rotor winding will be good, the copper loss per unit area of stator in this region will be less than that of the rotor by about the same fraction. Nevertheless, it is true that the stator loss per unit area in the region of the short-circuiting grid can be considerably higher than that in the active zone, but this is compensated by the fact that the short-circuit winding can be an uninsulated cage.

The effects shown in Fig. 11 are less marked when the machine is on load. With regard to the use of more than one conductor per slot, this is only possible with Gramme-ring-type coils, which are generally uneconomical. Any other type of coil arrangement on motor stator or transformer secondary would fix a pole pitch which was contrary to the fundamental idea. The T-joints between interconnectors and rings are doubtless capable of being treated in the same way as those of modern well-designed rotors.

We agree with Mr. Clapham's comments on magnetizing current, power factor and competitive drives.

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DEVELOPMENT AND DESIGN OF SPHERICAL INDUCTION MOTORS

By

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DEVELOPMENT AND DESIGN OF SPHERICAL INDUCTION MOTORS

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SUMMARY

The first major development described is concerned with the arrangement of stator blocks. A previous paper described a system in which four stator blocks were arranged around the rotor. The new layout involves the use of only two blocks containing pre-skewed slots. The arrangement of the laminations and slotting is such that the two blocks can be rotated as a unit to produce speed adjustment. This arrangement is shown to produce a more uniform effective field velocity over the active surface of the stator. It equalizes the magnetic pull between rotor and stator and has several other design advantages which are fully discussed. The earlier theory of the spherical motor is extended to the case of machines with pre-skewed slots. Test results obtained from several experimental machines are shown. In the light of these results a procedure for designing spherical motors is developed. Several aspects concerning manufacturing procedure, such as the required numbers of rotor bars and rings, have been investigated, and the practical results of these tests are included. A short section of the paper is devoted to the performance of spherical machines when used as induction generators.

(1) INTRODUCTION

In earlier publications[1, 2] the authors have described variable-speed induction motors based on a principle illustrated in Fig. 1. A stator block, A, is wound with a polyphase winding

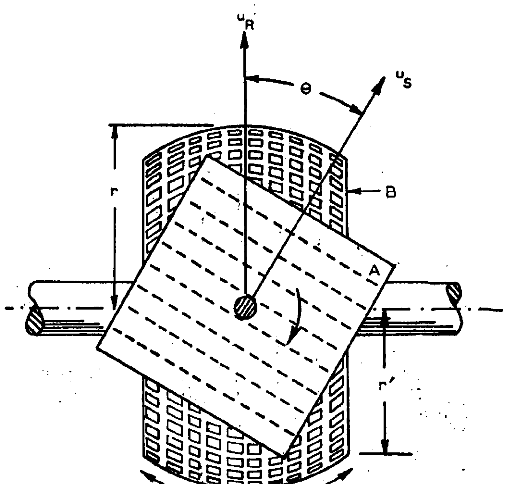

Fig. 1.—Arrangement of rotor and stator in a spherical machine.

designed to produce a field travelling across the block with velocity u_s. The inside surface of the block is part of a sphere and is concentric with the spherical surface of the rotor B, which can carry current in any surface direction. The rotor will approach the synchronous speed

$$u_r = u_s/\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

The authors are in the Department of Electrical Engineering, University of Manchester.

The stator block can be rotated so as to vary θ while maintaining a uniform air-gap between rotor and stator, and hence u_r can be controlled. Speed ranges as high as 6 : 1 have been achieved, and there is no theoretical reason why any desired speed range should not be possible.

The stator is essentially made up of several individual blocks. Unenergized rotor material is continuously entering the area of influence of each stator block, becoming energized, and then leaving the block in an energized condition. The system is never free from the influence of transients, and the flux density over the surface area of any block is non-uniform.

A theory accounting for these transient effects has been developed[2] for the case of $\theta = 0$. The theory assumes a travelling wave of current density $j_s = J_s \sin(\omega t - \pi x/p)$ in the stator block shown in plan in Fig. 2.

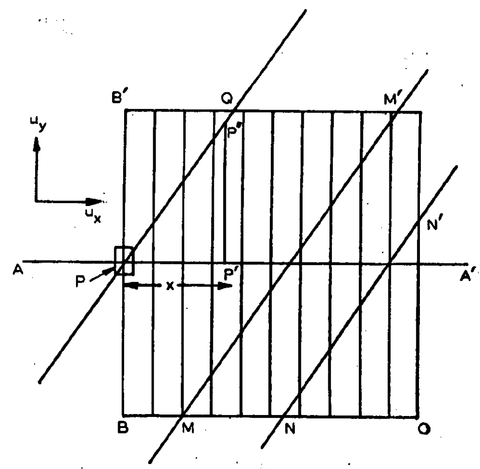

Fig. 2.—The skewed field principle.

The lines drawn in the Figure do not represent slots, but are lines of constant phase of the current distribution. The theory considers the factors influencing an element of rotor, P, as it passes under the block, along the axis of x, AA′. The line BB′ is taken as the origin of x, so that the position of P after time t is $x = u_x t$, shown at P′, where u_x is the velocity of P in the direction AA′.

The theory, which includes some approximations, enables the flux density at all points under the stator to be calculated for the particular case of $\theta = 0$ [eqns. (13) and (14) of Reference 2].

If now the rotor is given an additional velocity u_y in the direction BB′, the element will in fact arrive at P″, where x has the same value as before and $y = u_y t$. It may be seen, however, that the element is still embraced by the same pair of equiphase lines, and its behaviour is therefore unchanged by the introduction of u_y. The theory developed for the case of $\theta = 0$ can be applied to cases where $\theta \neq 0$, provided that x and p and all quantities derived therefrom are measured in the direction of motion of the current wave, and j_s is the current density in that direction.

The velocity u_y, however, will not be altogether without effect. Thus the element P will emerge from the block at point Q instead of A′, thereby reducing the effective length of the block and therefore the range of x over which the flux equations apply.

Furthermore, not all elements will now enter the block along BB′; some may enter at points such as M and leave at M′, or at points such as N and leave at N′. In both cases there is a reduction of effective block length, and for the theory to apply it will be necessary to treat x as zero at the entry point, whatever it is, and not as zero on the extension of B′B. Detailed analysis of these effects is not necessary here; it will suffice to note that the efficiency of the machine was shown to depend very much on the number of poles (n) of the travelling wave contained within the block length in the direction of motion of the rotor, good efficiencies being possible only for values of n in excess of about 4. The greater u_y is made relative to u_x, i.e. the greater θ becomes, the greater will be the effective reduction of block length, and therefore of pole number. Thus, the greater the speed range of the machine, the smaller will be the effective number of poles relative to the actual number on the block in the x-direction. Hence, if the effective number of poles at the highest speed is to be sufficient, say greater than 4, then the greater the speed range required, the greater will be the actual number of poles on the block and therefore the smaller will be the pole pitch for a given block length. Small pitches result in inefficient machines.

(2) THE 4-BLOCK TYPE OF CONSTRUCTION

In the machine described in a previous paper,[2] four sets of movable stator blocks were envisaged, each of which was a 'square' of side r, equal to the rotor radius. In the earlier machines the rotor arc α under a block was restricted to 60°. Having restricted the block width, the block length is now similarly restricted. If, for example, a 2 : 1 speed range is required, the stator blocks must be rotated through 60°. If the block dimension in the direction of the field velocity is greater than that in the direction of the slots, a large portion of the stator surface will have been turned clear of the rotor surface at top speed. The number of poles in the direction of motion of the rotor is now halved. The best efficiency at 50 c/s on a 4-block machine with a 14 in-diameter rotor was 33% at bottom speed, with much lower efficiencies at higher speeds.

The error made in designing this machine was to start from the minimum-speed condition and then see how far the performance deteriorated at higher speeds. By starting from $\theta = 0$ large angles of rotation are necessary before any useful speed range is obtained [see eqn. (1)].

(3) THE 2-BLOCK TYPE OF CONSTRUCTION

A new type of construction was designed to enable the block length in the direction of motion to be extended. Segmental stator punchings can be offset with respect to each other so as to produce pre-skewed slots as shown in Fig. 3, where the punching thickness has been exaggerated to illustrate the method. If all the punchings are identical, the edges of the block are serrated as shown. The individual punchings are arranged to lie in radial planes so that no machining of the inner surface is necessary. Furthermore, these planes are arranged to pass through the rotor axis when the block is in its central position, so that in terms of rotor movement the laminations could be said to lie at right angles to the direction of those in a conventional machine. The method of construction is illustrated in Fig. 4. Segmental punchings are stacked against a pair of accurately spaced rings. The serrated edges are not

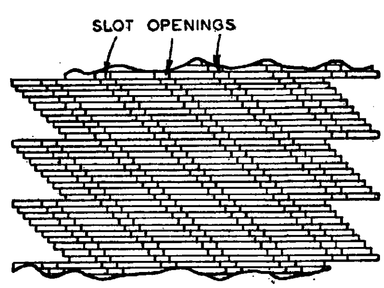

Fig. 3.—Assembly of laminations forming pre-skewed slots.

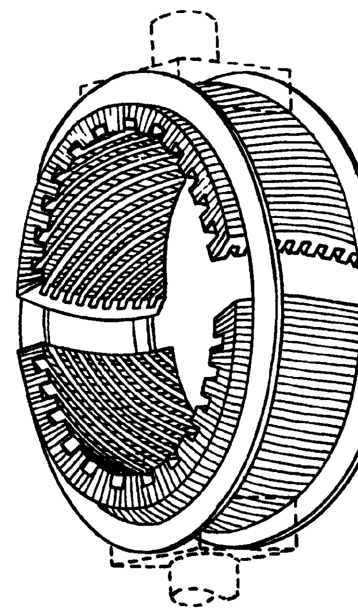

Fig. 4.—Two-block type of construction.

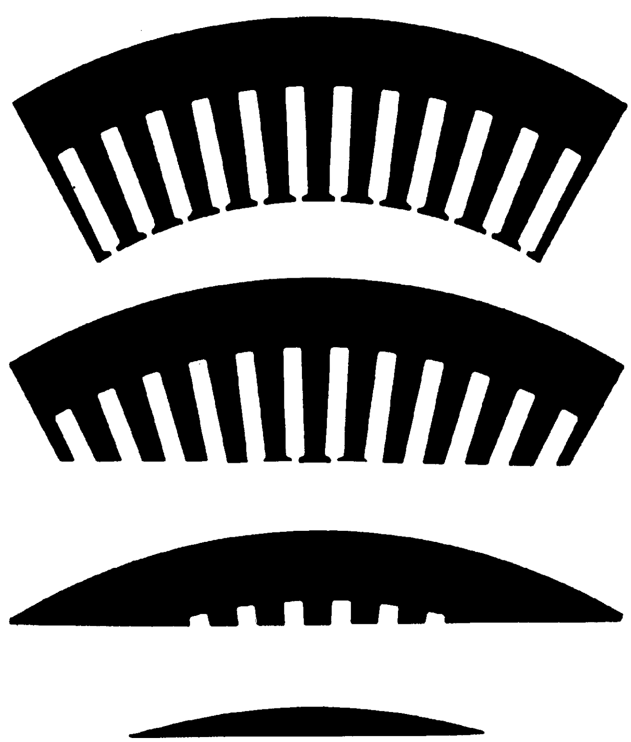

Fig. 5.—Types of stamping used in stator construction.

shown in this diagram for the sake of clarity. Each stamping is kept in a radial plane by suitable incorporation of truncated stampings in the right proportion (as shown in Fig. 5). To assist in stacking, a dummy rotor was made, being oversize by the amount of the air-gap. This can be seen in Fig. 6, which shows a machine in process of construction.

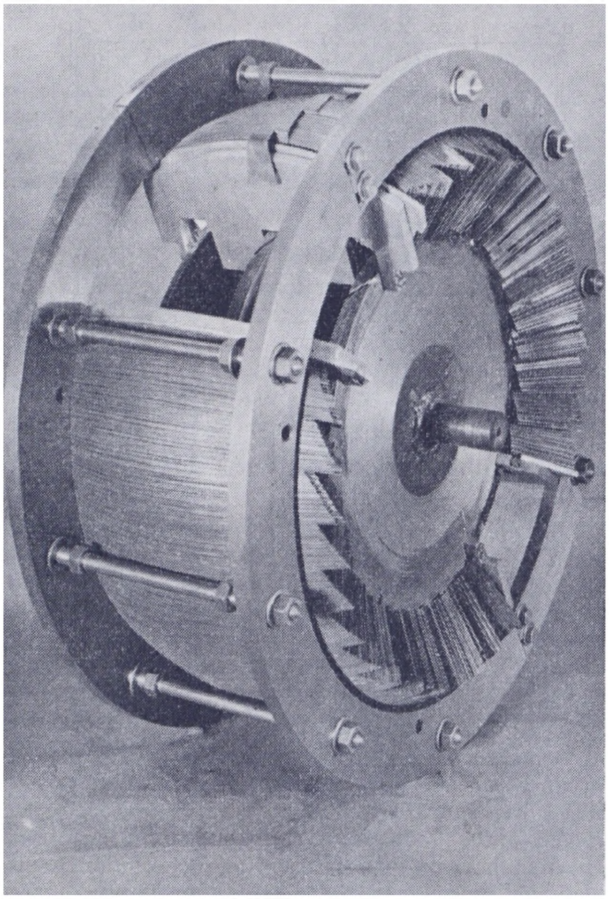

Fig. 6.—Method of stator construction.

Laminations were stacked to fill the first 5° with radially-disposed laminations, as shown.* This packet was given a skew of one slot pitch. The punchings were held together by clamps; the inner surface was pressed into contact with the dummy rotor, and a light weld was run along the back surface to hold the packet together. When sufficient packets had been made, they were stacked between end plates on the dummy rotor, with the slots aligned but advanced relative to each other by one slot per packet.

Two strips of thin steel were welded to the backs of the packets to form one stator block. These were used to anchor the block to the bolts joining the forming rings.

It is apparent from the geometry that each point on the rotor surface crosses the same number of slots as it traverses the block. In other words, there is no velocity discrepancy so long as the stator block is in the central position. Rotation of both blocks together as a unit produces an effective increase in θ in each block, or a reduction in θ in each.

The first 2-block machine built had a 45° pre-skew. With this rangement $\pm 18\frac{1}{2}°$ rotation of the stators is sufficient to give a speed range of $\cos 26\frac{1}{2}°/\cos 63\frac{1}{2}° = 2:1$. With such comparatively small stator displacements, very little of the stator surface is turned off the rotor edge even though each block subtends nearly 180°.

* This is the figure appropriate to about 45° skew with 12 slots per segment. The machine actually shown in Fig. 6 is a later model with $63\frac{1}{2}°$ skew which involves the use of 10° packets.

There are several advantages in such a form of construction, namely

(*a*) The increase in block length compared with a 4-block machine of the same speed range enables either an increase in pole numbers or an increase in minimum pole-pitch to be obtained on a given size of rotor. Both these features are known to give improved performance.

(*b*) At one speed setting, i.e. with the axis of the stator forming-rings coincident with the axis of the rotor, there is no velocity discrepancy.

(*c*) Since both blocks can be rotated as a unit, there is ideally no magnetic thrust on the stator bearings as there is in the 4-block type.

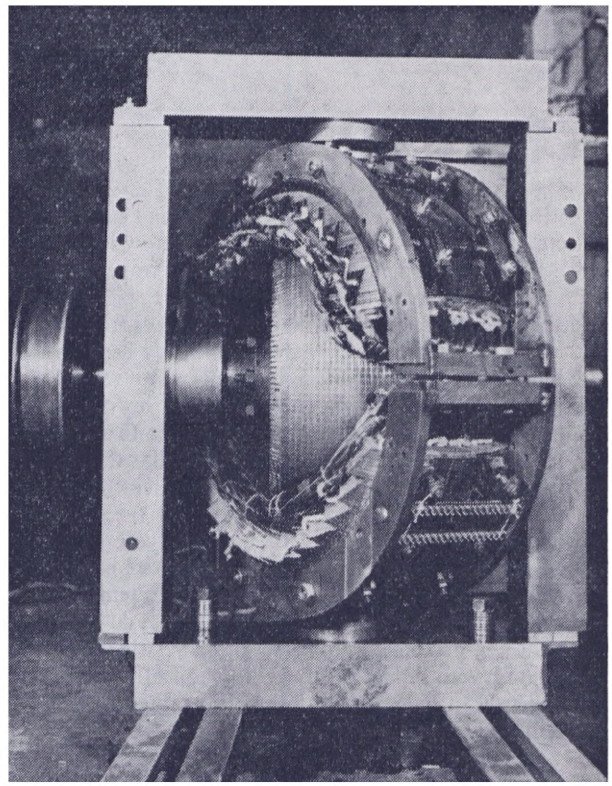

Fig. 7.—A complete 2-block machine ($63\frac{1}{2}°$ pre-skew).

The remaining mechanical details are summarized in Fig. 7 which shows one of the later complete machines with a $63\frac{1}{2}°$ pre-skew.

(3.1) Application of the Theory to Pre-Skewed Machines at Mid-Position

It has been pointed out that, when the stator blocks are centrally placed over the rotor, every element of the rotor spends the same time under the block and crosses the same number of slots. No rotor elements either enter or leave the sides of the block. Thus, for all rotor elements the effective number of poles on the block is the number of poles measured in the direction of motion of the rotor.

The theory developed in Reference 2 is thus fairly easily applied to the block in the central position where $\theta = \theta_0$, and θ_0 is the angle of pre-skew. In particular, the equations for the component of flux density B_p, which is in phase with the stator current J_s, and the component B_q, which is in quadrature with J_s, can be rewritten as

$$B_p = \frac{\rho_r J_s \pi}{p\omega} \frac{\sigma}{\sigma^2 + (1/\tau\omega)^2} \left\{1 - \exp\left[\frac{-\psi}{\tau\omega(1-\sigma)}\right]\right\}\left[\cos\frac{\sigma}{1-\sigma}\psi + \frac{1}{\sigma\tau\omega}\sin\frac{\sigma}{1-\sigma}\psi\right] \quad . \quad . \quad . \quad . \quad (2)$$

$$B_q = \frac{\rho_r J_s \pi}{p\omega} \frac{1/\tau\omega}{\sigma^2 + (1/\tau\omega)^2} \left\{1 - \exp\left[\frac{-\psi}{\tau\omega(1-\sigma)}\right]\right\}\left[\cos\frac{\sigma}{1-\sigma}\psi - \tau\omega\sigma\sin\frac{\sigma}{1-\sigma}\psi\right] \quad . \quad . \quad . \quad . \quad (3)$$

These equations have been obtained from eqns. (13) and (14) in Reference 2 by writing

$$\tau = 4p^2\mu_0/\pi\rho_r g \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$p\omega/\pi = u_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\psi = \pi s/p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Of these quantities, τ is rotor time-constant, and ψ is the distance in electrical radians which a rotor element, now situated at the point where the flux density is measured, has travelled under the stator block since entry. For example, in Fig. 2 for the flux density at P″ the relevant value of ψ is PP′ expressed in electrical radians of the current wave. $p\omega/\pi$ has been written for u_s to eliminate any confusion between wave velocity in the direction of the wave motion (u_s) and actual rotor synchronous speed, $u_s/\cos\theta$. It is important to remember that the symbols J_s, p, u_s and s relate to a direction perpendicular to the slots and not to the direction of rotor movement.

In Reference 2 the torque/slip curve and the rotor-copper-loss/slip curve were developed for the ideal case of a machine with no air-gap. A brief outline of the extra losses occurring at the exit edges of the stator blocks was attempted. These calculations are now extended, using eqns. (2) and (3) to include finite values of decrement, and a set of curves showing exit-edge loss variation with slip is added. Prediction of the performance of pre-skewed machines may now be attempted for the $\theta = \theta_0$ condition.

(3.2) Rotor Copper Loss

Rotor copper loss $= \frac{1}{2}\rho_r \int_0^{np} J_r^2 ds$, where J_r is the rotor current density, and may be written

$$J_r^2 = (J_s - J_{mp})^2 + J_{mq}^2$$

J_{mp} represents the component of J_s which sets up B_q, and J_{mq} the component of J_s setting up B_p.

As in Section 12.1 of Reference 2 it is assumed that J_{mp} and J_{mq} may be written

$$J_{mp} = gB_q/4p\mu_0 \text{ and } J_{mq} = gB_p/4p\mu_0$$

whence

$$\tfrac{1}{2}\rho_r \int_0^{np} J_r^2 ds = \tfrac{1}{2}\rho_r \int_0^{np}\left[J_s^2 - \frac{2J_s g}{4p\mu_0}B_q + \left(\frac{g}{4p\mu_0}\right)^2(B_p^2 + B_q^2)\right]ds \quad . \quad . \quad . \quad . \quad (7)$$

Evaluation of the rotor copper loss from this expression is completed in Section 10.1.1.

(3.3) Torque/Slip Curves

The total force, F, exerted tangentially by a stator block is given by

$$F = \frac{J_s}{2}\int_0^{np} B_p ds$$

Substituting for B_p from eqn. (2) gives

$$F = \frac{\rho_r J_s^2}{2u_s} \frac{\sigma}{\sigma^2 + (1/\tau\omega)^2}\int_0^{np}\left\{1 - \exp\left[-\frac{\psi}{\tau\omega(1-\sigma)}\right]\right\}\left[\cos\frac{\sigma}{1-\sigma}\psi + \frac{1}{\tau\sigma\omega}\sin\frac{\sigma}{1-\sigma}\psi\right]d\psi \quad . \quad . \quad (8)$$

The integration is completed in Section 10.1 and expressed as Fu_s.

(3.4) Magnetic Energy

Rotor elements enter the stator arc in an unenergized condition. As they pass under the block, magnetic flux is built up in the elements in accordance with eqns. (2) and (3). On reaching the end of the block the total flux density in the element is

$$B_x = \sqrt{(B_{px}^2 + B_{qx}^2)}$$

Associated with this flux density there is stored energy

$$\frac{1}{2}\left(\frac{B_x^2}{8\pi\mu_0}\right) \text{ per unit volume of gap}$$

This magnetic energy has been acquired during transit, and the rate of arrival of magnetic energy, non-existent at the entry edge, is

$$\frac{1}{2}\frac{(B_{px}^2 + B_{qx}^2)}{8\pi\mu_0}gu_r \text{ per strip of stator of unit width.}$$

It is necessary for this magnetic energy to be very largely dissipated before re-entry under the block. The phase of the 'carried over' flux as it re-enters will in general not be ideally disposed for the production of useful drive torque. It has been verified in practice that the carrying-over of flux results in undesirable undulations of the speed/torque curve, especially in the working range, since any flux carried over may either aid or oppose the wanted flux, depending on the speed setting (θ) and the slip, and tends always to favour synchronous speeds corresponding with even pole numbers, as would be expected by analogy with conventional machines.

It was suggested in the previous paper that the desired dissipation of exit-edge flux could be achieved by arranging for the rotor time-constant outside the block to have a value $\tau_0 = \tau/\lambda$, where $\lambda > 1$ and is chosen to dissipate the flux over the arc-length available between blocks. It was suggested that if this were done the magnetic energy being transported away immediately outside the block would be λ times that being transported under the block, i.e. the stator exit-edge loss P_x would be given by

$$P_x = \tfrac{1}{2}\lambda g\frac{(B_{px}^2 + B_{qx}^2)}{8\pi\mu_0}u_r \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where λg is the gap which would be required outside to give the time-constant τ_0, assuming no rotor leakage.

Eqn. (9) can be re-expressed as

$$P_x = \frac{1}{2}\frac{\tau}{\tau_0}g\frac{(B_{px}^2 + B_{qx}^2)}{8\pi\mu_0}u_r$$

It is convenient in the design procedure considered later to express $\tau\omega$, whenever it appears, in terms of the time t_s spent under the block by a point travelling at speed u_s, whence $t_s = np/u_s$.

Thus
$$\frac{P_x}{t_s/\tau_0} = \frac{1}{2}\,\frac{g}{8\pi\mu_0}(B_{px}^2 + B_{qx}^2)\frac{u_r}{t_s/\tau} \quad . \quad . \quad . \quad (10)$$

In a practical machine τ_0 must be chosen in appropriate relation to the time elapsing between leaving one block and entering the next, since it defines the time required to dissipate all but $(1/\varepsilon)^2$ of the transported energy. The quantity $P_x(\tau_0/t_s)$ is evaluated in Section 10.1. The general form of this exit-edge loss is shown by Fig. 8, which represents eqn. (26) plotted for $n = 4$ and various values of t_s/τ.

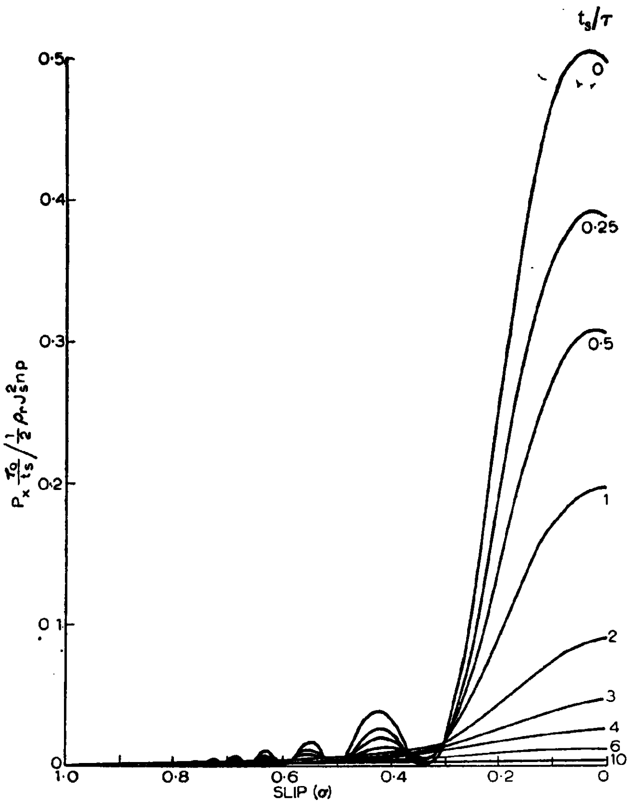

Fig. 8.—Theoretical curves of exit-edge loss.

(4) EXPERIMENTAL RESULTS

(4.1) The First Machine with Pre-Skewed Slots

Stator rotation was limited mechanically to $\pm 12°$. The stator blocks had a pre-skew angle of 45°, and the arc length in the direction of motion of the rotor was made as large as possible, the limiting factor being the space required for the end turns of the winding. The arc length was in fact 140°, whilst each segmental stator punching subtended 60° and carried 12 slots. The two slot per pole per phase, 5/6 chorded, double-layer winding therefore produced two pole-pitches of m.m.f. wave across every stator lamination, and therefore 4·67 pole-pitches around the 140° arc. The synchronous speed for the central position was therefore 500 r.p.m. when using 50 c/s supply.

The first series of experiments was designed to verify that the theory developed in Reference 2 was applicable to the pre-skewed machine in the manner outlined in Section 1. A flux analysis was carried out with the stator blocks central ($\theta = \theta_0$) in accordance with the method described in Reference 2, Section 4, due allowance being made for leakage flux. The results of this experiment are shown in Figs. 9 (*a*)–(*e*). The curves represent the calculated flux densities using eqns. (2) and (3).

The agreement between the calculated and observed values of B_p and B_q was taken to be ample evidence that the original theory could be relied upon, at least for values of n greater than 4.

(4.1.1) Load Test.

The torque/speed curves of this machine were obtained using an acceleration method at low voltage.[3]

A load test was performed at constant current so that direct comparison with the predicted curves of synchronous watts and input could be made. The calculation of stator exit-edge loss involves the determination of λ. In the absence of a calculated value, λ was derived by measuring the time-constant τ_0 outside the block. To do this, two search coils were mounted on non-magnetic supports at different distances from the exit edge, and readings of flux density in these coils were taken when the machine was running light, from which τ_0 was calculable. The magnetic energy arriving at the exit edge has been supplied directly by the stator. If the stator iron continues beyond the end of the stator winding, so that the gap outside equals the gap inside, there is no back-torque on the rotor. If the gap increases outside, as is the case in practice, then power $[B_x^2 u_r(\lambda - 1)g]/8\pi\mu_0$ per unit width is supplied as mechanical work by the rotor, i.e. there is an appropriate back-torque. The remaining power $B_x^2 u_r g/8\pi\mu_0$ is supplied as before by the stator directly.

The calculated performance curves were obtained as follows. The output/slip curve was plotted from the torque equation derived in Section 10.1 and computed for $n = 4{\cdot}67$. From the ordinates of this curve were subtracted $(\lambda - 1)/\lambda$ times the exit-edge losses plotted in a similar manner to those in Fig. 8 computed again for $n = 4{\cdot}67$. This is the real output curve. To this is added the calculated rotor copper loss as derived in Section 10.1, together with $1/\lambda$ times the exit-edge loss to give a synchronous-watts curve. A final addition of calculated stator copper loss gives the intake/slip curve. The full lines of Fig. 10 represent the calculated curves, while the points are the corresponding experimental values. The agreement is remarkably good considering the approximations made, one of the most satisfactory features being the ability to predict the failure of the machine to reach the calculated synchronous speed (in this case 1 000 r.p.m.).

(4.1.2) Variations in θ.

Load tests were carried out at minimum (33°) and maximum (57°) values of θ, and at $\theta = \theta_0$ using a 50 c/s supply. The intake and synchronous-watts curves, for constant-voltage operation, and power factor/slip and efficiency curves are shown in Fig. 11. The performance at 57° was disappointing. The efficiency had fallen considerably from the values at the other settings, the peak output at constant voltage was lower than at $\theta = \theta_0$, and the power factor was lower. Investigations into the reasons for the poor top-speed performance led to a detailed mathematical analysis to determine the effective speed of the field at different points on the rotor surface. A ratio $\gamma = u_r \cos\theta/u_s$ has been calculated for all points on the rotor surface, assuming that the velocity at the centre of the stator block is equal to $u_s/\cos\theta$. The analysis is performed in Section 10.2, and the lines on Fig. 12 are lines of constant γ.* The discrepancies are more serious in the case of $\theta > \theta_0$ than of $\theta < \theta_0$, for stator components of lower speed than the demanded value will produce back-torques in the working region, whilst components of higher speed, as appear when $\theta < \theta_0$, only come fully into effect when the slip is negative.

* The projection found most useful in describing spherical motors is the 'plate carée' projection in which degrees of longitude and latitude are represented as uniform scales of x and y respectively on a Cartesian co-ordinate system. The shaded areas in Fig. 12 correspond with regions of the stator which are traversed by rotor surface.

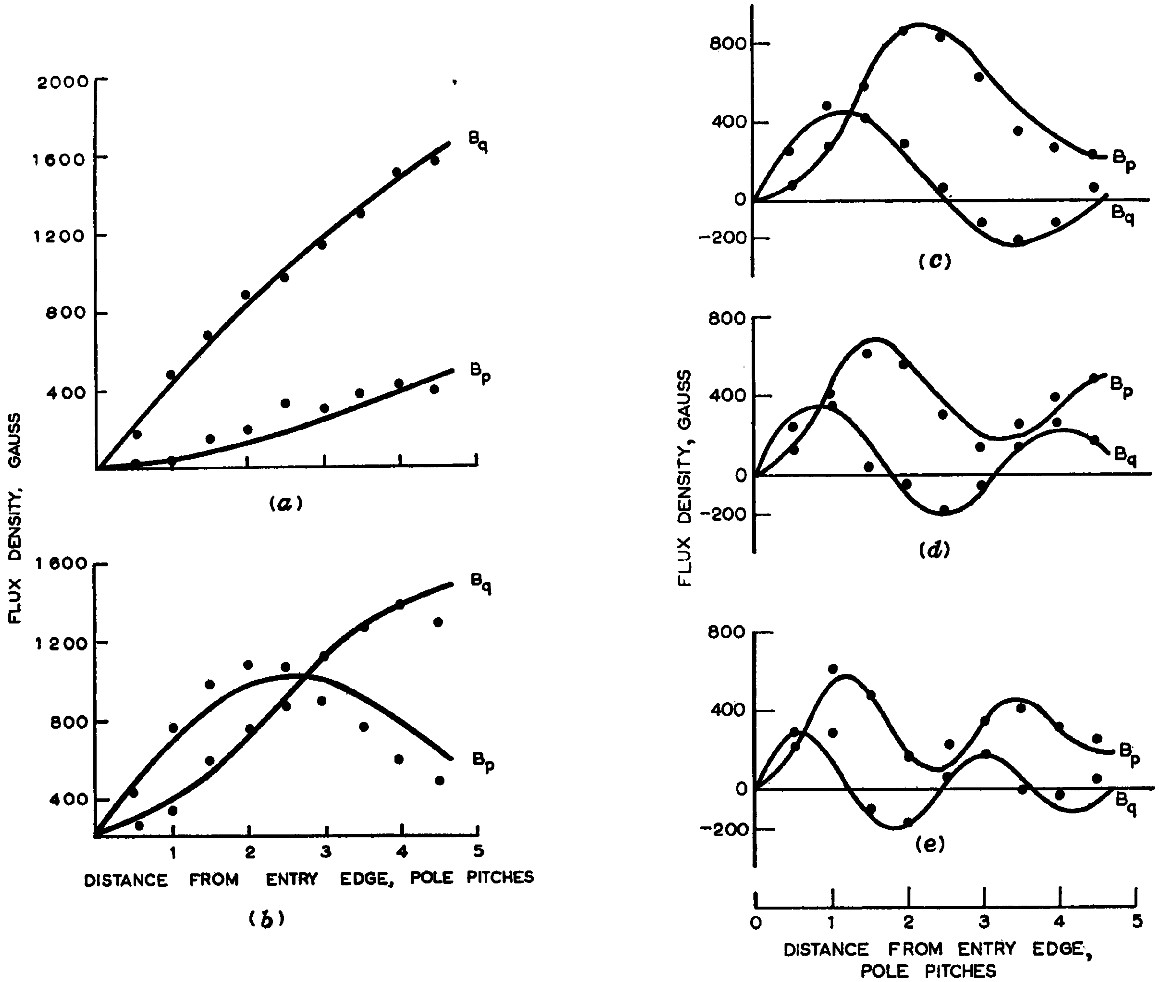

Fig. 9.—Flux distribution under a stator block.

(*a*) $\sigma = 4\%$.
(*b*) $\sigma = 1/(n + 1)$.
(*c*) $\sigma = 2/(n + 2)$.
(*d*) $\sigma = 3/(n + 3)$.
(*e*) $\sigma = 4/(n + 4)$.

(4.1.3) Conclusions Drawn from the First Machine.

The results of the first machine were quite pleasing in that the motor had reasonably good characteristics for the range $\theta = 33°$–$45°$ and would doubtless have been almost equally good below 33° had the mechanical arrangement permitted. The performance at $\theta > 45°$ was poor for several reasons.

(*a*) The peak output was reduced mainly because part of the stator block was turned off the rotor. This effect probably helped to lower the power factor also.

(*b*) The efficiency was lower because the number of poles was reduced.

(*c*) The efficiency and output were lower because of velocity discrepancies.

These facts could be summarized by saying that the machine was satisfactory for about 1·4 : 1 range only.

(4.2) Designing for Peak Performance at Top Speed

The next stage in the development is fairly obvious. The angle of pre-skew should be increased so that $\theta = \theta_0$ represents the top-speed condition where it is thought desirable to have maximum output, best efficiency, and so on, with the stator block centrally placed. All the stator surface is usefully employed, and there is no velocity discrepancy. The pole number can be arranged to be any desired value by design.

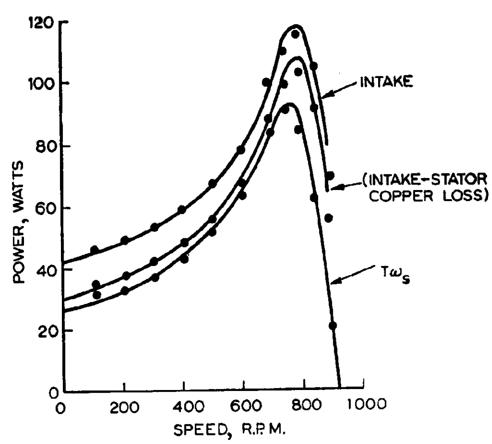

Fig. 10.—Load tests at 100 c/s on 45° pre-skewed machine.

——— Theoretical curves.
● ● ● Experimental results.

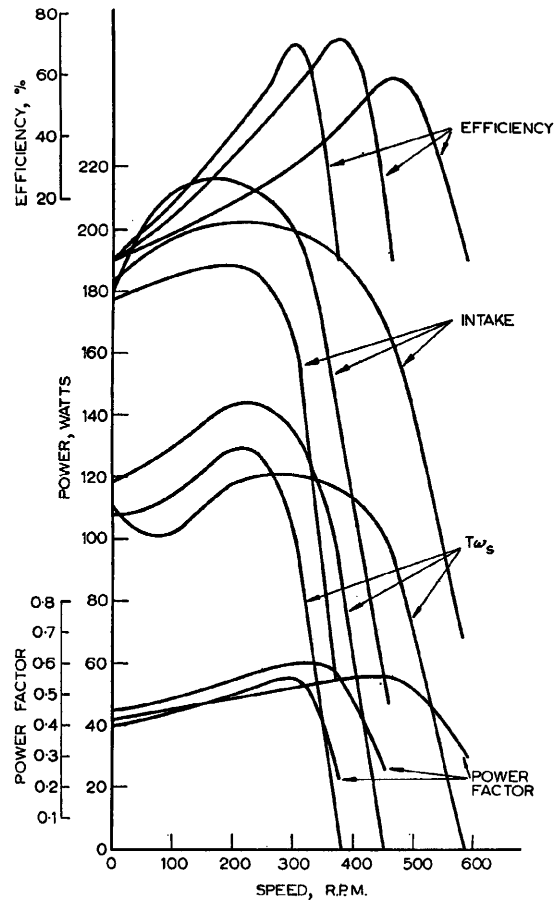

Fig. 11.—Performance curves for constant-voltage operation.

(4.2.1) The Second Pre-Skewed Motor.

Accordingly, a second motor was built with a view to a 2 : 1 speed range. The value of θ_0 was $63\frac{1}{2}°$ and θ could be varied down to 27° to give the desired range. The winding was designed to give 4·67 poles around 140° at top speed. This involved the use of four poles across the arc α. Using a 12-slot segment, the winding had one slot per pole per phase and was 2/3 chorded.

The main interest in this machine was the performance at top speed. A load test on 50 c/s supply at constant current yielded the results shown in Fig. 13(*a*). The cross at peak torque was obtained when the machine was delivering 5·1 h.p. The only difference in the high-power points is an increase in stator copper loss, and in rotor copper loss at standstill, both being due to the increased resistance at higher temperatures. Even though the mechanical construction was superior to that of the first machine and a smaller air-gap was obtained, the peak efficiency was reduced to 46%, with $n = 4·67$ as before. The theoretical calculations based on eqns. (2) and (3) enabled an explanation of the poor performance of the second machine to be given. Fig. 13(*a*) shows that the low efficiency is largely due to excessive stator copper loss. The ratio between the stator and rotor copper losses at standstill is seen to be about 0·82 and, had the coupling been good, the efficiency might have approached the value calculated from the simple theory[2] which showed that for $t_s/\tau = 0$, the output was equal to n times the rotor copper loss, and that rotor and stator current densities were equal. Using this theory, and neglecting iron loss and exit-edge loss, an efficiency of $4·67/(4·67 + 1 + 0·82) = 72\%$ would result. However, it is clear from Fig. 13 that the stator copper loss at full-load slip of $1/(n + 1)$ is a great deal bigger than $(0·82 \times \text{output})/4·67$.

Eqns. (7) and (8) may be used to predict the efficiency providing the value of t_s/τ is known. The value of τ was measured using the standstill method described in Section 4.1 and was

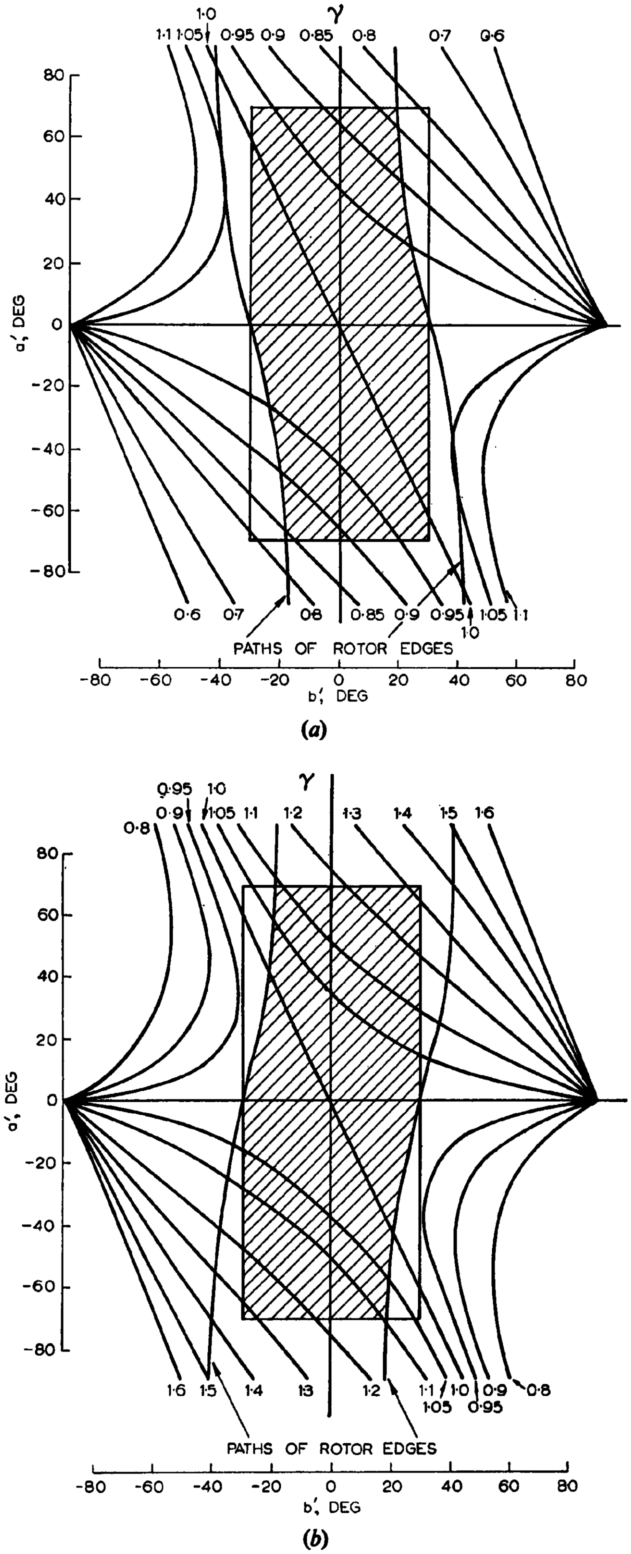

Fig. 12.—Velocity discrepancies for 45° pre-skewed machine.

(*a*) $\theta = 33°$.
(*b*) $\theta = 57°$.

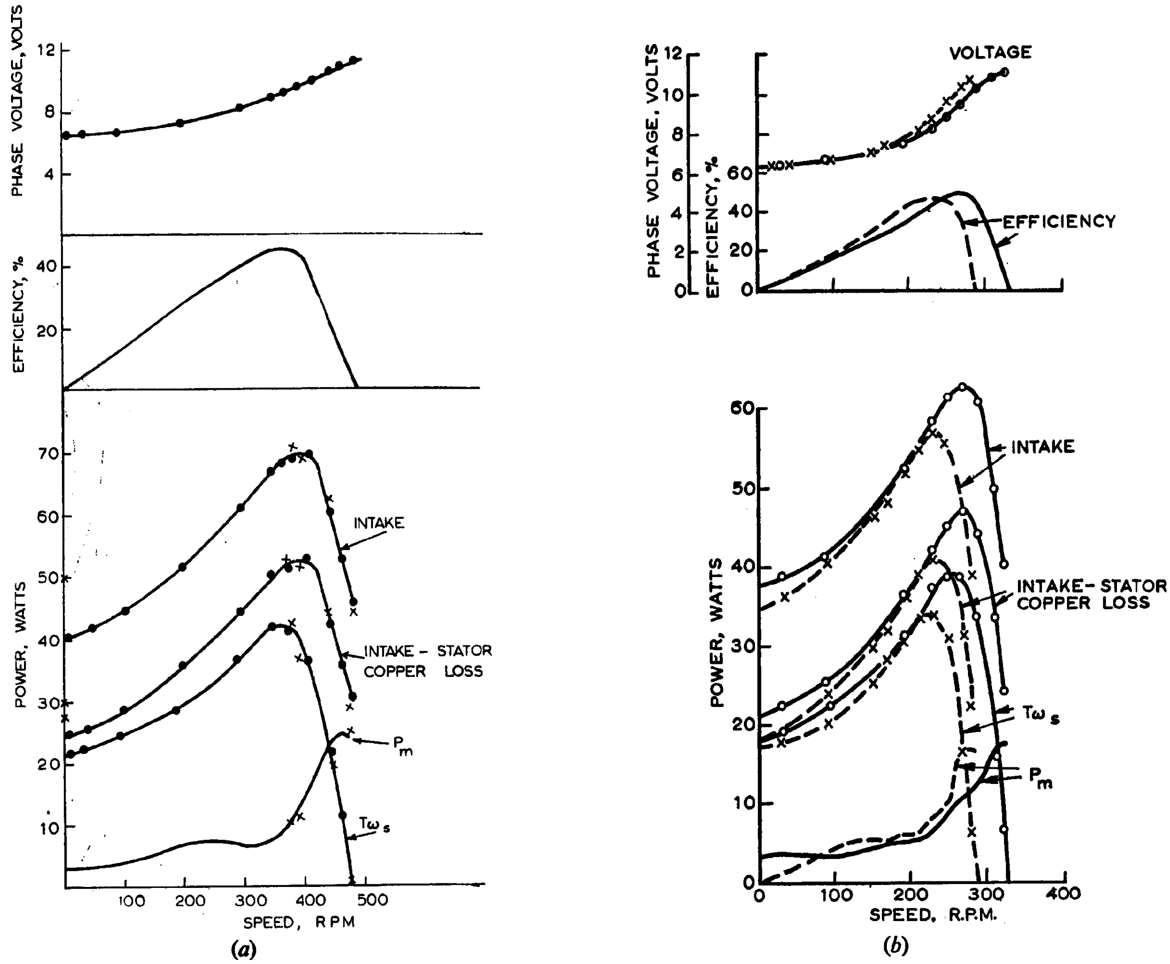

Fig. 13.—Load tests on a 63½° pre-skewed machine.

(a) $\theta = 63\frac{1}{2}°$.
● ● ● Results of constant-current test.
× × × Scaled results from constant-voltage test.
(b) —○—○— $\theta = 45°$.
---×---×--- $\theta = 27°$.

found to give a value of $t_s/\tau = 3 \cdot 1$. For this value, the peak torque occurs at approximately 25% slip.

Using eqns. (7) and (8) for $n = 4 \cdot 67$, the output per unit of stator copper loss at a slip of 25% is $1 \cdot 36\rho_r/\rho_s$ units. The corresponding rotor copper loss per unit of stator copper loss is $0 \cdot 51\, \rho_r/\rho_s$. Hence the efficiency predicted is

$$\frac{\dfrac{1 \cdot 36}{0 \cdot 82}}{\left(\dfrac{1 \cdot 36}{0 \cdot 82} + \dfrac{0 \cdot 51}{0 \cdot 82} + 1\right)} = 50\%$$

The difference between this value and the measured efficiency is presumed to be due to iron loss and stator exit-edge loss. The high value of t_s/τ in this second machine compared with the first is due entirely to the increase in pre-skew which was effected whilst maintaining the same number of poles at top speed. This reduced the pole pitch by $\cos 63\frac{1}{2}°/\cos 45° = 0 \cdot 63$. Had not the air-gap been reduced at the same time, the value of t_s/τ in the first machine would have been multiplied by $(1/0 \cdot 63)^2$ to become $5 \cdot 0$, and the efficiency would have been even lower than 46%. Comparison of the first two machines brought out clearly the fact that, on a given frame size, severe penalties were incurred by demanding too small a pole pitch, which in this case was the result of demanding too high a speed range.

When θ was changed, the efficiency remained fairly constant, and the properties of the machine at $\theta = \theta_0$, poor as they were, were made very little worse by the reduction of θ, as is illustrated by the results of load tests at angles of 45° and 27° shown in Fig. 13(b). The relevant velocity discrepancy diagram for $\theta = 27°$ is shown in Fig. 14.

(4.2.2) A Machine with a Small Pole Number.

One further experiment was performed on the second machine. The $4 \times 4 \cdot 67$ pole winding was replaced with a $2 \times 2 \cdot 33$ pole, 2-slot per pole per phase winding. The results showed that the peak efficiency of 45% was now much nearer that predicted by the simple theory, i.e. $2 \cdot 33/(2 \cdot 33 + 1 + 0 \cdot 82) = 56 \cdot 0\%$. The value of t_s/τ for this motor is one-quarter that of the $4 \times 4 \cdot 67$ pole machine.

(5) THE DESIGN EQUATIONS OF A PRE-SKEWED MACHINE

After the experiments on the 2-block type of machine had been completed, it was clear that certain parameters could be adjusted to give optimum performance for a given frame size and speed range. For example, it seemed likely that a machine with a 14 in-diameter rotor having a 2 : 1 speed range would have the best efficiency if n were chosen to be between $2 \cdot 33$ and $4 \cdot 67$.

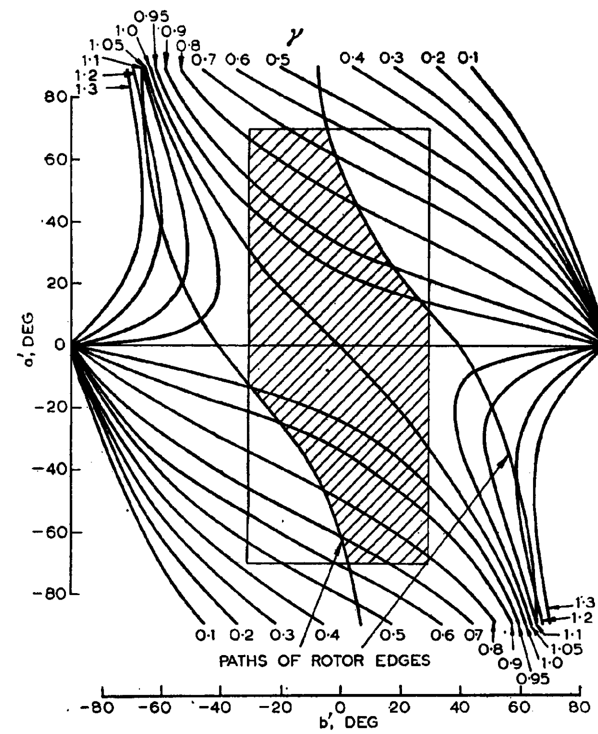

Fig. 14.—Velocity discrepancies for $63\frac{1}{2}°$ pre-skewed machine with $\theta = 27°$.

Having verified the theoretical arguments outlined in Sections 1 and 3 for the behaviour of pre-skewed machines, and being convinced that designing for optimum performance at top speed is advantageous, the authors set out the rules of short stator theory in the form of fundamental design equations for pre-skewed machines as follows, the full-load condition being taken to occur at a slip of $1/(n+1)$.

(5.1) Rotor Time-Constant

The arc length subtended by one stator block in the direction of motion of the rotor when $\theta = \theta_0$ may be expressed as $k_1 \times 2\pi r$, where r is the rotor radius.

The pole pitch in the direction of motion of the rotor in this position is $2\pi r k_1/n$, whence the pole pitch p in the direction of motion of the stator m.m.f. is $(2\pi r k_1/n)\cos\theta_0$. It follows that

$$\tau = \frac{4p^2\mu_0}{\pi\rho_r g} = \frac{16\pi r^2 k_1^2 \mu_0 \cos^2\theta_0}{n^2\rho_r g} \times 10^{-9} \text{ in C.G.S. units.}$$

The time t_s spent under the block at synchronous speed is $n/2f$, where f is the supply frequency. Hence

$$\frac{t_s}{\tau} = \frac{n^3\rho_r g 10^9}{32 f\mu_0 \pi r^2 k_1^2 \cos^2\theta_0} = K_3 \text{ (say)} \quad . \quad . \quad . \quad (11)$$

(5.1.1) Rotor Copper Loss.

For perfect coupling, the rotor copper loss would be $\frac{1}{2}\rho_r J_s^2 \times$ (block area). For $t_s/\tau \neq 0$, the rotor current is modified and the copper loss is calculated in accordance with eqn. (23) of Section 10.1. As this is a complicated function of n and K_3 it is convenient for the purposes of design to write the rotor copper loss as $\frac{1}{2}\rho_r J_s^2 k_\tau$ (block area), where k_τ can be read from a set of curves for different values of n and K_3 from eqn. (23) when desired. It is also convenient to calculate in terms of the current per slot, whence the chording factor k_5 must be introduced to give

$$\frac{\frac{1}{2}\rho_r J_s^2 k_\tau}{k_5^2} \text{ (block area)} = \text{rotor copper loss} = K_1$$

For a rotor of axial length equal to rotor radius (i.e. $\alpha = 60°$) the total spherical surface area $= 2\pi r^2$, whence the block area $= 4\pi r^2 k_1$ and

$$K_1 k_5^2 = 2\pi r^2 k_1 \rho_r J_s^2 k_\tau \quad . \quad . \quad . \quad . \quad . \quad (12)$$

(5.1.2) Flux Density.

Since the flux density is non-uniform it would be advantageous to utilize available space for winding where the flux density was low, i.e. at the entry edge. The method of construction proposed prohibits graded slot sizes, however. In any case a graded machine would be limited to rotation in one direction only. The punchings must, therefore, be designed to contain the peak flux density, which at full-load slip occurs at the exit edge. The fact that the density at the edge may rise above this value for slip less than $1/(n+1)$ is not thought to be serious, as it affects only a small fraction of the stator.

Eqn. (22) of Section 10.1 shows that at a slip of $1/(n+1)$ the flux density at the exit edge is

$$B_x = \frac{\rho_r J_s \pi(n+1)}{p\omega\sqrt{\left[1+\left(\frac{n+1}{\tau\omega}\right)^2\right]}}(1+\exp - t_s/\tau)10^8$$

If the ratio of slot width to slot pitch is k_2, the peak tooth density is $B_x/(1-k_2)$, which after substitution for p gives the flux density (K_2) as

$$K_2 = \frac{\rho_r J_s(n+1)n(1+\exp - t_s/\tau)10^8}{4\pi r k_1 f\cos\theta_0(1-k_2)\sqrt{\left[1+\left(\frac{n+1}{\tau\omega}\right)^2\right]}} \quad . \quad (13)$$

(5.1.3) Stator Copper Loss and Slot Leakage Reactance

To produce given values of J_s, stator copper loss and slot leakage reactance are interdependent, the deeper the slot the higher the leakage reactance, but the lower the copper loss.

$$\text{The stator resistivity } \rho_s = \frac{\rho_c \text{ (slot pitch)}}{\text{(area of copper section in a slot)}}$$

where ρ_c is the resistivity of copper.

For S slots/pole/phase, the slot pitch in the direction of motion of the field is $2\pi r k_1 \cos\theta_0/3Sn$; the slot area is $2\pi r k_1 k_2 d\cos\theta_0/3Sn$, where d is the slot depth; and the area of copper in a slot is $2\pi r k_1 k_2 k_3 d\cos\theta_0/3Sn$, where k_3 is the packing factor.

Thus the stator copper loss $K_4 = \dfrac{\frac{1}{2}\rho_s J_s^2}{k_5^2}$ (block area) or

$$K_4 = \frac{2\pi r^2 k_1 \rho_c J_s^2}{k_2 k_3 k_5^2 d} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

It is convenient to assess the slot leakage reactance in terms of the voltage drop it produces relative to the component V_p of the supply voltage in phase with J_s which produces the power output, so that we may write

$$\frac{\text{stator copper loss}}{\text{power output}} = \frac{\frac{1}{2}\rho_s J_s^2 \text{ (block area)}}{\frac{1}{2}V_p J_s \text{ (block area)}} = \frac{\rho_s J_s}{V_p}$$

Writing $k_4 = J_s X_s / V_p$, where $J_s X_s$ is the reactive voltage, then

$$\frac{\text{stator loss}}{\text{output}} = \frac{\rho_s J_s}{J_s X_s} k_4 = \frac{\rho_s k_4}{X_s} \quad . \quad . \quad . \quad . \quad (15)$$

X_s/ρ_s is a quantity which depends only on the geometry of a slot. As illustration, it is simply assumed here that an open slot is completely filled with a matrix of copper and insulation, evenly distributed, so that the slot permeance is $d/3w$, where w is the slot width, whence $X_s = 4\pi \, . \, d \, . \, 2\pi f 10^{-9}/3w$ whilst

$$\rho_s = \frac{\rho_c}{wdk_3} \quad \text{and} \quad \frac{X_s}{\rho_s} = \frac{8\pi^2 d^2 f k_3}{3\rho_c} 10^{-9}$$

Eqn. (15) now gives

$$k_4 = \frac{8\pi^2 d^2 f k_3 10^{-9} K_4}{3\rho_c \text{ (output)}}$$

Curves of output/n (rotor copper loss) are plotted in Fig. 15 against t_s/τ for various values of n for a slip of $1/(n+1)$. It

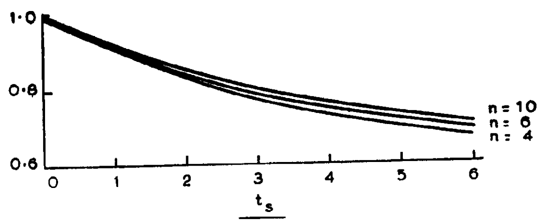

Fig. 15.—Theoretical curves of $\frac{\text{output}}{n \times \text{rotor copper loss}}$.

is apparent that, to a first order of approximation, for t_s/τ in the range $0{\cdot}5 - 2{\cdot}0$, the output may be expressed as $0{\cdot}9nK_1$, whence

$$k_4 = \frac{8\pi^2 d^2 f k_3 K_4 \, 10^{-9}}{2{\cdot}7\rho_c n K_1} \quad . \quad . \quad . \quad . \quad (16)$$

(5.2) Design Procedure

Eqns. (11)–(14) and (16) may be rearranged with the quantities which are being chosen and subsequently regarded as fixed on the right. The quantities ρ_r, J_s, k_2, K_4 and d are regarded as variables in a technique which appears to have advantages. Thus:

$$\rho_r = \frac{32\pi r^2 k_1^2 \mu_0 f \cos^2 \theta_0 K_3}{n^3 g} 10^{-9} \quad . \quad . \quad . \quad (17)$$

$$\rho_r J_s^2 = \frac{K_1 k_5^2}{2\pi r^2 k_1 k_\tau} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\frac{\rho_r J_s}{1 - k_2} = \frac{4\pi r k_1 f \cos \theta_0 K_2 \sqrt{\left[1 + \left(\frac{n+1}{n\pi} K_3\right)^2\right]}}{n(n+1)(1 + \exp - K_3)} 10^{-8} \quad . \quad (19)$$

$$k_2 d K_4 = \frac{2\pi r^2 k_1 \rho_c J_s^2}{k_3 k_5^2} \quad . \quad . \quad . \quad . \quad (20)$$

$$d^2 K_4 = \frac{2{\cdot}7 k_4 \rho_c n K_1}{8\pi^2 f k_3} 10^9 \quad . \quad . \quad . \quad . \quad (21)$$

The procedure suggested is as follows: The rotor copper loss is fixed for a particular value of r by the number of watts per unit area which can be dissipated by the cooling arrangement. For a machine of given output, the radius is therefore fixed by the fact that the output is nearly n times the rotor copper loss and that n has been chosen by experience*; $\cos \theta_0$ is fixed by the speed range demanded; g is set by considerations of mechanical construction and pole-face losses. By starting from the output and choosing r for maximum dissipation, an attempt is being made to obtain minimum frame size. The remainder of the design is aimed at optimizing the efficiency and minimizing the stator cooling problem. A value of K_3 is chosen. Then ρ_r is calculated from eqn. (17). Having obtained ρ_r, J_s can be calculated, and so on, until eqns. (20) and (21) remain with only d and K_4 as variables. Solution of this pair of equations for K_4 completes the first run. Now K_3 is changed and a repeat calculation indicates the trend in K_4. Then K_3 is varied until K_4 is a minimum. This minimum value is either acceptable or not as regards heating or efficiency. If it is unacceptable, n, r and K_1 are the parameters which may be changed.

The theory which would enable power factor to be predicted is incomplete. In the absence of an exact formula the expression

$$\frac{\int_0^{np} B_q ds}{\int_0^{np} B_p ds} + k_4 = \left[1 + \frac{\rho_s}{\rho_r}\right] \tan \zeta$$

is a rough guide, where the integrals represent the areas under the B_p/distance and B_q/distance curves respectively, and $\cos \zeta$ is the power factor. Substituting for B_p and B_q at $\sigma = 1/(n+1)$ from eqns. (2) and (3) and integrating yields

$$\tan \zeta = \frac{a_0(a_0^2 + \pi^2) - (a_0^2 - \pi^2)(1 + \varepsilon^{-a_0})}{\pi[a_0^2 + \pi^2 - 2a_0(1 - \varepsilon^{-a_0})](1 + \rho_s/\rho_r)} + \frac{k_4}{1 + \rho_s/\rho_r}$$

(5.3) Mechanical Construction

In specific designs due account should be taken of the unusual mechanical loading in a short-stator machine. To this end a calculation of $(B_p^2 + B_q^2)$, which is proportional to the magnetic force per unit area between rotor and stator, is shown in Fig. 16

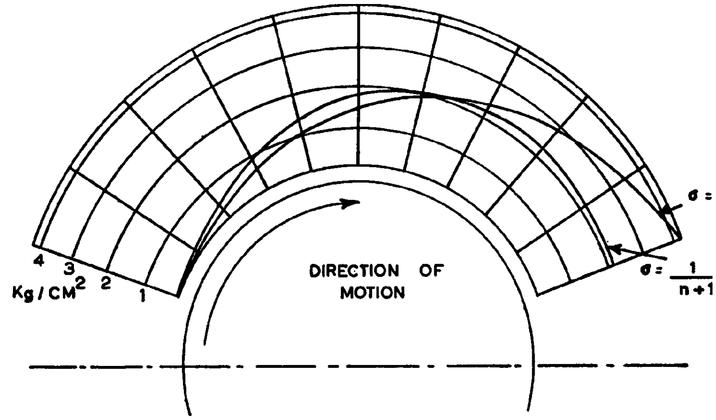

Fig. 16.—Mechanical loading in a 2-block machine.

for the conditions of full load and running light for $t_s/\tau = 1$, as examples of the type of loading to be expected. The peak flux density is taken to be 10 000 lines/cm^2 running light and the synchronous speed 1 000 cm/sec.

The torque required to rotate the stator has not been fully

* The choice of n has so far been limited by the belief that severe penalties are incurred by choosing values of n and θ_0 which demand anything but a whole number of pole pitches of m.m.f. across any given stator lamination. No experimental evidence is yet available to confirm or deny this. Trial designs for 4, 6, 8, etc., poles across a lamination rapidly reveals which is the best value.

investigated, but experiment has shown that its maximum value is about 40% of the full-load torque. A small pilot motor driving the stator shaft via a worm gear is considered to be a suitable arrangement.

(5.4) Rotor Construction

The interconnected conducting matrix of the rotor presents a special problem of its own. For small machines the obvious solution is a cast-aluminium rotor. For larger sizes the rotor conducting grid is probably easier to fabricate than to cast. The experimental rotors so far constructed were wound with tinned copper wire in both directions, together with wires of resin-cored solder. The whole mass was then heated to melt the solder. In larger machines it may be desirable to heat only a small section of the rotor at any one time and perhaps make the joints between bars and rings one at a time. In this case the total number of joints becomes a potent factor in the cost. The machines discussed in Section 4 had 150 rotor slots and 23 rings, making 3 450 joints. The rotors were entirely satisfactory from every point of view, and it was decided to build a further rotor for comparison which had only 54 slots and 7 rings (378 joints). The cross-sectional area of copper per centimetre of periphery was the same as before. The main external effect of a reduction in the number of joints was an apparent increase in rotor resistivity, in this case about 33%. Other evidence supporting this view was the reduction in the ratio peak/standstill torque values and the value of slip at peak torque. The other effect was a reduction of power factor, which is thought to be due to a combination of increased rotor slot leakage and zig-zag leakage. This theory is supported by evidence supplied by the stator exit-edge loss, which was about 15% smaller in the case of the rotor with the coarse structure. Increased rotor leakage increases the value of τ_0.

(6) OPERATION AS A GENERATOR

It was pointed out in Reference 2 that the spherical machine could be used as a generator by driving it above the demanded speed. This property could be utilized for such purposes as regenerative braking or for more continuous operation, such as providing a constant-frequency power supply from a variable-speed mechanical source.

Eqns. (2) and (3) may be used to predict the performance at negative values of slip. Table 1 gives a list of some of the important quantities in the design of a short-stator motor and their corresponding values for the machine used as a generator. The efficiency is lowered when the machine is run as a generator, and the electrical power output as a generator is less than the mechanical output as a motor for the same rotor loss.

Table 1

DESIGN QUANTITIES

	Motor	Generator
Full-load slip	$\frac{1}{n+1}$	$-\frac{1}{n-1}$
Torque × ω_s	$(n+1)$ × rotor copper loss	$(n-1)$ × rotor copper loss
Torque × ω	n × rotor copper loss	n × rotor copper loss
Efficiency at full-load slip	$\frac{n}{n+1+(\rho_s/\rho_r)}$	$\frac{n-1-(\rho_s/\rho_r)}{n}$

(6.1) Experimental Results

The 45° pre-skewed machine was tested as a generator. The results of a load test are shown in Fig. 17, and an experimental flux analysis is shown in Fig. 18. The efficiency as a motor is lower than that in the previous tests owing to the use of a larger air-gap. The lines in Fig. 18 indicate the flux distribution predicted by eqns. (2) and (3). Clearly the performance as a generator can be predicted in a similar manner to that for operation as a motor. No detailed design procedure has been

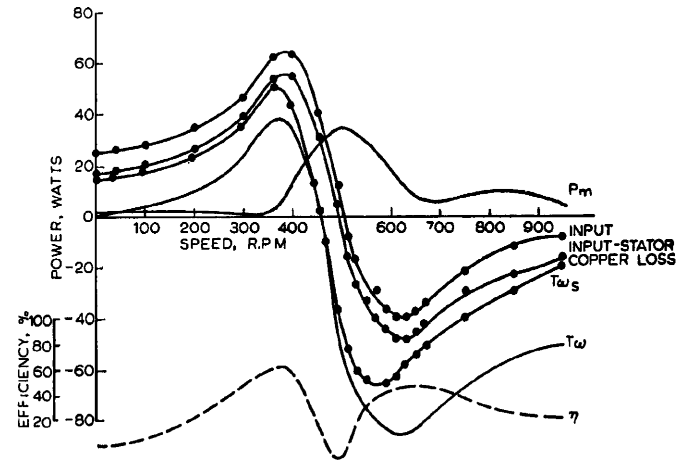

Fig. 17.—Performance curves of a machine used as a generator.

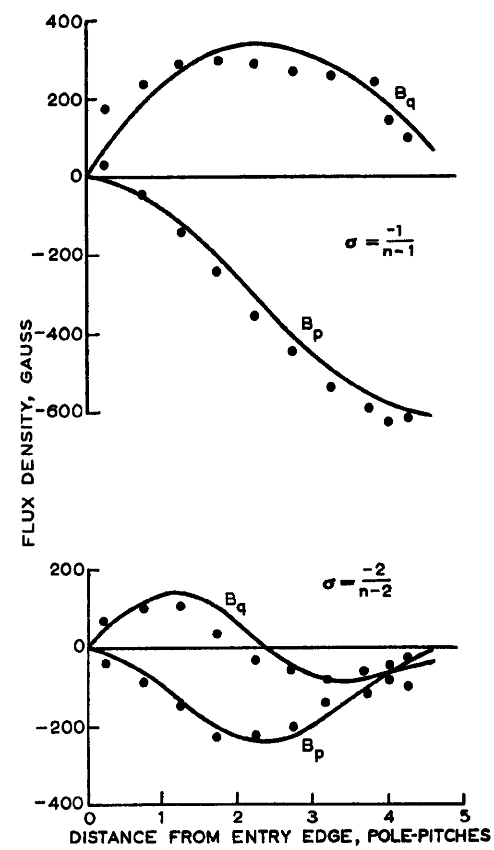

Fig. 18.—Flux distribution under a stator block when generating.

attempted for a machine specifically built for use as a generator, but it appears that for a given output a generator would be slightly larger and have rather more poles than a motor of the same output.

(7) CONCLUSIONS

The main feature of the development work described concerns the design of machines with an angle of pre-skew. The advantages of this type of construction are considerable. The experimental work on the machine with $\theta_0 = 45°$ showed convincingly that the theory developed for this type of machine was substantially correct. It appears that on a 14 in-diameter rotor, a 1·5 : 1 speed range is just about as large as can be obtained with reasonable performance. For larger machines, where n can be increased whilst maintaining an adequate pole pitch, higher speed ranges are clearly possible. The tests on the second machine with a 2 × 2·33-pole block reveal a rather exaggerated feature, which occurs to a lesser extent even in machines of superior design, namely, on a given frame size the best characteristics are not necessarily given by the greatest pole number which gives adequate pole pitch. The technique of designing for $\theta = \theta_0$ at top speed is clearly a good one. The efficiency of the 63½° skew machine, poor though it was, was substantially maintained at reduced angles. The properties of the 45° skew machine at reduced speed were certainly good. The first machine may, in fact, be regarded as a more or less correctly designed machine for 1·4 : 1 range.

Much work remains to be done on the design. The five equations at present used give a broad outlook as to the general effects of changes in various parameters. These equations may conveniently be modified to a 'per unit area' basis as suggested in Reference 4. Factors which have been neglected include rotor leakage, zig-zag leakage, end-turn leakage, and Carter's coefficient, all of which are very difficult to contemplate. The rotor leakage does not contribute any large effect, since the rotors of short-stator machines are of much higher resistivity than their conventional counterparts. Rotor slots in spherical motors tend to be very shallow. Neglect of the other leakages affects the design mostly in respect of the predicted power factor.

(8) ACKNOWLEDGMENTS

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(10) APPENDICES

(10.1) Evaluation of Torque and Losses

(10.1.1) Rotor Copper Loss.

To simplify the integration the following substitutions are made:

$$a_0 = \frac{n\pi}{\tau\omega(1-\sigma)}$$

$$b_0 = \frac{\pi\sigma n}{1-\sigma}$$

$$c_0 = \frac{1}{\tau\omega}$$

$$x = \frac{s}{np}$$

Eqns. (2) and (3) thus reduce to:

$$B_p = \frac{\rho_r J_s}{u_s}\left(\frac{\sigma}{\sigma^2 + c_0^2}\right)\left[1 - \varepsilon^{-a_0 x}\left(\cos b_0 x + \frac{c_0}{\sigma}\sin b_0 x\right)\right]$$

$$B_q = \frac{\rho_r J_s}{u_s}\left(\frac{c_0}{\sigma^2 + c_0^2}\right)\left[1 - \varepsilon^{-a_0 x}\left(\cos b_0 x - \frac{\sigma}{c_0}\sin b_0 x\right)\right]$$

$$(B_p^2 + B_q^2) = \frac{\rho_r^2 J_s^2}{(\sigma u_s)^2[1 + (c_0/\sigma)^2]}[1 - 2\varepsilon^{-a_0 x}\cos b_0 x + \varepsilon^{-2a_0 x}] \quad . \quad . \quad . \quad . \quad (22)$$

From eqn. (7),

rotor copper loss =

$$\tfrac{1}{2}\rho_r \int_0^{np}\left[J_s^2 - \frac{2J_s g}{4p\mu_0}B_q + \left(\frac{g}{4p\mu_0}\right)^2(B_p^2 + B_q^2)\right]ds$$

$$= \tfrac{1}{2}\rho_r J_s^2 np - \frac{\rho_r J_s^2 npc_0^2}{\sigma^2 + c_0^2}\int_0^1\left[1 - \varepsilon^{-a_0 x}\left(\cos b_0 x - \frac{\sigma}{c_0}\sin b_0 x\right)\right]dx$$

$$+ \quad \frac{\frac{1}{2}\rho_r^2 J_s^2 np}{\tau\omega}\int_0^1 (1 - 2\varepsilon^{-a_0 x}\cos b_0 x + \varepsilon^{-2a_0 x})dx$$

After integration and rearrangement the rotor copper loss is given by

$$\text{rotor copper loss} = \frac{\frac{1}{2}\rho_r J_s^2 np}{a_0^2 + b_0^2}$$

$$\left[b_0^2 + \frac{2a_0 b_0}{a_0^2 + b_0^2}(a_0 \sin b_0 + b_0 \cos b_0)\varepsilon^{-a_0} + \frac{a_0(a_0^2 - 3b_0^2)}{2(a_0^2 + b_0^2)} - \frac{a_0}{2}\varepsilon^{-2a_0}\right] \quad . \quad (23)$$

(10.1.2) Torque.

Rewriting eqn. (8) in terms of a_0, b_0 and c_0,

$$Fu_s = \tfrac{1}{2}\rho_r J_s^2 np\left(\frac{\sigma}{\sigma^2 + c_0^2}\right)$$

$$\int_0^1\left[1 - \varepsilon^{-a_0 x}\left(\cos b_0 x + \frac{c_0}{\sigma}\sin b_0 x\right)\right]dx$$

$$= \tfrac{1}{2}\rho_r J_s^2 np \frac{a_0}{c_0(a_0^2 + b_0^2)}\left\{b_0 + \frac{\varepsilon^{-a_0}}{a_0^2 + b_0^2}\right.$$

$$\left.[2a_0 b_0 \cos b_0 + (a_0^2 - b_0^2)\sin b_0] - \frac{2a_0 b_0}{a_0^2 + b_0^2}\right\} \quad . \quad . \quad . \quad (24)$$

(10.1.3) Exit-Edge Loss.

Using eqn. (10) and substituting for $B_p^2 + B_q^2$ from eqn. (22) with $x = 1$,

$$P_x \frac{\tau_0}{t_s} = \frac{\frac{1}{2}\rho_r J_s^2 g(1-\sigma)}{u_s 8\pi\mu_0 \frac{t_s}{\tau}(\sigma^2 + c_0^2)}(1 - 2\varepsilon^{-a_0}\cos b_0 + \varepsilon^{-2a_0})$$

$$= \frac{\frac{1}{2}\rho_r J_s^2 np}{2(a_0^2 + b_0^2)(1-\sigma)}(1 - 2\varepsilon^{-a_0}\cos b_0 + \varepsilon^{-2a_0}) \quad . \quad (26)$$

Eqn. (26) is plotted in Fig. 8 for $n = 4$.

(10.2) Velocity Discrepancy for 2-Block Pre-Skewed Construction

At the neutral position each part of the rotor surface crosses the same number of equally-spaced stator slots as it passes under one block. Therefore the slotting is everywhere such as to correspond with constant skew in an equivalent flat machine. This may be alternatively stated as follows:

When the rotor axis is used as the axis of a cylindrical projection in which equal angles of latitude and longitude are plotted as equal distances, the slots appear in this projection as straight lines. With the stator in the central position ($\theta = \theta_0$) the path of any point on the rotor also produces a straight line, since rotor paths are lines of latitude. If the rotor is turned through an angle ϕ with respect to the stator, the projection of the path of a point on the rotor is no longer a straight line. Fig. 19 illustrates the problem.

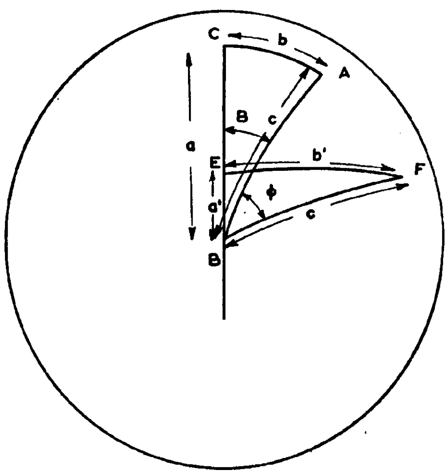

Fig. 19.—The problem of change of spherical axes.

A point A on the rotor is defined by spherical co-ordinates a and b with respect to a point B when the angle $\theta = \theta_0$.

The co-ordinates of this same point after the rotor has been turned through an angle ϕ about axis through B are a' and b'.

Then $\cos c = \cos a \cos b$ from $\triangle$ ABC . . . . . . (27)

$$\frac{\sin B}{\sin b} = \frac{1}{\sin c} = \frac{1}{\sqrt{(1 - \cos^2 a \cos^2 b)}} \text{ from } \triangle \text{ ABC} \quad . \quad (28)$$

$$\frac{\sin (B + \phi)}{\sin b'} = \frac{1}{\sin c} = \frac{\sin B}{\sin b} \text{ from } \triangle \text{ ABC} \quad . \quad (29)$$

$$\cos c = \cos a' . \cos b' \text{ from } \triangle \text{ BEF} \quad . \quad . \quad (30)$$

Substituting in eqn. (28) the value of $\sin b'$ from eqn. (29),

$$\sin b' = \sin b \cos \phi + \sin \phi \cos b \sin a \quad . \quad . \quad (31)$$

and using eqns. (27), (30) and (31)

$$\cos a' = \frac{\cos a \cos b}{\sqrt{[1 - (\sin b \cos \phi + \sin \phi \cos b \sin a)^2]}} \quad . \quad (32)$$

From eqns. (31) and (32) the projected co-ordinates of a point on the rotor surface can be calculated.

The path of the rotor edges is indicated and the area common to rotor and stator is shown shaded in Figs. 12 and 14.

The problem of calculating the field velocity in the direction of motion of the rotor at any position under the stator block at any angle ϕ is again a problem of two systems of co-ordinates. The problem is illustrated in Fig. 19. For a 45° pre-skew it will be apparent that if an observer starts from B and moves a' degrees along the direction of BE and then b' degrees along the direction of EF he crosses $a' \pm b'$ slots if there is one slot per degree, the sense of the slope of the slots governing the sign of b'. The rate of increase of $a' \pm b'$ with a' at fixed b, which is proportional to the stator field velocity in the direction of the rotor, is therefore $1 \pm \partial b'/\partial a'$. Note that b is fixed because a point on the rotor always moves along a line of latitude. When a point (a', b') on the stator block has been chosen, the latitude b of the point on the rotor which passes under (a', b') when the stator is in the position ϕ is defined by eqns. (31) and (32). The rate of increase of longitude a of this point, i.e. the actual rotor angular velocity, is given by $\partial a/\partial a'$ at fixed b, and the rate at which a point on the rotor cuts the stator slots is given by

$$K = \left(1 \pm \frac{\partial b'}{\partial a'}\right) \Big/ \frac{\partial a}{\partial a'} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

For the Figure as drawn, the sign of $\partial b'/\partial a'$ is negative; this is now used throughout for the sake of clarity.

The latitude b which passes under (a', b') for the position θ is found by solving eqns. (31) and (32) for $\sin b$.

Thus, substituting for $\sin a$ in eqn. (31),

$$\sin b' = \sin b \cos \phi + \sin \phi \cos b \sqrt{[1 - (\cos^2 a' \cos^2 b')/\cos^2 b]}$$

where $\phi = (\theta - \theta_0)$.

This yields the quadratic in $\sin b$ as follows:

$$\sin^2 b - (2 \sin b' \cos \phi) \sin b$$
$$- \sin^2 \phi(1 - \cos^2 a' \cos^2 b') + \sin^2 b' = 0$$

Whence $\quad \sin b = \sin b' \cos \phi$

$$\pm \sqrt{[\sin^2 b' \cos^2 \phi + \sin^2 \phi(1 - \cos^2 a' \cos^2 b' - \sin^2 b')]}$$

which reduces to

$$\sin b = \sin b' \cos \phi \pm \sin \phi \sin a' \cos b' \quad . \quad . \quad (34)$$

Consideration of the geometry will show that the relevant sign to the particular problem being studied is the negative one.

From eqn. (31)

$$\cos b' \frac{\partial b'}{\partial a} = \sin \phi \cos b \cos a \quad . \quad . \quad . \quad (35)$$

From eqn. (32)

$$\frac{\partial a'}{\partial a} = \frac{\sin a \cos b \cos^2 b' - \sin b' \cos^2 a \cos^2 b \sin \phi}{\sin a' \cos^3 b'} \quad . \quad (36)$$

From eqns. (35) and (36)

$$\frac{\partial b'}{\partial a'} = \frac{\sin\phi\cos b\cos a\cos^3 b'\sin a'}{\cos b'(\sin a\cos b\cos^2 b' + \sin b'\cos^2 a\cos^2 b\sin\phi)} \quad . \quad (37)$$

Using eqns. (33), (36) and (37)

$$K = \frac{\sin a\cos b}{\cos b'\sin a'} - \frac{\sin b'\cos^2 a\cos^2 b\sin\phi}{\cos^3 b'\sin a'} - \frac{\sin\phi\cos b\cos a}{\cos b'}$$

Substituting for $\sin a\cos b$ from eqn. (31), for $\cos a\cos b$ from eqn. (32), and for $\sin\phi$ from eqn. (34) and simplifying,

$$K = \tan b'\sin\phi\sin a' + \cos\phi - \sin\phi\cos a'$$

For given values of ϕ and K, this is the equation of the line of constant slip over the stator block.

By scaling K so that its value at $x = 0$, $y = 0$ is unity, lines of constant γ can be drawn as defined in Section 4.1.2. These are plotted for a 45° pre-skew in Figs. 12(*a*) and 12(*b*) for $\theta = 33°$ and 57°.

If the angle of pre-skew is not 45°, eqn. (33) is modified to read

$$K = \left(c \pm \frac{\partial b'}{\partial a'}\right)\Big/\frac{\partial a}{\partial a'}$$

where c is the number of lines of longitude crossed per line of latitude by a point moving along a slot. For a 63½° pre-skew, $c = \frac{1}{2}$. The plotted values of γ for $\theta = 27°$ for a 63½° pre-skewed block are shown in Fig. 14.

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