BINARY MODELS FOR HIGH FREQUENCY

RADIO SPECTRUM DATA

A Thesis submitted to

The University of Manchester Institute of Science and Technology

the degree of

DOCTOR OF PHILOSOPHY

by

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To

my father and mother

my wife

my brothers and sisters

my children

Acknowledgement

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I would also like to thank Professor G. F. GOTT and his students at the department of Electrical Engineering and Electronics at UMIST for their helpful discussion and for providing the data used in this work.

I also wish to express my appreciation to members of staff in the Mathematical Department

(statistical group).

I would like to thank the Secretariat of Education in Libya for financial support during my study.

Finally, special thanks to my parents for their encouragement and financial help and to my dear wife and children for their patience and understanding.

Declaration

I hereby declare that no part of this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

ABSTRACT

The work described in this thesis is divided into three parts:

Firstly: fitting multinomial models to de-concatenated HF (high frequency : 3-30 MHz) spectral multi-counts occupancy data for the years 1982 to 1989. The occupancy of the HF radio spectrum is known to depend in particular upon the solar cycle. We compared this multinomial fit to the previous work which is binomial fit, and we found out that for the data in concatenated occupancy form, the binomial fit is better than the multinomial fit, but the differences are small.

Secondly: Theoretical development of the ZHAO and PRENTICE (1990) model. Their model was for binary counts index n less than or equal 12 where as for our data, n is 650. So we have adapted their work to make it suitable for our binary counts data. Also we found a general expression for the correlation coefficient of this developed model. We found that the correlation coefficient is very small for our binary count data, with a dependence on one of the estimated parameters.

Lastly: fitting a binary model for each allocation for four thresholds and testing our binary count data for correlation. Also producing a transition matrix to find the probability of x_2 given x_1 i.e. if we know the previous signal what is the probability distribution of the next one, with this model it is now possible to predict the next signal frequency level given the previous level for congestion data.

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CHAPTER 1

Introduction To The High Frequency Radio Spectrum

1.1 Introduction:

The HF (high frequency) radio spectrum is defined to consist of radio frequencies lying between 3 and 30 MHz.

This is also known as " World Band Radio" and the demand for these particular channels has made the HF band possibly the most congested part of the radio spectrum.

The HF channel has certain advantages compared with other long distance communication systems. It functions by exploring the natural phenomenon of the ionosphere, which is a cloud of highly ionised electrons situated at a height of 80 - 300 Kilometers, above the earth surface. By a process of gradual refraction the ionosphere is able to reflect HF radio waves, Maslin (1987).

Since the ionosphere is a natural phenomenon, it provides us with an economical means of achieving worldwide long distance communication, and its naturality makes it very valuable particularly in the times of war when other systems are easy to disrupt.

Also due to the fact that the ionosphere is a natural phenomenon, the associated equipment is economical in comparison with other system such as satellites. Hence HF sky wave communication continues to be very popular.

Although the ionosphere can be altered by various modification techniques, such as chemical releases, radio frequency heating and atmospheric nuclear blasts, the effects are generally localised and temporary in duration, Goldberg (1975).

Propagation via the ionosphere at frequencies below 1.5 MHz suffers heavy absorption during the day, whilst frequencies above 30 MHz usually penetrate the ionosphere and dissipate into space.

The ionosphere consists of three layers D, E and F which have different effects on HF signals as illustrated in Figure (1.1)

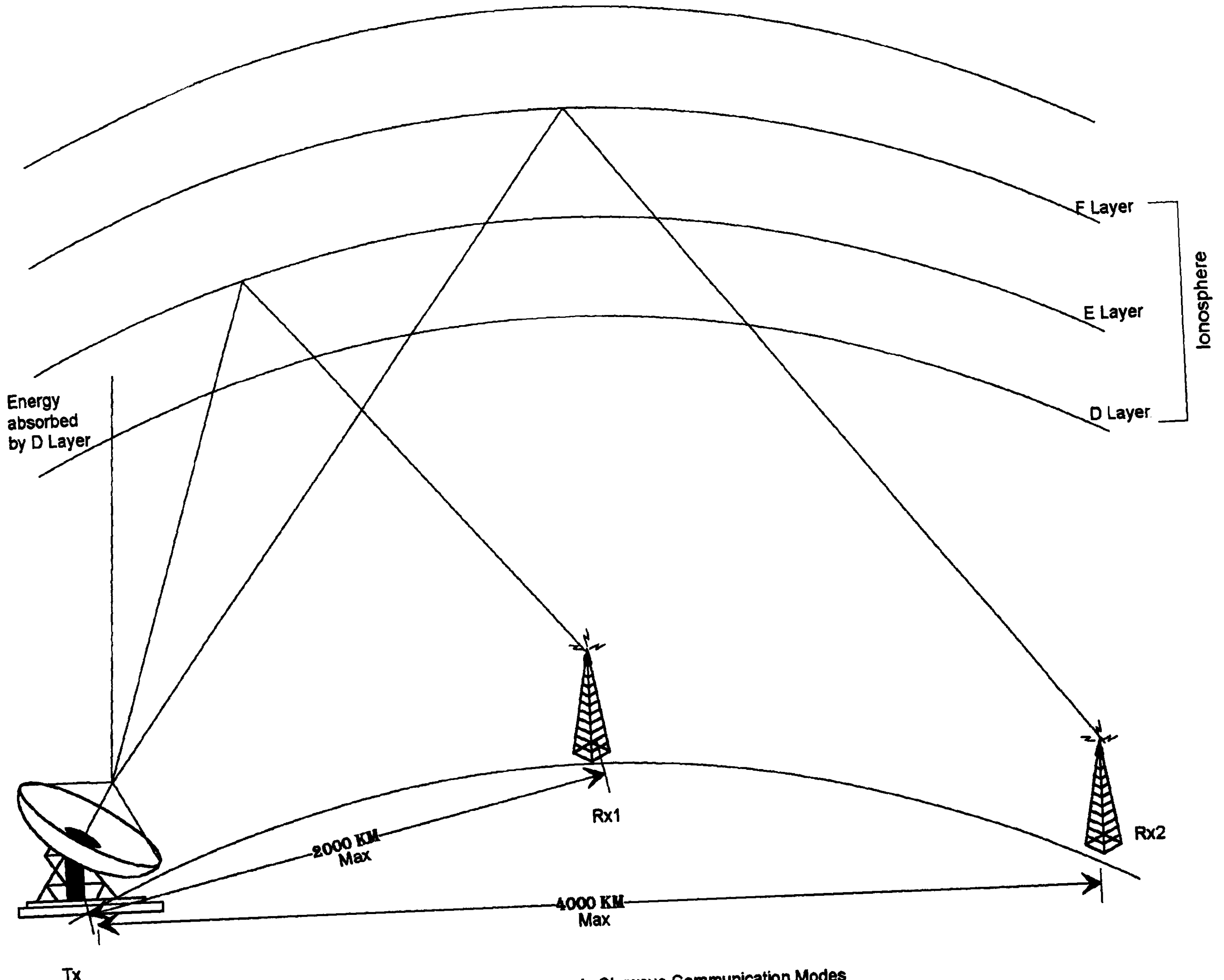


Figure (1.1) Example Skywave Communication Modes

The optimum angle of propagation and the frequency of transmission are the most important parameters in short wave communications.

The calculation of a suitable transmission frequency for a given set of conditions is complex, Picquenard (1974), and depends in particular on the time of day, time of year, sunspot number, place location of transmitter and intended receiver.

This process may be simplified by using modern technology and computer assistance.

It is necessary to divide the HF band into user allocations as defined by ITU (International Telecommunications Union) regulations. Modulation techniques, signal bandwidth requirements, transmitter power limitations, and operating producers may be different in these separate bands.

Since users in the HF band have different operational requirements, the HF band is shared between several radio services as follows:

1- **Amateur:-** This is a service of self-training communication or for technical investigations carried out for personal reasons, or as a hobby, rather than for business or profit.

It is one of the oldest services in the HF band and has made great contributions to the development and progress of radio. They are allocated frequency ranges at 3.5, 7.0, 10.0, 14.0, 18.0, 21.0, 24.0, and 28.0 MHz, so that frequencies of different characteristics will be available to perform the various kinds of communications and experimentations in which amateurs are interested. It also enables amateurs to communicate among themselves at different distances for most of the time, as conditions vary with time of day, time of year and sunspot number.

2- Broadcast:- This is a service in which the transmissions are intended for direct reception by the public.

This service may include sound, television and other types of transmissions. Several exclusive bands are allocated for HF AM broadcasting purpose. Some countries use this for domestic coverage, or in tropical areas where the atmospheric noise level may be too high for a significant part of the time in the low frequency and medium frequency bands. Thus the signals within broadcast allocations are typically high power, with side bands of about 4.5 kHz.

3- Fixed:- The fixed service provides for radio communications between fixed points on the earth's surface. It has quite a major share in the HF band and is often shared with mobile services.

4- Mobile:- It provides for communications between moving vehicles or between land stations and moving vehicles.

This service is divided into three main categories: land mobile, maritime mobile and aeronautical mobile (aeromobile).

These services are provided with generous allocations throughout the spectrum. Fixed and Mobile share quite generally, although there are exclusive allocations to each of them, as well as exclusive allocations to maritime mobile and aeromobile. Maritime mobile frequencies are used between ships or between ships and coast stations.

1.2 Study of Interference Characteristics at UMIST:-

The UMIST HF communication research group has undertaken a series of studies since 1971 in which HF interference structures and characteristics have been investigated. The aim of the work has been to achieve better understanding of interference from other users.

This may lead to better system designs and signal processing so that successful communication in the presence of interference may be possible.

1.2.1 Definition of Congestion:-

At any moment in time and at a particular place for any one band of frequencies selected from the 95 HF bands as defined by ITU regulations (e.g. band 7, 2.85 MHz - 3.155 MHz), the "occupancy" or "Congestion" at a given threshold (e.g. T=-97dBm) is defined as the percentage of that band which contains radio signals with power above the chosen threshold level. For measurement purposes, the whole HF spectrum to 30 MHz is divided into 30,000 contiguous 1 kHz steps, with signal power measured across each 1 kHz step.

For the measurement of congestion for the different HF users the receiver, with an IF filter bandwidth of 1 kHz was stepped in 1 kHz increments through each user defined allocation spending one second at each increment. Each 1 kHz window was defined as occupied at a particular threshold level if the RMS signal level in that window exceeded the selected threshold at the chosen 1 second observation time.

The occupancy for a whole allocation was calculated as a percentage combining the results for all the 1 kHz increments in that allocation. More generally, with this method of measurement, congestion is defined as follows :-

$$Q_{m,n} = \frac{W_{t,n}}{W_{m,n}} \times 100$$

where $Q_{m,n}$ is the congestion evaluated at threshold number m with a resolution filter bandwidth of n kHz.

$W_{t,n}$ is the sum of the signal band widths for which the signals exceed the chosen threshold level, t.

$W_{m,n}$ is the total width in units of n kHz at a time of the examined portion of the spectrum.

There is a very large number of actual or potential users within each frequency band. Their collective behaviour appears to have a very large random component in it, in that casual inspection of the data reveals very little in the way of "obvious patterns", after allowing for known physical effects across this spectrum.

A major purpose of the UMIST investigation has been to look for patterns in the collected data so as to aid prediction of where to find potentially useful, but quiet, bands for "interested parties". This stated aim has been achieved with remarkable success, despite what appeared at first sight to be highly random and unpredictable data.

To communicate in the presence of interference, it is important that operators and system designers know how interference behaves in different parts of the HF spectrum, at different times of day, at different seasons and in different parts of the world, so that they could change operating procedures and design a suitable system. It had been thought to be virtually impossible to give an exact description of interference characteristics due to the fact that it is largely man made and apparently unpredictable in general. The parameter 'congestion' has been found to be a useful indicator of the state of occupancy in HF band. Occupancy results could help the operator to determine which frequency gives the best chance of achieving reliable communication with their existing system in the presence of interference.

1.3 Measurement of High Frequency Spectral Occupancy:

The system of experimental measurement of the occupancy of the high frequency (HF) spectrum has been performed by Professor G. Gott of the Electrical Engineering and Electronics at UMIST, and co-workers, since 1982.

By international regulation, the HF spectrum is defined to be between 3 and 30 MHz. Signals in this frequency range have the important property that they may be reflected by the ionosphere and thus long range communications may be achieved using modest transmitter power.

As stated earlier, the ionosphere is a free phenomena and this has resulted in the HF spectrum being used very heavily for long range communications.

This has motivated the program in HF spectral measurement and analysis at UMIST, to study the characteristics of HF interference with the aim of developing models to assist in planning of frequency usage and management.

Although the HF spectrum is formally defined to extend from 3 to 30 MHz, signals outside this range may be reflected from the ionosphere, as sky waves, and hence this experiment has examined the spectral range of 1.606 to 30 MHz. This HF spectrum is divided into 95 frequency allocations used by 12 different types of users.

A flow chart showing the logical basis of the system used by Dr Gott and his co-workers for measuring occupancy can be seen in figure (1.2) overleaf. This has been implemented under remote computer control over telephone lines at various sites in the United Kingdom and abroad. The electronic equipment required on site, such as a radio receiver and a spectrum analyser is of the very highest quality and operates at the limits of technical feasibility.

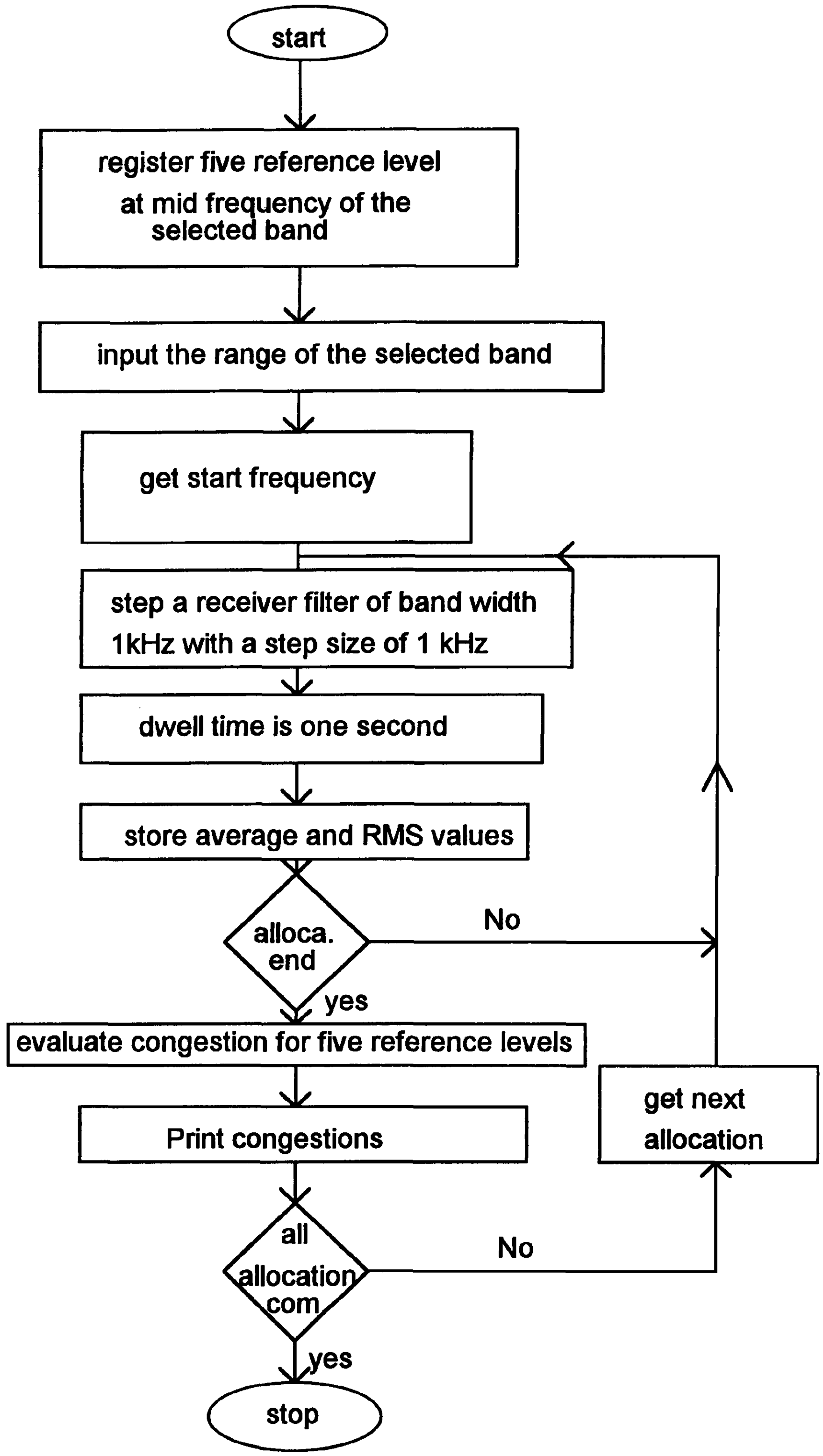


Figure (1.2) Occupancy Measurement across the HF Spectrum

1.3.1 HF Occupancy Data Collected Since 1982:-

Congestion has been measured twice yearly since 1982. These measurements have been made near the time of winter and summer solstices, corresponding to the time of maximum and minimum variation in ionospheric activity. During the summer or winter measurement sessions, congestions are measured once around midday and once around midnight for reasons which are explained below.

There is a band of frequency range over which sky waves can propagate over a given distance on earth. The upper limit of this range which is typically valid for 90% of the days of the month is known as the Optimum Working Frequency (OWF). The lower limit of the frequency range is the Lower Usable Frequency (LUF) which is largely dependent on the absorption of signals by the lower ionosphere.

Predictions of OWF and LUF for HF communications are published regularly by the Marconi Company on behalf of the British Admiralty.

These predictions assist in the reliable establishment of HF links.

The ionosphere is stable for about four hours around midday and midnight. Hence the researchers at UMIST measure congestion during these times, when the ionosphere is stable.

Using their present system, with a 1 second dwell time at each 1kHz step, a complete scan of the entire HF spectrum from allocations 1 to 95 takes about eight hours, since it involves approximately 30,000 separate 1 second measurements, in sequence. The measurements are spread over three days as shown below, with typically around 2 hours about midnight used on each day, to ensure stable ionospheric conditions and some consistency for the nomial "date". Much experimental effort has gave into checking the validity of this system for consistency, validity and interpretability in terms of congestion.

	Day 1≈MIDNIGHT	Day 2≈MIDNIGHT	Day 3≈MIDNIGHT
Allocations	1 - 35	36 - 68	69 - 95
Frequency (MHz)	1.606 - 10	10 - 20	20 - 30

1.3.2 Measurement of Occupancy Across The Entire HF Spectrum:

Wong (1983) divided the HF band into 95 user allocation as defined by the ITU. This was necessary since different services might have different modulation techniques, signal bandwidth requirements, transmitter power limitations, and operating procedures.

Two measuring systems were used by Wong (1983), one of which (the later one) is computer controlled and allowed analysis to be done in real time. This system has to be manually calibrated. The system block diagram is shown in figure (1.3).

The active antenna was a wideband omnidirectional monopole. This was calibrated so that a CW signal of -117 dBm at the aerial output corresponded to a received field strength of 0.6 μ $^{V}/_{m}$.

A switch at the receiver input allowed a frequency synthesizer to be connected for threshold calibration.

Five threshold levels from -117 dBm to -77 dBm in 10 dBm increments were set at the receiver input. These particular thresholds were chosen by the researchers for technical reasons.

Noise was measured by connecting a 50 Ω load to the receiver antenna input. The receiver was operated without AGC, and IF gain was manually adjusted so that the receiver noise was at least 10 dB below the lowest threshold (-117 dBm), and the dynamic range was at least 60 dB.

The spectrum analyser connected to the receiver acted only as a logarithmic amplifier and detector at the receiver IF. The spectrum analyser produces an output which was averaged via its video filter of 10 Hz bandwidth.

Congestion at any allocation was measured by stepping the receiver filter in 1 kHz increments through that allocation, staying at each step for a dewell time of one second

These measurements were done twice yearly in 1981 and 1982 near the winter and summer solstices. Winter solstice was the time of maximum diurnal variation in ionospheric activity while the summer solstice was the time of minimum variation. Propagation properties are different during the day from those during the night, so measurements were taken at midday and midnight since the ionospheric actives were most stable at these times.

Such a period of stability is about four hours, and the results for whole spectrum was done over three days and three nights. These measurements have been carried out year by year since 1981.

Ray (1985) continued measurements of the congestion, so congestion measurement for the period 1982 to 1985 are available.

Ray (1985) applied naive Chisquare tests to the underlying counts, in order to see whether there were any statistically significant changes between any of the data sets. He found that there were significant changes. He also found significant changes in congestion values measured from day to day compared to the changes of these values from year to year especially the night observations.

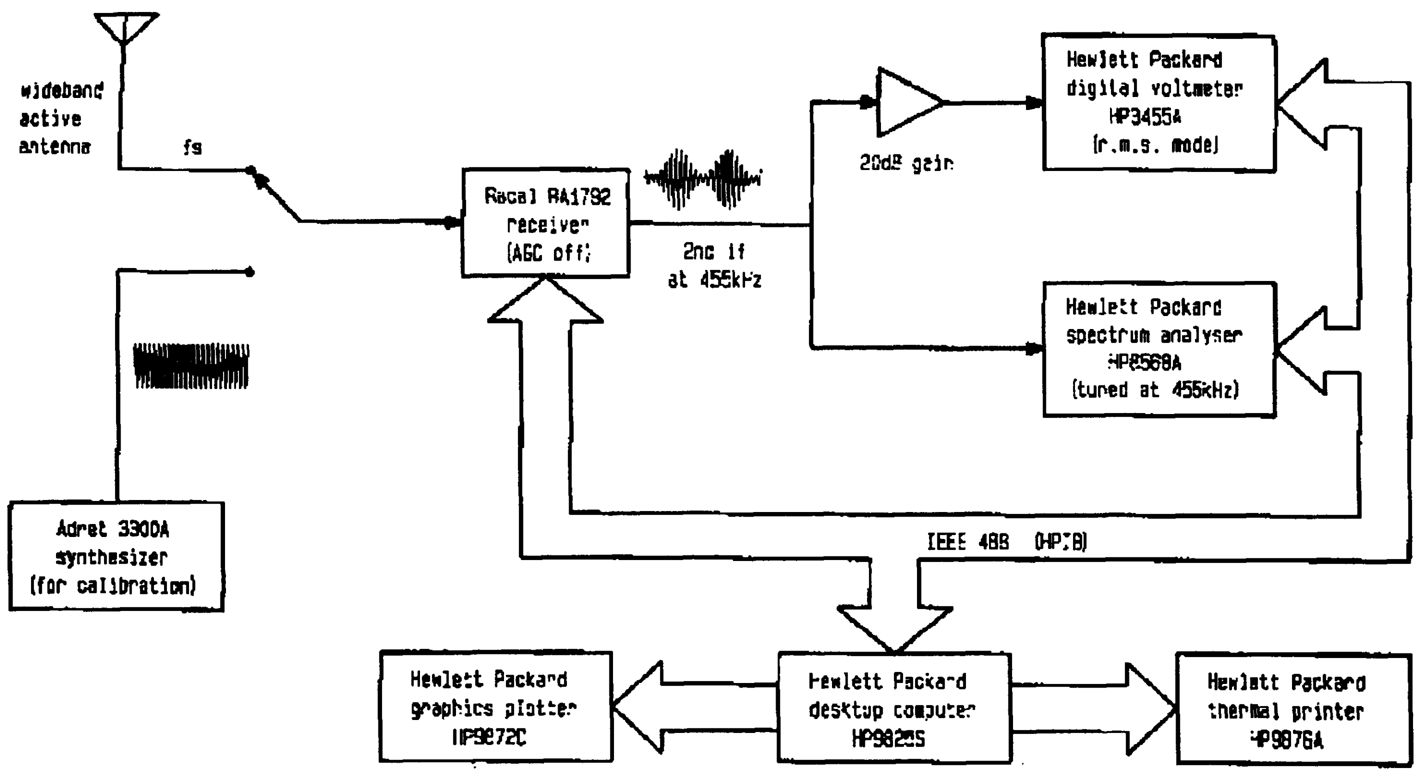

wideband
active
antenna
fs
Adret 3300A
synthesizer
(for calibration)
Racal RA1792
receiver
(AGC off)
2nd if
at 455kHz
20dB gain
Hewlett Packard
digital voltmeter
HP3455A
(r.m.s. mode)
Hewlett Packard
spectrum analyser
HP8568A
(tuned at 455kHz)
IEEE 488 (HPIB)
Hewlett Packard
graphics plotter
HP9872C
Hewlett Packard
desktop computer
HP9825S
Hewlett Packard
thermal printer
HP9876A

Figure (1.3)

1.3.3 Automated Measurement System:-

A fully automated measuring system was designed and implemented by Poole (1989) to replace Wong's semi-automated system. This new system, is still in use at the principal measurement site at Baldoch, Hertsfordshire, which is linked via the public telephone network to UMIST. Other systems have since been located in Sweden and Germany. The advantage of this system is that UMIST personnel no longer need to travel to the remote sites to make measurements. Hence extensive monitoring and studies of spectral occupancy can now be contemplated.

Moreover the sophisticated operating system afforded by the HP 310 micro computer enabled a modular approach in software implementation. This ensured ease of updating and adding to the software. The system is presented in figure (1.4).

The site at Baldock was carefully chosen to have optimum receiving conditions for the experiments within the UK.

In 1986 Morrel (1986) did some statistical analysis of the congestion data under the guidance of Dr P. J. Laycock for data collected in the years 1982 - 1986. A binomial model with logit transform was applied to the data, with remarkable success. See Morrel, Laycock, Gott and Ray (1988).

Dennigton (1990) continued this work fitting the data for 1982 to 1989.

In the next section the work carried out by Morrel and Dennington (1990) is summarized:

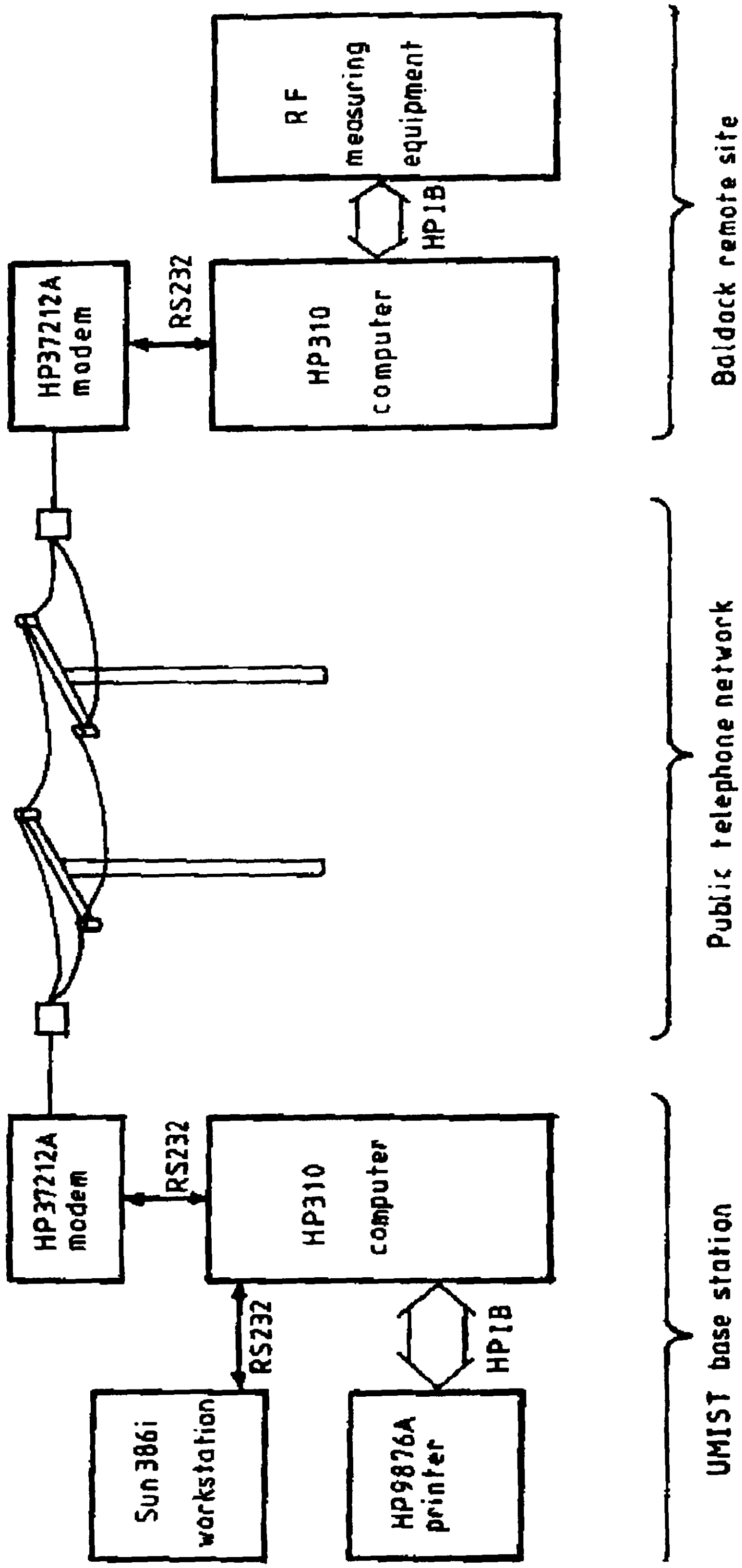

HP37212A modem
RS232
HP310 computer
HPIB
RF measuring equipment
Baldock remote site
Public telephone network
HP37212A modem
RS232
HP310 computer
RS232
HPIB
Sun386i workstation
HP9876A printer
UMIST base station

Figure (1.4)

1.4 The Occupancy Model:-

An effort has been made to fit a mathematical model to the occupancy data. The model used was suggested by Dr Laycock who has guided its application.

The experimental data in a mathematical model for congestion can be represented

$$Q = f(x_1, x_2, \ldots\ldots, x_n)$$

where $x_1, x_2, \ldots\ldots, x_n$ represent the parameters on which occupancy may be expected to depend, such as time, frequency, bandwidth, threshold level, sunspot number and geographical location.

Consider the choice of a linear function of selected variables i.e.

$$Q = \sum A_i X_i$$

Where X_i are in general functions of the parameters and A_i are coefficients.

The measured values of congestion can be regarded as estimated values of probability, and hence their values should lie in the range 0 to 1. Since a significant proportion of the observed values of congestion lies on or close to these boundary values, there is a risk that a linear model as defined above may give estimated values of congestion which lie outside this range. A logistic trasformation (logit) has been used to overcome this problem, that is

$$\operatorname{log}it(Q) = \ln\frac{Q}{1-Q} = \sum A_i X_i = Y$$

Hence

$$Q = \frac{\exp(Y)}{1+\exp(Y)} \qquad \text{where } 0<Q<1$$

This transformation has the advantage that it is easily invertible and its properties have been thoroughly studied in the statistical literature, Nelder (1989). It has been found to work extremely well for these radio frequency data. Other transformations are clearly possible (for example any cumulative density function will suffice).

In particular similar results have been obtained using the probit transformation: $Q = \Phi(Y)$, where Φ is the c.d.f. of standard normal variate this is a standard alternative to the logistic transformation which it is known to match very closely. Viewed as a c.d.f. these transformations imply a probability distribution for the underlying signal strength, S, with Q=Prob(S>t) with t the chosen threshold level. In particular, logistic and normal distributions for signal strength are implied by the above two transformations.

Extensive and very successful statistical analysis has resulted in the use of the following linear function.

$$Y = A_k + B \times threshold(dbm) + (C_o + C_1 f + C_2 f^2) \times S$$

Where $Y = \ln\frac{Q}{1-Q}$ is the logit of the theoretical occupancy Q, corresponding to a measured occupancy Q for a bandwidth of B steps, with r=bQ modelled as binomial, B(b,Q), where

T= threshold level in dbm including its sign.

f= centre frequency of user allocation in MHz.

S= sunspot number for that year.

B, C_o, C_1, C_2: estimated regression coefficients.

A_k: k=1...95: 95 constants corresponding to the 95 allocations.

This model has also been successfully extended in to allow for spatial, seasonal and step length (more than 1 kHz) variation.

This model has been fitted to the data using the statistical package GENSTAT, (developed at Rothamsted Experimental station and now marketed by NAG, Oxford). Initially run on an AMDAHL mainframe, then on a SUN workstation and lately on a 486 PC. Some experimental runs now take place on an IBM mainframe computer-intensive facility.

Four models were required: one each for summer day, summer night, winter day and winter night.

Dennington (1990) fitted models similar to the that above including the data of 1987 to 1989, and he also fitted this model using only single user allocations at the time.

Dennington found that the A_k constants produced for a given user could be described by a simple polynomial function.

In all cases the model has been used to predict new data and found to be remarkably accurate. It should be noted however that there is statistically significant extra-binomial variation in most cases. This implies that the underlying independence assumption for the binomial model is not completely true, although much experimental effort has gone into choosing the experimental parameters so as to ensure this assumption would hold. For prediction purposes this does not seem to matter, since the model fits the data, in the mean, extremely well. But the main purpose of this thesis has been to attempt to model this small but significant (serial) correlation.

1.4 1 Occupancy Model Variable:

Signal threshold:

In the HF band a channel is said to be occupied if during a one second observation period, the signal in that channel exceeds a reference threshold at the aerial input to the receiver. In the experiment five reference thresholds are used, these being:

-117dBm, -107dBm, -97dBm, -87dBm, -77dBm.

Frequency:

The electromagnetic HF radio spectrum ranges from 1.6 to 30.0 MHz. It is divided into 95 user-bands each of which is assigned a nomial frequency namely its mid-point frequency.

Sunspot number:

The model was fitted with (average, latterly individual) Belgian sunspot numbers supplied by the propagation group of the Marconi Company. Each season of each year yields a different sunspot number.

Time of Day:

Measurements were taken twice during each 24 hours period, at midday and midnight when the ionosphere is stable as stated before and a complete set of measurements across the 95 allocations takes about eight hours. Alternatives to this method have been studied in the past and are currently under review.

Time of The Year/Season: The experiments were performed at two times of the year. Once during the summer (July) and the other during winter (February) approximately at the times of the winter and summer solstice when the diurnal variation in the optimum working frequency is at a maximum and minimum respectively.

User Type:

The 12 user types listed below (with convenient abbreviations shown). Recall that each user is allocated a selection of the 95 available bands spread across the total bandwidth of 30 MHz.

1-	FIXED/MOBILE	FM
2-	AMATEUR	AM
3-	FXD./MOB./AMTR.	FMA
4-	FXD./MOB./BCST.	FMB
5-	AEROMOBILE	AE
6-	FIXED/BROADCAST	FB
7-	MARITIME/MOBILE	MM
8-	BROADCAST	B
9-	FIXED	F
10-	FIXED/AMATEUR	FA
11-	RADIO ASTRONOMY	RA
12-	FXD./MOB./METR.	FMM

Since congestion is given by :

$$Congestion = \frac{r}{b} \times 100$$

it can then be regarded to a first approximation as a binomial B(b,Q) variate where b is the bandwidth, and Q is P(Q>t) ,t is the threshold.

The data values (for congestion of any one season from 1982 - 1989), but not including the threshold -117dBm), Dennington (1990) can be usefully considered as independent random variables from a binomial distribution.

If the 3040 (8 years, 95 allocations and 4 threshold) observations are specified by the parameters $Q_1, Q_2, \ldots\ldots, Q_{3040}$, then these are related to a linear combination of the explanatory variables, Xβ through the link function g, i.e.

$$g(Q) = X\beta$$

The link function will be described in greater detail later.

The criteria used for fitting is Maximum Likelihood, Morrel (1988) and Dennington (1990) have explained the maximum likelihood function for this particular binomial distribution and this will be discussed briefly below.

1.5 Binomial Model:

The binomial distribution describes the probabilistic behaviour of the number of basic events occuring in a fixed number of independent trials of an experiment.

The basic event can have only two outcomes: it either occurred or did not. Most of the work carried out on these data so far has assumed that each of the 3040 (8 years, 95 allocation and 4 threshold) observations (measured congestion) as being derived from independent events. For each of these distributions the event is that the examined RMS "interference" in a 1 kHz bandwidth exceeds the specified threshold. The number of trials is the total number of 1 kHz steps in each allocation.

The binomial distribution for one allocation B(b,Q) takes the form

$$\binom{b}{r} Q^r (1-Q)^{b-r}, \ \mathrm{r} = 0,1,\ldots\ldots,b$$

There are two standard ways to estimate the linear predictor parameter vector β :

1- Least squares.

2- Maximum likelihood.

The method used, via GENSTAT, is maximum likelihood, achieved by an iterative least-squares technique as described by McCullagh and Nelder (1989).

The likelihood function for one allocation is

$$L(r;Q) = \binom{b}{r} Q^r (1-Q)^{b-r}$$

this can be written as

$$L(r;Q) = \exp[r\log(Q) - r\log(1-Q) + b\log(1-Q) + \log(\binom{b}{r})]$$

This is one member of the family of exponential distributions, see Nelder and McCullagh (1989) for more details.

Taking the logarithm of both sides

$$l(r;b) = r\log(\frac{Q}{1-Q}) + b\log(1-Q) + \log(\binom{b}{r})$$

Hence the log-likelihood function only for one allocation, for the whole N=3040 (8 years, 95 allocations and 4 threshold) allocations it is

$$l(r_1, r_2, \ldots\ldots, r_{3040}, Q_1, Q_2, \ldots\ldots, Q_{3040}) = \sum_{i=1}^{N}[r_i \log(\frac{Q_i}{1-Q_i}) + b_i \log(1-Q_i)]$$

The congestion

$$Q_i = \frac{r_i}{b_i}$$

in each allocation is then described in terms of the explanatory variables which characterise that allocation. This is done by modelling the probabilities Q_i as

$$g(Q_i) = X_i\beta$$

Where X_i is a vector of explanatory variables for that data item i.

β is a vector of parameters (common for all data items).

g is the link function.

As was explained before, to ensure that Q is restricted to the range 0 - 1, a logit transform has been used for the link function,

where

$$g(Q_i) = \log(\frac{Q_i}{1-Q_i}) = X_i\beta$$

In our case we have four variables and 95 constants.

So we can write

$$\log(\frac{Q_i}{1-Q_i}) = \beta_1 + \beta_2 + \ldots\ldots + \beta_{95} + \beta_{96}x_1 + \ldots\ldots + \beta_{99}x_4$$

$$\log(1-Q_i) = -\log[1 + \exp(\beta_1 + \beta_2 + \ldots\ldots + \beta_{95} + \beta_{96}x_1 + \ldots + \beta_{99}x_4)]$$

A probit transformation

$$g(Q_i) = \Phi^{-1}(Q_i)$$

where Φ is the c.d.f of the N(0,1) distribution was also tried, with almost identical results.

Thus the log likelihood becomes

$$l = \sum_{i=1}^{3040}[r_i(\beta_1 + \ldots.. + \beta_{95} + \beta_{96}x_1 + \ldots. + \beta_{99}x_4) - b_i \log(1 + \exp(1 + \exp(\beta_1 + \ldots.. + \beta_{95} + \beta_{96}x_1 + \ldots. + \beta_{99}x_4)))]$$

To obtain the maximum likelihood estimates for β's we find the partial derivatives of l with respect to each parameter. Each of these is equated to zero and they are then solved simultaneously to yield the estimates.

These simultaneous equations are in general non-linear and have to be solved using numerical iteration, which the GENSTAT package does automatically for the user.

This model has been fitted to all sets of data from 1982 to 1989 with four thresholds.

1.6 Markov Model:-

Laycock and Gott (1988) pointed out that for the underlying Bernoulli series which gave rise to the recorded "occupancy" counts, a Markov model might be thought to be more appropriate. This particular model was utilized rather than the more usual exchangeable distribution models typically used to explain extra - binomial variation, because of the underlying sequential nature of the data, through the frequency.

The stochastic transition matrix was written in the form:

$$\begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

where 1-α=prob (frequency i is occupied at threshold t, and subsequently frequency i is occupied at that same threshold).

α=prob (frequency i is occupied at threshold t, and subsequently frequency i+1 is occupied at that same threshold).

β=prob (frequency i+1 is occupied at threshold t, and subsequently frequency i is occupied at that same threshold).

1-β=prob (frequency i+1 is occupied at threshold t, and subsequently frequency i+1 is occupied at that same threshold).

It can be shown, Cox and Miller (1965), that for large n the binomial counts of such a Markov sequence has asymptotic mean $n\pi_o$ and variance $\phi n\pi_o\pi_1$ where

$$\pi_o = \frac{\alpha}{\alpha+\beta} \quad \text{and } \pi_1 = 1-\pi_o$$

$$\phi = \frac{2}{\alpha+\beta} - 1$$

The parameter ϕ is called the dispersion parameter and usually ϕ>1 in statistical practice, which increases the variance relative to that for a binomial distribution. This accounts for the name "extra binomial variation", Altham (1978) and Williams (1982).

The Markov model is one of four standard explanations for extra - binomial variation, the others being the beta, additive, and multiplicative models, Altham (1978).

The Markov model is an appropriate model for serial correlation.

We can show that the asymptotic correlation between adjacent states is ρ=1-(α+β).

Proof:-

Given two state Markov chain with transition matrix

$$P=\begin{bmatrix}1-\alpha & \alpha\\ \beta & 1-\beta\end{bmatrix}$$

Cox and Miller (1965) in their book showed that for large n

$$\lim_{n\to\infty} P^n=\frac{1}{\alpha+\beta}\begin{bmatrix}\beta & \alpha\\ \beta & \alpha\end{bmatrix}$$

so the limiting distribution is

$$\pi_\circ=\frac{\beta}{\alpha+\beta}$$

$$\pi_1=\frac{\alpha}{\alpha+\beta}$$

also in the limiting case

$$E(x)=E(y)=\pi_1 \quad \text{and} \quad V(x)=\pi_\circ\ ,V(y)=\pi_1$$

so

$$E(xy)=E(xy|x=1)P(x=1)+E(xy|x=0)P(x=0)$$

$$=1P(y=1|x=1)P(x=1)=(1-\beta)\pi_1$$

Since Cov(xy)=E(xy)-E(x)E(y)

therefore Cov(xy)=$(1-\beta)\pi_1-\pi_1^2$

$$=\pi_1(1-\pi_1)-\pi_1\beta$$

$$=\pi_1\pi_\circ-\pi_1\beta$$

$$=\pi_1(\pi_\circ-\beta)$$

$$\text{since } \rho=\frac{Cov(xy)}{V(x)V(y)}$$

$$\rho=\frac{\pi_1(\pi_\circ-\beta)}{\pi_\circ\pi_1}=1-\frac{\beta}{\pi_\circ}$$

$$\rho=1-\frac{\beta}{\pi_\circ}=1-\alpha-\beta$$

$$\text{hence } \rho=\frac{\phi-1}{\phi+1}$$

$$\phi=\frac{1+\rho}{1-\rho}$$

This leads immediately to the estimate

$$\hat{\rho}=\frac{\delta-1}{\delta+1}$$

where δ is the mean deviance, this result is given in Laycock and Gott (1988).

1.7 Multinomial Model:-

As pointed out by Brian and Bulter (1988), cumulative count data are more appropriately analysed using a multinomial model after de-segregation of the data.

For any one band on any one occasion the likelihood is proportional to

$$\prod \pi_i^{m_j}$$

Where the product is over K=6 or 5 categories depending on whether the lowest -117dbm level is included or not, m_j is the number falling in the jth category, and

$$\pi_j=F(T_j)-F(T_{j-1})$$

Where F is the cumulative distribution function of the chosen distribution and $-\infty = T_{\circ} < T_1 < \ldots\ldots\ldots < T_k = \infty$

This model is described in more detail with its applications in chapter four.

CHAPTER 2

Mathematical Statistics Background

2.1 Preamble:

In this chapter we give a brief summary of the mathematical methods underlying statistical techniques used in this thesis.

For the fitting of a simple linear relationship between several predictor variables $X_1, X_2, \ldots\ldots, X_p$ and a variable Y, the parameters are chosen so as to produce a fitted data set $\hat{y}_i$ that is close to the observed data Y_i.

In classical least squares analysis the closeness of a fit is determined by:

$$\sum_{i=1}^{p}(y_i - \hat{y}_i)^2$$

The use of this formula has two implications.

First, the straightforward summation of individual deviations, either $|y_i - \hat{y}_i|$ or $(y_i - \hat{y}_i)^2$, each depending on only one observation, implies that the observations are all made on the same physical scale and suggests that the observations are independent, or at least that they are in some sense exchangeable, so justifying an even-handed treatment of the components.

Second, the use of arithmetic differences $y_i - \hat{y}_i$ implies that a given model deviation carries the same weight irrespective of the value of $\hat{y}$. To ensure that variations in model residuals $y_i - \hat{y}_i$ are independent of the actual fitted values, mathematicians build into their models an explicit frequency distribution for residual variation as in, for example; the Generalized Linear Model based on the exponential family of distributions.

In classical least square analysis, we regard the x values as fixed or non-stochastic and the y values are assumed to have a normal (or Gaussian) distribution with mean μ in which the frequency of Y given μ is proportional to $\exp(-(y-\mu)^2/2\sigma^2)$, where σ is the standard deviation of the distribution.

We can look at this function in two ways:

(i) If we regard it as a function of y for fixed μ, the function specifies the probability density of the observations y_i's

(ii) For a given observation y, we may regard it as a function of μ, giving the relative plausibility of different values of μ for the observed value y. In this second form the distribution becomes the Likelihood function, (Nelder and McCullagh, 1989).

2.2 Maximum Likelihood:

To fit the model we usually choose the parameters so that the fitted values $\hat{Y}_i$ are 'close' to the actual values Y_i, and to estimate these parameters there are two standard techniques, which happen to be equivalent for the normal distribution:

1- Maximum Likelihood.

2- Least Squares.

2.2.1 Method Of Maximum Likelihood:

Let $Y_1, Y_2, \ldots\ldots, Y_n$ be n continous random variables with joint probability $f(y_1, y_2, \ldots\ldots, y_n; \theta_1, \theta_2, \ldots\ldots, \theta_p)$ where the parameters $\theta_1, \theta_2, \ldots\ldots, \theta_p$ are unknown. Then the probability density function can be denoted by $f(y;\theta)$. For discrete random variables we can equivalently take f to be the relevant probability distribution or mass function. Let Ω denote the set of all possible values of the parameter vector θ. Then θ is the value which maximizes the likelihood function i.e. the " maximum likelihood estimator".

This means

$$L(\hat{\theta};Y) \geq L(\theta;Y) \quad \text{for all } \theta \in \Omega$$

where $L(\theta;Y) = \prod_{i=1}^{n} f(y;\theta)$

and where the y_i's constitute a random sample from a distribution with density function, or more generally probability mass function (p.m.f.) $f(.|\theta)$.

Equivalently $\hat{\theta}$ is the value which maximizes the log-likelihood function (since the logarithmic function is monotonic).

Thus $l(\theta;Y) = \log(L(\theta;Y))$

Therefore $l(\hat{\theta};Y) \geq l(\theta;Y)$ for all $\theta \in \Omega$

It is usually better and easier to work with the log-likelihood function rather than with the likelihood function itself.

Note that, since the logarithm is a monotonically increasing function of its argument, the maximization problem is unchanged by the transformation.

Usually the estimator $\hat{\theta}$ for θ is obtained by differentiating the log-likelihood function with respect to the parameters and equating to zero:

$$\frac{\partial l(\theta;Y)}{\partial \theta_j} \qquad \text{for j} = 1, 2, \ldots, \text{p} \qquad (2.1)$$

These are called the "likelihood equations".

To check that this stationary value solution gives a maximum, we need to evaluate the second derivatives:

$$\Delta_{jk} = \frac{\partial^2 l(\theta;Y)}{\partial \theta_j \partial \theta_k} \qquad \text{evaluated at } \theta = \hat{\theta}$$

The Jacobian: $|\Delta_{jk}|$ must be negative for this to be a (local) maximum.

2.2.2 Solving The Likelihood Equations:

To solve the likelihood equations there are two standard methods.

1- Newton-Raphson method.

2- Scoring method.

A property of the exponential family of distribution is that they satisfy enough regularity conditions to ensure that the global maximum of the log-likelihood function $l(\theta;Y)$ is given uniquely by the solution of the equations $\frac{\partial l}{\partial \theta}=0$.

In general the equations (2.1) are non-linear and they have to be solved by numerical iteration. If the Newton-Raphson method is used, then the mth approximation is given by

$$b^{(m)} = b^{(m-1)} - \left(\frac{\partial^2 l}{\partial\beta_j \partial\beta_k}\right)^{-1}_{\beta=b^{(m-1)}} U^{(m-1)} \qquad (2.2)$$

where $\left(\frac{\partial^2 l}{\partial\beta_j \partial\beta_k}\right)$ is the matrix of second derivative of (l) evaluated at $\beta=b^{(m-1)}$, and $U^{(m-1)}$ is the "score" vector of first derivatives $U_j=\frac{\partial l}{\partial\beta_j}$ evaluated at $\beta=b^{(m-1)}$.

The second method of solving the equations is called "Fisher's scoring method" which is some times simpler than Newton-Raphson. It involves replacing the matrix of second derivatives in (2.2) by its expected values i.e.

$$E\left(\frac{\partial^2 l}{\partial\beta_j \partial\beta_k}\right)$$

This matrix is equal to "Fisher's information matrix".

2.3 Sampling Distribution For Maximum Likelihood Estimator:

Suppose the log-likelihood function has a unique maximum at b and that this estimator b is near the true value of the parameter β.

The first order Taylor approximation for the score vector $U(\beta)$ about the point $\beta = b$ is given by

$$U(\beta) \cong U(b) + H(b)(\beta - b)$$

where H(b) denotes the matrix of second derivatives of the log-likelihood function evaluated at $\beta = b$.

Asymptotically H is equal to its expected value which is related to the information matrix by

$$\xi = E(UU^T) = E(-H)$$

Therefore for large samples

$$U(\beta) = U(b) - \xi(\beta - b)$$

But U(b)=0 because b is the value where the given log-likelihood function will be the maximum, so its derivatives are zero:

$$\therefore (b - \beta) \cong \xi^{-1} U$$

this is provided that ξ is non-singular. Also if ξ is constant then

$$E(b - \beta) \cong \xi^{-1} E(U) = 0$$

Since E(U)=0, so b is an unbiased estimator of β.

The variance covariance matrix for b is

$$E[(b-\beta)(b-\beta)^T] \cong \xi^{-1}E(UU^T)\xi^{-1} = \xi^{-1}$$

Since $\xi = E(UU^T)$ and $(\xi^{-1})^T = \xi^{-1}$ since ξ is symmetric.

Thus for large samples

$$(b-\beta)^T \xi(b-\beta) \text{ is } \chi_p^2$$

from standard maximum likelihood theory, Dobson (1990).

2.4 The components of a generalized linear model:

Generalized linear models are an extension of classical linear models.

First we will simplify and consider the definitions of the classical linear models, and then will expand into generalised linear models.

2.4.1 Classical linear models:

A vector of observation y having n components is assumed to be a realization of a random variable Y whose components are independently distributed with means μ. The systematic part of the model is a specification for vector μ in terms of a number of unknown parameters $\beta_1, \beta_2, \ldots\ldots, \beta_p$. In the case of ordinary linear models, this specification takes the form

$$\mu = \sum_{j=1}^{p} \beta_j X_j$$

where X_j are the model covariates and β_j are the unknown parameters whose values to be estimated from the data.

For several observations, with i representing each observation this can be written:

$$E(Y_i) = \mu_i = \sum_{j=1}^{p} \beta_j x_{ij} \qquad\qquad i=1,2,\ldots\ldots,n$$

and in terms of matrix notation $\mu = X\beta$.

Where X is the model matrix and β is the vector of parameters.

The second part of a classical linear model is that the distribution of the errors follows a Gaussian or Normal distribution, with constant variance σ^2.

We may thus summarize the classical linear model in the form:

1- The random components: the components of Y have independent Normal distribution with mean μ and constant variance σ^2.

2- The systematic component: covariates $X_1, X_2, \ldots\ldots\ldots, X_p$ produce a linear predictor η given by

$$\eta = \sum_{j=1}^{p} \beta_j X_j$$

3- The link between the random and systematic components:

$$\mu = \eta$$

This specifies that the linear predictor and the expected value of the random component are identical.

More generally we can write

$$\eta_i = g(\mu_i)$$

Where g(.) will be called the link function.

In this formulation, classical linear models have a Normal (or Gaussian) distributed in component (1) and the identity for the link in component (3).

The Generalized linear models allow two extensions:

First the distribution in component (1) may come from an exponential family and be other than the Normal, and secondly the link function in component (3) may become any monotonic differentiable function, McCullagh and Nelder (1989).

2.4.2 Exponential Family:

For a distribution suppose we can write the probability density function (or probability mass function, for the discrete case) as

$$f_Y(y;\theta;\phi) = \exp[(y\theta - b(\theta))/a(\theta) + c(y,\phi)]$$

for some functions a( ), b( ) and c( ). If ϕ is known this is an exponential family with canonical parameters θ. It may or may not be exponential family if ϕ is unknown. (McCullagh and Nelder 1989)

Examples of exponential family

1- Discrete case (Binomial) suppose y is $B(m,\pi)$ and we have one observation then

$$P(Y=y) = P_y = \binom{m}{y}\pi^y(1-\pi)^{m-y} \quad y = 0,1,2,\ldots, m \text{ and } 0 \le \pi \le 1$$

$$\therefore P_y = \exp[(y\theta - m\ln(1+e^\theta)) + \ln\binom{m}{y}]$$

where $\theta = \ln(\frac{\pi}{1-\pi})$, $\phi = \frac{1}{m}$, $b(\theta) = \ln(1+e^\theta)$, and $c(y,\phi) = \log\binom{m}{y}$

2- Continuous case (Normal) suppose y is $N(\mu, \sigma^2)$ then

$$f_Y(y;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-m)^2}{2\sigma^2}}$$

$$= \exp[(y\mu - \frac{\mu^2}{2})/\sigma^2 - \frac{1}{2}(\frac{y^2}{\sigma^2} + \ln(2\pi\sigma^2))]$$

where $\theta = \mu$, $\phi = \sigma^2$, $b(\theta) = \frac{\theta^2}{2}$, and $c(y,\theta) = -\frac{1}{2}(\frac{y^2}{\phi} + \ln(2\pi\phi))$

2.4.3 Link Function:

The link function relates the linear predictor η to be expected value μ of datum y. In classical linear models the mean and linear predictor are identical, and the identity link is plausible in that both η and μ can take any value on the real line. However that when we are dealing with counts and the distribution is Poisson, we must have $\mu > 0$, so that the identity link is not suitable in this case, because η may be negative while μ must be positive, and we can avoid that by taking log link $\eta = \log(\mu)$, with inverse $\mu = \exp(\eta)$.

For the binomial distribution (with index n=1) we have $0 < \mu < 1$ and the link should satisfy this condition.

There are many link functions but we shall consider only three particular functions which are relevant to the models and data set examined in this thesis:

1- Logit

$$\eta = \log(\frac{\mu}{1-\mu})$$

2- Probit

$$\eta = \Phi^{-1}(\mu)$$

where ϕ is the Normal cumulative distribution function.

This is an alternative to the logit model, and usually indistinguishable from it in practice.

3- Complementary log-log

$$\eta = \log(-\log(1-\mu))$$

Figures (2.1), (2.2), (2.3) are the plot of each link function alone and figure (2.4) compares the four link functions.

The logistic (logit) and the probit function are almost linearly related over the interval $0.1 \le \mu \le 0.9$. For this reason, it is usually difficult to discriminate between these two functions on the grounds of goodness of fit, Chambers and Cox (1967).

For small values of μ, the complementary log-log function is close to the logistic. As μ approaches 1, the complementary log-log function approaches infinity much more slowly than either the logistic or the probit function.

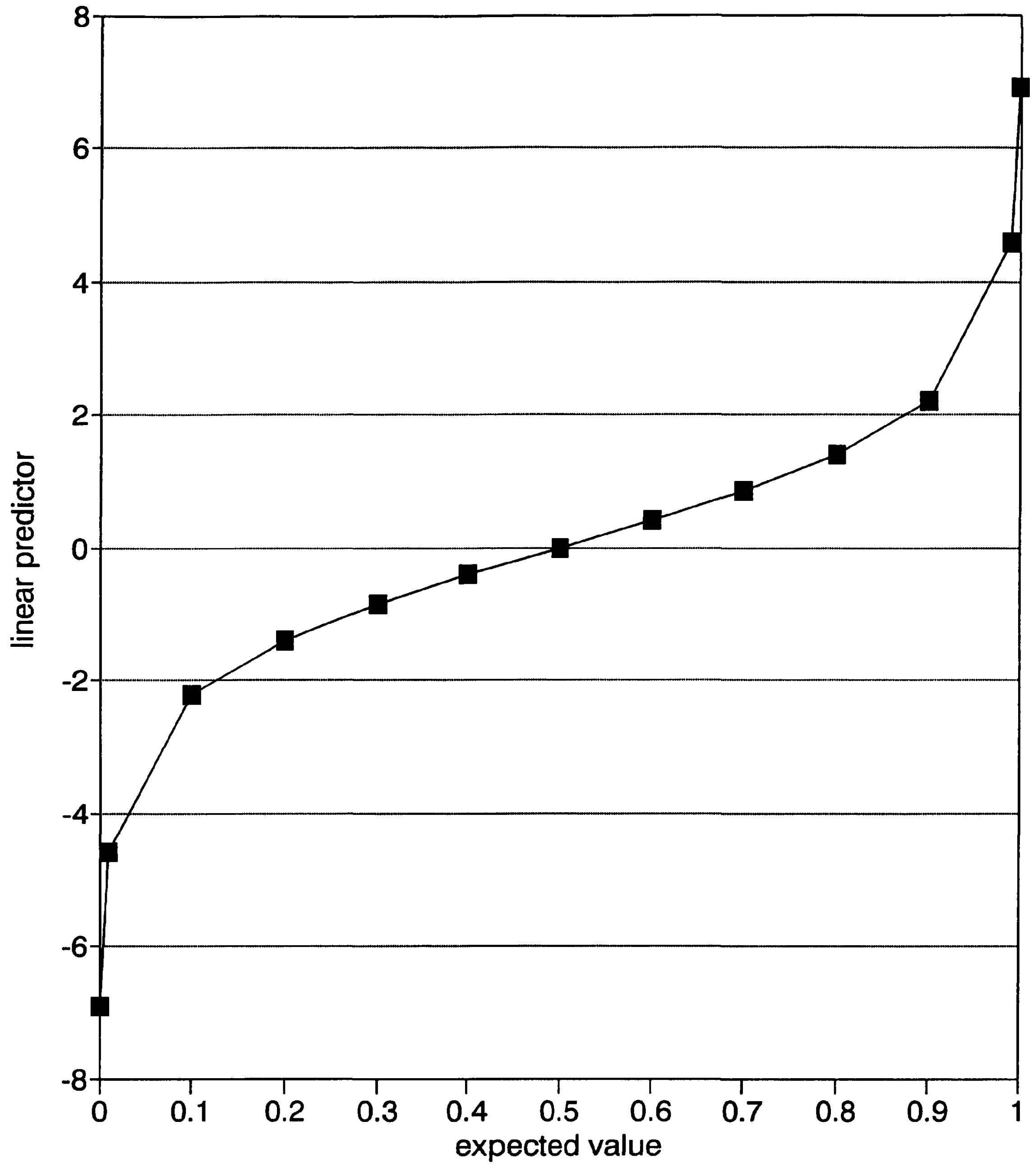


Figure (2.1) Logit Function

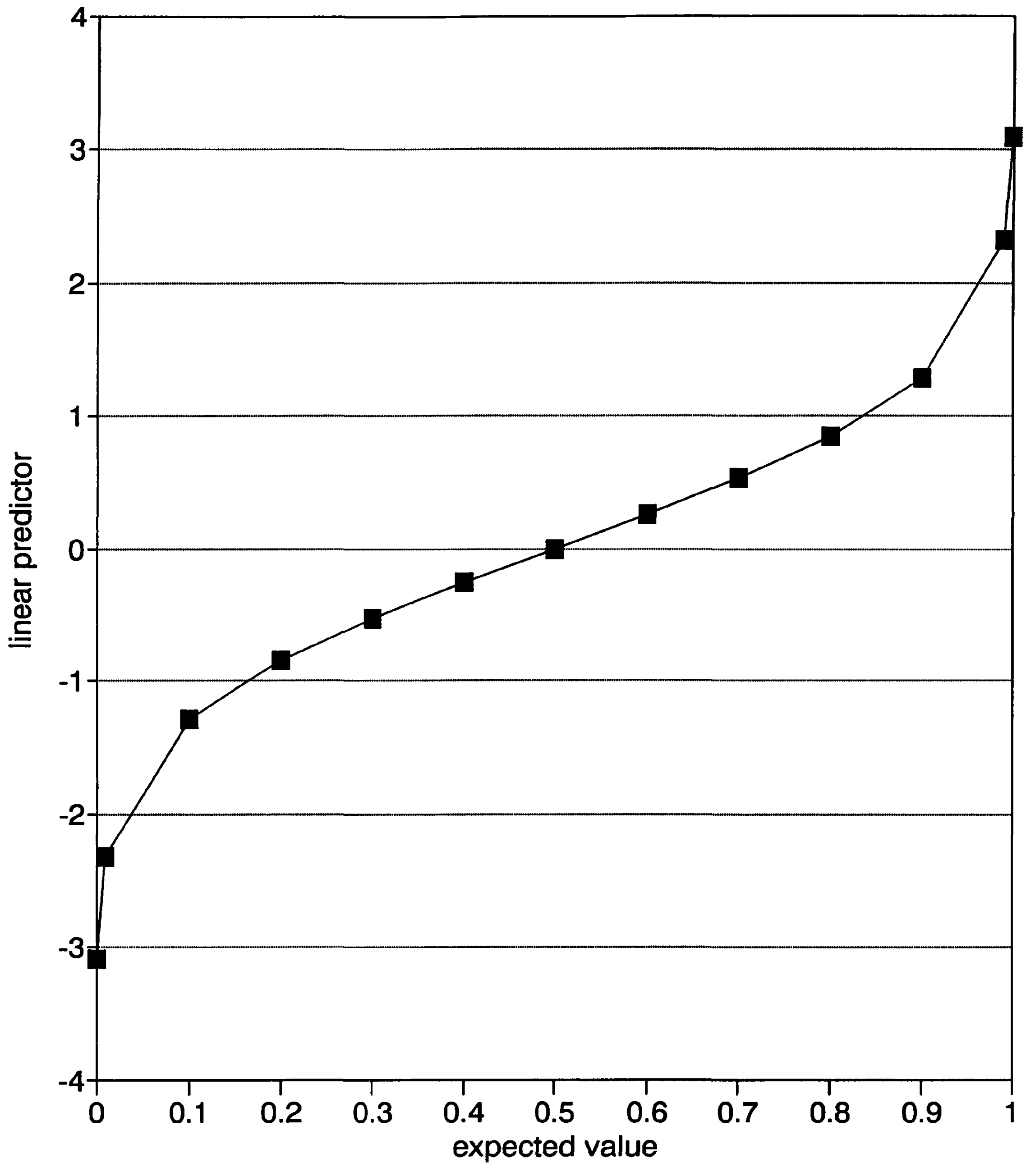


Figure (2.2) Inverse Normal Function

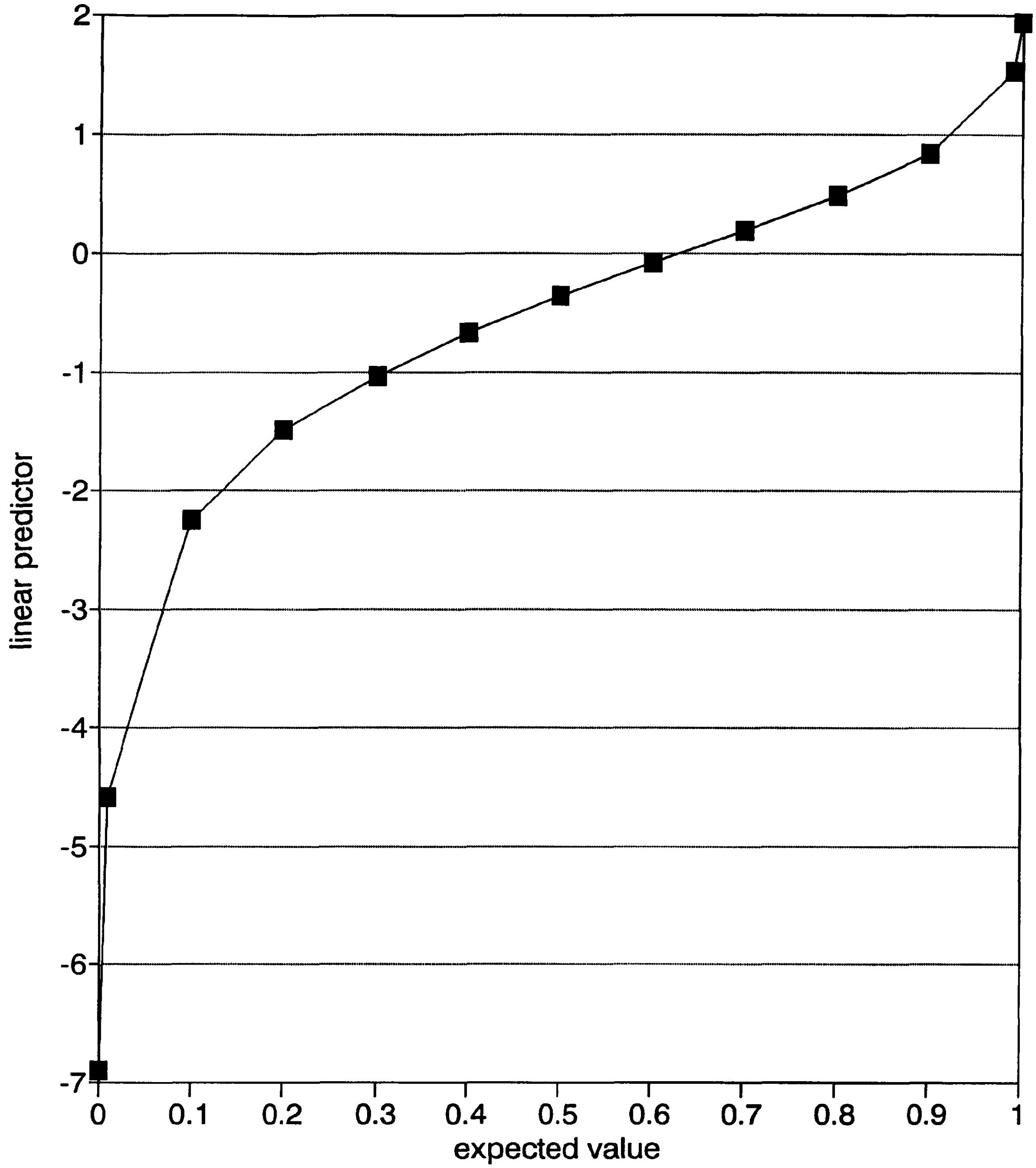


Figure (2.3) Complementary Log-Log Function

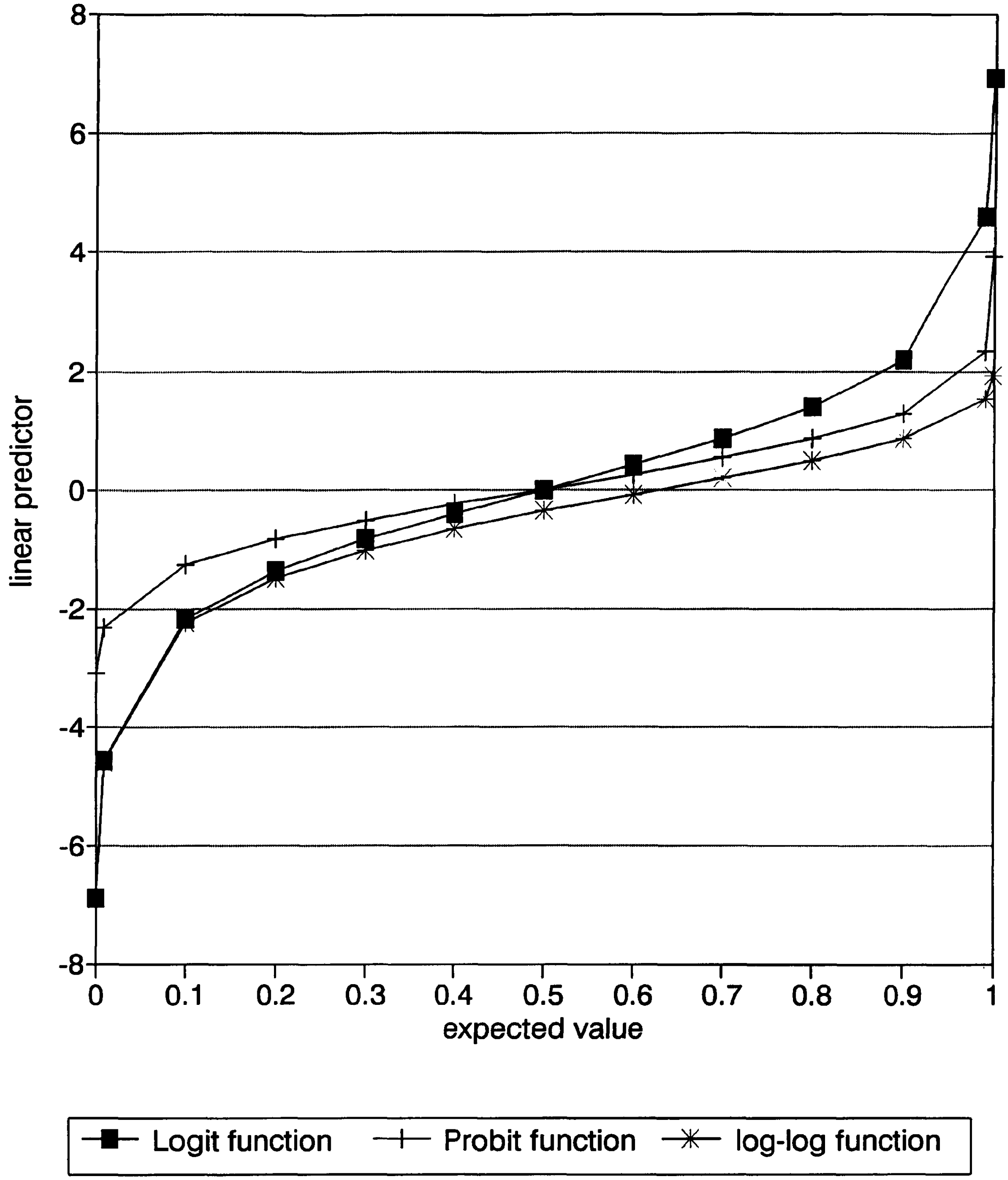


Figure (2.4) Comparison of The Three Link Functions

2.5 The Linear Logistic Model:

In many respects this is the simplest possible way of representing the dependence of a probability on explanatory variables, so that, the constraint $0 < \mu_i < 1$ is satisfied.

Let a_i be the vector of explanatory variables. Then set

$$\mu_i = \frac{e^{a_i\beta}}{1+e^{a_i\beta}}$$

$$\text{or } 1-\mu_i = \frac{1}{1+e^{a_i\beta}}$$

$$\mu_i(1+e^{a_i\beta}) = e^{a_i\beta}$$

$$\text{hence} \quad e^{a_i\beta} = \frac{\mu_i}{1-\mu_i}$$

$$\text{so} \qquad a_i\beta = \log(\frac{\mu_i}{1-\mu_i})$$

$$\text{and} \qquad \lambda_i = \log(\frac{\mu_i}{1-\mu_i}) = a_i\beta = \sum_{s=1}^{p} a_{is}\beta_s$$

we call λ_i the logistic transform since λ_i is the logit function.

$$\lambda = a\beta \qquad (2.3)$$

We call expresion(2.3) a linear logistic model.

Cox (1972) presents eight kinds of models related to multivariate binary models, one of which is the logistic model.

He states that the simplest, most flexible, and in many ways the most important models are probably the logistic representations of the probabilities.

Suppose that $z_i = 2y_i - 1$, so that Z's takes the values ±1, since $y_i = 0 \text{ or } 1$ using the standard representation for binary variables, and suppose that

$$\log[P(Z_1 = z_1, \ldots\ldots, Z_p = z_p)] = \alpha_1 z_1 + \ldots\ldots + \alpha_p z_p + \alpha_{12} z_1 z_2 + \ldots. + \alpha_{p-1} z_{p-1} z_p$$

$$\Rightarrow P(Z_1 = z_1, \ldots.., Z_p = z_p) = \exp(\alpha_1 z_1 + \ldots.. + \alpha_p z_p + \alpha_{12} z_1 z_2 + \ldots.. + \alpha_{p-1p} z_{p-1} z_p)\Lambda^{-1}$$

Where Λ is a normalizing constant: e^{Λ} is a sum of exponentials chosen to make the probabilities sum to unity.

The logistic model is implicit or explicit in a good deal of work on multivariate binary data, but only for small values of p, because of the large number of parameters involved.

2.6 Definition of Multinomial distribution:

The multinomial distribution is a generalization of the binomial to more than two categories. Suppose we have N independent identical trials. On each trial, we check to see which of q events occurs. In such a situation we assume that on each trial one of the q events must occur. Let n_i , $i = 1, \ldots, q$ be the number of times that the ith event occurs. Let p_i be the probability that ith event occurs on any trial. Note that the $p_i's$ must satisfy $p_1 + p_2 + \ldots + p_q = 1$. In this situation we say that $(n_1, n_2, \ldots, n_q)$ has a multinomial distribution with parameters $N, p_1, \ldots., p_q$.

The distribution is

$$\Pr(n_1 = r_1, \ldots\ldots, n_q = r_q) = \frac{N!}{r_1! \ldots\ldots r_q!} p_1^{r_1} \ldots.. p_q^{r_q}$$

2.6.1 The Multinomial Model:

Univariate models with response y_i given x_i have the form $\mu_i = h(z_i' \beta)$

For a dichotomous response variable $y_i \in \{0,1\}$ for example, the logistic model is given by

$$\pi_i = \frac{\exp(z_i'\beta)}{1+\exp(z_i'\beta)}$$

where $\pi_i = P(y_i = 1|x_i)$. In the multinomial case $\pi_i = \mu_i = E(y_i|x_i)$ is a $(q \times 1)$ vector, $\pi_i = (\pi_{i1}, \pi_{i2}, \ldots, \pi_{iq})$. Here the model has the form

$$\pi_i = h(Z_i\beta)$$

where h is a vector-valued response function.

Z_i is a $(q \times p)$ design matrix composed from $.x_i$ and β is a $(p \times 1)$ vector of unknown parameters.

2.7 Generating Function:

The generating functions reflect certain properties of distribution functions. They are transform of the density function (or probability function) defining the distribution.

They can be used to generate moments and cumulatnts. They also are particular usefulness in connection with sums of independent random variable.

Definition (1): The Moment Generating Function of a random variable X, denoted by M(t), is defined by

$$M(t) = E[e^{tx}] \qquad \text{for all real values of t.}$$

Definition (2): The characteristic Function of a random variables, denoted by $\phi(t)$, is defined by

$$\phi(t) = E[e^{itx}] \qquad \text{for all real values of t.}$$

Note that the moment generating function M(t) does not exist for every distribution for all values of t.

2.7.1 Relation Between Moment Generating Function And Moments:

If M(t) can be expanded in power of t, i.e. if

$$M(t) = \sum_{r=0}^{\infty} a_r \frac{t^r}{r!}$$

then $\mu'_r = a_r$ the coefficient of $\frac{t^r}{r!}$ in the expansion of M(t) as a power series in t.

$$\text{and } \mu'_r = \frac{\partial^r}{\partial t^r} M(t)|t = 0$$

$$\therefore \ \text{cov}(X,Y) = \frac{\partial^2}{\partial t_1 \partial t_2} M(t_1, t_2)|t_1 = 0, t_2 = 0$$

therefore $V(X) = \mu_2' - \mu_1'^2$

2.7.2 The Correlation Coefficient:

The correlation coefficient has the advantage of being independent of the units of measurement.

If weights are measured in kilograms instead of pounds, the covariance and variance are changed, but the correlation coefficient remains the same, because it measures the linear relationship between x and y. Note that $-1 \le \rho \le 1$ if it is +1 the slope of the line is positive, if it is -1 the slope of the line is negative.

The correlation coefficient between X and Y is a dimensionless quantity often denoted by ρ and defined by

$$\rho = \frac{\operatorname{cov}(x, y)}{\sqrt{V(x)V(y)}}$$

The correlation coefficient is widely used in the study in the interrelationship among dependent random variables.

2.7.3 Regression:

Regression analysis is designed to examine the relationship of a variable Y to a variable X.

Definition (3): Multiple regression:

In multiple regression, it is assumed to have a "dependent" variable Y which is dependent upon p "independent" variables $x_1, x_2, \ldots\ldots, x_p$. Or

$$Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots\ldots. + \beta_p x_p + \varepsilon$$

where $\alpha, \beta_1, \beta_2, \ldots\ldots, \beta_p$ are unknown parameters to be estimated, and $x_1, x_2, \ldots\ldots, x_p$ are covariates or known variables, ε_i is observation on a random variables, independent of x's.

Definition (4) Polynomial Regression:

If there are no theoretical reasons for expecting a curve a certain type to represent the relationship, polynomials are often selected because of their simplicity and flexibility. The lowest degree polynomial that will suffice can often be determined by an inspection of the scatter diagram. After the degree has been determined, the best-fitting polynomial of that degree may then be fitted by the method of least square or Likelihood.

It is usually unwise to use a power higher than degree three because of the inherent propensity of a polynomial to oscillate, and oscillation can occur unnoticed 'between' data values for high order polynomials.

CHAPTER 3

EXTENSION AND ADAPTATION OF THE ZHAO-PRENTICE MODEL

3.1 Introduction:

Regression analysis of binary data is frequently complicated by correlations between the outcomes; for example, in studies with repeated measures on individuals or in familial studies. Likelihood analysis of such data is hampered by a lack of realistic probability models for multivariate binary response vectors that allow the specification of the mean and correlation independently of one another.

Liang and Zeger (1986) and Zeger and Liang (1986) developed an extension to quasi-likelihood called the Generalized Estimating Equation (GEE). Also see Lipsitz (1991) and Fitzmaurice and Lipsitz (1995).

Since the (GEE) approach requires correct specification of only the mean values of the dependent outcomes, it leads to estimates of the regression parameters which are consistent and asymptotically normal even if the correlation structure is misspecified. Accurate specification of the correlation structure leads to increased efficiency in estimation.

Correlated binary data arise in many application areas including studies of disease occurrence among family members, studies involving repeated measurements on study subjects, longitudinal studies, and studies involving group randomization.

3.2 Binary Models:

Zhao and Prentice (1990) considered a sample of K, independent, multivariate binary observations

$$Y_K^T = (Y_{k1}, Y_{k2}, \dots\dots, Y_{kn_k}),\ (k = 1, 2, \dots, K)$$

They suppose that Y_k is distributed according to

$$\Pr(Y_k) = \Delta_k^{-1} \exp(y_k^T \theta_k + W_k^T \lambda_k + C_k(Y_k))$$

where $W_k^T = (y_{k1}y_{k2}, y_{k1}y_{k3}, \dots\dots, y_{k2}y_{k3}, \dots\dots)$, $\theta_k^T = (\theta_{k1}, \theta_{k2}, \dots\dots, \theta_{kn_k})$ and $\lambda_k^T = (\lambda_{k12}, \lambda_{k13}, \dots\dots, \lambda_{k23}, \dots\dots)$ are "canonical" parameters, and $\Delta_k = \Delta_k(\theta_k, \lambda_k)$ is the normalization constant defined by $\Delta_k = \sum \exp(Y_k^T \theta_k + W_k^T + C_k(Y_k))$.

However, they point out this model is impractical unless $n_k \le 12\ \forall k$. This arises from the difficulty in finding or calculating of the normalization constants, Δ_k. But for us $n_k \approx 700$, so we propose a modification and approximation to this model so as to make it suitable for our binary data.

This section is largely devoted to developing expressions and approximations for this normalization constant, so as to enable large values of n_k to be used in this type of model. Extensive numerical checks laid out in tables have been used to validate these alternative formula.

Our first proposal is to assume a strong regularity in the underlying process and use this to justify reducing the parameter space initially to two elements θ & λ fully specifying the linear and quadratic components respectively.

Taking $\theta_1 = \theta_2 = \dots\dots = \theta_k = \theta$, $\lambda_{k12} = \lambda_{k13} = \dots\dots = \lambda_{k23} = \dots\dots = \lambda$ and putting $C_k(Y_k) = 0$,

So $\Pr(Y) = \Lambda^{-1} e^{\theta y_\bullet + \lambda y_s}$

where $y_{.} = \sum_{i=1}^{n} y_i$ $y_s = \sum_{i=1}^{n-1} y_i y_{i+1}$ and $\Lambda = \sum_{\Omega_n} e^{\theta y_i + \lambda y_s}$,

with the last summation being over Ω_n={ all 2^n possible 0/1 sequences of length n}.

By assigning a nominal probability of 2^{-n} to each sequence in Ω_n , this definition of Λ can be interpreted as an expectation over a uniform distribution, scaled up by 2^n.

$$\text{i.e. } \Lambda = 2^n E(e^{\theta y_{.} + \lambda y_s})$$

For this distribution, we examine $\Pr(y_{.} = m)$:

we have m={# 1's in sequence $y_1, y_2, \ldots\ldots, y_n$} $\Rightarrow$ $\Pr(m) = \binom{n}{m} 2^{-n}$

Now from standard properties of expectation

$$E[e^{\theta y_{\bullet} + \lambda y_s}] = E[E[e^{\theta y_{\bullet} + \lambda y_s} | m]] = \sum_{m=0}^{n} E[e^{\theta y_{\bullet} + \lambda y_s} | m] \binom{n}{m} 2^{-n}$$

But $E[e^{\theta y_{\bullet} + \lambda y_s} | m] \approx e^{E[\theta y_{\bullet} + \lambda y_s | m]}$ from the standard linear approximation for non-linear function expectations. Also note that $E[e^x] \geq e^{E[x]}$, this is Jensen's inequality for convex functions, see Eggleston (1958) and Laycock (1972)

Now we need to find $E[y_i | m]$ and $E[y_s | m]$

(i) $E[y_i | m] = m$ by definition

(ii) $E[y_s | m] = \sum_{i=1}^{n-1} E[y_i y_{i+1} | m]$

but,

$$E[y_i y_{i+1} | m] = E[y_i y_{i+1} | y_i = 1, m] \Pr(y_i = 1 | m) + E[y_i y_{i+1} | y_i = 0, m] \Pr(y_i = 0 + m)$$

$=E[1.y_{i+1}|y_i=1,m]\frac{m}{n}+0$ since $\Pr(y_i=1|y_1+.....+y_n=m)$, by symmetry, since distribution is uniform and all possibilities are equally likely.

$=\frac{\text{m-1}}{\text{n-1}}\frac{m}{n}$ since $\Pr(y_{i+1}=1|y_1+y_2+....+y_{i-1}+y_{i+1}+.....+y_n=m-1)=\frac{m-1}{n-1}$, again by symmetry.

$\therefore E[y_a|m]=(m-1)\frac{m}{n}$ since $\sum_{i=1}^{n-1}k=(n-1)k$

Hence

$$E[e^{\theta y_\bullet+\lambda y_s}]\approx 2^{-n}\sum_{m=0}^{n}e^{\theta m+\lambda(m-1)\frac{m}{n}}\binom{n}{m}$$

$$\therefore \Lambda=\sum_{m=0}^{n}\binom{n}{m}e^{\theta m+\lambda(m-1)\frac{m}{n}}$$

So the model assumed for our data will be

$$\Pr(y)=\frac{e^{\theta\sum_{i=1}^{n}y_i+\lambda\sum_{i=1}^{n-1}y_iy_{i+1}}}{\sum_{m=0}^{n}\binom{n}{m}e^{\theta m+(m-1)\frac{m}{n}}}$$

3.2.1 Approximation checking:

In this section we demonstrate various checks on the validity and accuracy of this approximation, both numerically and theoretically

i.e. $\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}......\sum_{y_n}^{1}\exp(\theta\sum_{i=1}^{n}y_i+\lambda\sum_{i=1}^{n-1}y_iy_{i+1})\approx\sum_{m=0}^{n}\binom{n}{m}\exp(\theta m+\lambda(m-1)\frac{m}{n})$

and also check for a varying n that $\sum_{i=1}^{n}p_i=1$.

Checking that the left hand side is approximately equal to the right hand side we will take two parts of checking.

First for n is small theoretically checking, and second for some chosen values of estimating parameters checking the approximation numerically.

Taking n=2 then, the left hand side will be

$$\sum_{y_1=0}^{1}\sum_{y_2=0}^{1} e^{\theta(y_1+y_2)+\lambda(y_1y_2)} = 1+2e^{\theta}+e^{2\theta+\lambda}$$

The right hand side will be

$$\sum_{m=0}^{2}\binom{2}{m} e^{\theta m+\lambda(m-1)\frac{m}{n}} = 1+2e^{\theta}+e^{2\theta+\lambda}$$

So if n=2 then our approximation is exactly equal to the original one.

Taking n=3, the left hand side will be

$$\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\sum_{y_3=0}^{1} \exp[\theta(y_1+y_2+y_3)+\lambda(y_1y_2+y_2y_3)] = 1+3e^{\theta}+e^{2\theta}+2e^{2\theta+\lambda}+e^{2\theta+2\lambda}$$

the right hand side will be

$$\sum_{m=0}^{3}\binom{3}{m} \exp(\theta m+\lambda(m-1)\frac{m}{n}) = 1+3e^{\theta}+3e^{2\theta+\frac{2}{3}\lambda}+e^{3\theta+2\lambda}$$

we require $3e^{2\theta+\frac{2}{3}\lambda} = e^{2\theta}+2e^{2\theta+\lambda}$

or $3e^{2\theta+\frac{2}{3}\lambda} = e^{2\theta}(1+2e^{\lambda})$

or $3e^{\frac{2}{3}\lambda} = 1+2e^{\lambda}$

or $3(1+\frac{2}{3}\lambda+\frac{4}{9}\frac{\lambda^2}{2!}+........) = 1+2(1+\lambda+\frac{\lambda^2}{2!}+......)$

if λ is small then both sides are equal, so still the approximation is accurate.

Taking n=4, then the left hand side will be

$$\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\sum_{y_3=0}^{1}\sum_{y_4=0}^{1}\exp[(y_1+y_2+y_3+y_4)\theta+\lambda(y_1y_2+y_2y_3+y_3y_4)]$$

$$=1+4e^{\theta}+3e^{2\theta}+3e^{2\theta+\lambda}+2e^{3\theta+2\lambda}+e^{4\theta+3\lambda}$$

the right hand side will be

$$\sum_{m=0}^{4}\binom{4}{m}\exp(\theta m+\lambda(m-1)\frac{m}{n})=1+4e^{\theta}+6e^{2\theta+\frac{1}{2}\lambda}+4e^{3\theta+\frac{3}{2}\lambda}+e^{4\theta+3\lambda}$$

Again we require

$$3e^{2\theta}+3e^{2\theta+\lambda}+2e^{3\theta+\lambda}+2e^{3\theta+2\lambda}=6e^{2\theta+\frac{1}{2}\lambda}+4e^{3\theta+\frac{3}{2}\lambda}$$

or $3e^{2\theta}(1+\lambda+\frac{\lambda^2}{2!}+......)+2e^{3\theta}(e^{\lambda}+e^{2\lambda})=6e^{2\theta}(1+\frac{1}{2}\lambda+\frac{(\frac{1}{2}\lambda)^2}{2!}+...)+4e^{3\theta}(1+\frac{3}{2}\lambda+...)$

or $3e^{2\theta}(1+\lambda+\frac{\lambda^2}{2!}+....)+2e^{3\theta}[(1+\lambda+\frac{\lambda^2}{2!}+....)+(1+2\lambda+\frac{(2\lambda)^2}{2!}+....)]$

$=6e^{2\theta}(1+\frac{1}{2}\lambda+....)+4e^{3\theta}(1+\frac{3}{2}\lambda+.......)$

Again if λ is small then both sides are approximately equal.

The second part of the checking is to take typical values for the estimated parameters θ and λ, then substitute in both sides and see if they are approximately equal for realistic values of n.

We will take one of the allocations which is allocation 47

After writing the FORTRAN program see appendix (E) we fitted that the parameters

$\theta_j=\alpha+\beta T_j$ where $T_j=-107,-97,-87,-77$

$\lambda_j=\lambda_1+\lambda_2 T_j$ j=1,2,3,4 and the estimates for the parameters are:

α=-11.736 , β=-0.086 , λ_1=11.6843 , λ_2=0.0855

(1) If T=-107

then θ=-2.534 and λ=2.5358

We know that if n=3 we require

$$1+3e^{\theta}+2e^{2\theta+\lambda}+e^{\theta}+e^{3\theta+2\lambda}\approx 1+3e^{\theta}+3e^{2\theta+\frac{2}{3}\lambda}+e^{3\theta+2\lambda}$$

or $$\sum P=\frac{1+3e^{\theta}+2e^{2\theta+\lambda}+e^{2\theta}+e^{3\theta+2\lambda}}{1+3e^{\theta}+3e^{2\theta+\frac{2}{3}\lambda}+e^{3\theta+2\lambda}}$$

$$\sum P=\frac{1.4829}{1.4201}=1.044=1$$

(2) If T=-97 then θ=-3.394 and λ=3.3908

$$\sum P=\frac{1.2021}{1.1665}=1.0305=1$$

(3) If T=-87 then θ=-4.2544 and λ=4.2458

$$\sum P=\frac{1.08493}{1.06682}=1.01697=1$$

(4) If T=-77 then θ=-5.114 and λ=5.1008

$$\sum P=\frac{1.03579}{1.02714}=1.0084=1$$

We concluded that our approximation is accurate even if n is large.

3.2.2- Calculating the percentage error for various data sets:

By taking various allocations and from estimating the parameters we compute θ, and λ for ten allocations and we plot the numerator against the denominator.If the developed approximate formula for the normalization constant is correct then these ratios should equal one, and we will calculate the mean error and variance error for the allocations 47, 68, 65, 63, 58, 53, 50, 48, 36, and 29.

As can be seen from these tables the approximation for the normalization constant is accurate across and wide range of parameter values.

In the following table we will calculate theta and lambda for each allocation for four thresholds.

	θ	λ	θ	λ	θ	λ	θ	λ
47	-2.534	2.536	-3.394	3.391	-4.254	4.246	-5.114	5.101
68	-3.479	3.478	-4.041	4.037	-4.602	4.596	-5.165	5.155
65	-2.694	2.694	-3.474	3.473	-4.253	4.252	-5.034	5.031
63	-2.635	2.636	-3.137	3.137	-3.638	3.638	-4.141	4.139
58	-2.427	2.428	-2.818	2.807	-3.208	3.186	-3.599	3.565
53	-2.399	2.400	-2.514	2.510	-2.629	2.620	-2.744	2.730
50	-2.514	2.511	-3.374	3.371	-4.234	4.231	-5.094	5.091
48	-2.548	2.554	-3.043	3.045	-3.538	3.536	-4.033	4.027
36	-2.996	2.989	-3.563	3.542	-4.130	4.094	-4.697	4.646
29	-2.349	2.360	-2.519	2.525	-2.689	2.690	-2.859	2.855

Table (3.1) The estimating parameters.

In the following four tables we will present the calculation of the numerator and denominator for ten allocations for four thresholds

Table (3.2) Calculation of the numerator and the denominator and the percentages error for the case T=-107

Allocation	Numerator	Denominator	Percentages error
47	1.48	1.42	4.225%
68	1.19	1.16	2.586%
65	1.41	1.35	4.444%
63	1.44	1.38	4.348%
58	1.54	1.47	4.762%
53	1.55	1.49	4.027%
50	1.49	1.42	4.93%
48	1.48	1.42	4.225%
36	1.3	1.25	4%
29	1.59	1.52	4.605%

To find the C.V. (error) we need to calculate the mean error and the standard deviation error.

mean error=0.058

S.D. error=0.01135

So C. V.$=\frac{\sigma}{\mu}\times 100=\frac{0.01135}{0.058}\times 100=19.57\%$

Table (3.3) Calculation of the numerator and the denominator and the percentages error for the case T=-97

Allocation	Numerator	Denominator	Percentage error
47	1.2	1.17	2.564%
68	1.11	1.08	2.778%
65	1.19	1.15	3.478%
63	1.26	1.22	3.279%
58	1.36	1.31	3.817%
53	1.49	1.43	4.196%
50	1.49	1.47	1.361%
48	1.29	1.24	4.032%
36	1.17	1.14	2.632%
29	1.49	1.43	4.196%

Again we will calculate C.V. for T=-97

Mean error=0.041

S.D.=0.0099

$$C.V.=\frac{0.0099}{0.041}\times 100=24.15\%$$

Table (3.4) Calculation of the numerator and the denominator and the percentages error for the case T=-87

Allocation	Numerator	Denominator	Percentage error
47	1.08	1.07	0.935%
68	1.06	1.05	0.952%
65	1.09	1.07	1.869%
63	1.16	1.13	2.655%
58	1.24	1.20	3.333%
53	1.44	1.38	4.348%
50	1.09	1.07	1.869%
48	1.17	1.14	2.632%
36	1.09	1.08	0.926%
29	1.41	1.36	3.676

C.V. for t=-87 is

Mean error=0.028

S.D.=0.0148

$$C.V.=\frac{0.0148}{0.028}\times 100=52.86\%$$

Table (3.5) Calculation of the numerator and the denominator and the percentages error for the case T=-77

Allocation	Numerator	Denominator	Percentage error
47	1.04	1.03	0.971%
68	1.03	1.03	0%
65	1.04	1.03	0.971%
63	1.09	1.08	0.926%
58	1.16	1.13	2.655%
53	1.38	1.33	3.759%
50	1.04	1.03	0.971%
48	1.1	1.08	2.0%
36	1.05	1.04	0.962%
29	1.34	1.30	3.077%

C.V. for T=-77 is

Mean error=0.023

S.D.=0.0195

C.V.=$\frac{0.0195}{0.023} \times 100 = 84.78\%$

3.3- Estimating Likelihood Equation For Threshold Crossing Model in One Band:

We shall now assume that we can write

$$\Pr(y_{ij}) = \frac{e^{\theta_j \sum_{i=1}^{n} y_i + \lambda \sum_{i=1}^{n-1} y_i y_{i+1}}}{\sum_{m=0}^{n} \binom{n}{m} e^{\theta_j m + \lambda(m-1)(\frac{m}{n})}} \qquad i=1\ldots n \quad j=1\ldots 4$$

where y_{ij} is the indicator variate for frequency i at threshold j in the selected band.

Hence

$$L = \frac{e^{\sum_{j=1}^{4}\sum_{i=1}^{n} \theta_j y_{ij} + \sum_{j=1}^{4}\sum_{i=1}^{n-1} \lambda y_{ij} y_{i+1j}}}{\sum_{j=1}^{4}\sum_{m=0}^{n} \binom{n}{m} e^{\theta_j m + \lambda(m-1)(\frac{m}{n})}}$$

Therefore

$$\ln L = \sum_{j=1}^{4}\sum_{i=1}^{n} \theta_j y_{ij} + \lambda \sum_{j=1}^{4}\sum_{i=1}^{n-1} y_{ij} y_{i+1j} - \sum_{j=1}^{4} \ln \sum_{m=0}^{n} \binom{n}{m} e^{\theta_j m + \lambda(m-1)(\frac{m}{n})}$$

$$-\ln L = l = \sum_{j=1}^{4} \ln \sum_{m=0}^{n} e^{(\alpha+\beta T_j)m + \lambda(m-1)(\frac{m}{n})} - \sum_{j=1}^{4} (\alpha+\beta T_j) \sum_{i=1}^{n} y_{ij} - \lambda \sum_{j=1}^{4}\sum_{i=1}^{n-1} y_{ij} y_{i+1j}$$

We now assume the linear regression $\theta_j = \alpha + \beta T_j$ where

$T_1 = -107 \quad ,T_2 = \text{-}97 \quad ,T_3 = \text{-}87 \quad ,T_4 = \text{-}77$ and

this assumption is based on the finding for previous models in Gott and Laycock (1989).

Hence $l = \sum_{j=1}^{4} \ln \sum_{m=0}^{n} \binom{n}{m} e^{(\alpha+\beta T_j)m + \lambda(m-1)(\frac{m}{n})} - \sum_{j=1}^{4} (\alpha+\beta T_j) S(1,j) - \lambda \sum_{j=1}^{4} S(2,j)$ (3.1)

where $\sum_{j=1}^{4} S(1,j) = \sum_{j=1}^{4}\sum_{i=1}^{n} y_{ij}$ and $\sum_{j=1}^{4} S(2,j) = \sum_{j=1}^{4}\sum_{i=1}^{n-1} y_{ij} y_{i+1j}$

To find the likelihood equations for the parameters α, β, λ, differentiate (3.1) with respect to α, β, λ, and equate to zero:

$$\frac{\partial}{\partial \alpha} = \sum_{j=1}^{4} \frac{\sum_{m=0}^{n} m \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}}{\sum_{m=0}^{n} \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}}$$

$$= \sum_{j=1}^{4} \frac{\Lambda_{1j}}{\Lambda_j} - \sum_{j=1}^{4} S(1,j) \qquad \text{(say)}$$

where $\Lambda_{1j} = m \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}$ and $\Lambda_j = \sum_{m=0}^{n} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}$

$$\frac{\partial}{\partial \beta} = \sum_{j=1}^{4} T_j \frac{\sum_{m=0}^{n} m \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}}{\sum_{m=0}^{n} \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}} - \sum_{j=1}^{4} T_j S(1,j)$$

$$= \sum_{j=1}^{4} T_j \frac{\Lambda_{1j}}{\Lambda_j} - \sum_{j=1}^{4} T_j S(1,j)$$

$$\frac{\partial}{\partial \lambda} = \sum_{j=1}^{4} \frac{\sum_{m=0}^{n} (m-1)(\frac{m}{n}) \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}}{\sum_{m=0}^{n} \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}}$$

$$= \sum_{j=1}^{4} \frac{\Lambda_{2j}}{\Lambda_j} - \sum_{j=1}^{4} S(2,j)$$

where $\Lambda_{2j} = \sum_{m=0}^{n} (m-1)(\frac{m}{n}) \binom{n}{m} e^{(\alpha+\beta T_j)m+\lambda(m-1)(\frac{m}{n})}$

We first consider calculating Λ_j, Λ_{1j}, and Λ_{2j}. Because of potential overflow or underflow problems with these expressions (arising because our "n" is large) great care must be taken in evaluating the summations.

We know that

$$\binom{n}{m} = \binom{n}{n-m}$$

consider

$$\Lambda = \sum_{m=0}^{n} \binom{n}{m} e^{\theta m + \lambda(m-1)(\frac{m}{n})}$$

If $\theta > 0$, then put r=n-m, so m=n-r

$$\text{therefore } \Lambda = \sum_{r=0}^{n} \binom{n}{r} e^{\theta(n-r) + \lambda\frac{(n-r-1)(n-r)}{n}}$$

$$\frac{(n-r-1)(n-r)}{n} = \frac{(n-1)n-(n-1)r-r(n-r)}{n}$$

$$= \frac{(n-1)n-(n-1)r-r(n-r)}{n}$$

$$=(n-1)-\frac{r}{n}(2n-r-1)$$

$$\text{then } \Lambda = e^{n\theta+\lambda(n-1)} \sum_{r=0}^{n} \binom{n}{r} e^{-r(\theta+\frac{\lambda}{n}(2n-r-1))}$$

$$= e^{n\theta+\lambda(n-1)} \sum_{r=0}^{n} T_r$$

$$\text{where } T_r = \binom{n}{r} e^{-r(\theta+\frac{\lambda}{n}(2n-r-1))} \quad \text{and} \quad T_{r+1} = \binom{n}{r+1} e^{-(r+1)(\theta+\frac{\lambda}{n}(2n-r-2))}$$

$$\text{therefore } \frac{T_{r+1}}{T_r} = \frac{n!}{(r+1)!(n-r-1)!} \frac{r!(n-r)!}{n!} e^{-(r+1)(\theta+\frac{\lambda}{n}(2n-r-2))+r(\theta+\frac{\lambda}{n}(2n-r-1))}$$

$$= \frac{n-r}{r+1} e^{-E_r}$$

$$\text{where } e^{-E_r} = \exp[-\theta - \frac{2\lambda}{n}(n-r-1)]$$

$$\text{So } \Lambda = e^{n\theta+\lambda(n-1)} \sum_{r=0}^{n} T_r$$

$$\text{where } T_{r+1} = \Psi_r T_r \qquad \Rightarrow \Psi_r = \frac{T_{r+1}}{T_r}$$

But $E_r = \theta + \frac{2\lambda}{n}(n-r-1)$ and $E_{r+1} = E_r - \frac{2\lambda}{n}$

therefore $\ln\Lambda = n\theta + \lambda(n-1) + \ln\sum_{r=0}^{n} T_r$

Now if $\theta < 0$, then

$$\Lambda_n = \sum_{m=0}^{n}\binom{n}{m}e^{\theta m + (m-1)(\frac{m}{n})}$$

$$= \sum_{m=0}^{n} T_m \qquad \text{(say)}$$

Where $T_m = \binom{n}{m}e^{\theta m + \lambda(m-1)(\frac{m}{n})}$

Hence $T_{m+1} = \binom{n}{m}e^{\theta(m+1) + \lambda m(\frac{m+1}{n})}$

Define $\Psi_m = \frac{T_{m+1}}{T_m}$

$$= \frac{n!}{(m+1)!(N-m-1)!}\frac{m!(n-m)!}{n!}e^{\theta(m+1)+\lambda\frac{(m+1)m}{n}}e^{-\theta m-\lambda(m-1)\frac{m}{n}}$$

$$= \frac{n-m}{m}e^{\theta+2\lambda\frac{m}{n}} = \frac{n-m}{m}e^{\theta}e^{2\lambda\frac{m}{n}}$$

$$\Psi_1 = ne^{\theta}e^{\frac{2\lambda}{n}} \quad \text{and} \quad \Psi_{n-1} = \frac{1}{n-1}e^{\theta}e^{2\lambda\frac{n-1}{n}}$$

where

$$T_\circ = 0 \ , \ T_1 = ne^{\theta} \ , \ T_2 = \Psi_1 T_1 \ , \ldots\ldots\ldots, T_n = \Psi_{n-1}T_{n-1}$$

We will use the same procedure to calculate Λ_{1j} and Λ_{2j}

setting

$$\Lambda_1 = \sum_{m=0}^{n} m\binom{n}{m}e^{\theta m + \lambda(m-1)\frac{m}{n}}$$

We distinguish the two cases $\theta < 0$ and $\theta > 0$:

If $\theta > 0$ then since $\binom{n}{m} = \binom{n}{n-m}$,

$$\Lambda_1 = \sum_{r=0}^{n} (n-r)\binom{n}{r} e^{\theta(n-r)+\lambda\frac{(n-r-1)(n-r)}{n}}$$

where r=n-m and m=n-r

$$\therefore \Lambda_1 = e^{n\theta+\lambda(n-1)} \sum_{r=0}^{n} (n-r)\binom{n}{r} e^{-r(\theta+\frac{\lambda}{n}(2n-r-1))}$$

$$= e^{n\theta+\lambda(n-1)} \sum_{r=0}^{n} T_r$$

$$\Psi_r = \frac{T_{r+1}}{T_r} = \frac{n!(n-r-1)}{(r+1)!(n-r-1)!} \frac{r!(n-r)!}{n!(n-r)} e^{-(r+1)[\theta+\frac{\lambda}{n}(2n-r-2)]+r[\theta+\frac{\lambda}{n}(2n-r-1)]}$$

$$= \frac{n-r-1}{r+1} e^{-E_r}$$

where $E_r = \theta + \frac{2\lambda}{n}(n-r-1)$ and $E_{r+1} = E_r - \frac{2\lambda}{n}$

$$\Psi_r = \frac{T_{r+1}}{T_r} \qquad \Rightarrow \Psi_r = \frac{n-r-1}{r+1} e^{-E_r}$$

where $T_\circ = n$, $T_1 = \Psi_\circ T_\circ = n(n-1)e^{\theta+\frac{2\lambda}{n}(n-2)}$

and so on $T_n = \Psi_{n-1} T_{n-1}$

If $\theta < 0$ then

$$\Lambda_1 = \sum_{m=0}^{n} m\binom{n}{m} e^{\theta m+(m-1)\frac{m}{n}}$$

$$\Lambda_1 = \sum_{m=0}^{n} T_m \qquad \text{(say)}$$

where $T_m = m\binom{n}{m} e^{\theta+\lambda(m-1)\frac{m}{n}}$, $T_{m+1} = (m+1)\binom{n}{m+1} e^{\theta(m+1)+\lambda\frac{m(m+1)}{n}}$

$$\text{and } \Psi_m = \frac{T_{m+1}}{T_m}$$

$$\therefore \Psi_m = (m+1)\frac{n!}{(m+1)!(n-m-1)!}\frac{m!(n-m)!}{n!m}e^{\theta(m+1)+\frac{\lambda m(m+1)}{n}}e^{\theta m+\lambda(m-1)\frac{m}{n}}$$

$$\text{Hence } \Psi_m = \frac{n-m}{m}e^{\theta+\frac{2\lambda m}{n}} = \frac{n-m}{n}e^{\theta}e^{\frac{2\lambda m}{n}}$$

So $T_\circ = 0 \quad , \quad T_1 = ne^{\theta}$

$$T_2 = \Psi_1 T_1 = ne^{\theta+\frac{2\lambda}{n}} \times ne^{\theta} = n^2 e^{2\theta+\frac{2\lambda}{n}}$$

$$\text{therefore } T_n = \Psi_{n-1}T_{n-1}$$

Finally calculating Λ_2 in the same way

$$\Lambda_2 = \sum_{m=0}^{n}\frac{m(m-1)}{n}\binom{n}{m}e^{\theta m+\lambda(m-1)\frac{m}{n}}$$

Again we will allow for two situations:

Since if $\theta > 0$ then $\binom{n}{m} = \binom{n}{n-m}$

$$\Lambda_2 = \sum_{r=0}^{n}\frac{(n-r)(n-r-1)}{n}\binom{n}{r}e^{\theta(n-r)+\lambda\frac{(n-r-1)(n-r)}{n}}$$

$$\Lambda_2 = \sum_{r=0}^{n}T_r \qquad \text{(say)}$$

$$\text{where } T_r = \frac{(n-r)(n-r-1)}{n}\binom{n}{r}e^{\theta(n-r)+\lambda\frac{(n-r-1)(n-r)}{n}}$$

$$\text{and } T_{r+1} = \frac{(n-r-1)(n-r-2)}{n}\binom{n}{r+1}e^{\theta(n-r-1)+\frac{(n-r-2)(n-r-1)}{n}}$$

$$\text{Hence } \Psi_r = \frac{T_{r+1}}{T_r} = \frac{n-r-2}{r+1}e^{-E_r} \quad \text{where } E_r = \theta + \frac{2\lambda}{n}(n-r-1)$$

$$\text{So } T_\circ = (n-1)e^{\theta n+\lambda(n-1)}$$

$$T_1 = \Psi_{\circ} T_{\circ} \quad , \quad T_2 = \Psi_1 T_1 \quad , \dots\dots\dots , T_n = \Psi_{n-1} T_{n-1}$$

If $\theta < 0$ then

$$\Lambda_2 = \sum_{m=0}^{n} \frac{m(m-1)}{n} \binom{n}{m} e^{\theta m + \lambda(m-1)\frac{m}{n}}$$

$$\Lambda_2 = \sum_{m=0}^{n} T_m \qquad \textbf{(say)}$$

where $T_m = \dfrac{m(m-1)}{n} \dbinom{n}{m} e^{\theta m + \lambda(m-1)\frac{m}{n}}$ and $T_{m+1} = \dfrac{(m+1)m}{n} \dbinom{n}{m+1} e^{\theta(m+1) + \lambda\frac{(m+1)m}{n}}$

$$\text{Hence } \Psi_m = \frac{T_{m+1}}{T_m} = \frac{n-m}{m-1} e^{\theta + \frac{2\lambda m}{n}}$$

So $T_{\circ} = 0$, $T_1 = 0$, $T_2 = (n-1)e^{2\theta + \frac{2\lambda}{n}}$ and $T_n = \Psi_{n-1} T_{n-1}$

When we calculate Λ_j , Λ_{j1} and Λ_{2j} using these formulas, the under flow or overflow of terms can be easily controlled in the programming so as to enable correct evaluation of the overall likelihood value.

3.4 Finding a General Expression For Expectation of Y, Variance of Y, Covariance of Y and Correlation Coefficient of Y:

To find general expression of the moments, we will use the Moment Generating Function.

By definition M. G. F.$= E(e^{t'y}) = M(t)$

and we also have

$$f(y_1, y_2, \dots, y_n) = \frac{e^{\theta \sum_{i=1}^{n} y_i + \lambda \sum_{i=1}^{n-1} y_i y_{i+1}}}{\sum_{m=0}^{n} \binom{n}{m} e^{\theta m + \lambda \frac{m(m-1)}{n}}}$$

$M_y(t) = E(e^{t'y})$

$$M_y(t) = \frac{1}{D_n}\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\cdots\cdots\cdots\sum_{y_n=0}^{1}\exp[t'Y + \theta'Y + \lambda Y_1' \bullet Y_2]$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{bmatrix} \qquad Y_1 = \begin{bmatrix} y_1 \\ y_2 \\ \\ . \\ y_{n-1} \end{bmatrix} \qquad Y_2 = \begin{bmatrix} y_2 \\ y_3 \\ . \\ . \\ y_n \end{bmatrix}$$

and $D_n = \sum_{m=0}^{n}\binom{n}{m} e^{\theta m + \lambda\frac{m(m-1)}{n}}$

so $E(y_i) = (\frac{\partial M}{\partial t_i})_{t_i=0} = M'_{y_i}(0)$

$$= \frac{1}{D_n} = \sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\cdots\sum_{y_i=0}^{1}\cdots\sum_{y_n=0}^{1} y_i \exp(\theta'Y + \lambda Y_1 Y_2)$$

$$= \frac{e^{\theta+\lambda(y_{i-1}+y_{i+1})}}{D_n}\sum_{y_1=0}^{1}\cdots\sum_{y_{i-1}=0}^{1}\sum_{y_{i+1}=0}^{1}\cdots\sum_{y_n=0}^{1} e^{\theta\sum_{i=1}^{i-1} y_i + \lambda\sum_{i=1}^{i-2} y_i y_{i+1}}\, e^{\theta\sum_{i=i+1}^{n} y_i + \lambda\sum_{i=i+1}^{n-1} y_i y_{i+1}} \quad ,2<i<n-1$$

$$E(Y_i) = \frac{e^{\theta+\lambda}}{D_n} D_{i-1} D_{n-i} \qquad (3.2)$$

Ex: take n=5, i=3

$$E(y_3) = \frac{1}{D_5}\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\sum_{y_3=0}^{1}\sum_{y_4=0}^{1}\sum_{y_5=0}^{1} y_3 e^{\theta\sum_{i=1}^{5} y_i + \lambda\sum_{i=1}^{4} y_i y_{i+1}}$$

$$= \frac{e^{\theta+\lambda(y_3+y_4)}}{D_5}\sum_{y_1=0}^{1}\sum_{y_2=0}^{1} e^{\theta(y_1+y_2)+\lambda(y_1 y_2)} \times \sum_{y_4=0}^{1}\sum_{y_5=0}^{1} e^{\theta(y_4+y_5)+\lambda(y_4 y_5)}$$

$$= \frac{e^{\theta+\lambda}}{D_5} D_2 D_2$$

Putting n=5, i=3 and substituting in equation (3.2) we get $E(y_3)$ immediately

$$E(y_3)=\frac{e^{\theta+\lambda}}{D_5}D_2D_2$$

using the same procedure we can find that:

$$E(y_j)=\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j} \qquad \text{for } i\neq j$$

$$E(y_iy_j)=\frac{\partial M_y(0)}{\partial y_i\partial y_j}=\frac{1}{D_n}\sum_{y_1=0}^{1}\cdots\sum_{y_i=0}^{1}\cdots\sum_{y_j=0}^{1}\cdots\sum_{y_n=0}^{1}y_iy_j\exp(\theta'Y+\lambda Y_1'Y_2)$$

$$=\frac{e^{2\theta+\lambda(y_{i-1}+y_{i+1}+y_{j-1}+y_{j+1})}[\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\cdots\sum_{y_{i-1}=0}^{1}e^{\theta\sum_{i=1}^{i-1}y_i+\lambda\sum_{i=1}^{i-2}y_iy_{i+1}}]\times[\sum_{y_{i+2}=0}^{1}\sum_{y_{i+3}=0}^{1}\cdots\sum_{y_{j-1}}^{1}e^{\theta\sum_{i=i+1}^{j-1}y_i+\lambda\sum_{i=i+1}^{j-2}y_iy_{i+1}}]}{}$$

$$\times[\sum_{y_{j+1}=0}^{1}\sum_{y_{j+2}=0}^{1}\cdots\cdots\sum_{y_{n-1}=0}^{1}e^{\theta\sum_{i=j+1}^{n}y_i+\lambda\sum_{i=j+1}^{n-1}y_iy_{i+1}}]$$

$$\therefore E(y_iy_j)=\frac{e^{2\theta+2\lambda}}{D_n}D_{i-1}D_{j-1}D_{n-j} \tag{3.3}$$

Example: if n=9, i=4, j=7 then

using the previous formula, we can write

$$E(y_4y_7)=\frac{e^{2\theta+2\lambda}}{D_9}D_3D_2D_2$$

Proof:

$$\text{since } E(y_4y_7)=\frac{1}{D_9}\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\cdots\cdots\sum_{y_9=0}^{1}y_4y_7e^{\theta(y_1+y_2+\cdots+y_9)+\lambda(y_1y_2+y_2y_3+\cdots+y_8y_9)}$$

$$=\frac{e^{2\theta+\lambda(y_3+y_5+y_6+y_8)}}{D_9}[\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\sum_{y_3=0}^{1}\sum_{y_5=0}^{1}\sum_{y_6=0}^{1}\sum_{y_8=0}^{1}\sum_{y_9=0}^{1}e^{\theta(y_1+y_2+y_3+y_5+y_6+y_8+y_9)}e^{\lambda(y_1y_2+y_2y_3+y_5y_6+y_8y_9)}]$$

$$=\frac{1}{D_9}e^{2\theta+2\lambda}\{[\sum_{y_1=0}^{1}\sum_{y_2=0}^{1}\sum_{y_3=0}^{1}e^{\theta(y_1+y_2+y_3)+\lambda(y_1y_2+y_2y_3)}]\times[\sum_{y_5=0}^{1}\sum_{y_6=0}^{1}e^{\theta(y_5+y_6)+\lambda(y_5y_6)}]$$

$$\times[\sum_{y_8=0}^{1}\sum_{y_9=0}^{1}e^{\theta(y_8+y_9)+\lambda(y_8y_9)}]\}$$

$$=\frac{1}{D_9}e^{2\theta+2\lambda}D_3D_2D_2$$

Finding $Cov(y_i,y_j)$, $V(y_i)$, $V(y_j)$ and $Corr(y_i,y_j)$

Since $Cov(y_i,y_j)=E(y_iy_j)-E(y_i)E(y_j)$

$$=\frac{e^{2\theta+2\lambda}}{D_n}D_{i-1}D_{j-i-1}D_{n-j}-\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-j}$$

$$=\frac{e^{2\theta+2\lambda}}{D_n}D_{i-1}D_{n-j}[D_{j-i-1}-\frac{1}{D_n}D_{n-i}D_{j-1}] \tag{3.4}$$

and $V(y_i)=E(y_i^2)-(E(y_i))^2$

$$=E(y_i)-(E(y_i))^2 \qquad \text{since } y_i=y_i^2=0 \text{ or } 1$$

$$=\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}-\frac{e^{2\theta+2\lambda}}{D_n^2}D_{i-1}^2D_{n-i}^2$$

therefore $V(y_i)=\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}[1-\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}]$ (3.5)

and $V(y_j)=\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j}[1-\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j}]$ (3.6)

$$Corr(y_iy_j)=\frac{\text{cov}(y_iy_j)}{\sqrt{V(y_i)V(y_j)}}$$

Now by substituting (3.4), (3.5) and (3.6) in the above equation then

$$Corr(y_iy_j)=\frac{\frac{e^{2\theta+2\lambda}}{D_n}D_{i-1}D_{n-j}[D_{j-i-1}-\frac{1}{D_n}-D_{n-i}D_{j-1}]}{\sqrt{\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}[1-\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}]\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j}[1-\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j}]}}$$

$$=\frac{\frac{e^{2\theta+2\lambda}}{D_n}D_{i-1}D_{n-j}[D_{j-i-1}-\frac{1}{D_n}D_{n-i}D_{j-1}]}{\frac{e^{\theta+\lambda}}{D_n}\sqrt{D_{i-1}D_{n-i}[1-\frac{e^{\theta+\lambda}}{D_n}D_{i-1}D_{n-i}]D_{j-1}D_{n-j}[1-\frac{e^{\theta+\lambda}}{D_n}D_{j-1}D_{n-j}]}}$$

$$\therefore Corr(y_i y_j) = \frac{e^{\theta+\lambda} D_{i-1} D_{n-j} [D_{j-i-1} - \frac{1}{D_n} D_{n-i} D_{j-1}]}{\sqrt{D_{i-1} D_{n-i} [1 - \frac{e^{\theta+\lambda}}{D_n} D_{i-1} D_{n-i}] D_{j-1} D_{n-j} [1 - \frac{e^{\theta+\lambda}}{D_n} D_{j-1} D_{n-j}]}} \tag{3.7}$$

Checking the value of correlation if $\lambda = 0$

since $E(y_i) = \frac{e^{\theta+\lambda}}{D_n}[D_{i-1} D_{n-i}]$ where $D_n = \sum_{m=0}^{n} \binom{n}{m} e^{\theta m + \lambda \frac{m(m-1)}{n}}$

If $\lambda = 0$

$$\text{then } E(y_i) = \frac{e^{\theta}}{D_n}[D_{i-1} D_{n-i}]$$

$$= \frac{e^{\theta}}{(1+e^{\theta})^n}[(1+e^{\theta})^{i-1}(1+e^{\theta})^{n-i}]$$

$$= \frac{e^{\theta}}{1+e^{\theta}}$$

if $\lambda = 0$ then $D_n = \sum_{m=0}^{n} \binom{n}{m} e^{\theta m} = (1+e^{\theta})^n$ since $\{(1+t)^n = \sum_{j=0}^{n} \binom{n}{j} t^j\}$

$$\text{therefore } E(y_j) = \frac{e^{\theta}}{1+e^{\theta}}$$

$$\therefore E(y_i) = E(y_j)$$

$$V(y_i) = \frac{e^{\theta}}{1+e^{\theta}}[1 - \frac{e^{\theta}}{1+e^{\theta}}] = V(y_j)$$

$$E(y_i y_j) = \frac{e^{2\theta}}{D_n}(1+e^{\theta})^{n-2} = \frac{e^{2\theta}}{(1+e^{\theta})^2}$$

$$\therefore Cov(y_i y_j) = \frac{e^{2\theta}}{(1+e^{\theta})^2} - \frac{e^{2\theta}}{(1+e^{\theta})^2} = 0$$

$$\text{Since } Corr(y_i y_j) = \frac{Cov(y_i y_j)}{\sqrt{V(y_i) V(y_j)}}$$

$$\therefore Corr(y_i y_j) = \frac{0}{V(y_i)} = 0$$

therefore if $\lambda = 0$ then $Corr(y_i y_j)$=0.

It is possible then to Plot the correlation for some values of theta and lambda of various correlations for different thresholds.

Figures (3.1), and (3.2), present the correlation between some Y with the rest for four thresholds to see the correlation in the four thresholds.In these plots, it is intersting to observe that the correlation typically falls, at lag of 1 and 2 kHz, before rising to zero. A practical explanation for this has not been obtained, although the the experimenters are very interseted in this phenomenon and intend to examine their own data experiments and models in the light of this result.

Figure (3.3) presents the different correlations for one allocation for one threshold, in this plot we see that the correlation falling from 0.8 to zero as the lag getting bigger.

Figure (3.4) presents all the correlations between one and the rest.

3.4.1 Relation between the correlation coefficient and lambda:

Figures (3.5), (3.6), (3.7), (3.8) show the graph of the correlation against lambda for some theta values for separate threshold at some allocations.

and from the various graphs we find that if lambda equal zero the correlation between y_5 and y_6 is equal to zero, also as lambda getting bigger the correlation getting bigger, where theta is constant, figures (3.9), (3.10), (3.11), (3.12) show the behaviour of the correlation when $1 \prec \lambda \prec 3$, and from the graph we see that the correlation is filling down from zero as lambda is small to -0.7 as lambda getting bigger.

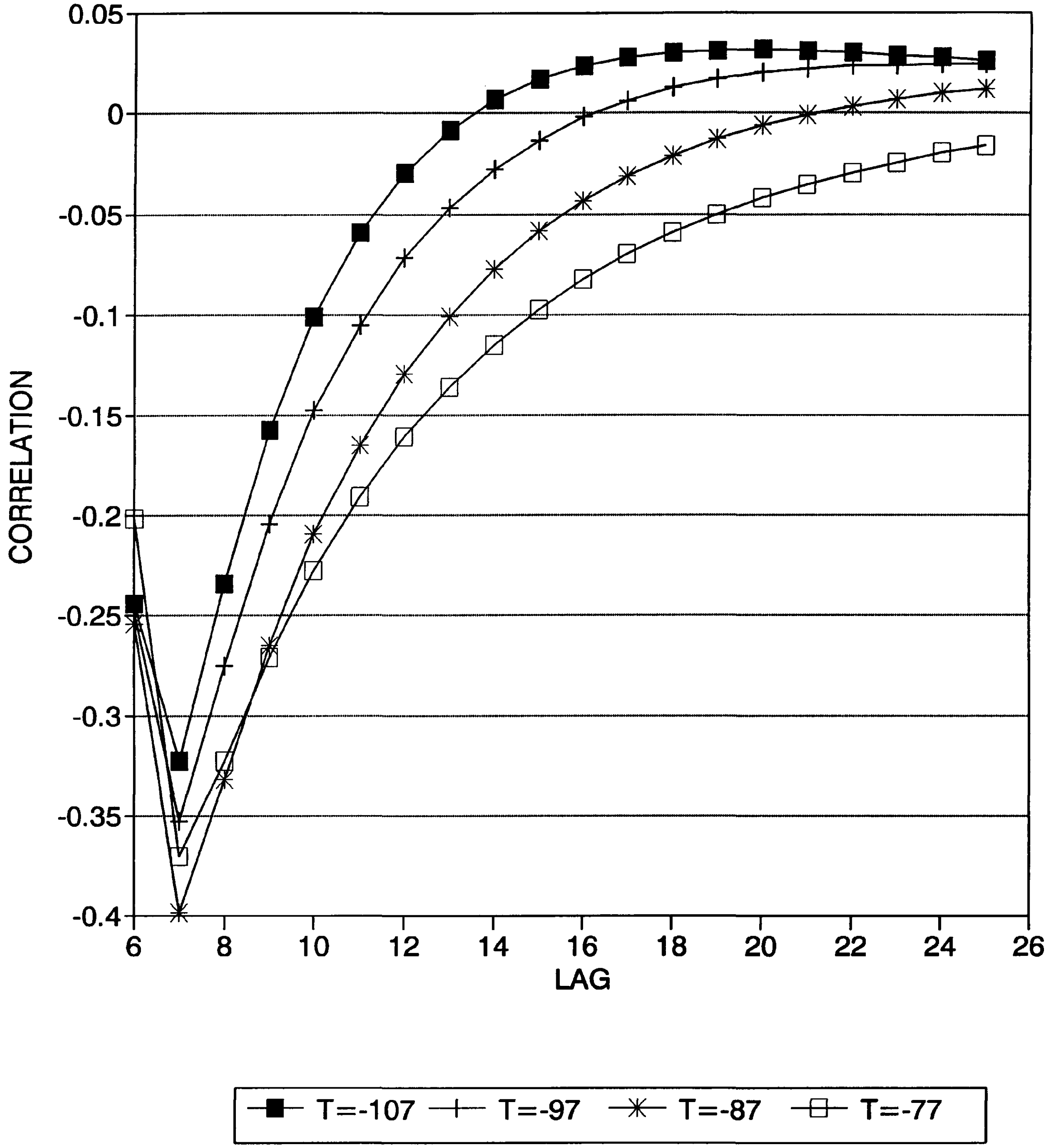


Figure (3.1) Correlation coefficient between Y5 and the rest for allocation seven

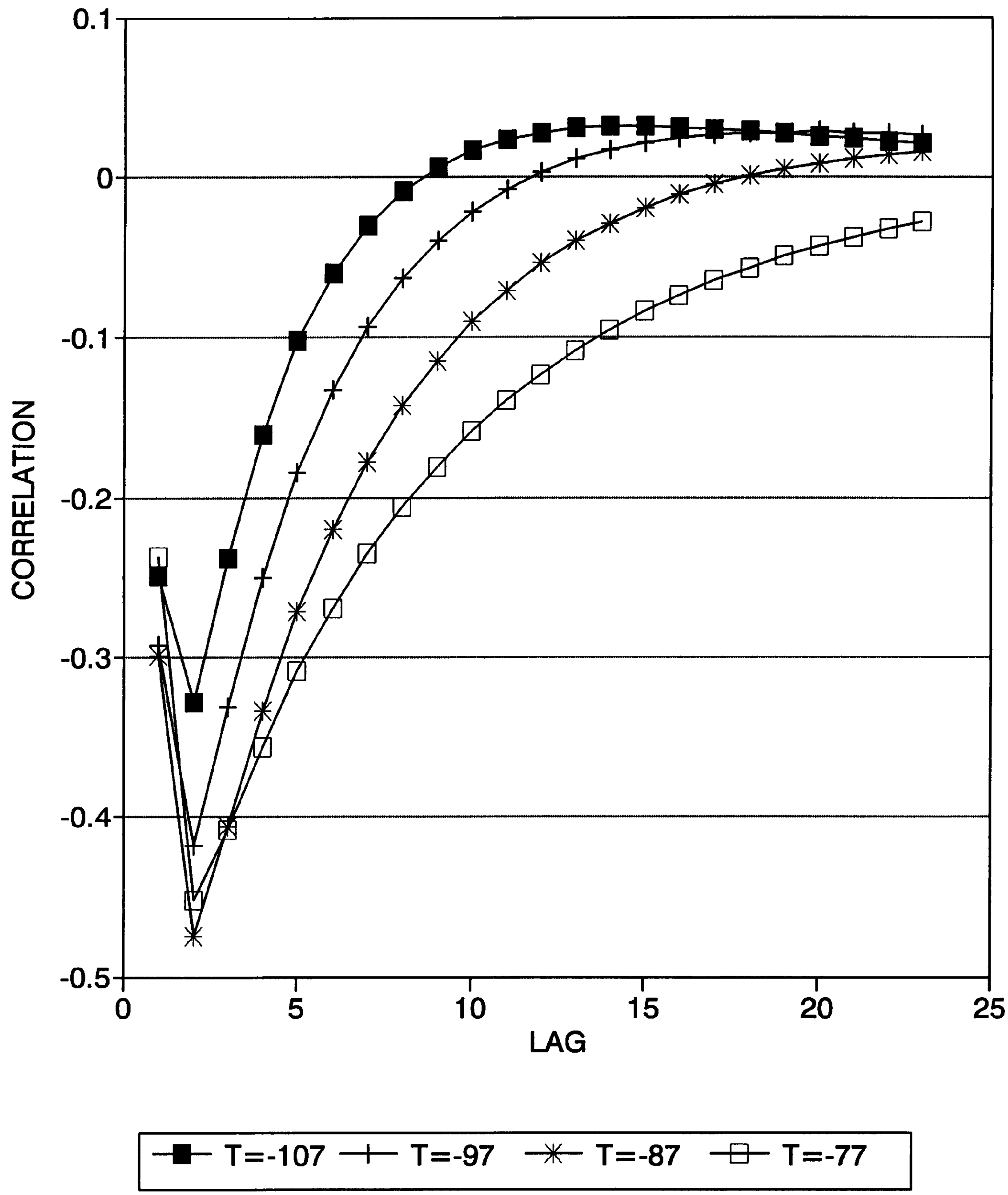


Figure (3.2) Correlation coefficient between Y5 and the rest for allocation 32

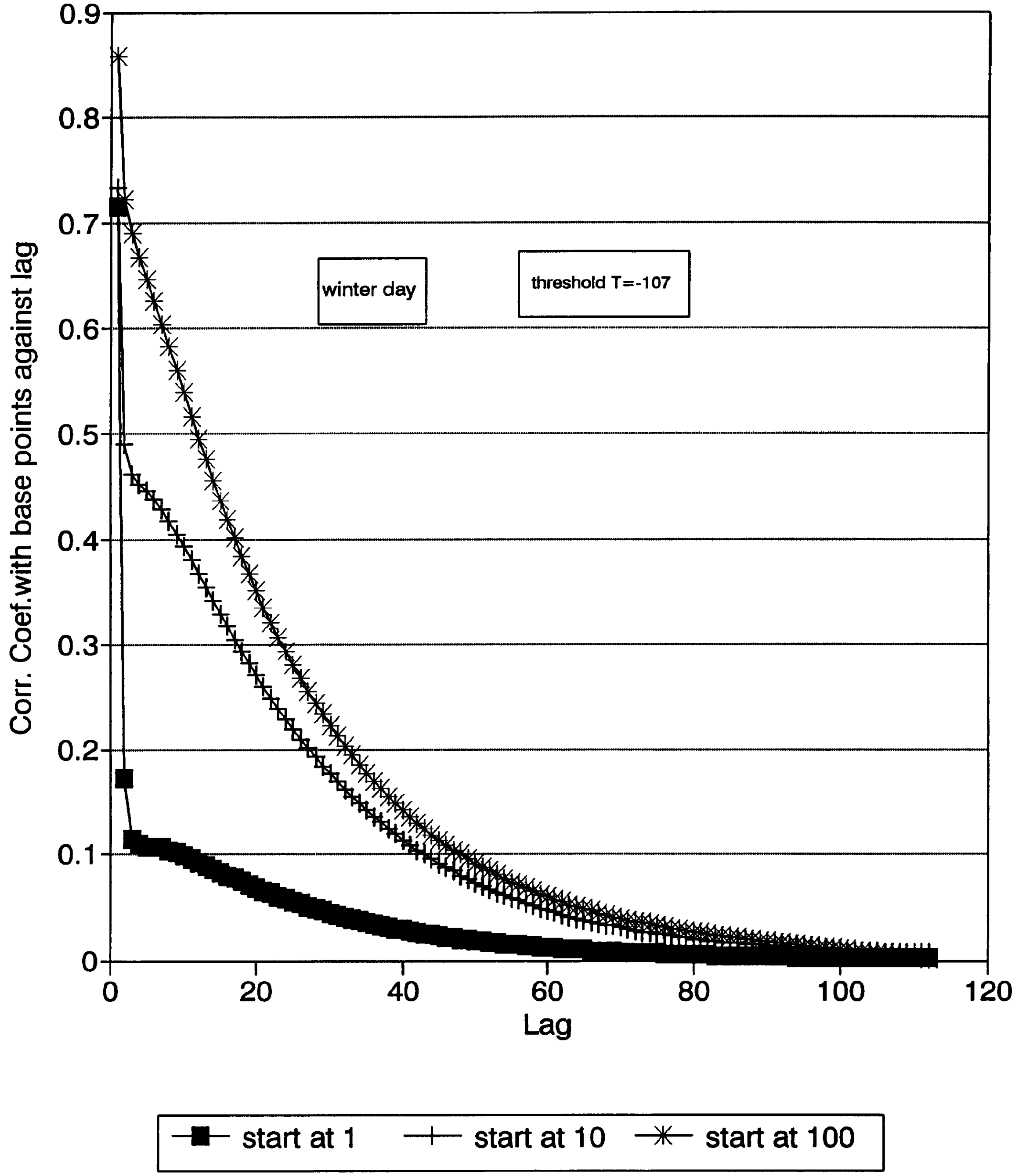


Figure (3.3)Auto correlation function with alternative start point for allocation 16

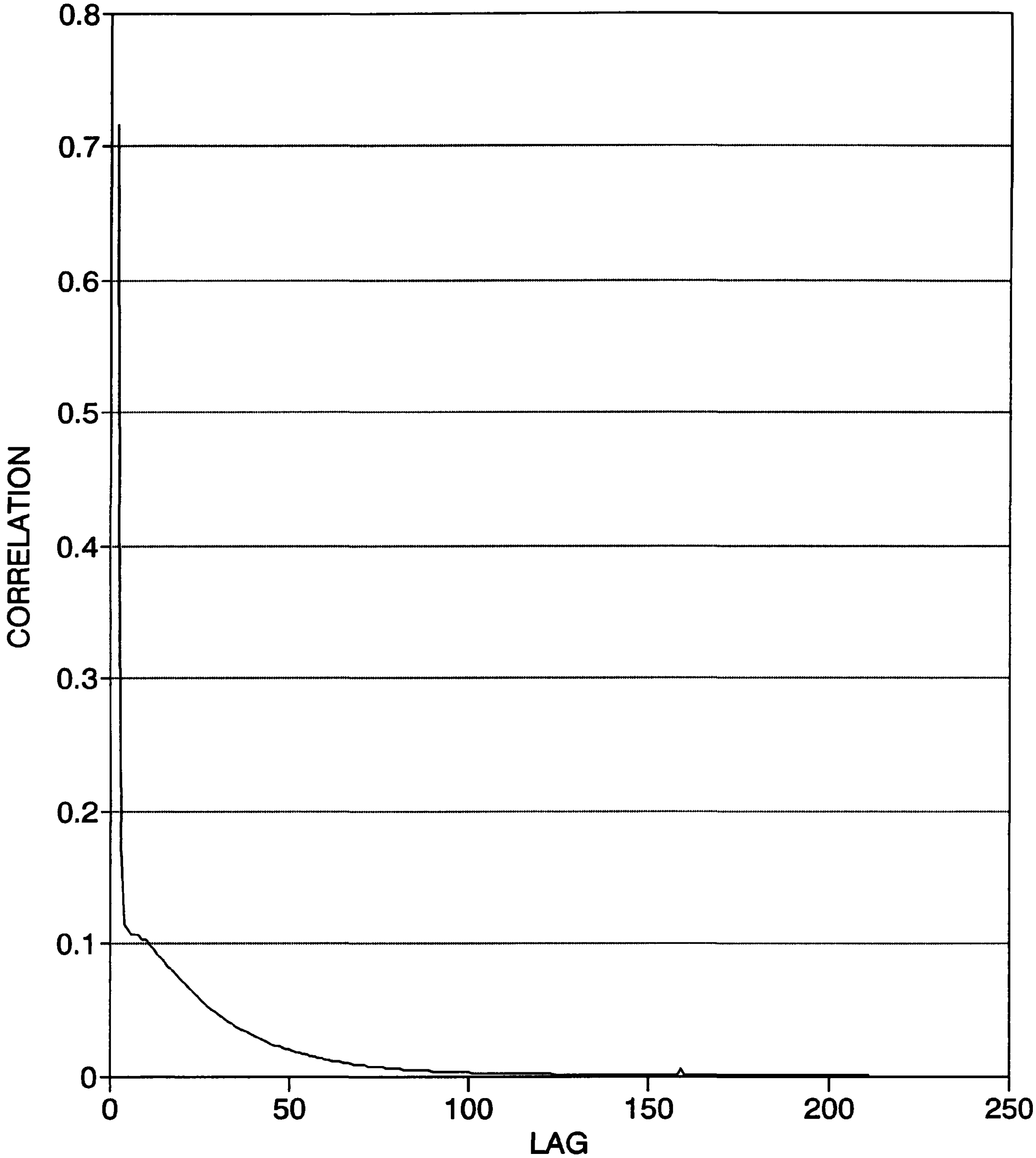


Figure (3.4) Correlation between Y1 and the rest

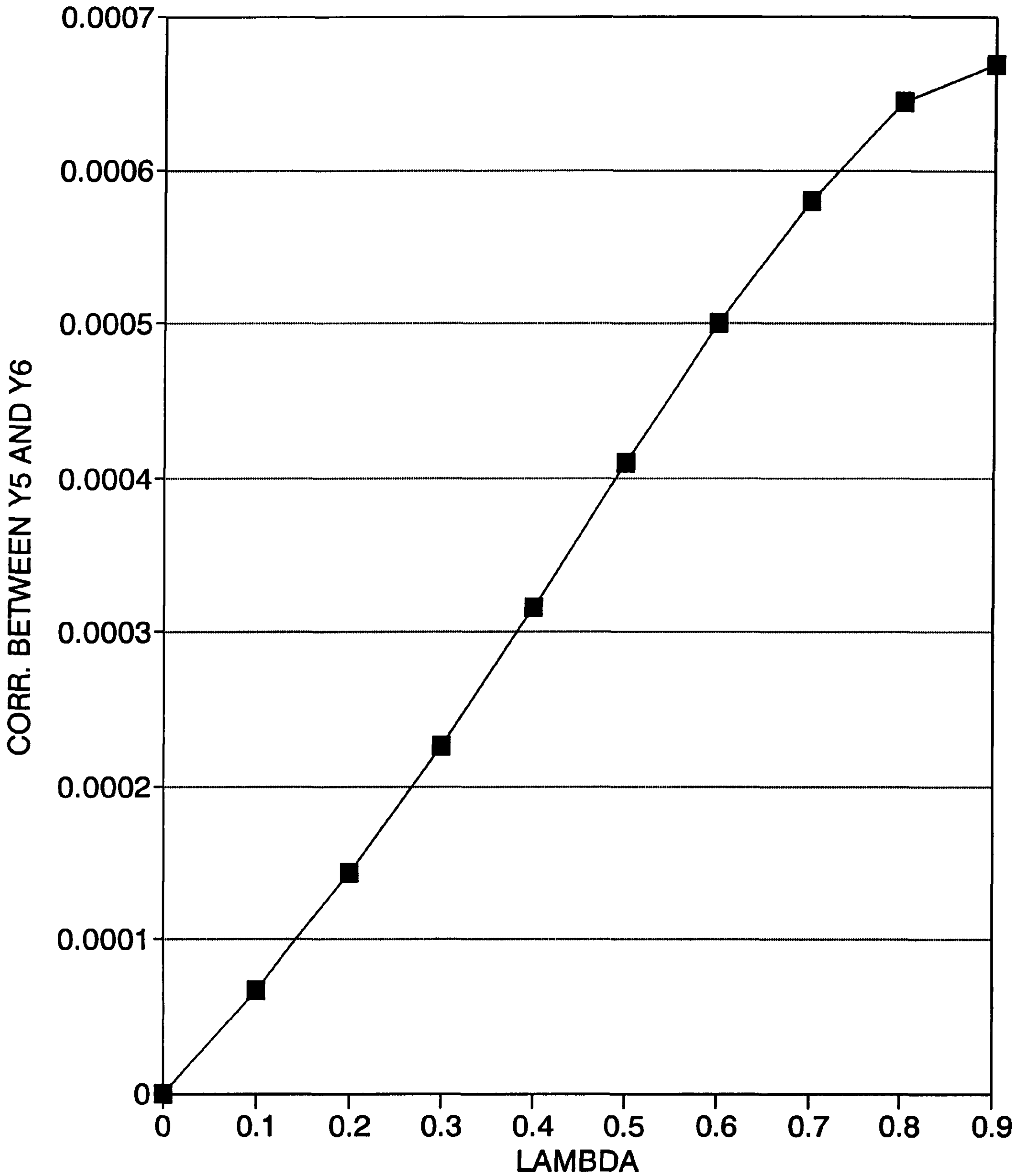


Figure (3.5) Relation between Correlation and Lambda for Threshold =-107

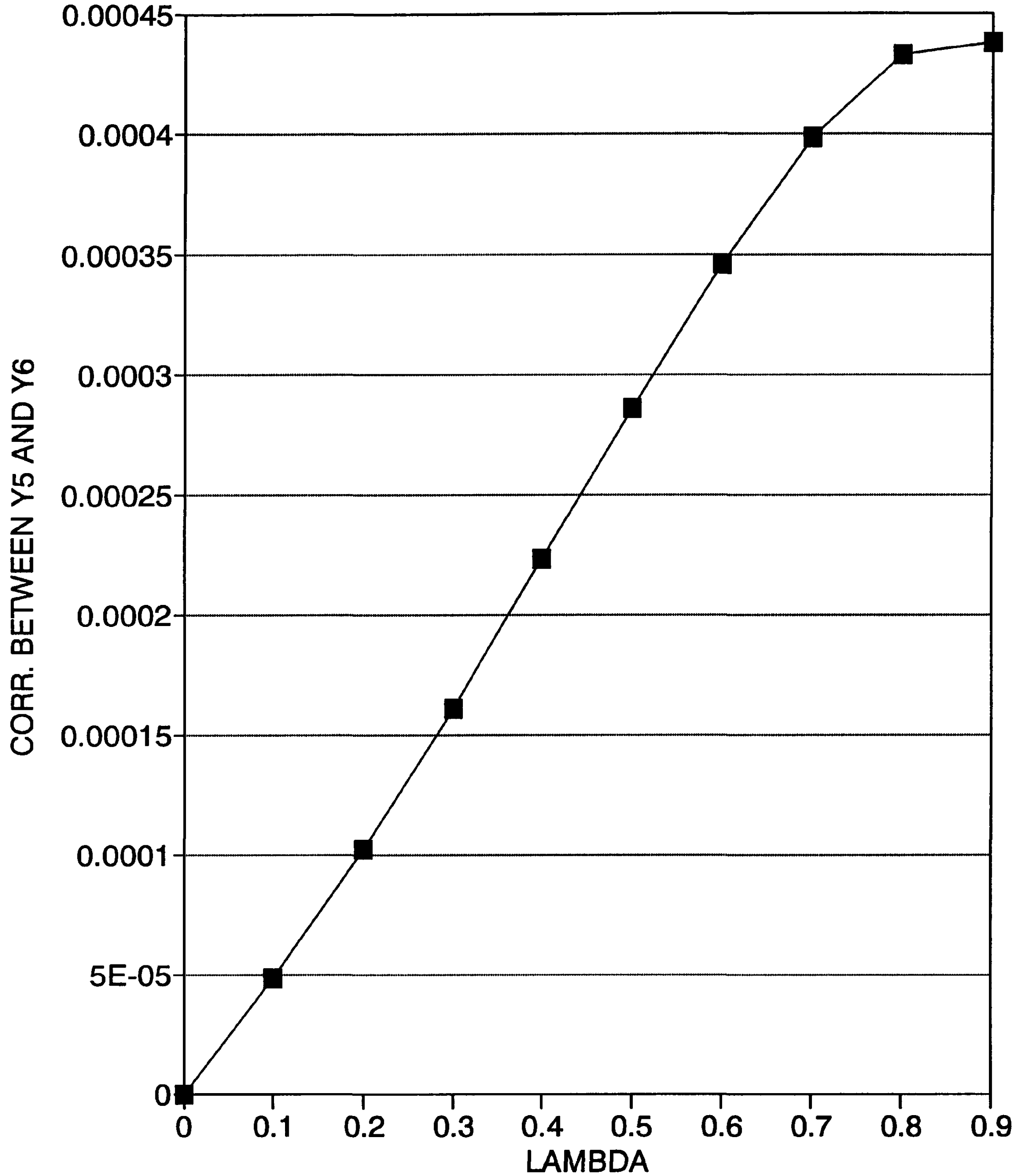


Figure (3.6) Relation between Correlation and Lambda for Threshold =-97

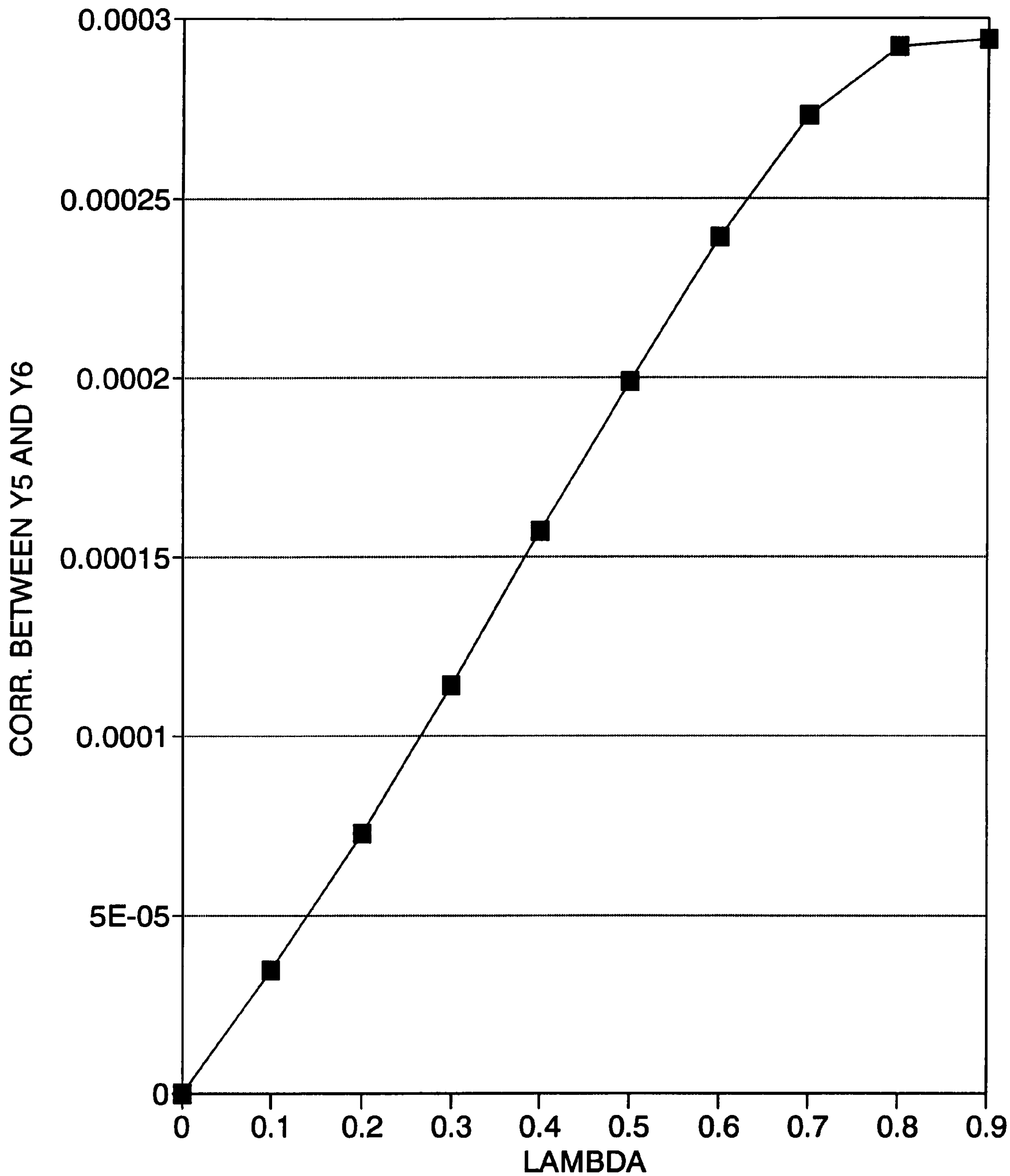


Figure (3.7) Relation between Correlation and Lambda for Threshold =-87

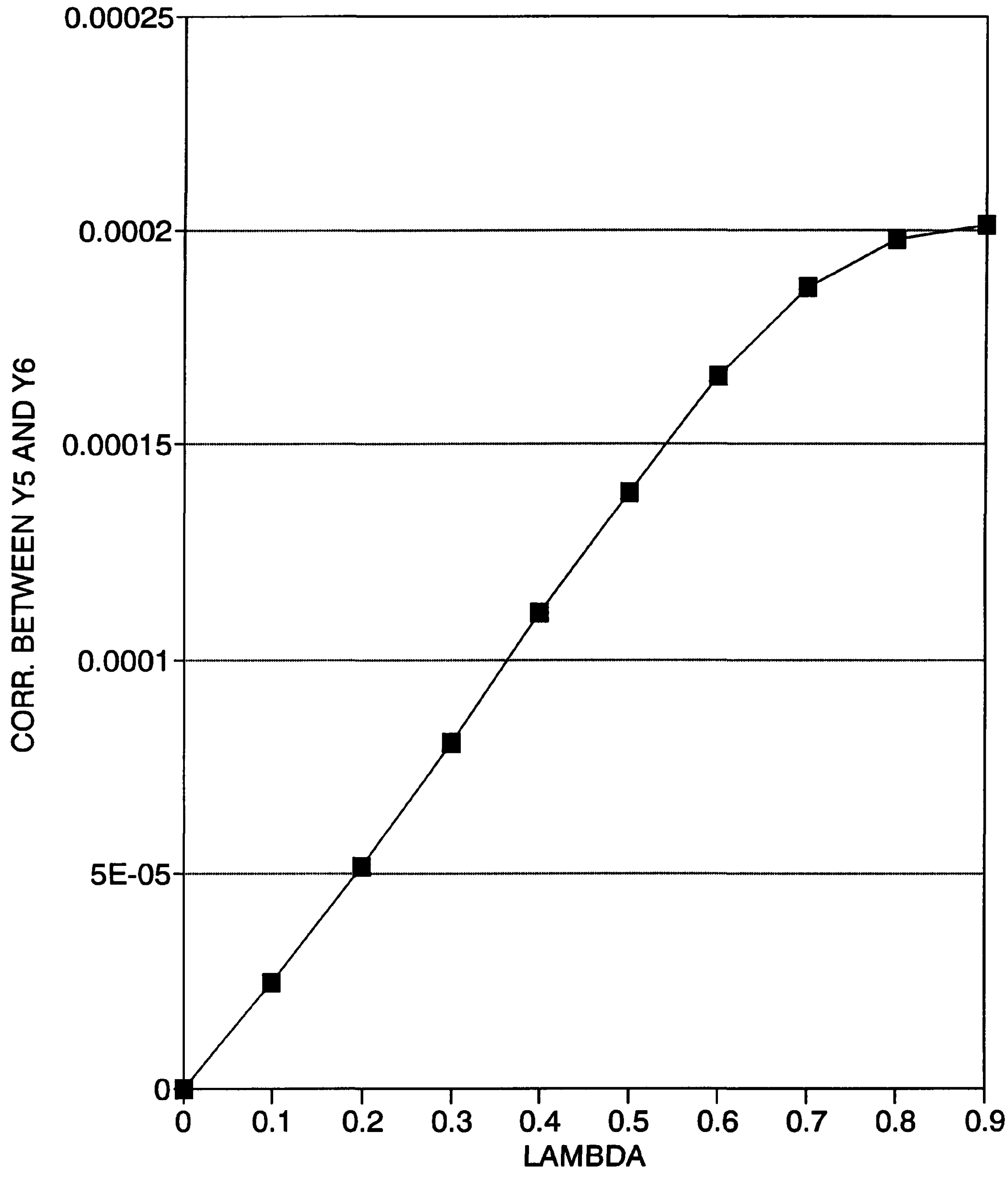


Figure (3.8) Relation between Correlation and Lambda for Threshold =-77

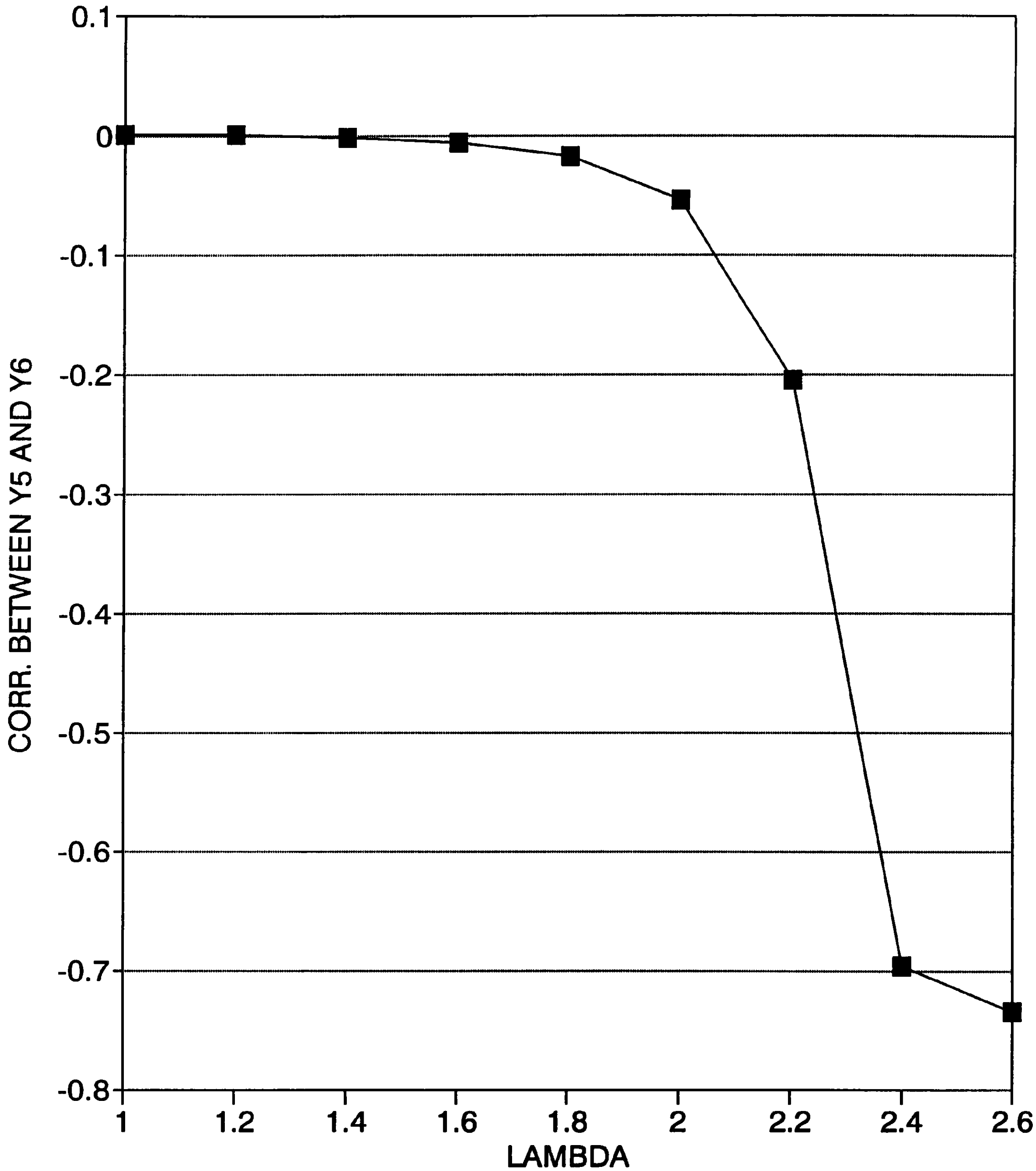


Figure (3.9) Relation between Correlation and Lambda for Threshold =-107

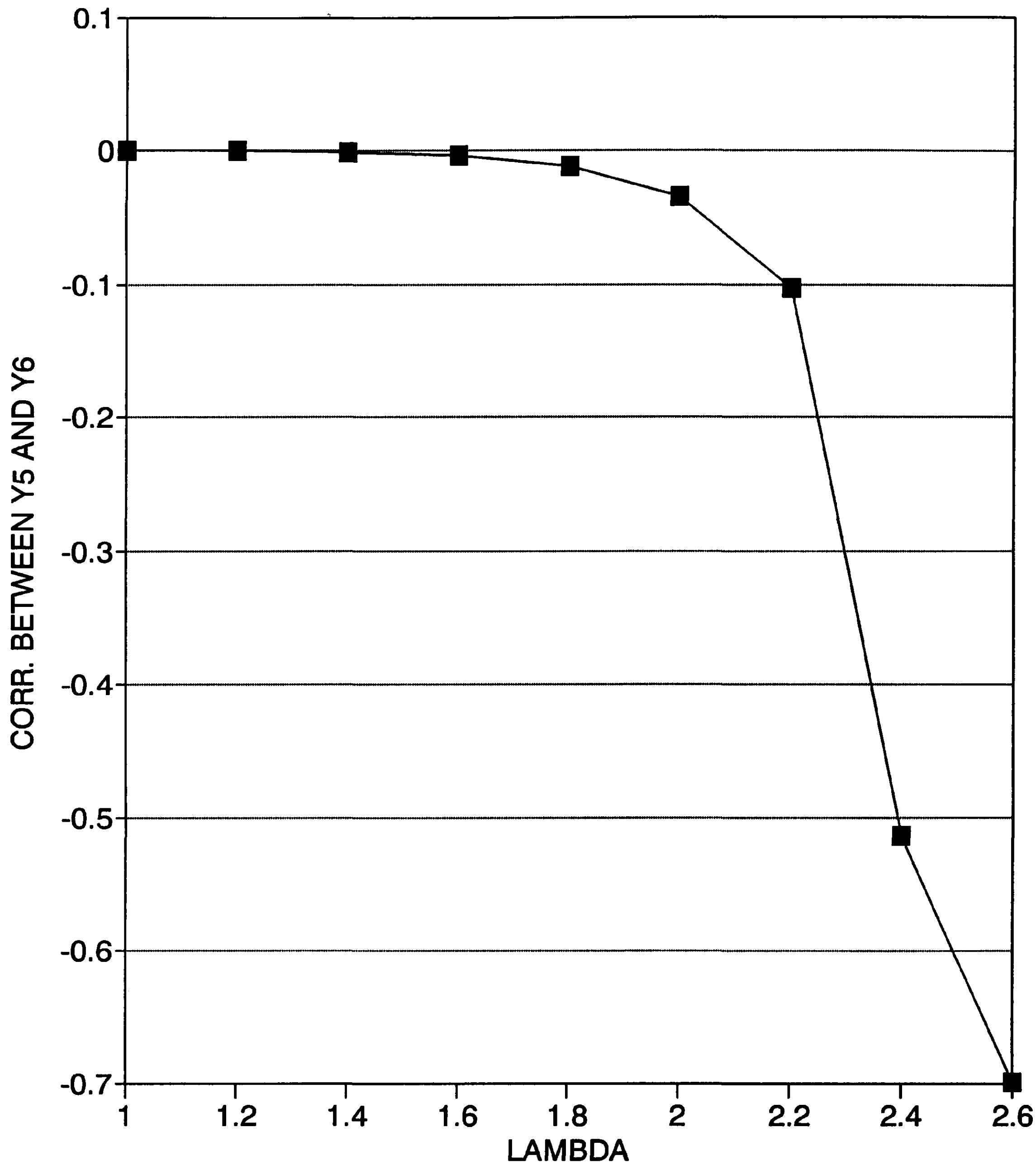


Figure (3.10) Relation between Correlation and Lambda for Threshold =-97

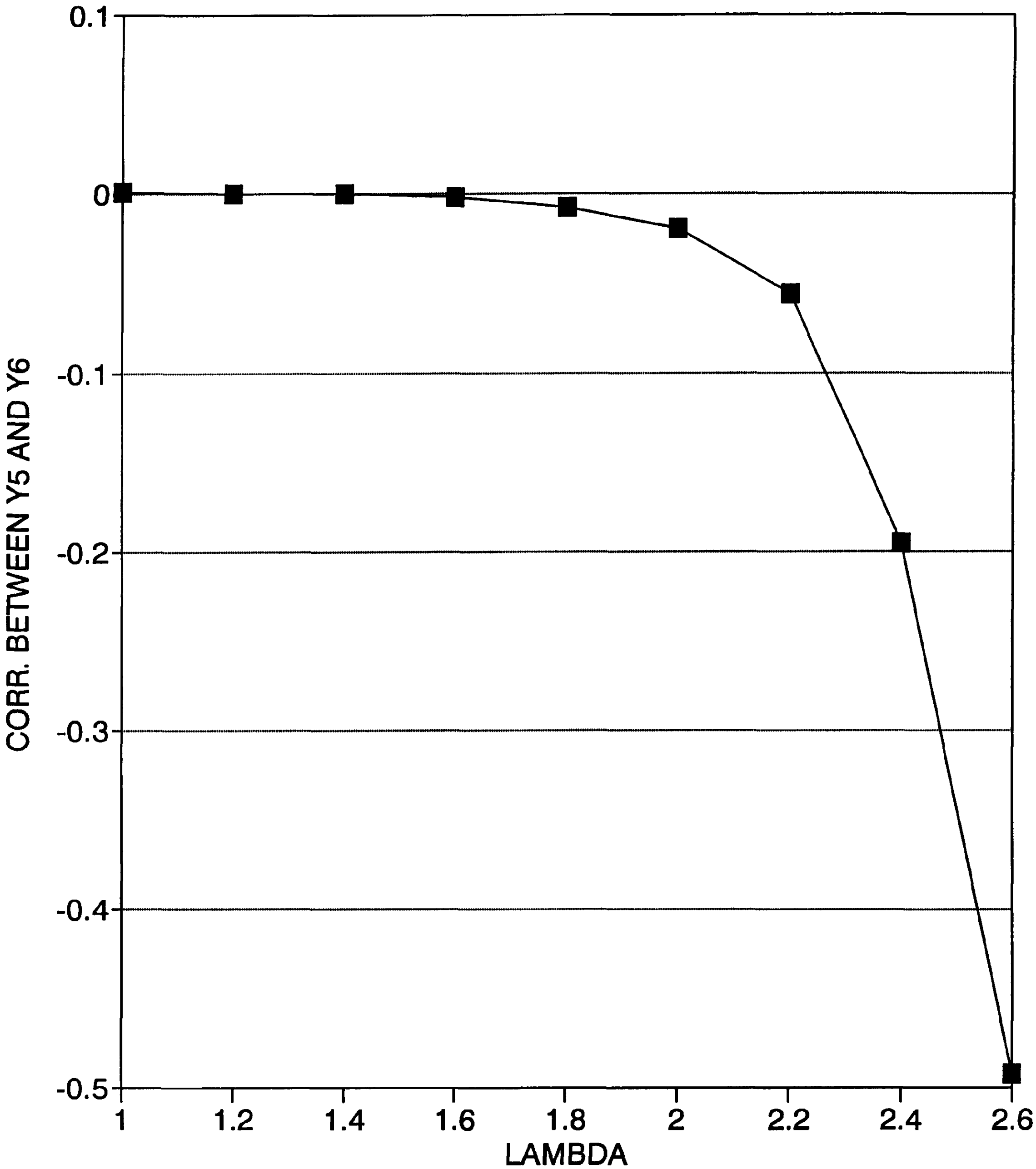


Figure (3.11) Relation between Correlation and Lambda for Threshold =-87

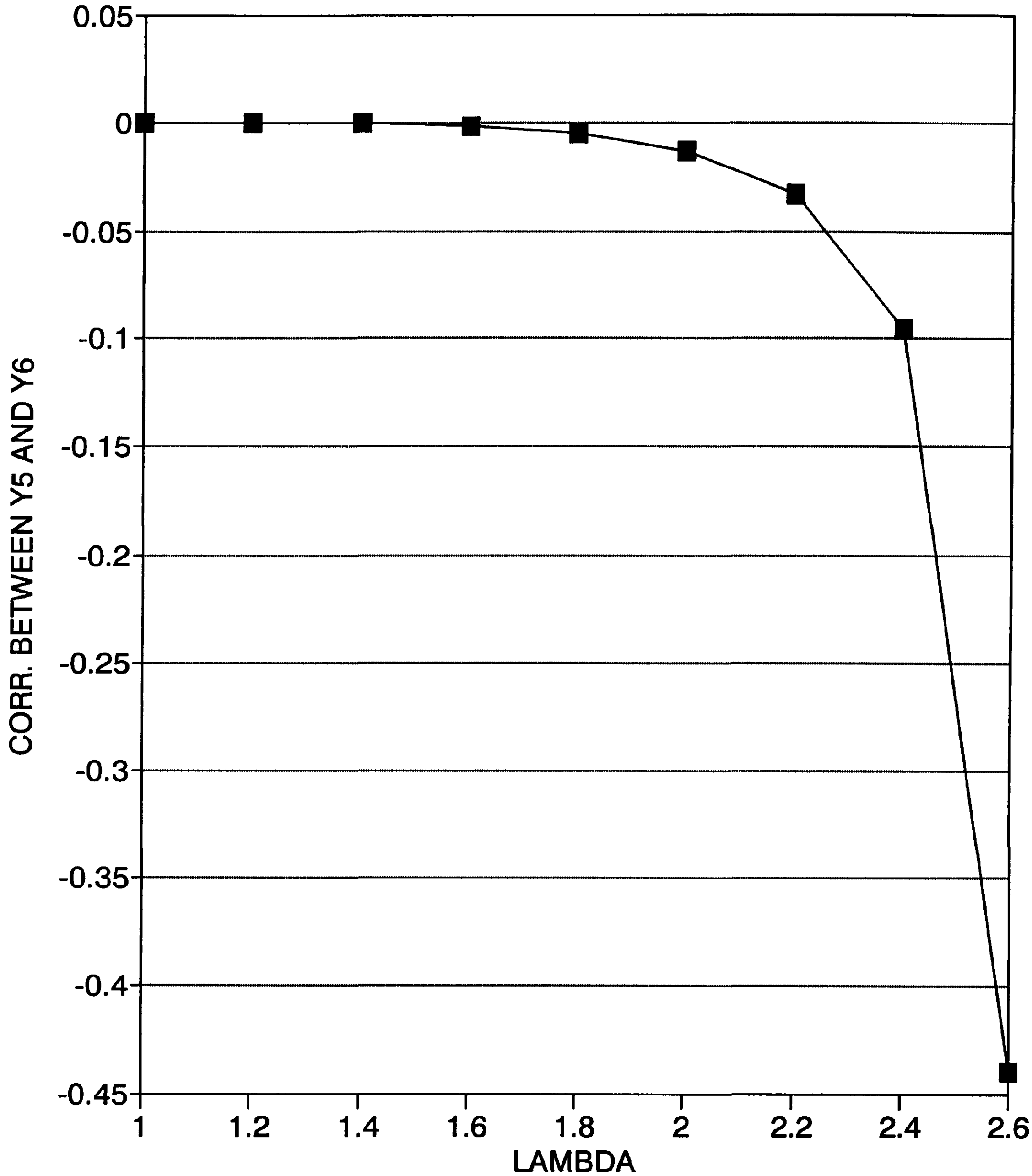


Figure (3.12) Relation between Correlation and Lambda for Threshold =-77

Chapter 4

Application Of Multinomial Models To HF Radio Spectrum Data

4.1 Introduction:

We fitted separate models to the marginal binomial counts of the derived multinomial data. This model was fitted for "all users" using Genstat Statistical Package. With the version of GENSTAT available at the time of doing the work in this section it was not possible to fit full models to our ordered categorical multinomial data.

After that we fitted a full multinomial model to "each user" separately instead of "all users" at the same time, using the statistical package T. S. P.,which had the required facilities for ordered categorical multinomial data. It was not feasible with either package to fit "all users" with the full multinomial model.

The raw occupancy values, Q as recorded can be converted (back) into signal exceedancy counts by multiplying with relevant bandwidth, b, and dividing by 100. The resultant variable :

$$R=(Qb)/100$$

can then be regarded, to a first approximation, as a binomial $B(b,Q\times10^{-2})$ variate.

However the occupancy values "Q" overlap to some extent, since any signal which exceeds a given level must necessarily exceed all levels below that one.

For example:

Q_1= that percentage of signals in a given band on a specified occasion, which exceed -107dbm.

Q_2= that percentage of signals in the same band on the same specified occasion which exceed -97 dbm.

Then all those signals which are signals counted into Q_2 must be counted into Q_1 ,and $Q_1 \geq Q_2$ necessarily.

This clearly violates the standard statistical assumption of independence which in particular was used in the maximum likelihood producers for fitting the binomial logistic model. This problem has been recognized by Laycock and Gott (1988). The principal effect will be in an incorrect estimation of the associated standard errors.

The true situation is more appropriately fitted by a multinomial model after suitable differencing of the transformed (to R) Q values. The lowest signal level -117 dbm, is known to be close to the inherent noise level of the whole system and this fact is reflected in a relatively poor fit of the binomial model to this portion of the data. Hence it was decided to ignore this signal level when constructing the multinomial counts.

These therefore refer to the following five continous intervals of signal strength, S, for any one band on any one occasion:

Interval	$I_1 = (-\infty, -107]$	$I_2 = (-107, -97]$	$I_3 = (-97, -87]$	$I_4 = (-87, -77]$	$I_5 = (-77, \infty]$
Multi-count	R_1	R_2	R_3	R_4	R_5
Probability	P_1	P_2	P_3	P_4	P_5

If the cumulative distribution function of S is F_θ, so that

$\Pr(S \le s) = F_\theta(s)$ then we can write

$$P_1 = \Pr(S \in I_1) = F_\theta(-107) - F_\theta(-\infty)$$

and similarly for the other intervals.

Next, if we assume that successive signal strength observations are independent (and the experiment was designed to produce this effect if possible) then we can write down a multinomial expression for the probability of any one vector **R** of these counts for a particular band on a particular occasion:

$$P = \Pr(R_1, R_2, R_3, R_4, R_5) = \Pr(R) = \frac{N!}{R_1! R_2! R_3! R_4! R_5} P_1^{R_1} P_2^{R_2} P_3^{R_3} P_4^{R_4} P_5^{R_5}$$

And finally for a set of **n** independent observation on R_1 , R_2 , R_3 , R_4 ,R_5

we can write

$$P=\prod_{i=1}^{n}\frac{N_i!}{R_{1i}!R_{2i}!R_{3i}!R_{4i}!R_{5i}!}P_{1i}^{R_{1i}}....P_{5i}^{R_{5i}}$$

and hence the log likelihood function is

$$l=\ln P=\text{constant}+\sum_{i=1}^{n}\sum_{j=1}^{5}R_{ji}\ln P_{ji}$$

where the constant depends only on the data and

$P_{ji}=\Pr(S\in I_j \text{ on the ith occasion})=F_{\theta ji}(-107)-F_{\theta ji}(-\infty)$ for j=1 say

and the parameters to be estimated are contained in the θ_{ji} coefficients.

In particular:

Case 1: S is normally distributed

$$\theta=(\mu,\sigma)\ \&\frac{dF_\theta}{ds}=f_\theta=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}(\frac{s-\mu}{\sigma})^2}$$

and we set

$$\mu=\beta_0+\beta_1x_1+........+\beta_px_p$$

for a direct linear model dependency of the mean function on the predictors:- such as frequency and sunspot number.

Case 2: S is logistic

$$F(s)=[1+\exp(-\frac{(s+\alpha)}{\beta})]^{-1}\quad ,\beta>0\qquad ,-\infty<s<\infty$$

This is the implied signal strength distribution for the binomial logistic fit to the occupancy data as described in chapter one .As stated previously the work in this chapter is divided into two parts.

1- First part:- Fitting binomial model to appropiate subsets of the multi-counts using Genstat program.

2-Second part:- Fitting multinomial model to all counts simultaneously using T. S. P. package.

The use of Genstat fitting for a binomial model to these data will be described first:

Extensive data analysis, based on the binomial distribution (see Genstat Manual) of the congestion counts within each frequency allocation has resulted in the use of the following linear predictor to represent the experimental congestion values.

This model has been fitted to various subsets of the original undifferenced data by students under the supervision of Dr Gott and Dr Laycock. The linear predictor is given by

$$\mu = G + A_k + B(dBm) + (c_o + c_1 f_k + c_2 f_k^2) \times \text{sunspot number} \qquad (4.1)$$

This is the original model fitted to the percentage occupancy using a link to the binomial mean.

A_k has 95 values corresponding to the 95 frequency allocation or user band.

B is a single coefficient to be multiplued by the threshold (dBm) including its signals.

c_o, c_1, c_2 are coefficient in a quadatic expression for frequency to be multiplied by sunspot number, and is a constant term.

f_k is the mid point frequency of the Kth user band.

This was the original model with all threshold levels fitted for the percentage occupancy using Genstat with binomial distribution.

But in our case we will exclude the threshold levels term since we are taking the differences between the counts and then fitting a (marginal) binomial model to each separate difference at the given threshold partitions e.g. (-107,-97)dBm.

Hence B is a constant for each of these separate subsets of the data.

So the model becomes:

$$\mu = G + A_k + (c_o + c_1 f_k + c_2 f_k^2) \times \text{sunspot number} \qquad (4.2)$$

where all the coefficients are as before except that the constant "B(dBm)" has been absorbed into G.

It was necessary to make changes to the original Genstat program used by the Gott research group (in the department of Electrical engineering and electronic at UMIST) to make it suitable for our data. A listing of this program and a sample set of results can be found in appendix A. We ran this "binomial model" using logistic link i.e.

$B(n_k, p_j)$ $\qquad q_j = 1 - p_j \qquad j=1.....5 \qquad k=1.....95$

$P = (1+e^{-\mu})^{-1} \qquad Q = 1 - P = (1+e^{\mu})^{-1}$

$$= \frac{e^{\mu}}{1+e^{\mu}} \quad \text{where } \mu = \sum b_j' x_j' \quad \text{which is defined in (4.2)}$$

for every separate multi-count for summer day, summer night, winter day and winter night with all users (i.e. all 95 bands at once).

The ANOVA table below gives the marginal analysis for each term of the model, and for the model as a whole (less the constant) for "summer day" threshold C_5 =$(-77,\infty)$.

Source	d.f.	deviance	mean deviance	P value
Bands	95	6619.8	70.423	< 0.5 %
Sunsopt	1	4.409	4.409	< 5 %
Fsun	1	8.681	8.681	< 0.5 %
F_2sun	1	16.774	16.774	< 0.5 %
Regression	97	9338.9	96.227	< 0.5 %
Residual	662	961.2	1.452	> 50 %
Total	759	10300.1	13.571	

Assuming each deviance is asymptotically a chi-squared variate on the corresponding degrees of freedom under the corresponding null hypothesis McCullagh and Nelder (1989), we see that each term in the model is statistically significant (on the margin), as is the whole regression.

Also the residual deviance is consistent with such a χ^2.

Since the congestion model is a generalized linear model with identity link, congestion is represented by $\mu = \sum \beta_j x_j$ where x_j are the covariates and β_j are the parameters.

The mean deviance of this model for C_5 is 1.452 and since a good fit in this context has been found to be indicated by a mean deviance less than 10, the model fit for summer day C_5 =$(-77,\infty)$ is more than adequate for all practical purposes, as can be seen by examining the histogram of residuals, in appendix 1.

The terms "constant" and "uband" can used to calculate the 95 band constants A_k. The "constant" term defines A_1 and the remaining band constants A_n are found by adding "constant" and "band". For example:

For the band constant :

A_2=-14.3(constant)+1.6(uband2)

=-12.7

Histograms for occupancy and errors are produced by the program, where "error" is defined as a percentage by:

$$\frac{C-\hat{C}}{\#\text{Channels in allocation}}\times 100$$

Where C is the multi-count under $(-77,\infty)$, and $\hat{C}$ is the fitted number for the multi-count.

There are 760 observations in the total error histogram showing that the percentage of all fitted values within 0.01 of the measured data is 74.7% for "summer day" C_5.

In the following section which describe all the model fits, tables will be given which quote mean deviance and estimation of ρ, the "1st order markov chain serial correlation" coefficient for each subset of the data calculated by:

$$\hat{\rho} = \frac{\delta-1}{\delta+1}$$

where δ is the mean deviance.

For more details on estimating this correlation coefficient,Laycock and Gott (1989), and also chapter one of this thesis.

TABLE 4.1 RESULTS SUMMARY FOR THE GENSTAT RUN COVERING THE DATA SET FOR ALL USERS AND ALL MULTI-COUNTS. SUMMER-DAY.

MULTI-COUNT	RESIDUAL MEAN DEVIANCE	1ST ORDER MARKOV CHAIN SERIAL CORR.
$C_1 = (-\infty, -107)$	6.504	0.7335
$C_2 = (-107, -97)$	3.813	0.5845
$C_3 = (-97, -87)$	2.156	0.3663
$C_4 = (-87, -77)$	1.683	0.2546
$C_5 = (-77, \infty)$	1.452	0.1843

TABLE 4.2 RESULTS SUMMARY FOR THE GENSTAT RUN COVERING THE DATA SET FOR ALL USERS AND ALL MULTI-COUNTS. SUMMER NIGHT.

MULTI-COUNT	RESIDUAL MEAN DEVIANCE	1ST ORDER MARKOV CHAIN SERIAL CORR.
$C_1 = (-\infty, -107)$	17.64	0.8927
$C_2 = (-107, -97)$	7.455	0.7635
$C_3 = (-97, -87)$	6.252	0.7242
$C_4 = (-87, -77)$	2.437	0.4181
$C_5 = (-77, \infty)$	3.702	0.5746

TABLE 4.3 RESULTS SUMMARY FOR THE GENSTAT RUN COVERING THE SET FOR ALL USERS AND ALL MULTI-COUNTS. WINTER-DAY.

MULTI-COUNT	RESIDUAL MEAN DEVIANCE	1ST ORDER MARKOV CHAIN SERIAL CORR.
$C_1 = (-\infty, -107)$	10.93	0.8324
$C_2 = (-107, -97)$	5.508	0.6927
$C_3 = (-97, -87)$	3.78	0.5816
$C_4 = (-87, -77)$	2.822	0.4767
$C_5 = (-77, \infty)$	2.536	0.4344

TABLE 4.4 RESULTS SUMMARY FOR THE GENSTAT RUN COVERING THE SET FOR ALL USERS AND ALL MULTI-COUNTS. WINTER-NIGHT.

MULTI-COUNT	RESIDUAL MEAN DEVIANCE	1ST ORDER MARKOV CHAIN SERIAL CORR.
$C_1 = (-\infty, -107)$	6.216	0.7228
$C_2 = (-107, -97)$	3.95	0.5959
$C_3 = (-97, -87)$	3.231	0.5273
$C_4 = (-87, -77)$	1.81	0.2883
$C_5 = (-77, \infty)$	2.642	0.4509

With the degrees of freedom available for these data, a residual mean deviance of 5 or more shows a statistically significant deviance away from the fitted model.

However, examination of the error histograms confirms a commonly observed phenomena namely, that χ^2 tests with large degrees of freedom are too sensitive.

A value of 10 or less typically gives a more than satisfactory fit of the model to the data for our situation.

From the previous tables for residual mean deviance, note that only C_1 has a value which exceeds ten. This happens for winter day and summer night.

So we conclude that just the "C_1" counts in winter day and summer night are not accurate, and all the other multi-counts are accurate i.e fitted very well.

This "rule of thumb" for binomial fits with large data sets can be checked by inspection of the histograms, see for instance Appendix (C).

<u>Second part:-</u> **Fitting a full multinomial model to all counts [summer day only].**

At the begining we tried to fit the model as before, when we used a Genstat package, with all the users and all 95 bands.

Since there are 95 parameters it is not easy to estimate with the T. S. P (Time Series Package) because of the capability of the package itself, so we changed the model to a polynomial model:

$\eta = \beta_{i1} f + \beta_{i2} f^2 + \beta_{i3} f^3 + (\beta_{i4} f + \beta_{i5} f^2 + \beta_{i6} f^3) \times$ sunspot number

Where f is the mid frequency, and β_{i1} to β_{i6} are parameters (i=1,...,5 for the five thresholds bands)

This polynomial model was fitted using logistic response function by Maximum Likelihood Estimator via "NEWTON-RAPHSON" using the package T. S. P.

Separate models were produced for each user type.

The models that were fitted where the same as the all user model for the Multinomial model, with

$$P_j = \frac{e^{\eta_j}}{(1+H)} \qquad j=2,3,4,5$$

where $H = e^{\eta_2} + e^{\eta_3} + e^{\eta_4} + e^{\eta_5}$

$\eta_i = \beta_{i1} f + \beta_{i2} f^2 + \beta_{i3} f^3 + (\beta_{i4} f + \beta_{i5} f^2 + \beta_{i6} f^3) \times$ sunspot number

where f is the mid frequency, and β_{i1} to β_{i6} are parameters (i=1,...,5 for the five threshold bands).

So we fit this model for all multi-counts summer day for each user.

This model for multi-counts summer day produced good fit for some users like (MM,FM,AE,B,F,FMB) and for the users with a single band allocated, the frequency will be constant, so for this reason we remove the terms which depend on frequency only e.g. f^2 and f^3 to make the model suitable for such data, so the model will then be

$$\eta = G + \beta_{i1} \times \text{sun} + \beta_{i2} \times \text{f sun} + \beta_{i3} f^2 \text{ sun}$$

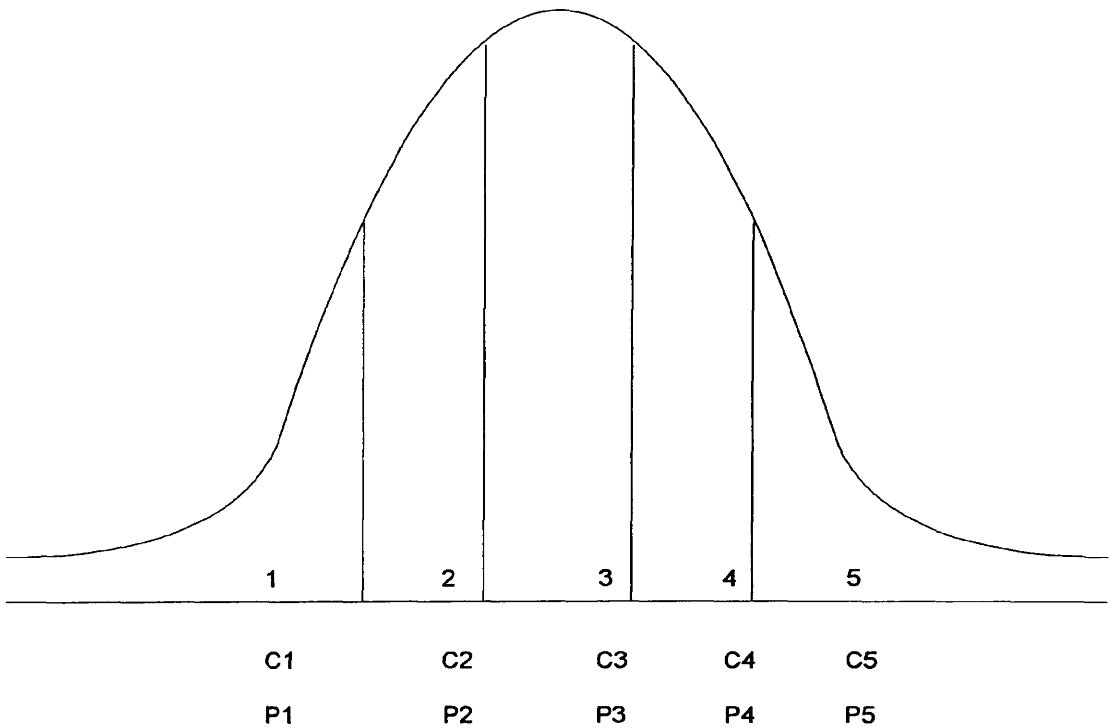
Next we fitted a multinomial "signal strength" model using a normal distribution for signal strength, ie

$$P_j = F_{\theta_j}(s_j) - F_{\theta_j}(s_{j-1})$$

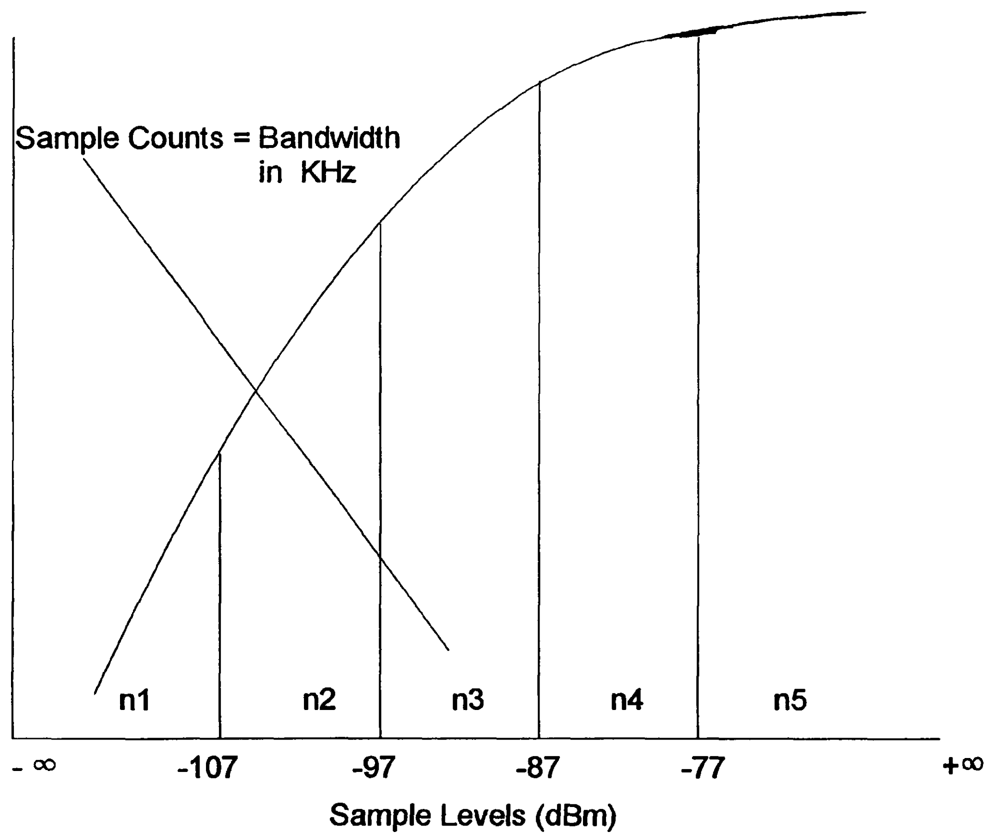
$F = \Pr(S \leq s_j)$ from normal distribution

where $s_0 = -\infty$, $s_1 = -107$, $s_2 = -97$, $s_3 = -87$, $s_4 = -77$, $s_5 = \infty$

S has normal distribution with mean μ and variance σ^2 say.



Normal distribution function and cumulative Normal function



$\eta_j = \beta_1 \times s_j + \beta_2 \times sun + \beta_3 \times fsun + \beta_4 \times f_2 sun$ [identity link] and j=1,2,3,4

$P_1 = [F(-107) - F(-\infty)]^{n_1} = [F(-107) - 0]^{n_1}$

$P_2 = [F(-97) - F(-107)]^{n_2}$

$P_3 = [F(-87) - F(-97)]^{n_3}$

$P_4 = [F(-77) - F(-87)]^{n_4}$

$P_5 = [F(\infty) - F(-77)]^{n_5} = [1 - F(-77)]^{n_5}$

where

n_1, n_2, n_3, n_4, n_5 are the sample counts = band width.

and $F(-107) = F[(-107) \times \beta_{21} + \beta_{22} \times sun + \beta_{23} fsun + \beta_{24} f_2 sun + \beta_{25} \times f]$

where $fsun = f \times sunspot\ number$

$f_2 sun = f^2 \times sunspot\ number$

So $\log l = n_1 \log(P_1) + n_2 \log(P_2) + n_3 \log(P_3) + n_4 \log(P_4) + n_5 \log(P_5)$

Again fitting this model to our data for each user we obtained good fit for some users and again we reduced the terms because we faced the same problem with the previous model. Also with this model we used the same procedure except in this program we changed the log likelihood function.

To find the mean and the variance of the distribution we change the model, so the model became

$$\mu = B_1(x + B_2 \times sunspot\, no. + B_3 \times Fsun + B_4 \times F_2 sun + B_5 \times F_1)$$

which is a more complicated model since the first cumulative model was a purely linear model but this one is not linear in the parameter B, T.S.P had difficulty to achieve the maximum likelihood , from an arbitrary starting value. So we put in initial values of the parameters deduced from the previous fit.

How do we choose the values of the parameters?

Since from the linear cumulative model we estimate the parameters for each user but in this model B_2 is multiplied by every parameter.

Therefore $B_1 \times B_2 = B_2^*$ and hence

$$\therefore B_2 = \frac{B_2^*}{B_1}$$

where B_2^* is a parameter which is estimated directly by the fitted model.

Then from these parameters we can estimate the mean and the variance for each user. Since if x is $N(\mu,\sigma^2)$ then $\frac{x-\mu}{\sigma}$ is $N(0,1)$ or standard normal.

So in this program B_{21} is $\dfrac{x-\mu}{\sigma}$ and x takes values -107, -97,-87,-77 and $\mu = -(B_{22} + B_{23} + B_{24} + B_{25})$

Table (4.5) gives the associated mean and the variance for each user.

Fitting this model to each user we obtained bad fit for some users.

TABLE (4.5) THE MEAN AND VARIANCE

FOR EACH USER FOR SUMMER DAY

USER	MEAN	VARIANCE
FIXED/MOBILE	-16.139	85.186
AMATEUR	-11.297	45.364
FXD./MOB./AMTR.	-34.618	12.877
FXD./MOB./BCST	-52.738	62.152
AEROMOBILE	-25.573	101.202
FIXED/BROADCAST	-36.374	18.936
BROADCAST	-8.366	34.127
FIXED	-11.454	34.173
FIXED/AMATEUR	-11.186	17.272
RADIO/ASTRONOMY	-6.303	31.846
FXD./MOB./METR.	-4.677	14.875
MARITIME/MOBILE	-12.876	58.058

Looking at the output it's clear that the results are similar to the previous C.D.F. model (linear model).

Again fitting this model to the data we obtained good fit for some users and again we removed some of the terms from the model to make it suitable for some of the users,Appendix (C)

4.2 Comparing the Multinomial Fit with the Binomial fit:

It was stated in Chapter Two that Morrel (1988) fitted a binomial model to the occupancy data for the years 1982 - 1986 covering a period of the solar cycle when sunspot numbers were falling, then Dennigton (1990) fitted the same model to the years 1982 - 1989 since during the period 1987 - 1989 sunspot numbers were rising .

The form of the model was

$$\mu = A_k + b \text{ threshold(dBm)} + (C_\circ + C_1 f_k + C_2 f_k^2) \times \text{sunspot number}$$

where A_k are constants and b,c's are parameters to be estimated

sunspot numbers are 115, 65, 44, 17, 14, 31, 104, 158 for the years 1982 to 1989 respectively .

He fitted this model for the occupancy data for the years 1982 to 1989 for all user and for single user and then produced estimated occupancy data.

A multinomial model was fitted to the multicount occupancy data for the years 1982 to 1989, and an estimated occupancy data was produced.

The main aim of this section is to compare the binomial fit (using single user) and multinomial fit (using single user) to see the difference.

The comparison is considered in two ways:

The first comparison is by taking a random year to see the absolute difference between the binomial fit and the multinomial fit i.e. $|\hat{M}-\hat{B}|$ to see the variation of the estimated occupancy data for both multinomial fit and binomial fit, also the absolute difference between the congestion occupancy data and the multinomial fitted occupancy data i.e. $|C-\hat{M}|$, and finally taking the absolute difference between the binomial fit and congestion occupancy data i.e. $|C-\hat{B}|$ we present these three absolute differences in one graph for each threshold for user Aeromobile for year 1989, we present each threshold in one graph.

Figure (4.1) presents threshold -107, we can see from the graph that the difference is varies between zero and 6% for the three comparisons but for the absolute difference between the binomial fit and multinomial fit we can see that the difference is varies between zero and 3.5% and only the difference in allocation 52 was 3% but the rest of the allocations the difference is varies between zero and 2%.

Figure (4.2) presents threshold -97 , we can see clearly that the difference is nearly varies between zero and 3.3% for the the three comparisons, but for the comparison between binomial fit and multinomial fit the difference is between zero and 2.4%.

For threshold -87, from figure (4.3) we can see that the difference is varies between zero and 2.1% for the three comparisons, but the comparison between the binomial fit and multinomial fit the difference is varies between zero and 1.4%.

Finally threshold -77. it is clearly from figure (4.4) that the difference is varies between zero and one for the three comparisons, but for the comparison between the binomial fit and multinomial fit the difference is varies between zero and 0.5%.

From the figures (4.1), (4.2), (4.3) and (4.4) we conclude that the binomial fit model and the multinomial fit model are very close to each other specially for T=-87 and T=-77.

The second comparison:- by taking the absolute difference between the multinomial fit and the binomial fit for each year for user Aeromobile for four thresholds, and for eight years of summer solstice day data. Figures (4.5) to (4.12) represent the difference between binomial fit and multinomial fit.

For year 1982, we can see from figure (4.5) that the difference in threshold -107 is slightly bigger up to 10% and only for two allocations 60 and 73, but for threshold -97 the difference is varies between zero and 1.9%, for threshold -87 the difference is getting smaller which is varies between zero and one, and finally the difference in threshold -77 it's nearly zero.

For year 1983, it's clear from figure (4.6) that the difference is varies between zero and three for threshold -107, for threshold -97 the difference is varies between zero and 1.7% for threshold -87 the difference is varies between zero and 0.6% and finally for threshold -77 the difference is nearly zero.

And so on for the rest of the figures which produce the difference between binomial fit and multinomial fit for each year.

Conclusion:- we conclude that the binomial fit and the multinomial fit are very close to each other specially in thresholds -87 and -77, and also in threshold -107 except for some allocations.

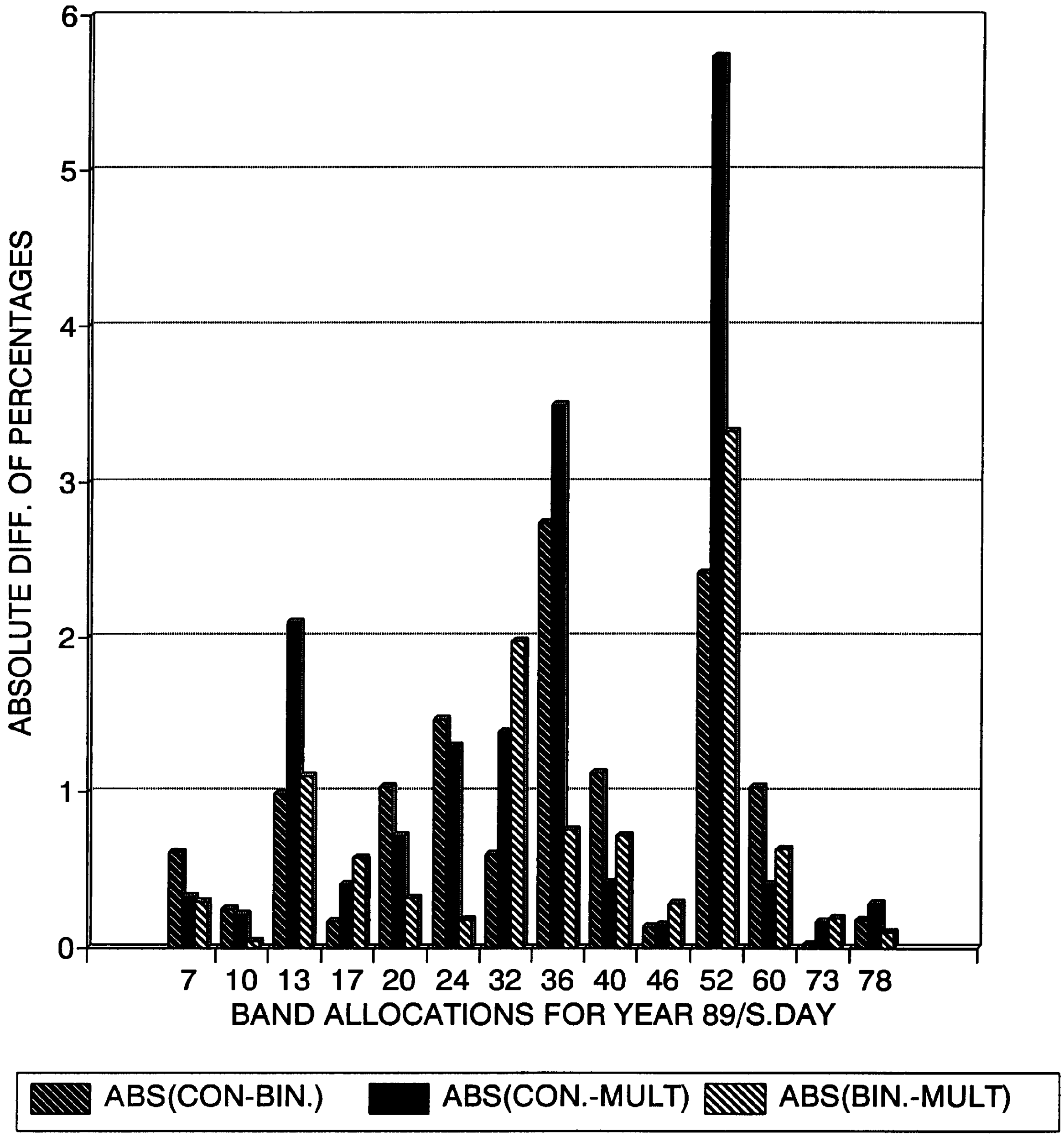


Figure (4.1) Comparison between Multinomial fit , Binomial fit and Congestion values for user (AEROMOBILE) for Threshold =-107

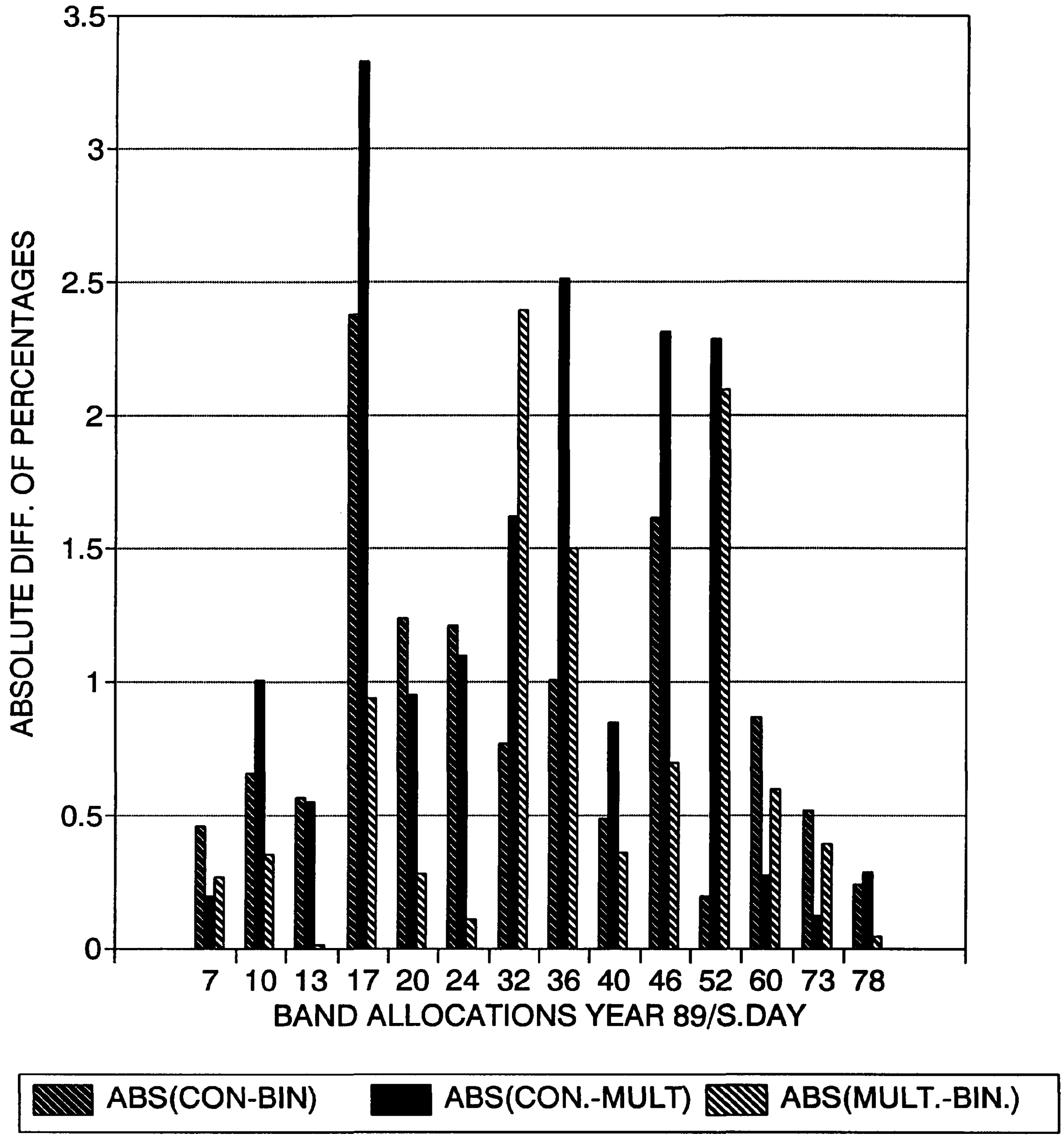


Figure (4.2) Comparison between Multinomial fit , Binomial fit and Congestion values for user (AEROMOBILE) for Threshold =-97

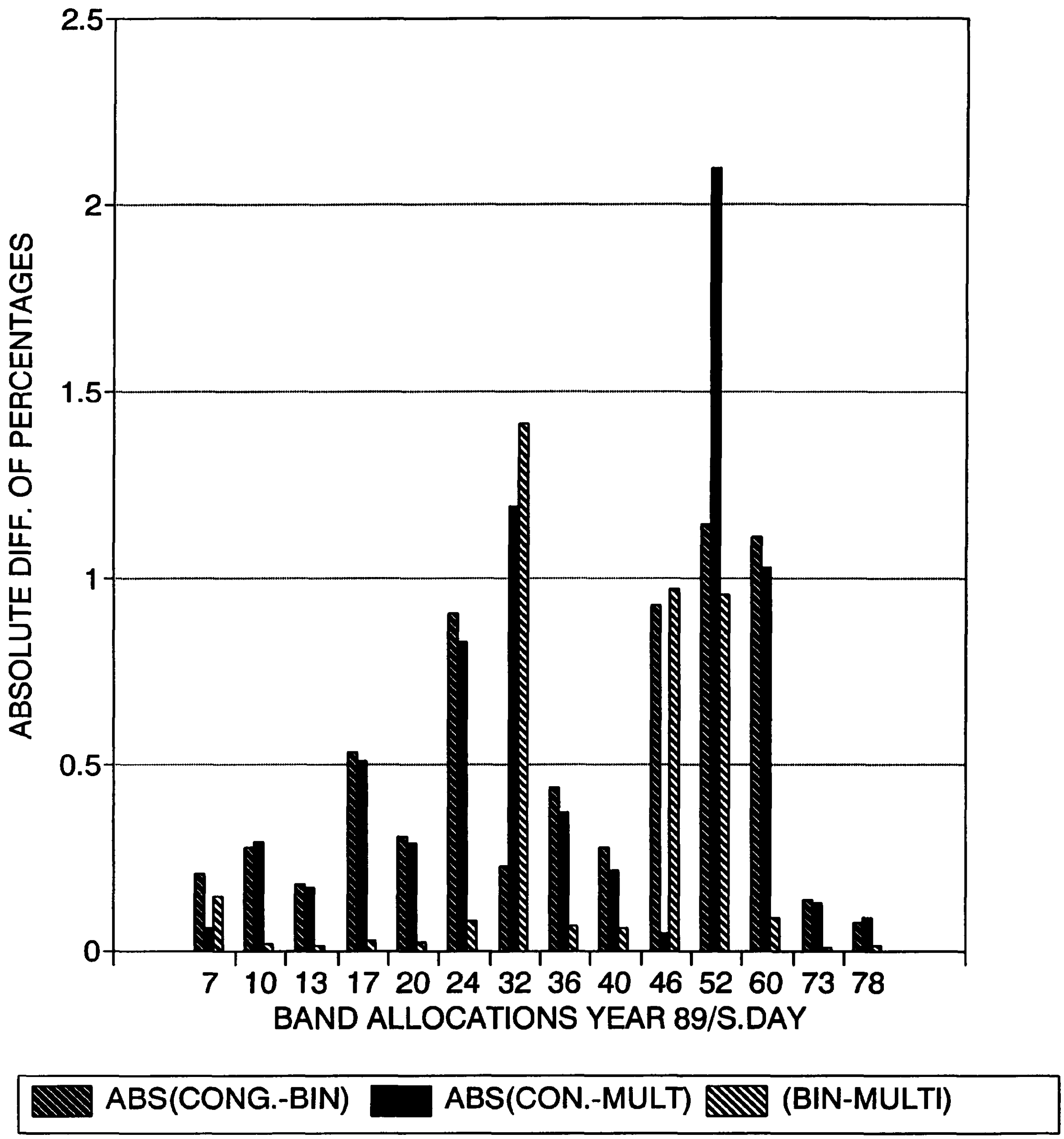


Figure (4.3) Comparison between Multinomial fit Binomial fit and Congestion values for user (AEROMOBILE) for Threshold =-87

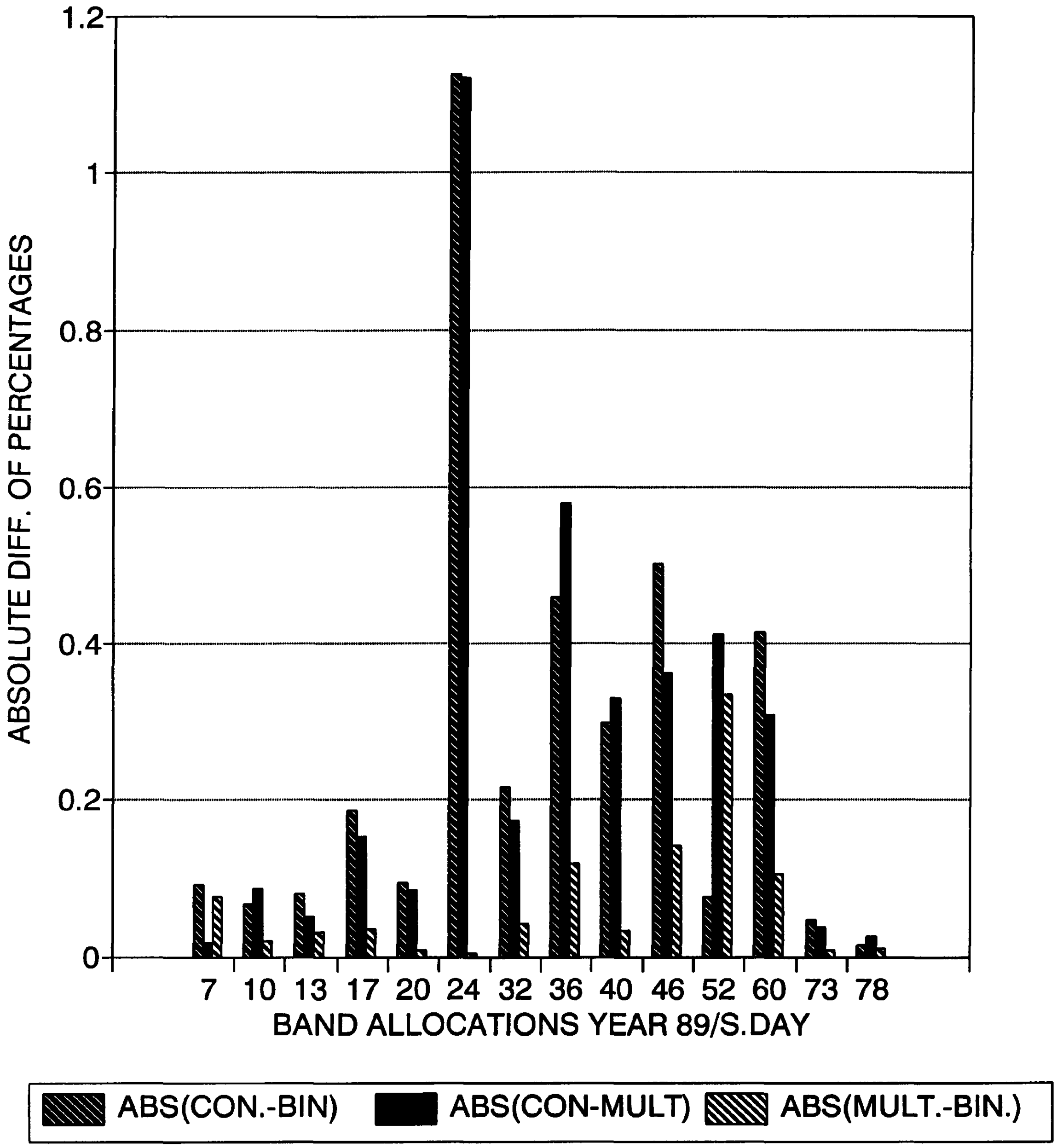


Figure (4.4) Comparison between Multinomial fit Binomial fit and Congestion valuesfor user (AEROMOBILE) for Threshold =-77

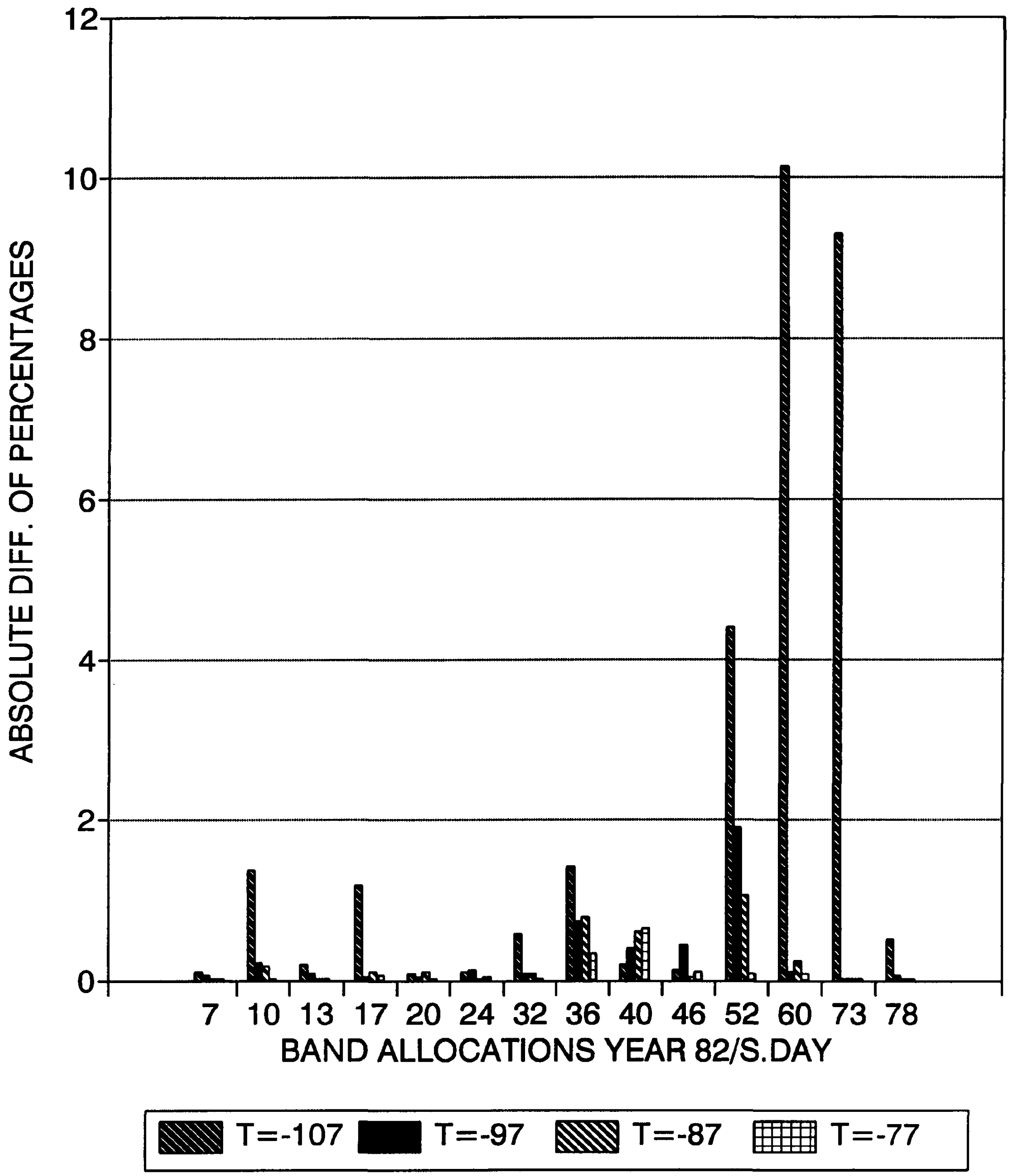


Figure (4.5) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1982

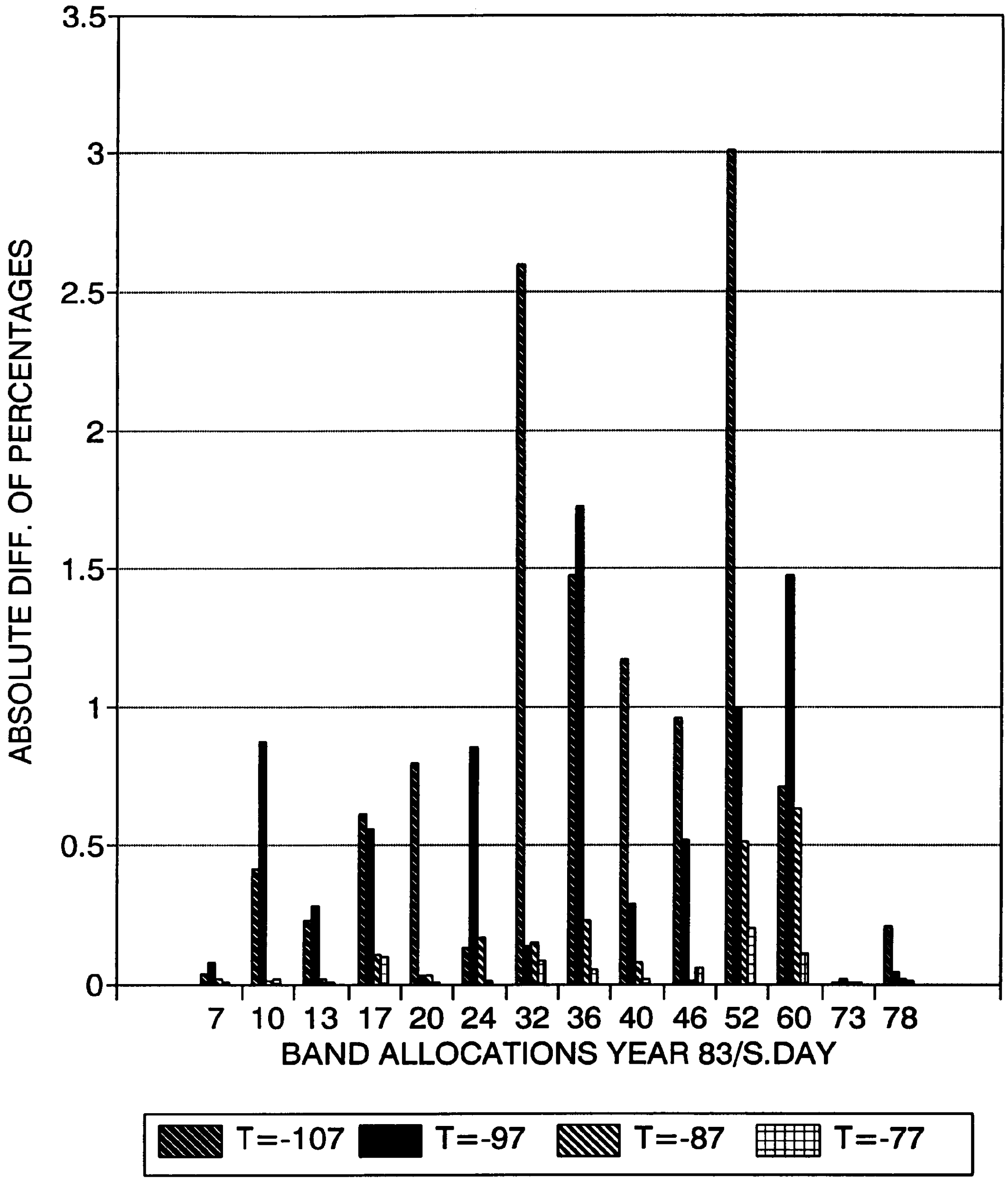


Figure (4.6) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1983

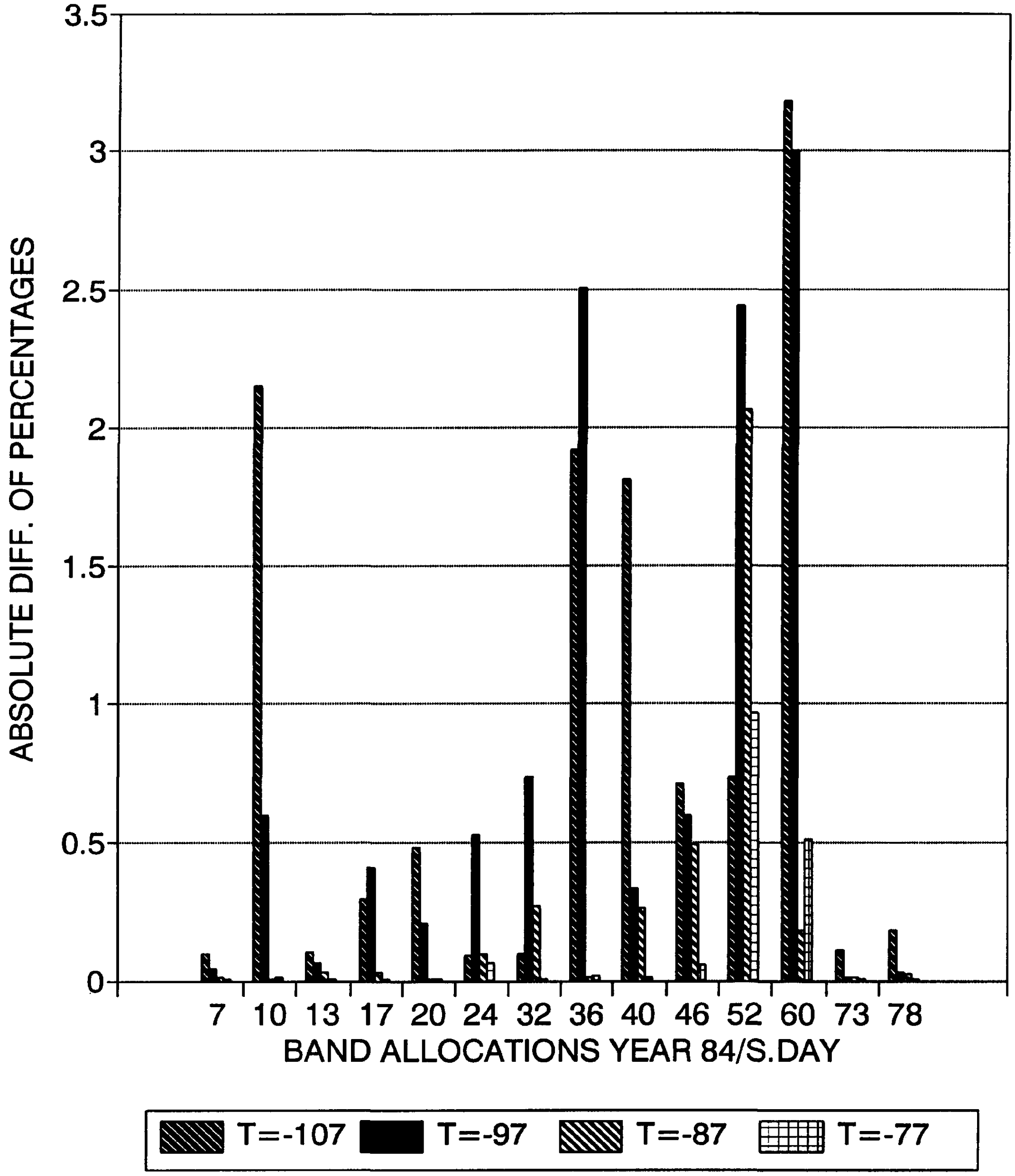


Figure (4.7) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1984

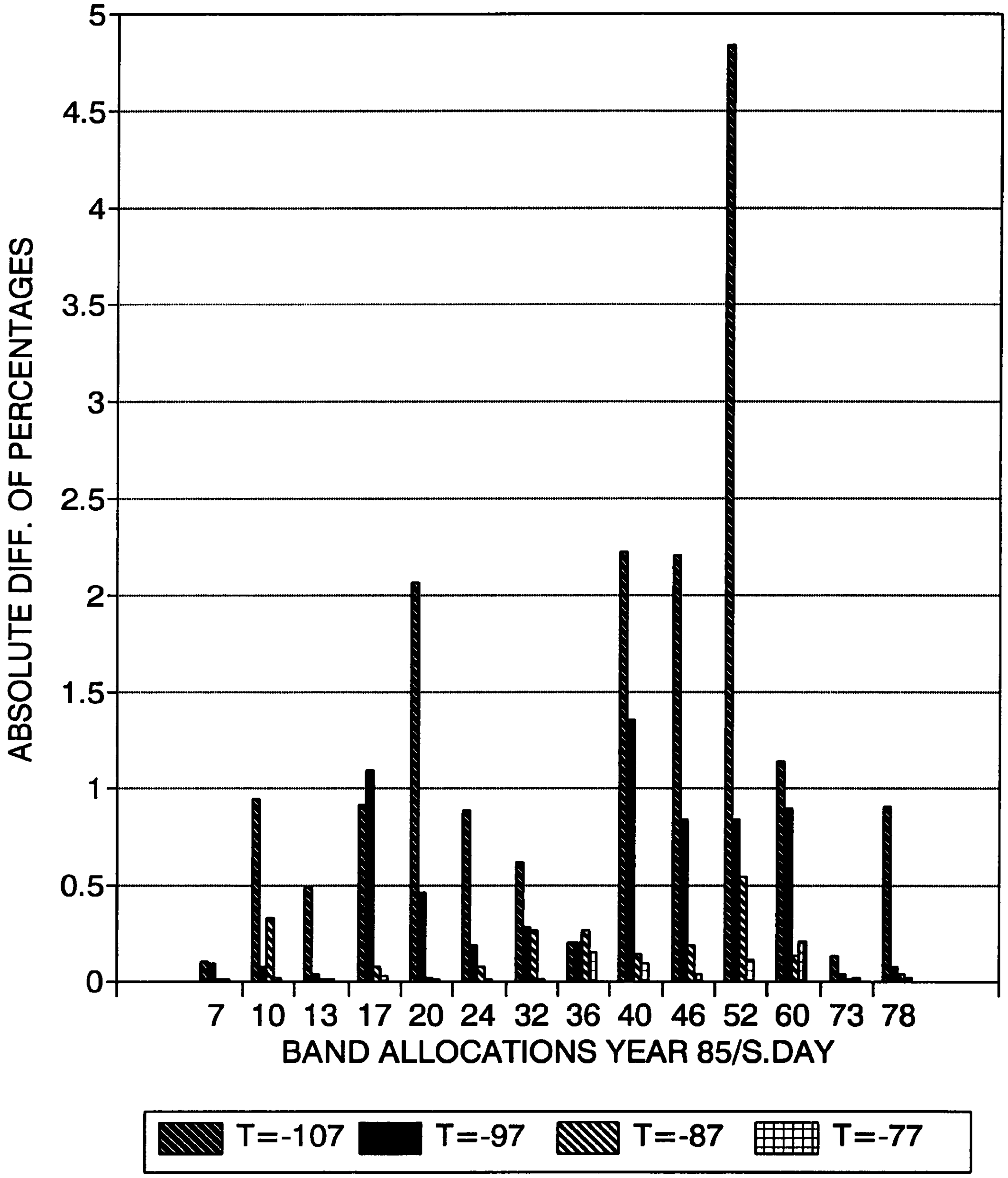


Figure (4.8) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1985

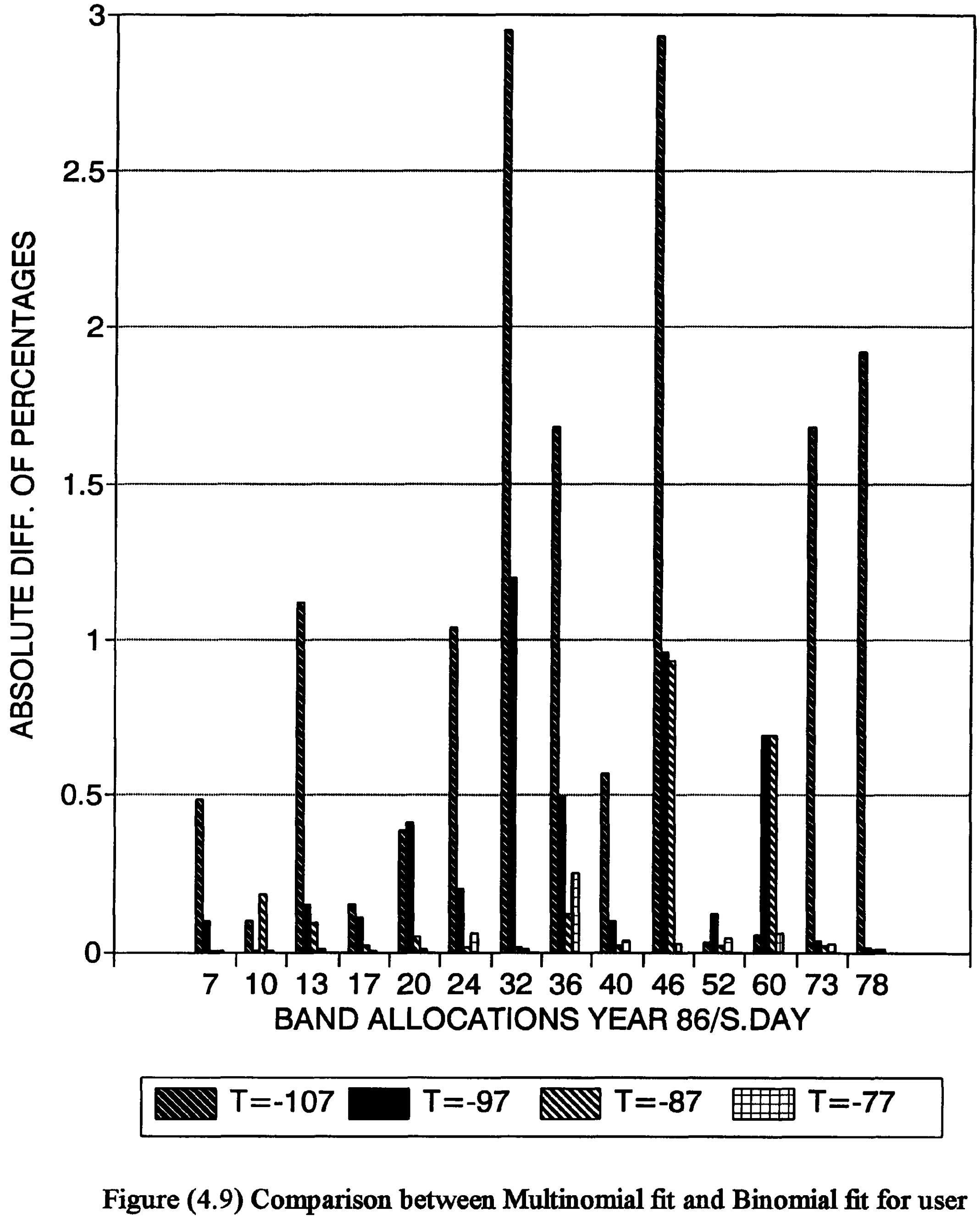


Figure (4.9) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1986

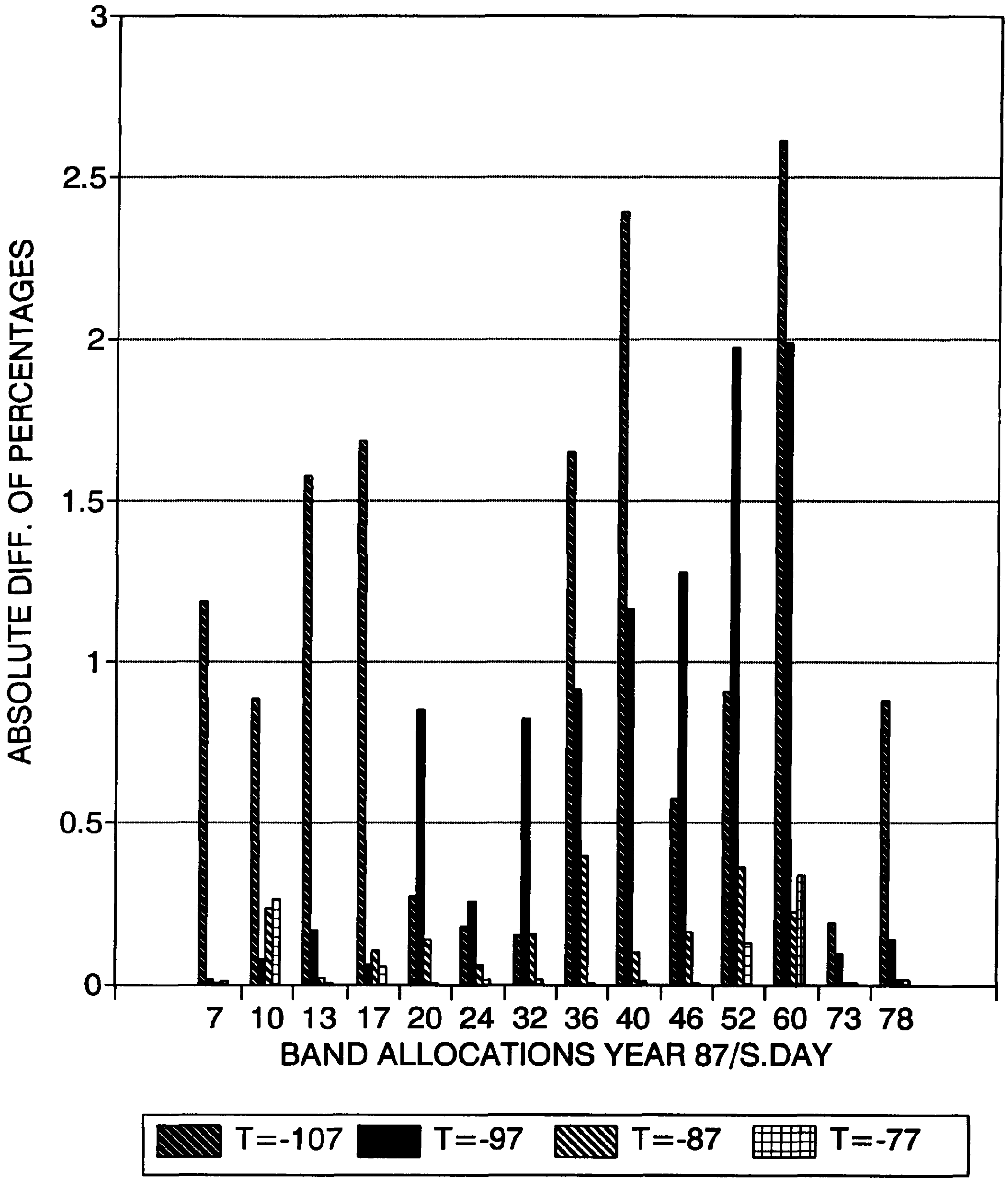


Figure (4.10) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1987

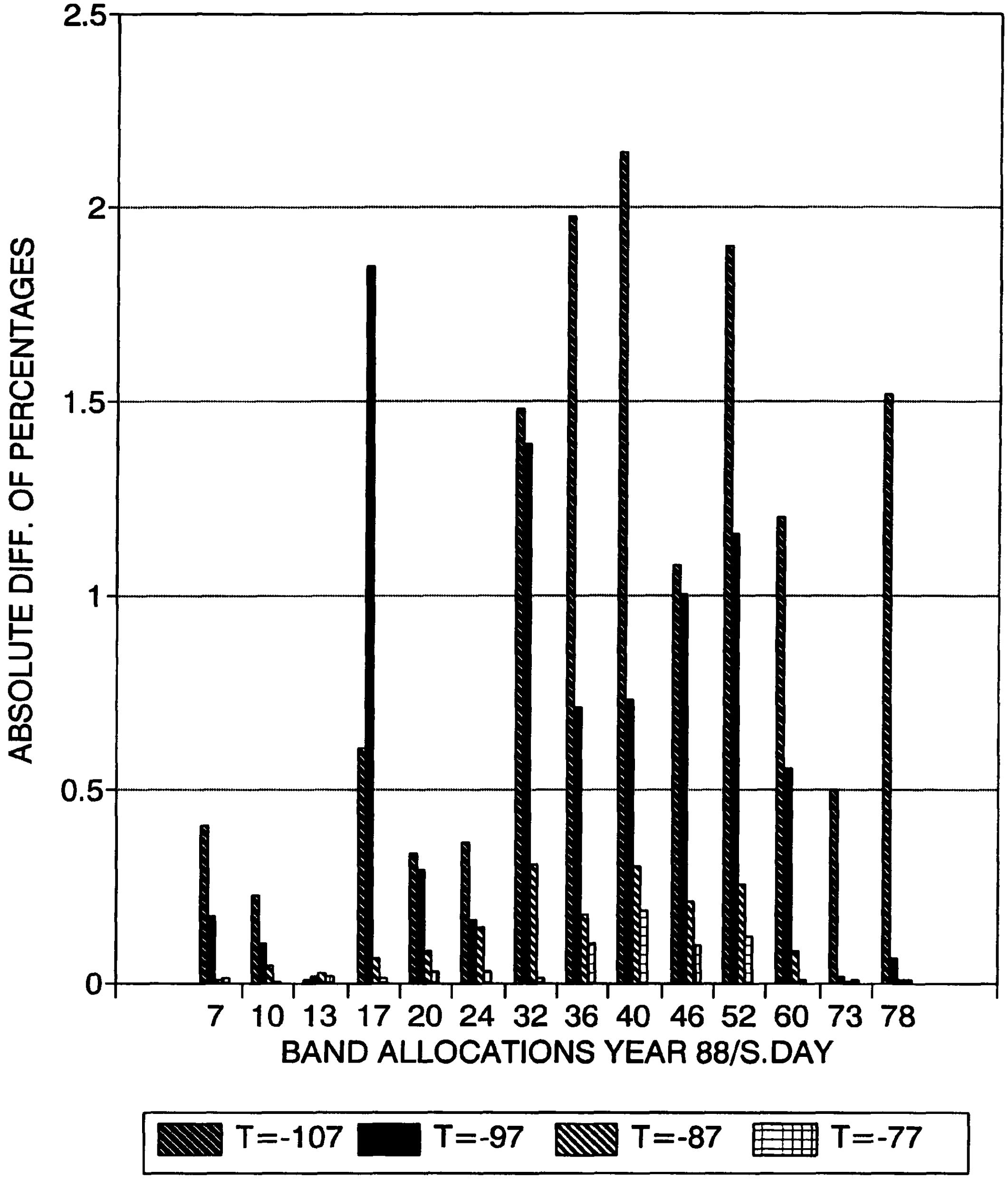


Figure (4.11) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1988

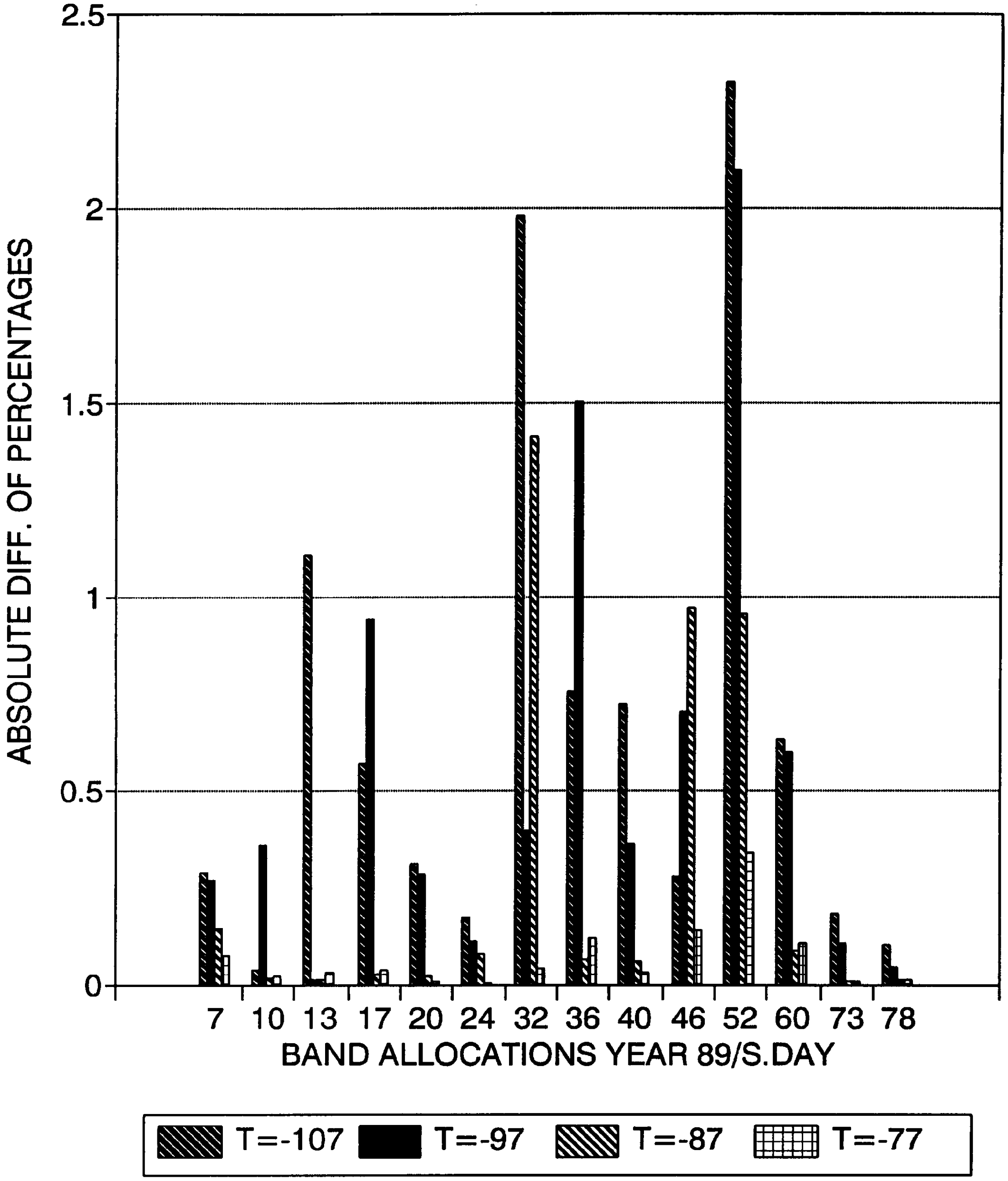


Figure (4.12) Comparison between Multinomial fit and Binomial fit for user (AEROMOBILE) for year 1989

CHAPTER 5

APPLICATION OF BINARY MODELS TO HF RADIO DATA

5.1- Introduction:

In this chapter the data presented is modelled by first or higher order Markov chains, and also the data is treated to the maximum likelihood estimation on the adjustable probability function taken from Zhao paper.(1990).

Subsequently the dependency of our data was checked by testing λ is equal to zero or not, since in Chapter Three we proved that if λ is zero then the correlation coefficient is zero.

Also included in this chapter an important results which predict the signal frequency level instead of predicting the congestion data as done in previous work and as was noted in chapter four.

Finally, the profile likelihood function was plotted to see the confidence interval for our estimated likelihood function.

5.2- The uncertainty and Redundancy of Happenings in a Sequence:-

A fundamental idea in information theory is that the stochastic process is characterized by some degree of redundancy between 0 and 100 percent. The simplest kind of complete redundancy occurs when one symbol has a probability of one, and others have zero probability , so that for example in our case of the sequence of binary data of the form {1,1,1,....,1} this state of affairs was approached when the signal is higher than the threshold everywhere.

Redundancy which thus depends upon unequal probabilities of individual symbol is said to be of the first order. Another form of complete redundancy is {1,0,1,0,1,0,.....,1,0} even though 1's and 0's occur with equal frequency such a sequence is said to have second order redundancy. A double alternation of form {1,1,0,0,1,1,0,0,...,1,1,0,0} has third order redundancy, since prediction of a given symbol depends upon a knowledge of the two preceding ones.

It was shown earlier that information, in the binary digits (bits) persymbol, is equal to the log of the number of alternative symbols when all symbols are independent and equiprobable, which was called a zero-order estimate of H: Attneave (1959).

$$H_0 = \sum p \log \frac{1}{p} = m(\frac{1}{m} \log m) = \log m$$

in our case m=4 which is the four thresholds

$$H_0 = \log_2 (4) = \log_2 2^2 = 2 \log_2 2 = 2$$

likewise , a first order estimate of H.

$$\hat{H}_i = \sum \hat{P}_i \log(\frac{1}{\hat{P}_i})$$

$$=\sum \hat{P}_i \log \frac{1}{\hat{P}_i} \qquad (5.1)$$

To calculate a second order estimate H_2 first the average information in all pairs (just as if each pair were a separate symbol).

$$\hat{H}(pairs) = \sum \hat{P}(pairs) \log \frac{1}{\hat{P}(pairs)}$$

where the $\hat{P}$ (pairs)'s are the proportions of all (overlapping) pairs falling into the various possible classes of pairs, for example in our case (binary sequence) there are four classes : 00, 01, 10, 11, A second order of estimate of H is then obtained by taking the difference between $\hat{H}$ (pairs) and H_1, i.e.

$$\hat{H}_2 = \hat{H}(pairs) - \hat{H}_1$$

A third order estimate is calculated in a manner entirely analogous to that discussed above:

$$\hat{H}_3 = \hat{H}(triplets) - \hat{H}(pairs)$$

So the general formula will be

$$\hat{H}_N = \hat{H}(N-gram) - \hat{H}((N-1)-gram)$$

$$\text{where } \hat{H}(N-gram) = \sum \hat{P}(N-gram) \log \frac{1}{\hat{P}(N-gram)}$$

With a sequence of observation, the usual estimate of $\hat{H}_1$ is obtained by taking $\hat{P}_i = \frac{n_1}{N_1}$ in the (5.1). Also an estimate of H(pairs) is given by

$$\hat{H}(pairs) = \log(N_2) - N_2^{-1} \sum_{i,j} n_{ij} \log n_{ij}$$

$$\hat{H}(pairs) = \sum \hat{p}(pairs)\log(\frac{1}{\hat{P}(pairs)})$$

$$= \sum \frac{n_2}{N_2}\log(\frac{N_2}{n_1})$$

$$\hat{H} = \sum \frac{n_1}{N_1}\log(\frac{N_1}{n_1})$$

A similar procedure can be adopted to find the estimation of $\hat{H}$(triplets), $\hat{H}$(tetragram) and so on.

5.2.1 Test for Higher Order Markov Chain:-

By using the results obtained from Chatfield, (1970), (1975) and Yarmohammadi, (1988), given that the difference between successive values is

$$D_i = H_i - H_{i+1} \quad (\mathrm{H_o} = 1)$$

then $\Lambda_i = 2(\log_e 2)N_{i+1}\hat{D}_i$ where Λ_i is the likelihood ratio to test the null hypothesis is:

H_{i-1}: The sequence is (i-1)th order markov chain against the alternative hypothesis:

H_i:The sequence is ith order markov chain.

Since $$N_1 = N_{2+1} = N_{3+2} = \dots\dots = N_{i+(i-1)}$$

$$N_i = N_1 - (i-1)$$

and hence $$\Lambda_i = 2(\log_e 2)N_{i+1}\hat{D}_i$$

$$= 2\log_e 2(N_1 - i)(\hat{H}_i - \hat{H}_{i+1}) \qquad (5.2)$$

is asymptotically χ^2 distributed with the following d.f.

i	d.f.
0	1
1	1
≥ 2	2^{i-1}

It would be possible to test the following hypothesis (i-1) order against (i) order which means

H_o	against	H_1
-1 order	vs	0 order
0 order	vs	1st order
1st order	vs	2nd order
2nd order	vs	3rd order
3rd order	vs	4th order

The result of values Λ, degrees of freedom, and critical χ^2 values for winter (January 1991) allocation 6 is given in the table below.

i	$\Lambda_i(-117)$	$\Lambda_i(-107)$	$\Lambda_i(-97)$	$\Lambda_i(-87)$	$\Lambda_i(-77)$	d.f.	$\chi^2_{\alpha=0.05}$
1	408.237	1.415	26.159	156.81	348.034	1	3.84
2	1.94	43.603	39.719	43.967	7.627	2	5.99
3	15.672	2.723	2.071	0.201	4.712	4	9.49
4	7.847	5.372	2.597	1.463	8.749	8	15.5
5	-0.668	5.283	8.241	7.406	1.724	16	26.3

The test hypothesis can be found in the following table. Note that in this table R means "Reject" the specified hypothesis and A "Accept" it.

Our conclusion on the order of Markov Chain has been arrived at by application of the following rule.

Rule: " If two acceptance are found in succession, stop and accept current H_0, otherwise last acceptance found is used, if none of the null hypothesis are accepted, then order greater than four assumed."

H_0 vs H_1	-117 dBM	-107 dBM	-97dBM	-87 dBM	-77 dBM
-1 vs 0	R	A	R	R	R
0 vs 1	A	R	R	R	R
1 vs 2	R	A^*	A	A	A
2 vs 3	A^*	A	A^*	A^*	A^*
3 vs 4	A	A	A	A	A
Conclusion	2	1	2	2	2

Final Conclusion: It seems that this example of Winter (1991) may be modelled by a second order Markov Chain, although the dependence is weak.

This may be contrasted with the experiment's original assumption of serial independence for these observations.

5.3. Constrained Monte Carlo Maximum Likelihood Dependent Data:-

Geyer and Thompson (1992) state that maximum likelihood estimates in autologistic models and other exponential family models for dependent data can be calculated with Markov Chain Monte Carlo method "Gibbs sampler".

The maximum likelihood estimate in the closure of the exponential family may and can be calculated by a two phase algorithm, first finding the support of the MLE by linear programming and then finding the distribution within the family conditioned on the support by maximizing the likelihood for the family.

They used Monte Carlo Maximum Likelihood instead of the exact maximum likelihood to estimate the parameters, they assumed that

$$f_{\theta}(x) = \frac{1}{c(\theta)} \exp(t(x), \theta) \qquad \text{where}\, c(\theta) = \sum_{\mu} \exp(t(x), \theta)$$

They call $C(\theta)$, the Laplace transformation of the measure $t(\mu)$ of the exponential family or "normalizing constant" and they point out that the difficulty with the exact likelihood calculation for many exponential family models for dependent data is that the normalizing constant C can't be calculated exactly nor are there analytic approximations available.

In our case we have found a good approximation to the normalizing constant, so we have applied exact maximum likelihood estimation to estimate the parameters in our model.

5.3.1 Maximum Likelihood Estimate of The HF data:-

We know from chapter 3 that the probability density function is

$$\Pr(y_i) = \frac{e^{\theta\sum y_i + \lambda\sum y_i y_{i+1}}}{\sum_{m=0}^{n} {}^{n}C_{m} e^{\theta m + \frac{\lambda(m-1)m}{n}}}$$

where $\theta = \alpha + \beta T_j$ and T_j=-107, -97, -87, -77

and $\lambda = \lambda_1 + \lambda_2 T_j$ and T_j=-107, -97, -87, -77

By writing a Fortran program using Nag - library to estimate the parameters using Maximum Likelihood estimation, we estimate the parameters from allocation 1 to allocation 68. The following table shows the estimation of the parameters:

allocation	α	β	λ_1	λ_2
1	-6.915	-0.0443	6.8864	0.044
2	-13.2212	-0.1043	13.2198	0.1039
3	-8.9715	-0.0629	8.9216	0.0623
4	-8.847	-0.061	8.8288	0.0608
5	-8.1442	-0.0552	8.134	0.0551
6	-5.585	-0.0315	5.545	0.031
7	-9.2974	-0.0648	9.2662	0.0645
8	-4.342	-0.0262	2.2652	0.0021
9	-3.296	-0.009	3.1808	0.0077
10	-7.9306	-0.053	7.6474	0.0502

allocation	α	β	λ_1	λ_2
11	-3.9858	-0.0148	3.8782	0.0136
12	-3.1924	-0.0081	2.9962	0.0057
13	-6.644	-0.0445	5.6515	0.0347
14	-0.574	0.0247	-0.01	-0.0325
15	-3.1302	-0.0076	3.093	0.0071
16	-2.1746	0.0036	2.0923	-0.0046
17	-5.1321	-0.0257	5.0001	0.0242
18	-3.3569	-0.0095	3.1769	0.0076
19	-2.4404	-0.0004	2.3949	-0.0002
20	-6.4042	-0.04	6.3625	0.0396
21	-1.21	0.015	1.1399	-0.016
22	-1.8458	0.0097	-0.01	-0.034
23	-2.3565	0.0015	2.3165	-0.002
24	-9.8747	-0.0751	6.4481	0.0397
25	-2.9587	-0.0034	2.8377	0.002
26	-5.7419	-0.0355	2.78	0.0044
27	-0.0788	0.0333	-0.01	-0.0346
28	-2.3043	0.0036	2.2799	-0.0039
29	-4.1683	-0.017	4.1255	0.0165
30	-5.2439	-0.0279	5.2029	0.0275
31	-2.6582	-0.0021	2.6171	0.0016
32	-9.2901	-0.0657	9.265	0.0654
33	-3.29	-0.0062	3.2707	0.0059

allocation	α	β	λ_1	λ_2
34	-2.298	0.0026	-0.01	-0.0327
35	-1.9393	0.0082	1.7469	-0.0106
36	-9.0632	-0.0567	8.8959	0.0552
37	-9.5352	-0.0666	9.3761	0.065
38	-10.1398	-0.0675	10.136	0.0675
39	-13.1148	-0.0952	13.1034	0.0951
40	-14.2743	-0.1044	14.2437	0.1041
41	-8.6006	-0.0565	8.5735	0.0562
42	-4.2164	-0.015	4.1876	0.0146
43	-9.2272	-0.0619	9.1832	0.0615
44	-10.7864	-0.0702	10.7791	0.0702
45	-7.9495	-0.0515	7.9397	0.0514
46	-17.3464	-0.1271	17.3464	0.1271
47	-11.6843	-0.086	11.6843	0.0855
48	-7.8448	-0.0495	7.8073	0.0491
49	-9.5822	-0.0663	9.5572	0.0661
50	-11.7162	-0.086	11.7129	0.086
51	-9.171	-0.0605	9.1622	0.0604
52	-6.0729	-0.0303	5.9099	0.0287
53	-3.6295	-0.0115	3.5772	0.011
54	-8.5407	-0.0553	8.52	0.055
55	-10.5545	-0.0756	10.547	0.0756
56	-11.4511	-0.0806	11.438	0.0805

allocation	α	β	λ_1	λ_2
57	-7.6236	-0.0493	7.6093	0.0492
58	-6.6106	-0.0391	6.483	0.0379
59	-2.0875	0.0057	2.0136	-0.0065
60	-15.6783	-0.1153	15.6874	0.1154
61	-8.4693	-0.0541	8.4098	0.0535
62	-9.9242	-0.0683	9.8207	0.0673
63	-8.0059	-0.0502	7.9968	0.0501
64	-16.9335	-0.1299	16.9321	0.1299
65	-11.0395	-0.078	11.0288	0.0779
66	-13.5741	-0.099	13.5759	0.099
67	-14.9882	-0.1139	14.9788	0.1138
68	-9.4922	-0.0562	9.4594	0.0559

5.3.2 Testing of Hypothesis:-

The second part of the analysis is to test whether λ equal to zero or not.Setting $\lambda = \lambda_1 + \lambda_2 T$ we will try to test first if λ_2 equals to zero or not. Then if it is zero we will run the program to estimate 3 parameters instead of 4, and the parameters will be $\theta = \alpha + \beta T_i$ and λ, but after performing the test we found that some of the allocations tested λ_2 zero and the rest test $\lambda_2$is not zero, so for this reason I will present some examples of the allocations which test zero, and for the rest will test the hypothesis $\lambda = \lambda_1 + \lambda_2 T$ equals to zero or not.

Theory:- It often occurs that we do not know the distribution of estimators. Fortunately some of the standard methods of estimation automatically lead to estimators with known approximate distributions for large samples. The maximum likelihood estimators for example are approximately normally distributed with expectation equal to the true value. Their variance can be found from a quantity called the Fisher information.

This is defined by $$I(\theta)=V(\frac{\partial L}{\partial \theta})$$

where L is the log-likelihood. If the maximum likelihood estimator is $\hat{\theta}$, then for large n

$$\hat{\theta} \to N(\theta, \frac{1}{I(\theta)})$$

As the variance of $\hat{\theta}$ is $\frac{1}{I(\theta)}$ the larger the information the better, the more efficient the maximum likelihood estimator. An alternative expression for $I(\theta)$ that emphasizes this point is

$$I(\theta)=-E(\frac{\partial^2 L}{\partial \theta^2})$$

The second derivative is a measure of the curvature so $I(\hat{\theta})$ indicates the expected sharpness of the peak of the log likelihood curve, Gilchrist (1984).

In our case, since we have four parameters so $I(\theta)$ will be 4×4 matrix, and to find the variance of α ,β ,λ_1 ,λ_2 we need to find the inverse of the $I(\theta)$ as mentioned above.

$$A^{-1}=\frac{1}{\mathrm{I}(\theta)}=\begin{bmatrix} \mathrm{var}(\alpha) & \mathrm{cov}(\alpha,\beta) & \mathrm{cov}(\alpha,\lambda_1) & \mathrm{cov}(\alpha,\lambda_2) \\ \mathrm{cov}(\alpha,\beta) & \mathrm{var}(\beta) & \mathrm{cov}(\beta,\lambda_1) & \mathrm{cov}(\beta,\lambda_2) \\ \mathrm{cov}(\alpha,\lambda_1) & \mathrm{cov}(\beta,\lambda_1) & \mathrm{var}(\lambda_1) & \mathrm{cov}(\lambda_1,\lambda_2) \\ \mathrm{cov}(\alpha,\lambda_2) & \mathrm{cov}(\beta,\lambda_2) & \mathrm{cov}(\lambda_1,\lambda_2) & \mathrm{var}(\lambda_2) \end{bmatrix}$$

By writing a Fortran program to produce the Hessian matrix which is the second partial derivative, and then by taking the inverse of the Hessian matrix we can find an approximation to the variance of λ_1 and the variance of λ_2.

So the hypothesis will be

$H_0:\lambda_2=0$ VS $H_1:\lambda_2\neq 0$

where Z (cal.) will be $$Z_c = \frac{\hat{\lambda}_2 - 0}{\sqrt{\operatorname{var}(\lambda_2)}} = \frac{\hat{\lambda}_2}{\sigma_{\lambda_2}}$$

Examples which accepted the null hypothesis:

Allocation 8 n=45

$$A^{-1} = \begin{bmatrix} 9.5323 & 1.1589\times10^{-1} & -5.2662 & -6.6271\times10^{-2} \\ 1.1589\times10^{-1} & 1.4236\times10^{-3} & -6.0447\times10^{-2} & -7.7988\times10^{-4} \\ -5.2662 & -6.0447\times10^{-2} & 4.6182 & 5.1845\times10^{-2} \\ -6.627\times10^{-2} & -7.7988\times10^{-4} & 5.1845\times10^{-2} & 6.0229\times10^{-4} \end{bmatrix}$$

therefore Z(cal.)= $\frac{0.0021}{\sqrt{6.0229\times10^{-4}}}$ =0.0855

Z(tab.)=1.96

Since Z(cal.)<Z(tab.) we accept the null hypothesis and we reject the alternative hypothesis, which means that $\lambda = \lambda_1 + \lambda_2 T$ will reduce to λ, again we run the program to estimate 3 parameters and to produce 3×3 Hessian matrix and then the inverse of the Hessian matrix.

$H_0 : \lambda = 0$ vs $H_1 : \lambda \neq 0$

$$A^{-1} = \begin{bmatrix} 2.1724 & 2.9169\times10^{-2} & 4.2766\times10^{-1} \\ 2.9169\times10^{-2} & 4.0147\times10^{-4} & 6.5301\times10^{-3} \\ 4.2766\times10^{-1} & 6.5301\times10^{-3} & 1.5326\times10^{-1} \end{bmatrix}$$

$\hat{\lambda} = 2.0827$

Z(cal.)= $\frac{2.0827}{\sqrt{1.5326\times10^{-1}}} = 5.32$

Since Z(cal.) .> Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, which means the data in allocation 8 is dependent.

Next we are going to produce a 2×2 transition matrix.

we know that $P(x_2 \mid x_1) = \dfrac{p(x_1, x_2)}{p(x_1)}$ $\qquad x_1 = x_2 = 0 \text{ or } 1$

$$p(x_1, x_2) = \frac{1}{\sum_{m=0}^{n} C_m^n e^{\theta m + \lambda(m-1)\frac{m}{n}}} e^{\theta(x_1 + x_2) + \lambda x_1 x_2}$$

$$= \frac{1}{D_n} e^{\theta(x_1 + x_2) + \lambda x_1 x_2}$$

$$p(x_1) = \sum_{x_2 = 0}^{1} p(x_1, x_2)$$

$$= \frac{1}{D_2} \sum_{x_2 = 0}^{1} e^{\theta(x_1 + x_2) + \lambda x_1 x_2}$$

$$p(x_1) = \frac{1}{D_2} [e^{\theta x_1} + e^{\theta(x_1 + 1) + \lambda x_1}]$$

so $p(x_2 = 0 | x_1 = 0) = \dfrac{p(x_1 = 0, x_2 = 0)}{p(x_1 = 0)} \qquad = \dfrac{1}{1 + e^{\theta}}$

$$p(x_2 = 0 | x_1 = 1) = \frac{p(x_1 = 1, x_2 = 0)}{p(x_1 = 1)}$$

$$= \frac{e^{\theta}}{e^{\theta} + e^{2\theta + \lambda}} = \frac{1}{1 + e^{\theta + \lambda}}$$

$$p(x_2 = 1 | x_1 = 0) = \frac{p(x_1 = 0, x_2 = 1)}{p(x_1 = 0)}$$

$$= \frac{e^{\theta}}{1 + e^{\theta}}$$

$$p(x_2 = 1 | x_1 = 1) = \frac{p(x_1 = 1, x_2 = 1)}{p(x_1 = 1)}$$

$$= \frac{e^{2\theta + \lambda}}{e^{\theta} + e^{2\theta + \lambda}} = \frac{e^{\theta + \lambda}}{1 + e^{\theta + \lambda}}$$

so the transition matrix will be

$$p(\theta,\lambda)=\begin{bmatrix}\frac{1}{1+e^{\theta}} & \frac{e^{\theta}}{1+e^{\theta}}\\ \frac{1}{1+e^{\theta+\lambda}} & \frac{e^{\theta+\lambda}}{1+e^{\theta+\lambda}}\end{bmatrix}$$

In allocation 8

$\theta=\alpha+\beta T_j$ and $\lambda=\lambda$ for four different thresholds

if T=-107

θ=-4.1112+(-0.0235)(-107)=-1.5967

λ=2.0827

$$p(\theta,\lambda)=\begin{bmatrix}0.83 & 0.17\\ 0.38 & 0.62\end{bmatrix}$$

Comparing $P(x_2|x_1)$ and $P(x_2)$

Since $P(x_1,x_2)=\dfrac{1}{\sum_{m=0}^{n}\binom{n}{m}e^{\theta m+\lambda(m-1)\frac{m}{n}}}e^{\theta(x_1+x_2)+\lambda x_1x_2}$ where in this case n=2

$$\text{So } P(x_2)=\sum_{x_1=0}^{1}P(x_1,x_2)$$

$$=\frac{1}{D_2}\sum_{x_1=0}^{1}e^{\theta(x_1+x_2)+\lambda x_1x_2}$$

$$P(x_2)=\frac{1}{D_2}[e^{\theta x2}+e^{\theta(x_2+1)+\lambda x_2}]$$

$$\therefore P(x_2=0)=\frac{1}{D_2}(1+e^{\theta})$$

$$\text{and since } D_2=\sum_{m=0}^{2}\binom{2}{m}e^{\theta m+\lambda(m-1)\frac{m}{n}}=1+2e^{\theta}+e^{2\theta+\lambda}$$

$$\therefore P(x_2=0)=\frac{1+e^{\theta}}{1+2e^{\theta}+e^{2\theta+\lambda}} \qquad (5.1)$$

$$\text{and } P(x_2 = 1) = \frac{e^{\theta} + e^{2\theta+\lambda}}{1 + 2e^{\theta} + e^{2\theta+\lambda}} \qquad (5.2)$$

So for the above allocation we have θ=-1.5967 and λ=2.0827

$P(x_2 = 0 | x_1 = 0)$=0.83 and $P(x_2 = 0)$=0.693 which is nearly the same.

and $P(x_2 = 1 | x_1 = 0)$=0.38 and $P(x_2 = 1)$=0.31 which again are nearly equal.

Allocation 15 n=438

$H_0 : \lambda_2 = 0$ VS $H_1 : \lambda_2 \neq 0$

$$A^{-1} = \begin{bmatrix} 1.7755 & 1.9031 \times 10^{-2} & -1.7390 & -1.8593 \times 10^{-2} \\ 1.9031 \times 10^{-2} & 2.0436 \times 10^{-4} & -1.8633 \times 10^{-2} & -1.9960 \times 10^{-4} \\ -1.7390 & -1.8633 \times 10^{-2} & 1.7050 & 1.8225 \times 10^{-2} \\ -1.8593 \times 10^{-2} & -1.9960 \times 10^{-4} & 1.8225 \times 10^{-2} & 1.9518 \times 10^{-4} \end{bmatrix}$$

var(λ_2)=1.9518×10^{-4}

$\lambda_2 = 0.0071$

Z(cal.)=$\dfrac{0.0071}{\sqrt{1.9518 \times 10^{-4}}} = 0.5082$

Since Z(cal.)<Z(tab.) we accept the null hypothesis and reject the alternative hypothesis, again we need to produce 3×3 variance covariance matrix to test the hypothesis

$H_0 : \lambda = 0$ VS $H_1 : \lambda \neq 0$

$$A^{-1} = \begin{bmatrix} 4.3906 \times 10^{-3} & 1.7364 \times 10^{-5} & -2.7693 \times 10^{-3} \\ 1.7364 \times 10^{-5} & 2.2558 \times 10^{-7} & 3.5704 \times 10^{-6} \\ -2.7693 \times 10^{-3} & 3.5704 \times 10^{-6} & 3.1094 \times 10^{-3} \end{bmatrix}$$

$\hat{\lambda} = 2.4334$

Z(cal.)=$\dfrac{2.4334}{\sqrt{3.1094 \times 10^{-3}}} = 43.639$

Since Z(cal.) >Z(tab.) we reject the null hypothesis and accept the alternative hypothesis. Next we will produce the transition matrix for T=-97

$$\theta=-2.4744+(-0.0005)(-97)=-2.4259$$

$$\lambda=2.4334$$

$$P(\theta,\lambda)=\begin{bmatrix} 0.92 & 0.08 \\ 0.5 & 0.5 \end{bmatrix}$$

Again we need to see the comparison between $P(x_2|x_1)$ and $P(x_2)$ for Allocation 15 for T=-97.

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.92 $P(x_2=1|x_1=0)$=0.08 and by substituting in equation (5.1) $P(x_2=0)$=0.86 which is nearly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) $P(x_2=1)$=0.14

Allocation 18 n=250

To test the hypothesis

$H_o:\lambda_2=0$ VS $H_1:\lambda_2\neq 0$

$$A^{-1}=\begin{bmatrix} 5.5411 & 5.8498\times10^{-2} & -5.9125 & -6.2081\times10^{-2} \\ 5.8498\times10^{-2} & 6.1826\times10^{-4} & -6.2374\times10^{-2} & -6.5563\times10^{-4} \\ -5.9125 & -6.2374\times10^{-2} & 6.2374 & 6.6663\times10^{-2} \\ -6.2081\times10^{-2} & -6.5563\times10^{-4} & 6.6663\times10^{-2} & 7.0\times10^{-4} \end{bmatrix}$$

$\hat{\lambda}$ =0.0076

Z(cal.)=$\dfrac{0.0076}{\sqrt{7.0\times10^{-4}}}=0.2873$

Since Z(cal.)<Z(tab.) we accept the null hypothesis and reject the alternative hypothesis, which means we going to run the program with 3 parameters to produce the variance covariance matrix to test the hypothesis

$H_0 : \lambda = 0$ VS $H_1 : \lambda \neq 0$

$$A^{-1} = \begin{bmatrix} 4.913\times10^{-2} & 5.2283\times10^{-4} & 2.4116\times10^{-3} \\ 5.2283\times10^{-4} & 6.3147\times10^{-6} & 9.9298\times10^{-5} \\ 2.4116\times10^{-3} & 9.9298\times10^{-5} & 7.4769\times10^{-3} \end{bmatrix}$$

$\lambda = 2.4635$

$$Z(\text{cal.}) = \frac{2.4635}{\sqrt{7.4769\times10^{-3}}} = 28.489$$

Z(cal.)>Z(tab.) therefore we reject the null hypothesis and accept the alternative hypothesis, to produce the transition matrix we have to calculate θ and λ for T=-87

$$\theta = -3.3569 + (-87)(-0.0095) = -2.5304$$

$$\lambda = 3.1769 + (-87)(0.0076) = 2.5157$$

$$p(\theta, \lambda) = \begin{bmatrix} 0.93 & 0.07 \\ 0.5 & 0.5 \end{bmatrix}$$

Again we need to see the comparison between $P(x_2 | x_1)$ and $P(x_2)$ for Allocation 18 for T=-87.

From the above transition matrix we can find that $P(x_2 = 0 | x_1 = 0)$=0.93 ,$P(x_2 = 1 | x_1 = 0)$=0.07 and by substituting in equation (5.1) then $P(x_2 = 0)$=0.87 which is nearly equal to $P(x_2 = 0 | x_1 = 0)$, also $P(x_2 = 0 | x_1 = 1)$=0.5 , $P(x_2 = 1 | x_1 = 1)$ =0.5 and by substituting in equation (5.2) then $P(x_2 = 1)$=0.13

The last example will be allocation 59 with n=350

$H_0: \lambda_2 = 0$ VS $H_1: \lambda_2 \neq 0$

$$\text{variance covariance matrix} = \begin{bmatrix} 1.6327\times10^{-1} & 1.7509\times10^{-3} & -1.6219\times10^{-1} & -1.7395\times10^{-3} \\ 1.7509\times10^{-3} & 1.9382\times10^{-5} & -1.7331\times10^{-3} & -1.9199\times10^{-5} \\ -1.6219\times10^{-1} & -1.7331\times10^{-3} & 1.6751\times10^{-1} & 1.7852\times10^{-3} \\ -1.7395\times10^{-3} & -1.9199\times10^{-5} & 1.7852\times10^{-3} & 1.9648\times10^{-5} \end{bmatrix}$$

$\lambda_2 = -0.0065$

$$Z(\text{cal.}) = \frac{-0.0065}{\sqrt{1.9648\times10^{-5}}} = 1.4664$$

So we accept the null hypothesis and reject the alternative hypothesis, which means we will run the program with 3 parameters to test the hypothesis

$H_0: \lambda = 0$ VS $H_1: \lambda \neq 0$

$$A^{-1} = \begin{bmatrix} 8.5743\times10^{-3} & 4.3526\times10^{-5} & -4.4257\times10^{-3} \\ 4.3526\times10^{-5} & 5.4198\times10^{-7} & 8.0190\times10^{-6} \\ -4.4257\times10^{-3} & 8.0190\times10^{-6} & 5.2105\times10^{-3} \end{bmatrix}$$

$\lambda = 2.6131$

$$Z(\text{cal.}) = \frac{2.6131}{\sqrt{5.2105\times10^{-3}}} = 36.201$$

Since Z(cal.)>Z(tab.) therefore we reject the null hypothesis and accept the alternative hypothesis, next we will estimate the transition matrix for T=-77

$$\theta = -2.0875 + (-77)(0.0057) = -2.5264$$

$$\lambda = 2.0136 + (-77)(-0.0065) = 2.5141$$

$$p(\theta, \lambda) = \begin{bmatrix} 0.93 & 0.07 \\ 0.5 & 0.5 \end{bmatrix}$$

Again we need to see the comparison between $P(x_2|x_1)$ and $P(x_2)$ for Allocation 59 for T=-97.

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.93, $P(x_2=1|x_1=0)$=0.07 and by substituting in equation (5.1) then $P(x_2=0)$=0.93 which is exactly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.14

Those are some examples of the data which accept the hypothesis $\lambda_2=0$, also the allocations 12, 13, 14, 16, 17, 19, 21, 23, 25, 26, 28, 33, 35, 48, 52.

The rest of the allocations reject the null hypothesis and accept the alternative hypothesis, and in the following section we will present some examples to test the hypothesis

$H_0:\lambda_T=0$ VS $H_1:\lambda_T\neq 0$

where $\lambda_T=\lambda_1+\lambda_2 T_j$ where $T_j=-107.-97,-87,-77$

So $\mathrm{var}(\lambda)=\mathrm{var}(\lambda_1)+T^2\,\mathrm{var}(\lambda_2)+2T\,\mathrm{cov}(\lambda_1,\lambda_2)$

Again by running the same program to produce estimated Hessian matrix and then by taking the inverse of Hessian matrix we can find variance covariance matrix.

Allocation 5 n=200

Estimated Hessian matrix is

$$A=\begin{bmatrix} 5.8203\times10^{3} & -5.6185\times10^{5} & 5.6489\times10^{3} & -5.4458\times10^{5} \\ -5.6185\times10^{5} & 5.4765\times10^{7} & -5.4459\times10^{5} & 5.3016\times10^{7} \\ 5.6489\times10^{3} & -5.4459\times10^{5} & 5.5669\times10^{3} & -5.363\times10^{5} \\ -5.4458\times10^{5} & 5.3016\times10^{7} & -5.363\times10^{5} & 5.2175\times10^{7} \end{bmatrix}$$

variance covariance matrix will be the inverse of Hessian matrix

$$A^{-1}=\begin{bmatrix} 1.7064 & 1.6882\times10^{-2} & -1.7189 & -1.7012\times10^{-2} \\ 1.6882\times10^{-2} & 1.6814\times10^{-4} & -1.7013\times10^{-2} & -1.6952\times10^{-4} \\ -1.7189 & -1.7013\times10^{-2} & 1.7499 & 1.7333\times10^{-2} \\ -1.7012\times10^{-2} & -1.6952\times10^{-4} & 1.7333\times10^{-2} & 1.7288\times10^{-4} \end{bmatrix}$$

if T=-107

$$\text{var}(\lambda_{-107}) = 1.7499+(-107)^2(1.7288\times10^{-4})+2(-107)(1.7333\times10^{-2}) = 0.0199$$

$$\lambda=\lambda_1+\lambda_2 T$$

$$=8.134+(-107)(0.0551)=2.2383$$

$$Z(\text{cal.})=\frac{2.2383}{\sqrt{0.0199}}=15.8669$$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis.

Next we will estimate transition matrix for T=-107

θ=-8.1442+(-0.0552)(-107)=-2.2378

$$p(\theta,\lambda)=\begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.9, $P(x_2=1|x_1=0)$=0.1 and by substituting in equation (5.1) then $P(x_2=0)$=0.93 which is exactly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.18

if T=-97 then

$\text{var}(\lambda_{-97}) = 1.7499 + (-97)^2(1.7288\times10^{-4}) + 2(-97)(1.7333\times10^{-2}) = 0.0139$

$\lambda = 8.134 + (-97)(0.0551) = 2.7893$

$\theta = -8.1442 + (-0.0552)(-97) = -2.7898$

$$Z(\text{cal.}) = \frac{2.7893}{\sqrt{0.0139}} = 23.659$$

Again since Z(cal.)>Z(tab.) so we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-97 will be

$$p(\theta,\lambda) = \begin{bmatrix} 0.94 & 0.06 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.94, $P(x_2=1|x_1=0)$=0.06 and by substituting in equation (5.1) then $P(x_2=0)$=0.65 which is nearly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.35

if T=-87 then

$\text{var}(\lambda_{-87}) = 1.7499 + (-87)^2(1.7288\times10^{-4}) + 2(-87)(1.7333\times10^{-2}) = 0.0425$

$\lambda = 8.134 + (-87)(0.0551) = 3.3403$

$\theta = -8.1441 + (-0.0552)(-87) = -3.3418$

$$Z(\text{cal.}) = \frac{3.3403}{\sqrt{0.0425}} = 16.203$$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-87 will be

$$p(\theta,\lambda)=\begin{bmatrix} 0.97 & 0.02 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.97, $P(x_2=1|x_1=0)$=0.03 and by substituting in equation (5.1) then $P(x_2=0)$=0.97 which is exactly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.067

if T=-77 then

$\text{var}(\lambda_{-77})= 1.7499+(-77)^2(1.7288\times10^{-4})+2(-77)(1.7333\times10^{-2})= 0.1056$

$\lambda = 8.134+(-77)(0.0551)= 3.8913$

$\theta = -8.1442+(-0.0552)(-77)= -3.8938$

$\text{Z(cal.)}=\dfrac{3.8913}{\sqrt{0.1056}}=11.97$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-77 will be

$$p(\theta,\lambda)=\begin{bmatrix} 0.98 & 0.02 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.98, $P(x_2=1|x_1=0)$=0.02 and by substituting in equation (5.1) then $P(x_2=0)$=0.98 which is exactly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.04

The second example will be allocation 6 to test the hypothesis

$H_0:\lambda=0$ VS $H_1:\lambda\neq 0$

Estimated Hessian matrix is

$$A=\begin{bmatrix} 3.8314\times10^{4} & -3.5647\times10^{6} & 3.835\times10^{4} & -3.5716\times10^{6} \\ -3.5647\times10^{6} & 3.3437\times10^{8} & -3.5716\times10^{6} & 3.3536\times10^{8} \\ 3.835\times10^{4} & -3.5716\times10^{6} & 3.8597\times10^{4} & -3.5992\times10^{6} \\ -3.5716\times10^{6} & 3.3536\times10^{8} & -3.5992\times10^{6} & 3.3843\times10^{8} \end{bmatrix}$$

The variance covariance matrix will the inverse of the above matrix

$$A^{-1}=\begin{bmatrix} 2.0316\times10^{-1} & 2.0306\times10^{-3} & -2.0065\times10^{-1} & -2.0021\times10^{-3} \\ 2.0306\times10^{-3} & 2.0804\times10^{-5} & -1.9976\times10^{-3} & -2.0429\times10^{-5} \\ -2.0065\times10^{-1} & -1.9976\times10^{-3} & 2.0143\times10^{-1} & 2.0041\times10^{-3} \\ -2.0021\times10^{-3} & -2.0429\times10^{-5} & 2.0041\times10^{-3} & 2.0431\times10^{-5} \end{bmatrix}$$

$\mathrm{var}(\lambda_T)=\mathrm{var}(\lambda_1)+T^2\,\mathrm{var}(\lambda_2)+2T\,\mathrm{cov}(\lambda_1,\lambda_2)$

if T=-107

$\mathrm{var}(\lambda_{-107})=2.0143\times10^{-1}+(-1\times07)^2(2.0431\times10^{-5})+2(-107)(2.0041\times10^{-3})=6.4671\times10^{-3}$

$\lambda=5.545+(0.031)(-107)=2.228$

$\theta=-5.585+(0.0315)(-107)=-2.2145$

$$Z(\text{cal.})=\frac{2.228}{\sqrt{6.4671\times10^{-3}}}=27.705$$

Since Z(cal.)>Z(tab.) so we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-107 will be

$$p(\theta,\lambda)=\begin{bmatrix}0.9 & 0.1\\ 0.5 & 0.5\end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.9, $P(x_2=1|x_1=0)$=0.1 and by substituting in equation (5.1) then $P(x_2=0)$=0.83 which is nearly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.18

if T=-97

$\mathrm{var}(\lambda_{-97})= 2.0143\times10^{-1}+(-97)^2(2.0431\times10^{-5})+2(-97)(2.0041\times10^{-3})= 4.8699\times10^{-3}$

$\lambda=5.545+(0.031)(-97)= 2.538$

$\theta=-5.585+(-0.0315)(-97)=-2.5295$

$$Z(\text{cal.})=\frac{2.538}{\sqrt{4.8699\times10^{-3}}}=36.369$$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-97 will be

$$p(\theta,\lambda)=\begin{bmatrix}0.93 & 0.07\\ 0.5 & 0.5\end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.93, $P(x_2=1|x_1=0)$=0.07 and by substituting in equation (5.1) then $P(x_2=0)$=0.87 which is nearly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.13

if T=-87 then

$\mathrm{var}(\lambda_{-87})= 2.0143\times10^{-1}+(-87)^2(2.0431\times10^{-5})+2(-87)(2.0041\times10^{-3)}=7.3588\times10^{-3}$

$\lambda_{-87}=5.545+(0.031)(-87)= 2.848$

$\theta = -5.585 + (-0.0315)(-87) = -2.8445$

$$Z(cal.) = \frac{2.848}{\sqrt{7.3588 \times 10^{-3}}} = 33.199$$

Since Z(cal.)>Z(tab.) so we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-87 will be

$$p(\theta, \lambda) = \begin{bmatrix} 0.95 & 0.05 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2 = 0 | x_1 = 0)$=0.95, $P(x_2 = 1 | x_1 = 0)$=0.05 and by substituting in equation (5.1) then $P(x_2 = 0)$=0.9 which is nearly equal to $P(x_2 = 0 | x_1 = 0)$, also $P(x_2 = 0 | x_1 = 1)$=0.5 , $P(x_2 = 1 | x_1 = 1)$=0.5 and by substituting in equation (5.2) then $P(x_2 = 1)$=0.1

if T=-77, then

$var(\lambda_{-77}) = 2.0143 \times 10^{-1} + (-77)^2(2.0431 \times 10^{-5}) + 2(-77)(2.0041 \times 10^{-3}) = 0.0139$

$\lambda = 5.545 + (0.031)(-77) = 3.158$

$\theta = -5.585 + (-0.0315)(-77) = -3.159$

$$Z(cal.) = \frac{3.158}{\sqrt{0.0139}} = 26.785$$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-77 will be

$$p(\theta, \lambda) = \begin{bmatrix} 0.96 & 0.04 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2=0|x_1=0)$=0.96, $P(x_2=1|x_1=0)$=0.04 and by substituting in equation (5.1) then $P(x_2=0)$=0.92 which is nearly equal to $P(x_2=0|x_1=0)$, also $P(x_2=0|x_1=1)$=0.5 , $P(x_2=1|x_1=1)$=0.5 and by substituting in equation (5.2) then $P(x_2=1)$=0.08

The last example to test if λ equal zero or not is allocation 53 with n=500

Estimated Hessian matrix is

$$A=\begin{bmatrix} 2.5529\times10^{4} & -2.6565\times10^{6} & 2.5034\times10^{4} & -2.6091\times10^{6} \\ -2.6565\times10^{6} & 2.7718\times10^{8} & -2.6091\times10^{6} & 2.7262\times10^{8} \\ 2.5034\times10^{4} & -2.6091\times10^{6} & 2.4846\times10^{4} & -2.5905\times10^{6} \\ -2.6091\times10^{6} & 2.7262\times10^{8} & -2.5905\times10^{6} & 2.7082\times10^{8} \end{bmatrix}$$

The variance covariance matrix will be the inverse of the Hessian matrix

$$A^{-1}=\begin{bmatrix} 1.6147\times10^{-1} & 1.6903\times10^{-3} & -1.5087\times10^{-1} & -1.589\times10^{-3} \\ 1.6903\times10^{-3} & 1.8062\times10^{-5} & -1.5717\times10^{-3} & -1.6932\times10^{-5} \\ -1.5087\times10^{-1} & -1.5717\times10^{-3} & 1.5609\times10^{-1} & 1.6217\times10^{-3} \\ -1.589\times10^{-3} & -1.6932\times10^{-5} & 1.6217\times10^{-3} & 1.7252\times10^{-5} \end{bmatrix}$$

$\text{var}(\lambda_T)=\text{var}(\lambda_1)+T^2\,\text{var}(\lambda_2)+2T\,\text{cov}(\lambda_1,\lambda_2)$

$\text{var}(\lambda_{-107})=1.5609\times10^{-1}+(-107)^2(1.7252\times10^{-5})+2(-107)(1.6217\times10^{-3})=6.5643\times10^{-3}$

$\lambda=3.5772+(0.011)(-107)=2.4002$

$\theta=-3.6295+(-0.0115)(-107)=-2.399$

Therefore $Z(\text{cal.})=\dfrac{2.4002}{\sqrt{6.5643\times10^{-3}}}=29.6728$

Since Z(cal.)>Z(tab.) we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-107 will be

$$p(\theta,\lambda)=\begin{bmatrix} 0.91 & 0.09 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2 = 0|x_1 = 0)$=0.91, $P(x_2 = 1|x_1 = 0)$=0.09 and by substituting in equation (5.1) then $P(x_2 = 0)$=0.86 which is nearly equal to $P(x_2 = 0|x_1 = 0)$, also $P(x_2 = 0|x_1 = 1)$=0.5 , $P(x_2 = 1|x_1 = 1)$=0.5 and by substituting in equation (5.2) then $P(x_2 = 1)$=0.14

ifT=-97, then

$\text{var}(\lambda_{-97})= 1.5609\times10^{-1} +(-97)^2(1.7252\times10^{-5})+2(-97)(1.6217\times10^{-3})= 3.8043\times10^{-3}$

$\lambda = 3.5772+(0.011)(-97)= 2.5102$

$\theta = -3.6295+(-0.0115)(-97)= -2.514$

$$Z(\text{cal.})=\frac{2.5102}{\sqrt{3.8043\times10^{-3}}} = 40.197$$

Since Z(cal.)>Z(tab.) so we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-97 will be

$$p(\theta,\lambda)=\begin{bmatrix}0.92 & 0.08\\ 0.5 & 0.5\end{bmatrix}$$

From the above transition matrix we can find that $P(x_2 = 0|x_1 = 0)$=0.92, $P(x_2 = 1|x_1 = 0)$=0.08 and by substituting in equation (5.1) then $P(x_2 = 0)$=0.87 which is nearly equal to $P(x_2 = 0|x_1 = 0)$, also $P(x_2 = 0|x_1 = 1)$=0.5 , $P(x_2 = 1|x_1 = 1)$=0.5 and by substituting in equation (5.2) then $P(x_2 = 1)$=0.13

if T=-87

$\text{var}(\lambda_{-87})= 1.5609\times10^{-1} +(-87)^2(1.7252\times10^{-5})+2(-87)(1.6217\times10^{-3})= 4.4946\times10^{-3}$

$\lambda = 3.5772+(0.011)(-87)= 2.6202$

$\theta = -3.6295 + (0.0115)(-87) = -2.629$

$$Z(\text{cal.}) = \frac{2.6202}{\sqrt{4.4946 \times 10^{-3}}} = 39.083$$

Since Z(cal.)>Z(tab.) therefore we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-87 will be

$$p(\theta,\lambda) = \begin{bmatrix} 0.93 & 0.07 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2 = 0 | x_1 = 0)$**=0.93,** $P(x_2 = 1 | x_1 = 0)$**=0.07 and by substituting in equation (5.1) then** $P(x_2 = 0)$**=0.88 which is nearly equal to** $P(x_2 = 0 | x_1 = 0)$**, also** $P(x_2 = 0 | x_1 = 1)$**=0.5 ,** $P(x_2 = 1 | x_1 = 1)$**=0.5 and by substituting in equation (5.2) then** $P(x_2 = 1)$**=0.12**

if T=-77, then

$\text{var}(\lambda_{-77}) = 1.5609 \times 10^{-1} + (-77)^2(1.7252 \times 10^{-5}) + 2(-77)(1.6217 \times 10^{-3}) = 8.6353 \times 10^{-3}$

$\lambda = 3.5772 + (0.011)(-77) = 2.7302$

$\theta = -3.6295 + (-0.0115)(-77) = -2.744$

$$Z(\text{cal.}) = \frac{2.7302}{\sqrt{8.6353 \times 10^{-3}}} = 29.38$$

Since Z(cal.)>Z(tab.) so we reject the null hypothesis and accept the alternative hypothesis, the transition matrix for T=-77 will be

$$p(\theta,\lambda) = \begin{bmatrix} 0.94 & 0.06 \\ 0.5 & 0.5 \end{bmatrix}$$

From the above transition matrix we can find that $P(x_2 = 0|x_1 = 0)$=0.94, $P(x_2 = 1|x_1 = 0)$=0.06 and by substituting in equation (5.1) then $P(x_2 = 0)$=0.89 which is nearly equal to $P(x_2 = 0|x_1 = 0)$, also $P(x_2 = 0|x_1 = 1)$=0.5 , $P(x_2 = 1|x_1 = 1)$=0.5 and by substituting in equation (5.2) then $P(x_2 = 1)$=0.11

So after these examples we conclude that all the allocation test significant ($\lambda \neq 0$), and I proved in chapter 3 that if $\lambda = 0$ then the correlation is zero, so we conclude that our data is correlated data.

From the previous analysis one can see that $P(x_2 = 0|x_1 = 1) = P(x_2 = 1|x_1) = 0.5$, and also $P(x_2 = 0|x_1 = 0) > P(x_2 = 1|x_1 = 0)$.

We conclude may be that most of the data is zero, with occasional sequence of 1's in which the (number of 1's) is geonetric with parameter $\frac{1}{2}$, i.e

00001111..10000.....

$\Pr = \left(\frac{1}{2}\right)^{r+1}, r = 0,1,2,.....$ where r=number of 1's after a 1.

Mean of the geomatric parameter p is $\frac{1}{p}$

$\therefore$ mean number of 1's $= 1 + \frac{1}{0.5} = 3$

5.4 Profile Likelihood:-

In those instances where they exist, marginal and conditional likelihoods work well, often with little sacrifice of information. However marginal and conditional likelihoods are available only in very special problems. The profile log-likelihood, while less satisfactory from several points of view, does have the important virtue that it can be used in all circumstances.

Let $\hat{\varphi}_{\lambda}$ be the maximum likelihood estimate of φ for fixed λ. This maximum is assumed here to be unique, as it is for most generalized linear models. The partially maximized log-likelihood function

$$l(\lambda;y) = l(\lambda,\hat{\phi}_{\lambda};y) = \sup_{\varphi} l(\lambda,\varphi;y)$$

is called the profile log-likelihood for φ. Under certain conditions the profile log-likelihood may be used just like any other log-likelihood. In particular the maximum of$l(\lambda;y)$ coincides with the overall maximum likelihood estimate. Further approximate confidence sets for λ may be obtained in the usual way

$$\{\lambda: 2l(\hat{\lambda};y) - 2l(\lambda;y) \le \chi^2_{p,1-\alpha}\}$$

where P=dim(λ). Alternatively though usually less accurately intervals may be based on $\hat{\lambda}$ together with the second derivatives of $l(\lambda;y)$ at the maximum. Such confidence intervals are often satisfactory if dim(λ) is small in relation to the total Fisher information, but are liable to be misleading otherwise, Nelder and McCullagh (1989).

To find the confidence interval for the likelihood functionin our situation we will use the previous result

$$2(l(\hat{\lambda};y) - l(\lambda;y)) \le \chi^2_{1,1-\alpha}$$

$$l(\hat{\lambda};y) \leq l(\lambda;y) + \frac{1}{2}\chi^2_{1,1-\alpha}$$

where $l(\lambda;y)$ is the log likelihood function and we can find the profile likelihood function by substituting in the log likelihood function with different values of λ.

The next example shows how we calculate 90% , 95% confidence intervals and profile likelihood.

Estimated $\lambda = 6.483$ for allocation 58, and minus log likelihood function =159.598 then to find the confidence interval

$90\% = l < 159.598 + \frac{1}{2}(2.706) = 160.951$

$95\% = l < 159.598 + \frac{1}{2}(3.84) = 161.518$

The profile likelihood will be

Lambda	Minus log likelihood function
6.32	164.003
6.33	163.713
6.34	163.424
6.35	163.135
6.36	162.847
6.37	162.560
6.38	162.274
6.39	161.988
6.40	161.703
6.41	161.419

6.42	161.137
6.43	160.857
6.44	160.579
6.45	160.308
6.46	160.048
6.47	159.816
6.48	159.646
6.49	159.607
6.50	159.827
6.51	160.477
6.52	161.746
6.53	163.769
6.54	166.598

The following figures show the profile likelihood and confidence interval for some allocations with the percentage of the one's for each threshold printed inside the figure.

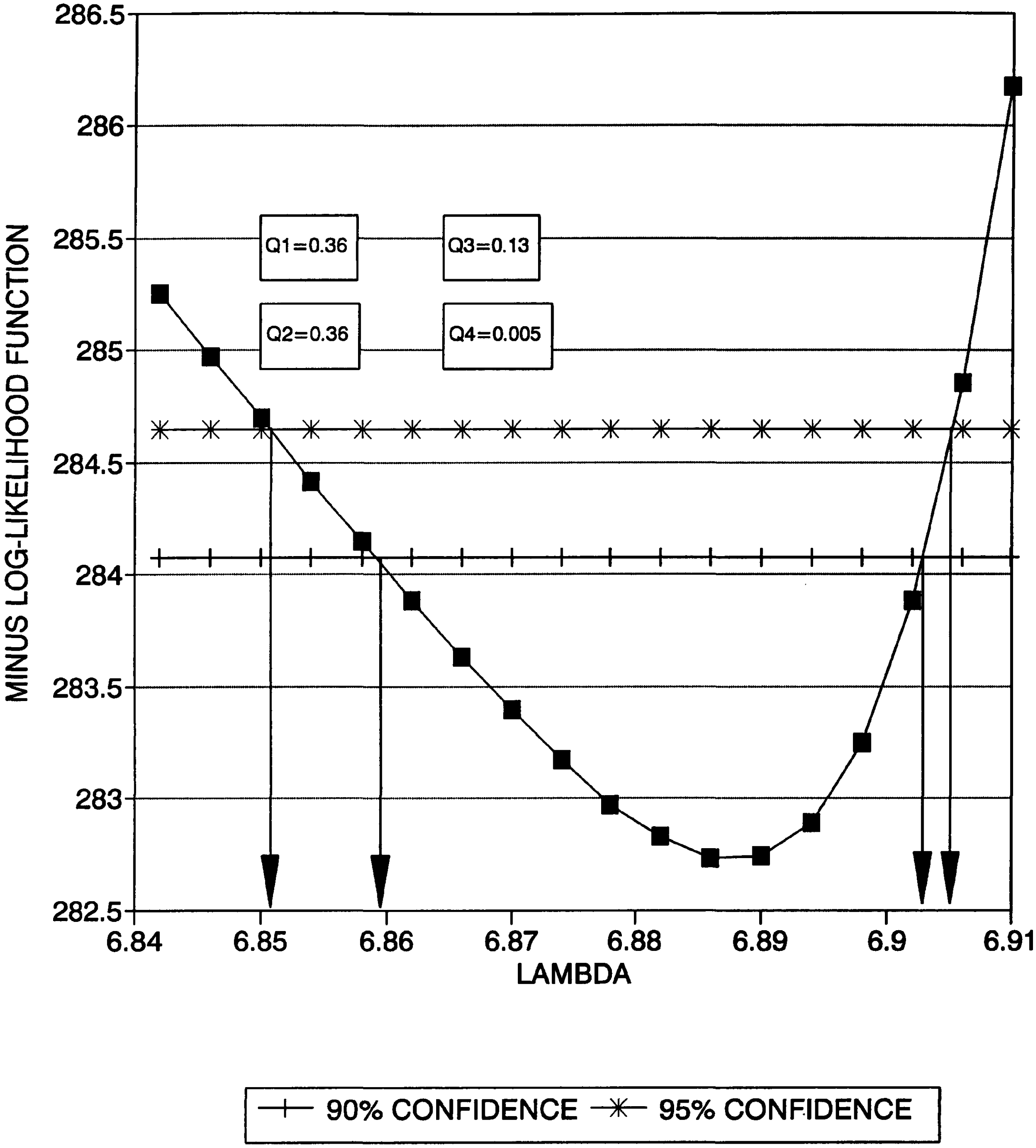


Figure (5.1) Profile Likelihood Function For Allocation (1)

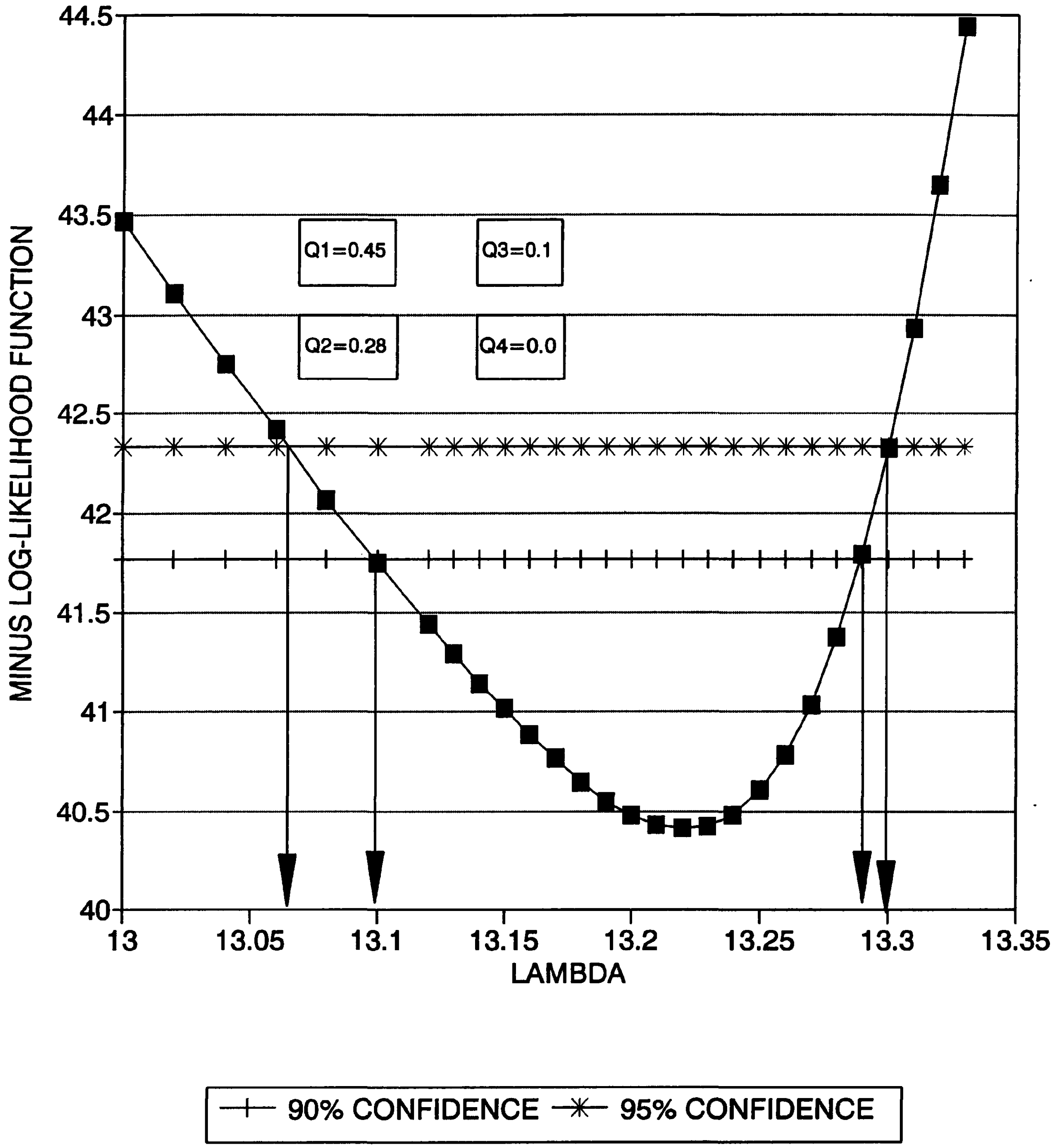


Figure (5.2) Profile Likelihood Function For Allocation (2)

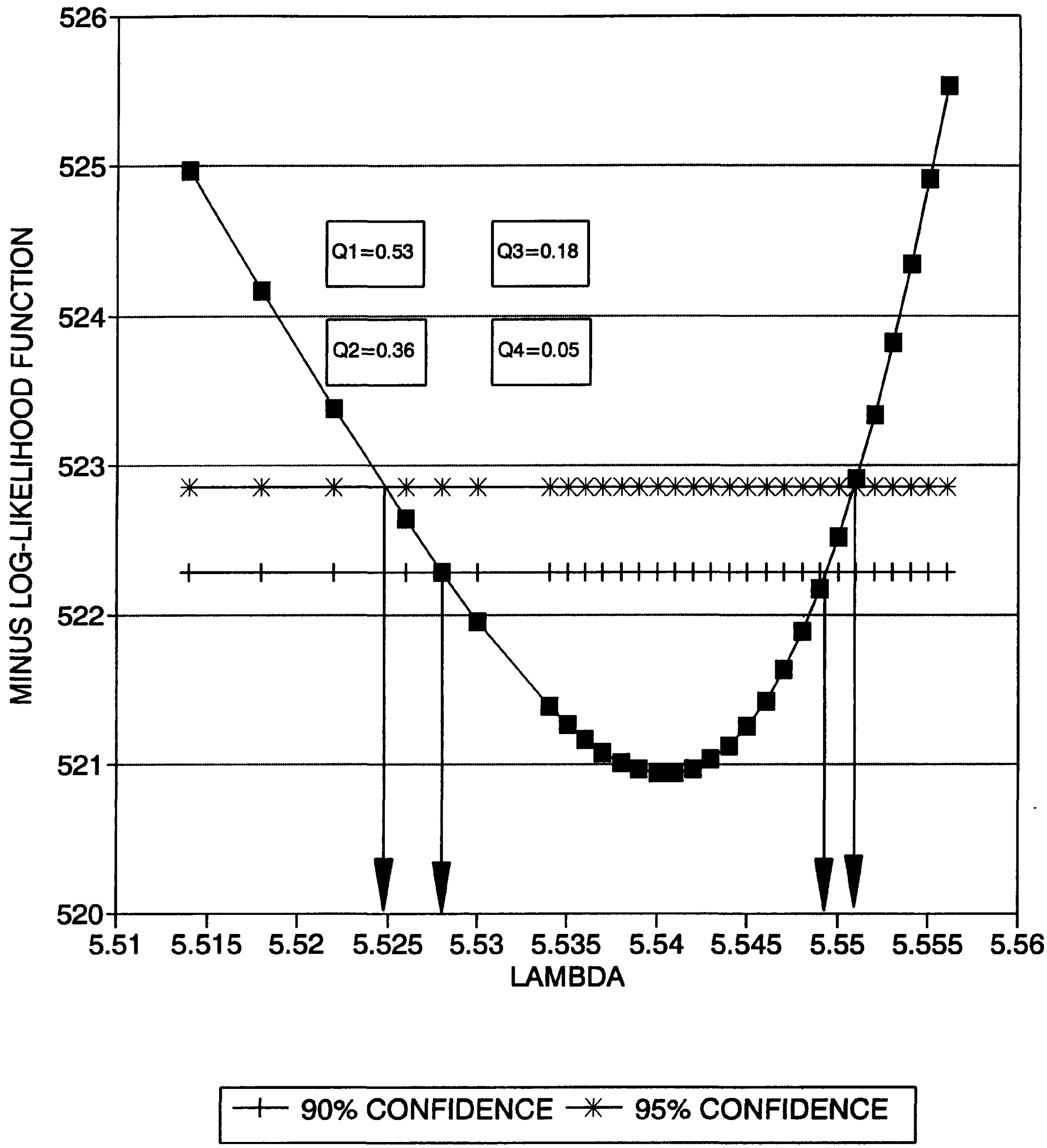


Figure (5.3) Profile Likelihood Function For Allocation (6)

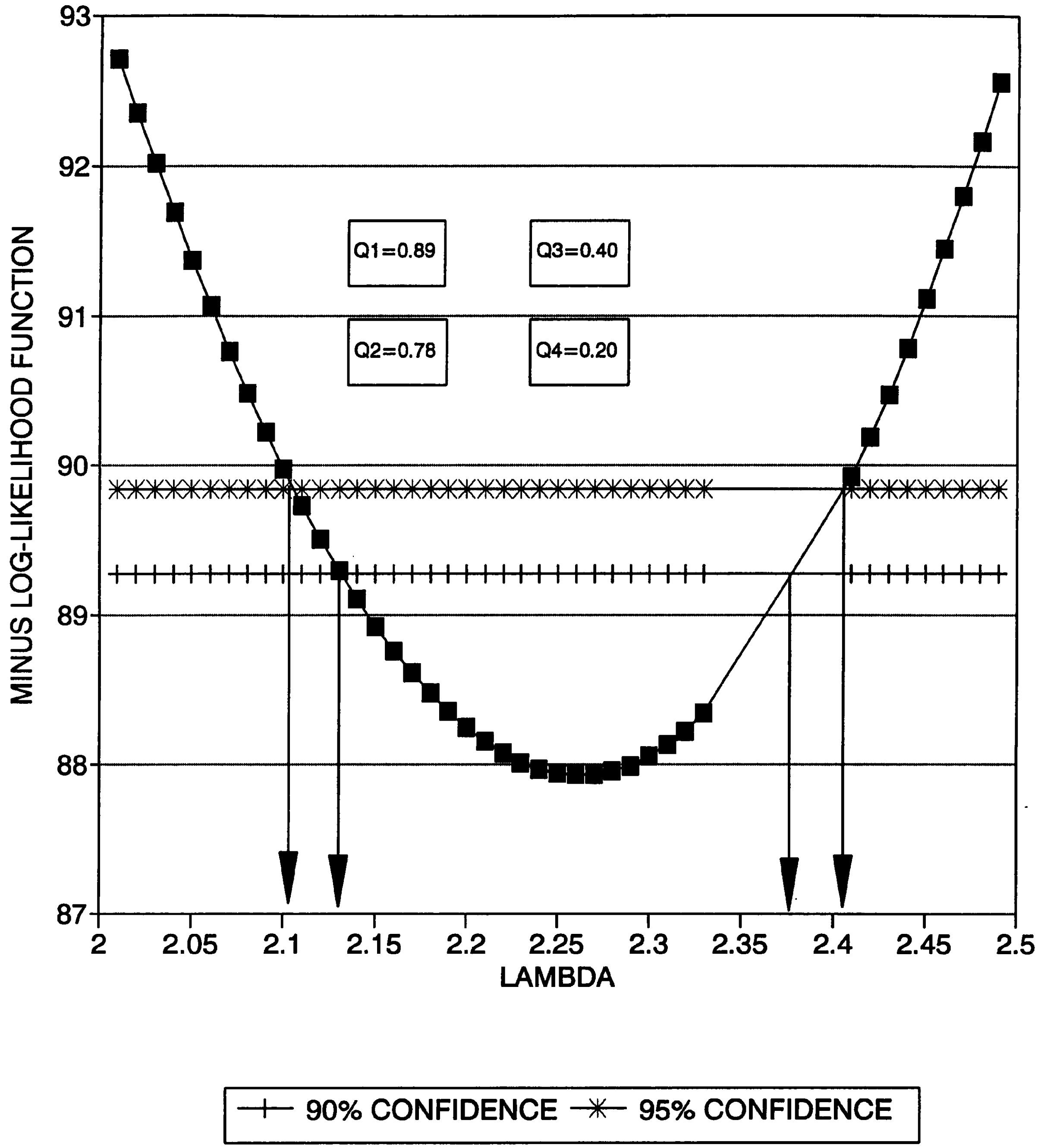


Figure (5.4) Profile Likelihood Function For Allocation (8)

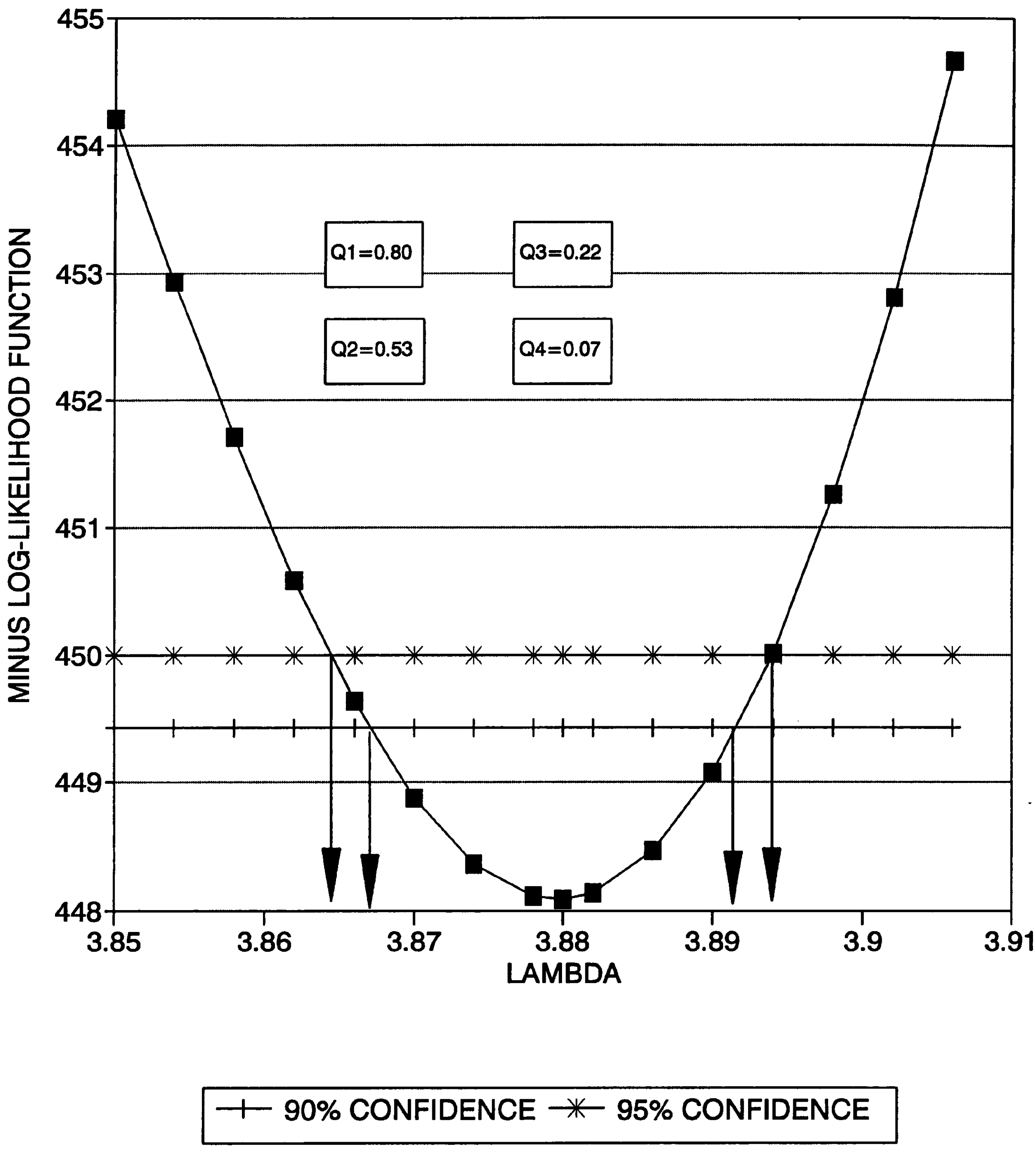


Figure (5.5) Profile Likelihood Function For Allocation (11)

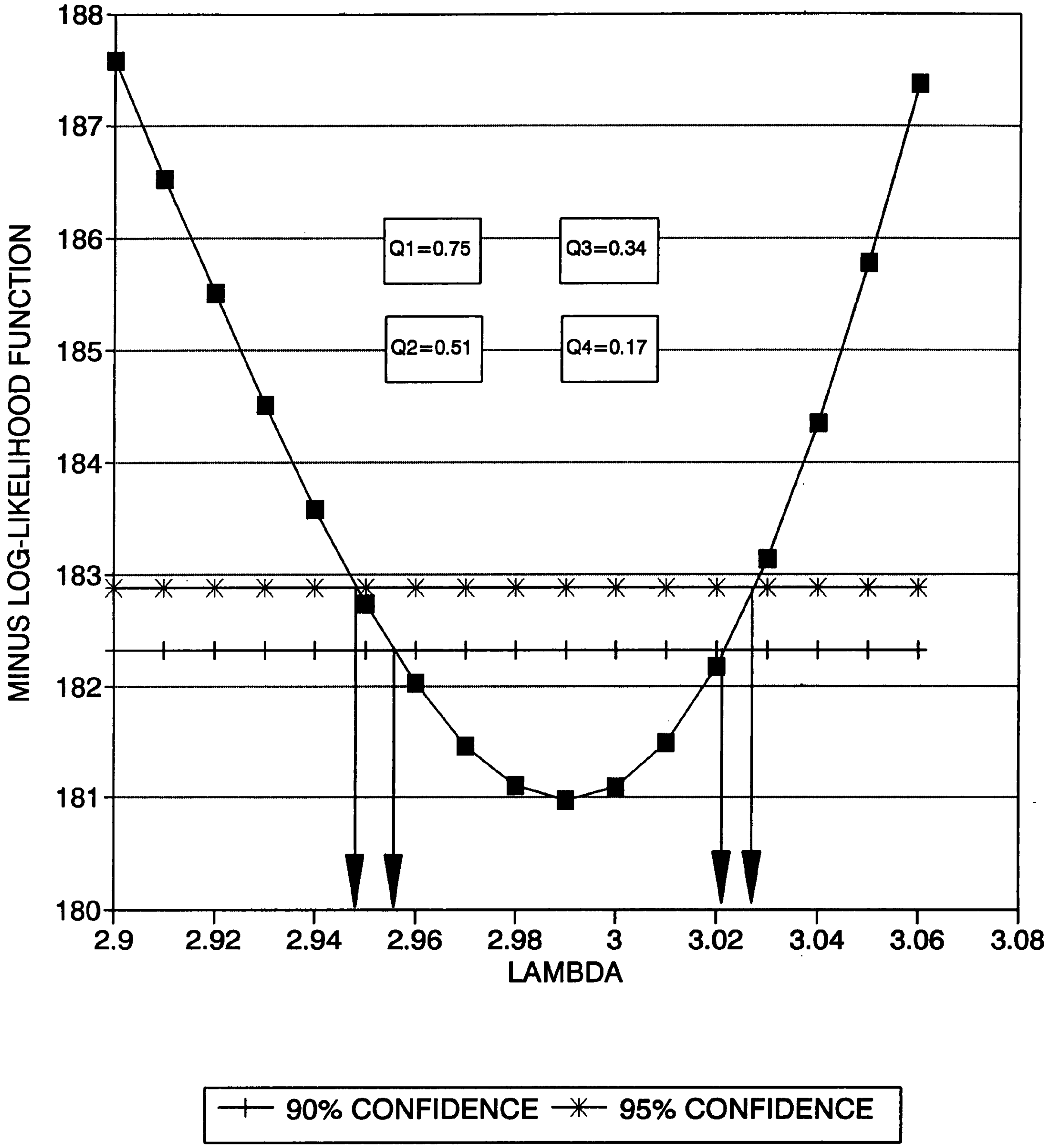


Figure (5.6) Profile Likelihood Function For Allocation (12)

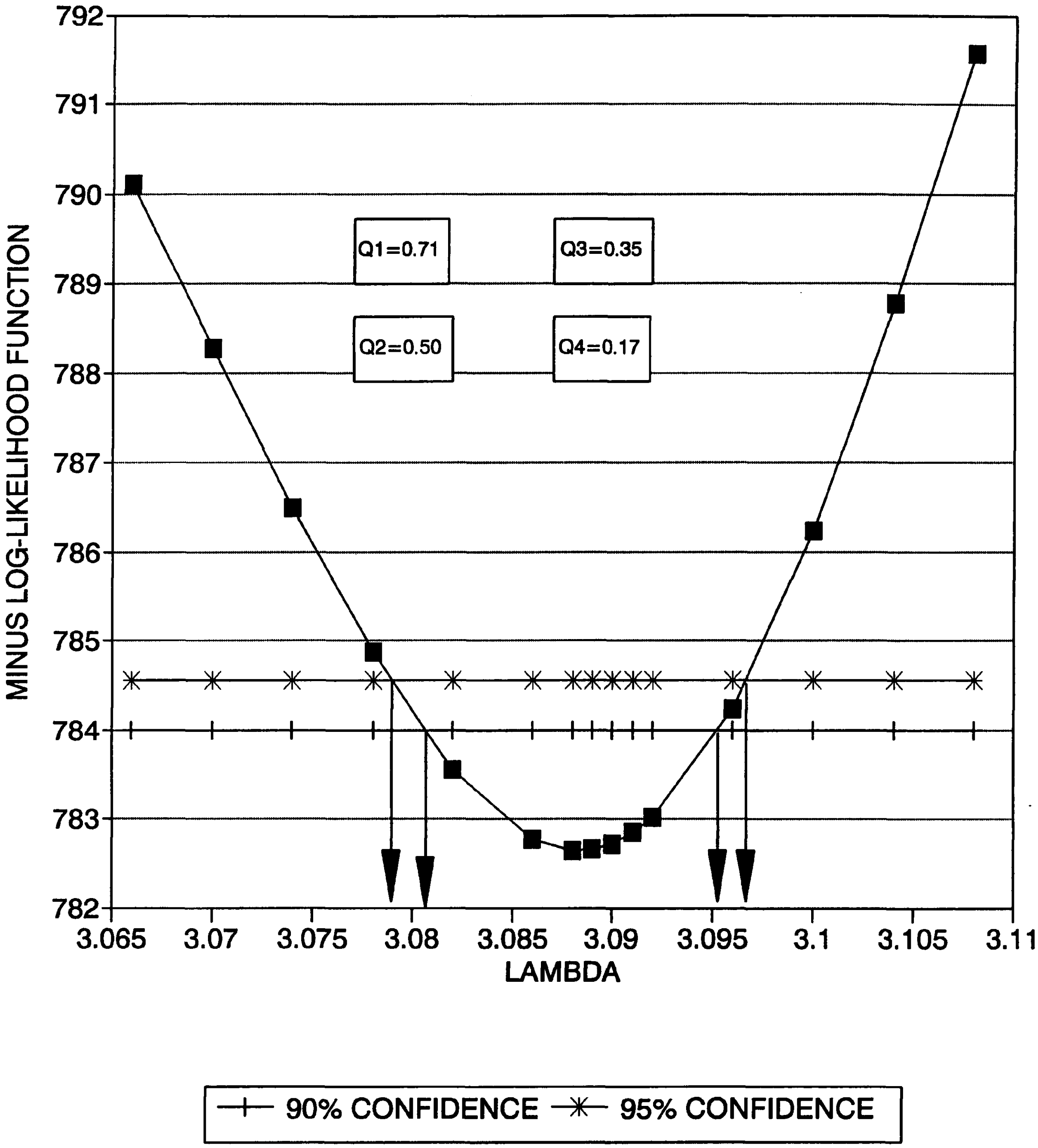


Figure (5.7) Profile Likelihood Function For Allocation (15)

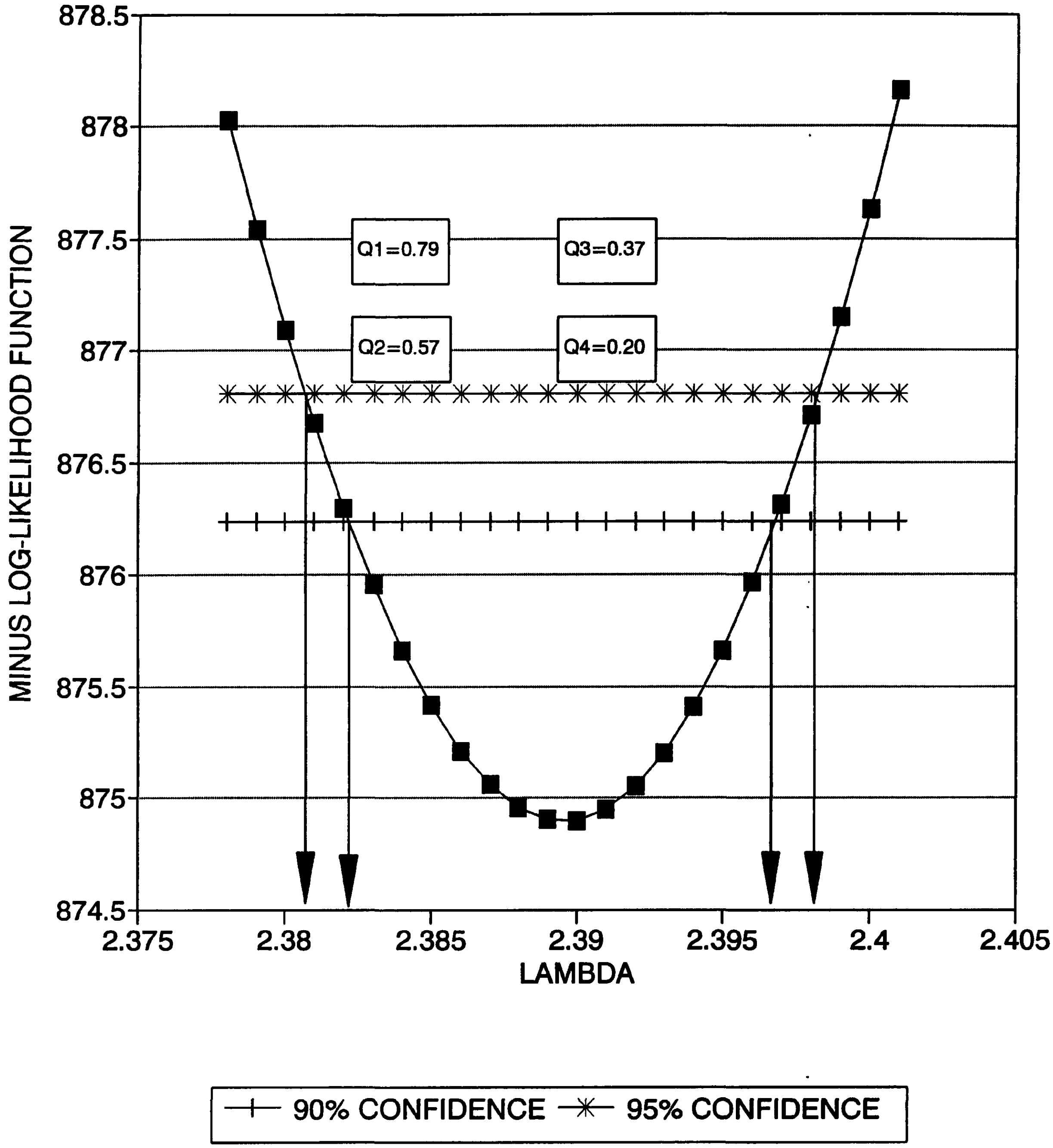


Figure (5.8) Profile Likelihood Function For Allocation (19)

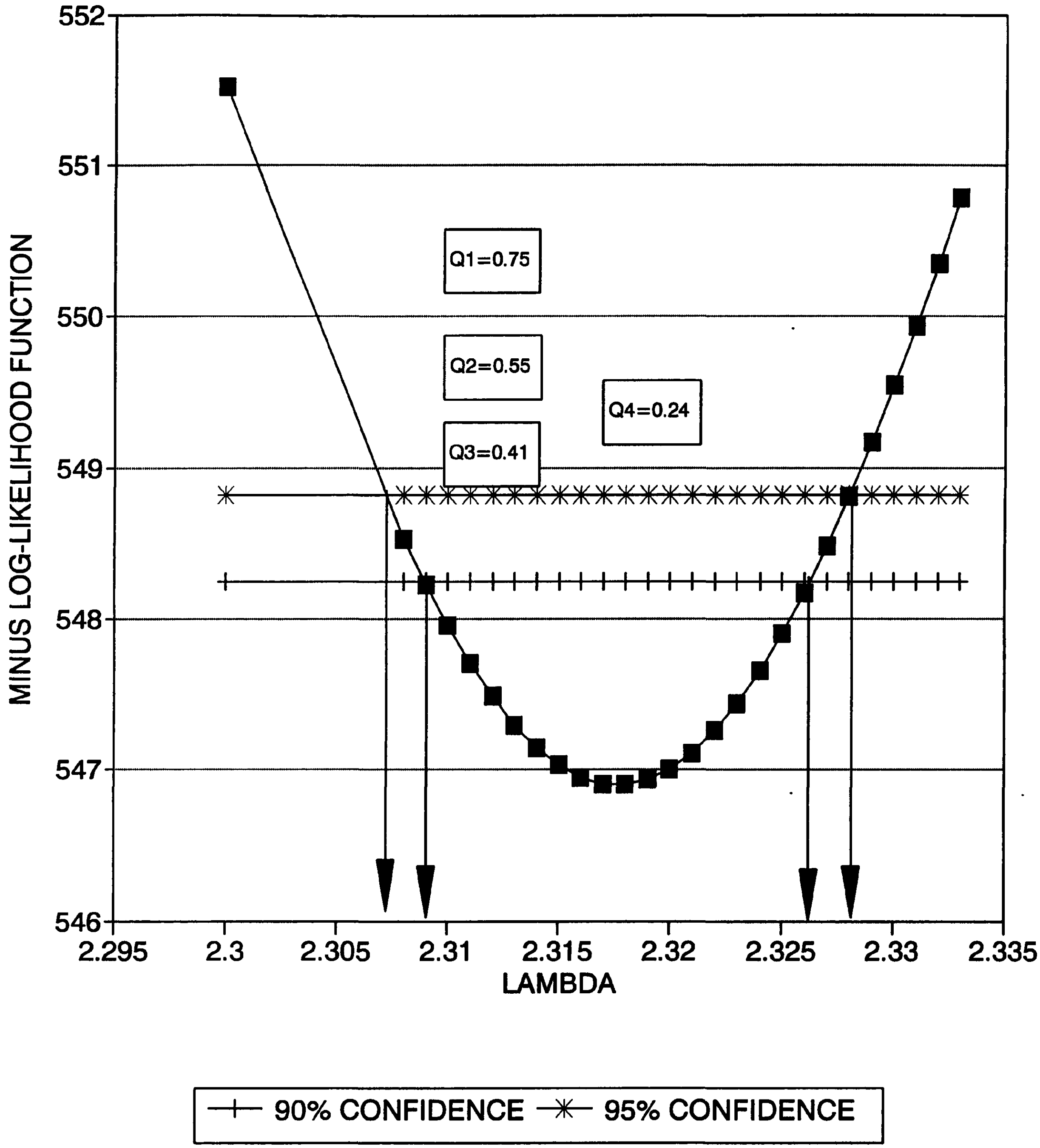


Figure (5.9) Profile Likelihood Function For Allocation (23)

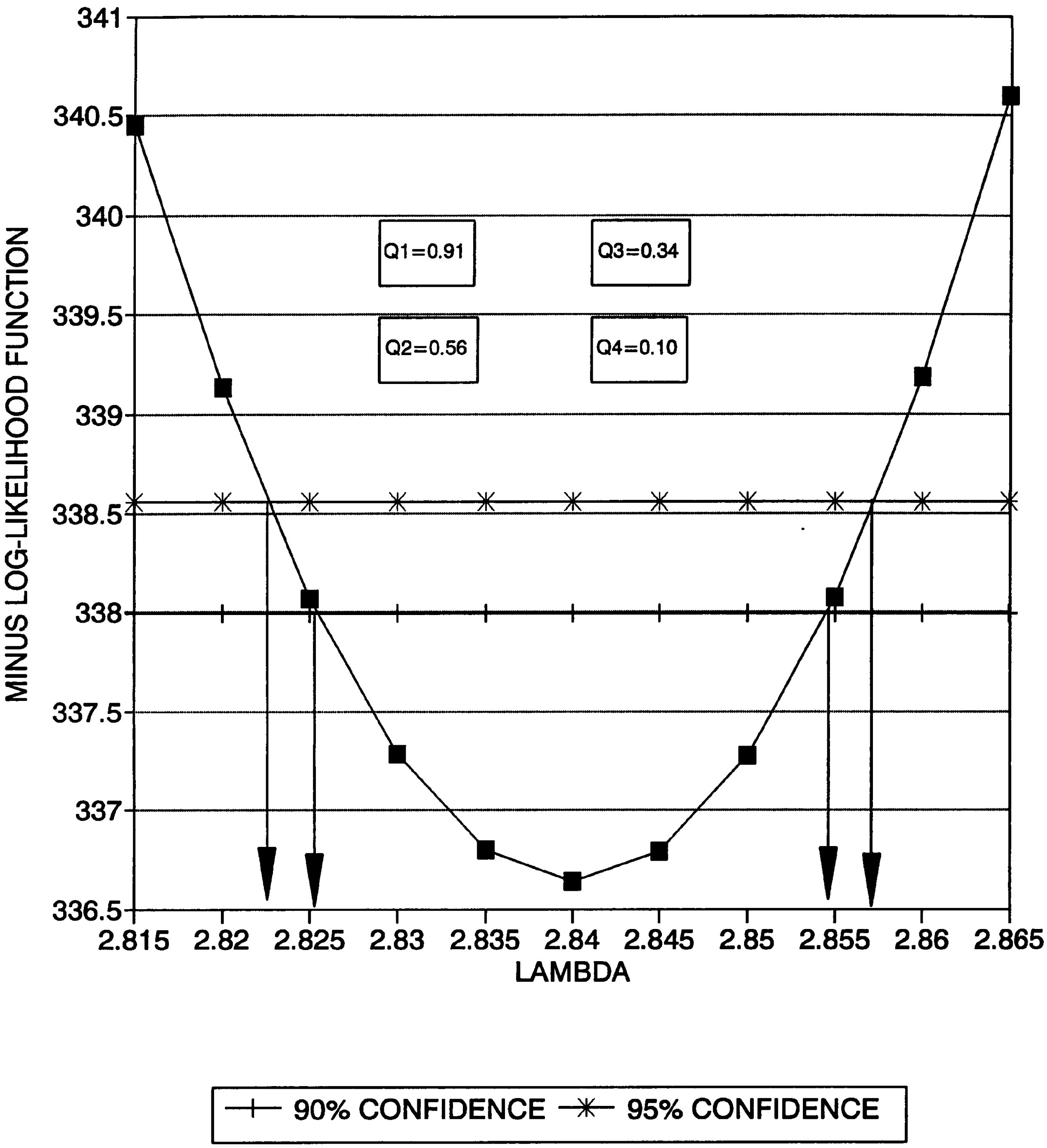


Figure (5.10) Profile Likelihood Function For Allocation (25)

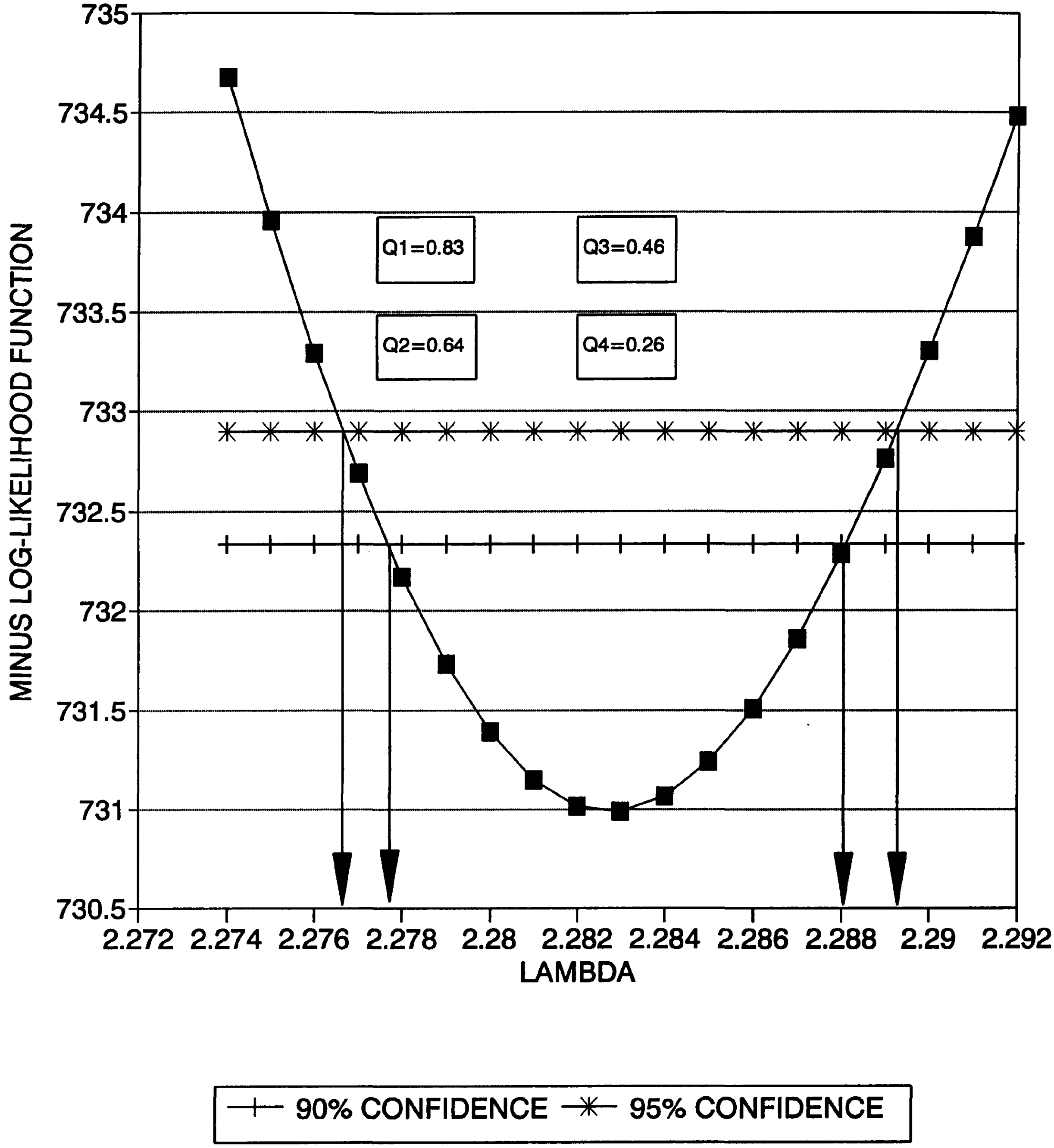


Figure (5.11) Profile Likelihood Function For Allocation (28)

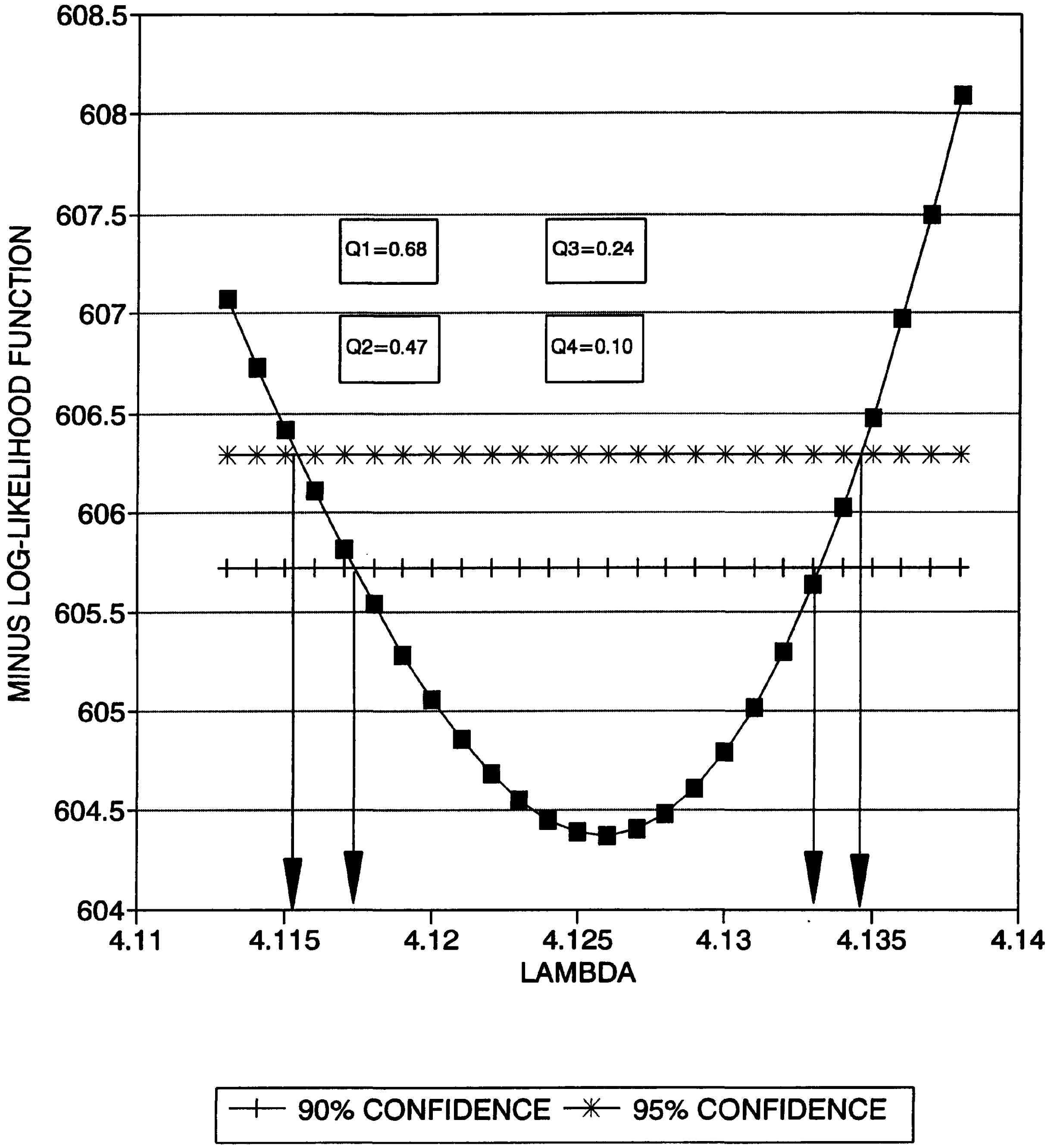


Figure (5.12) Profile Likelihood Function For Allocation (29)

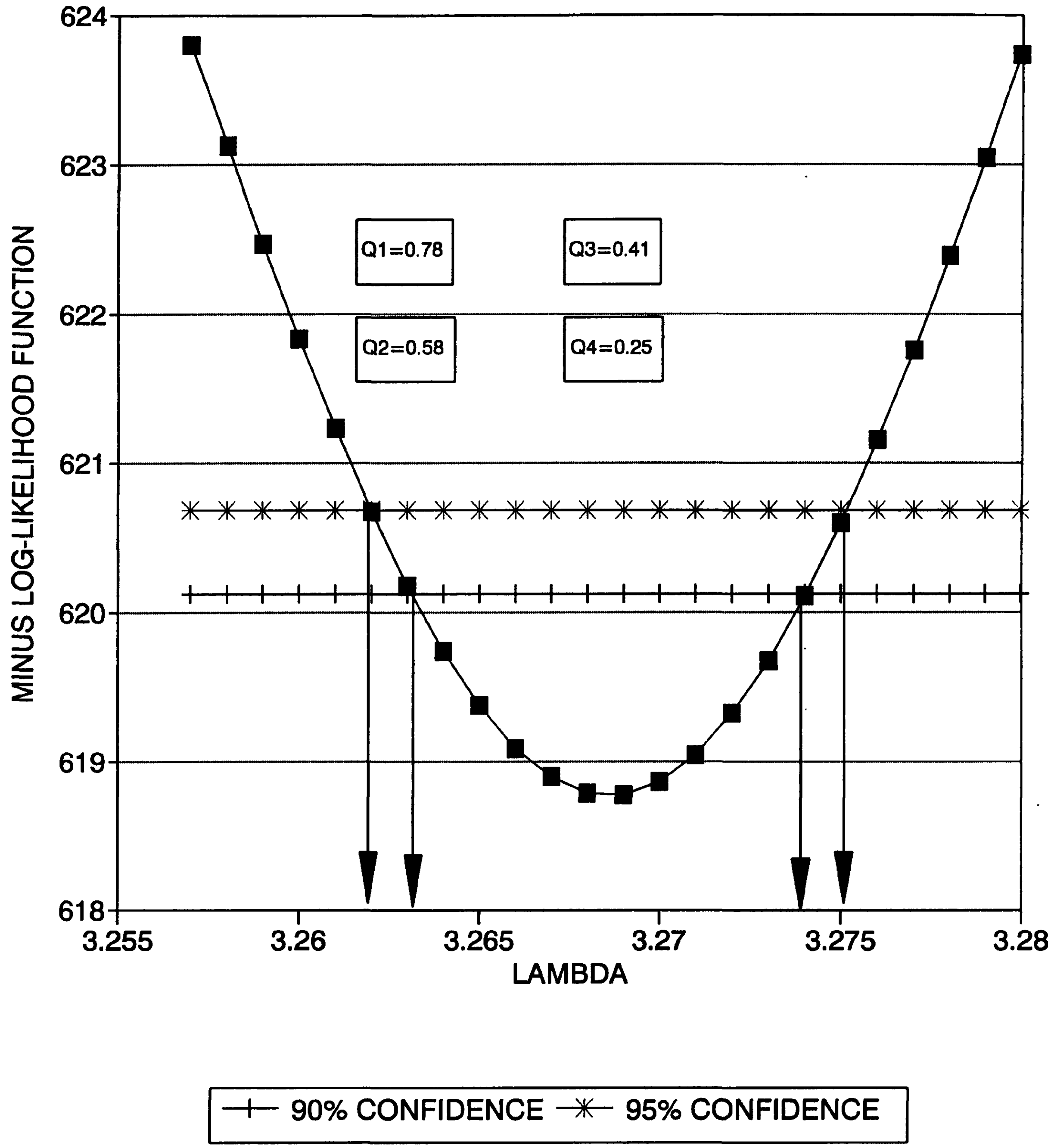


Figure (5.13) Profile Likelihood Function For Allocation (33)

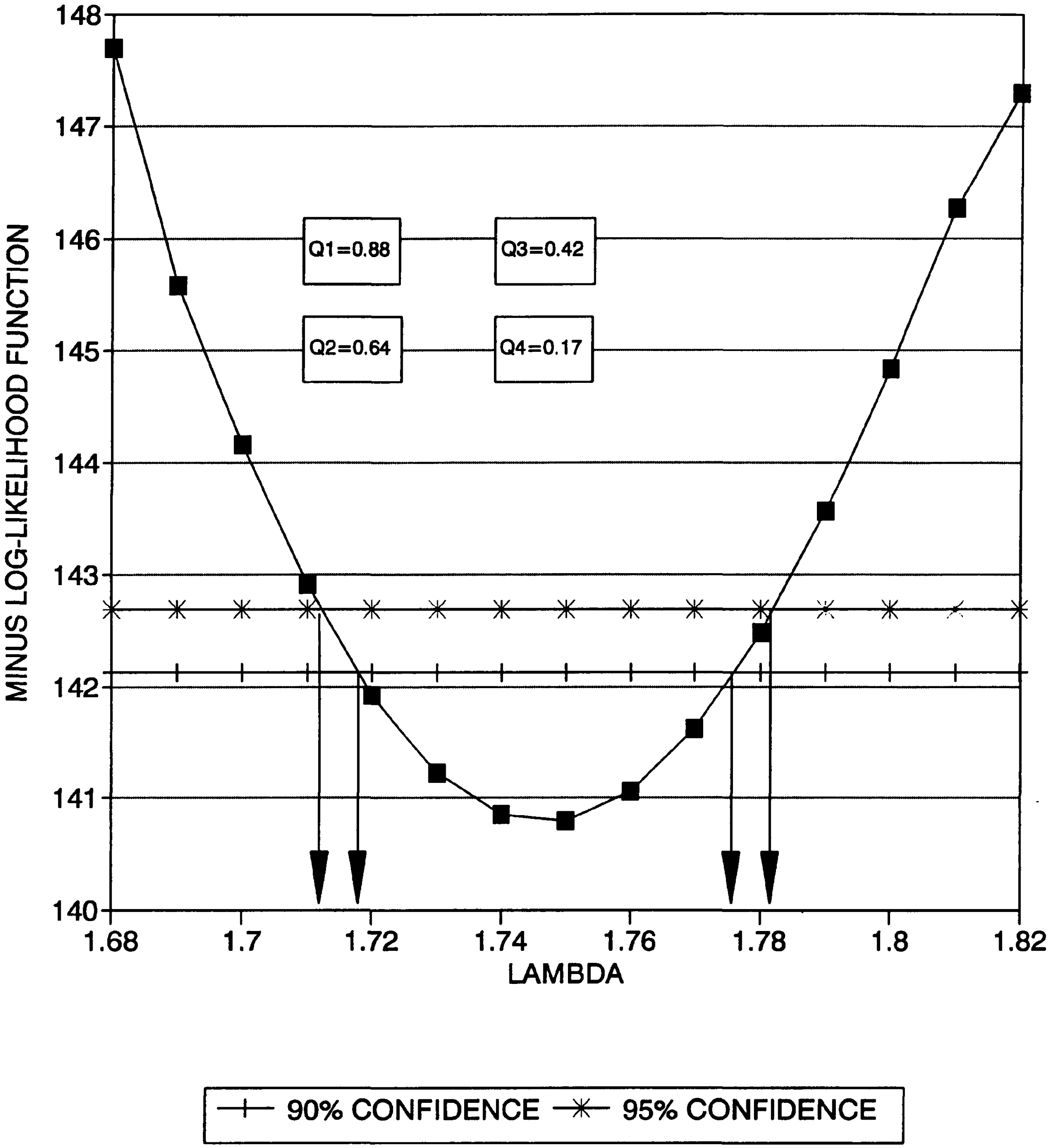


Figure (5.14) Profile Likelihood Function For Allocation (35)

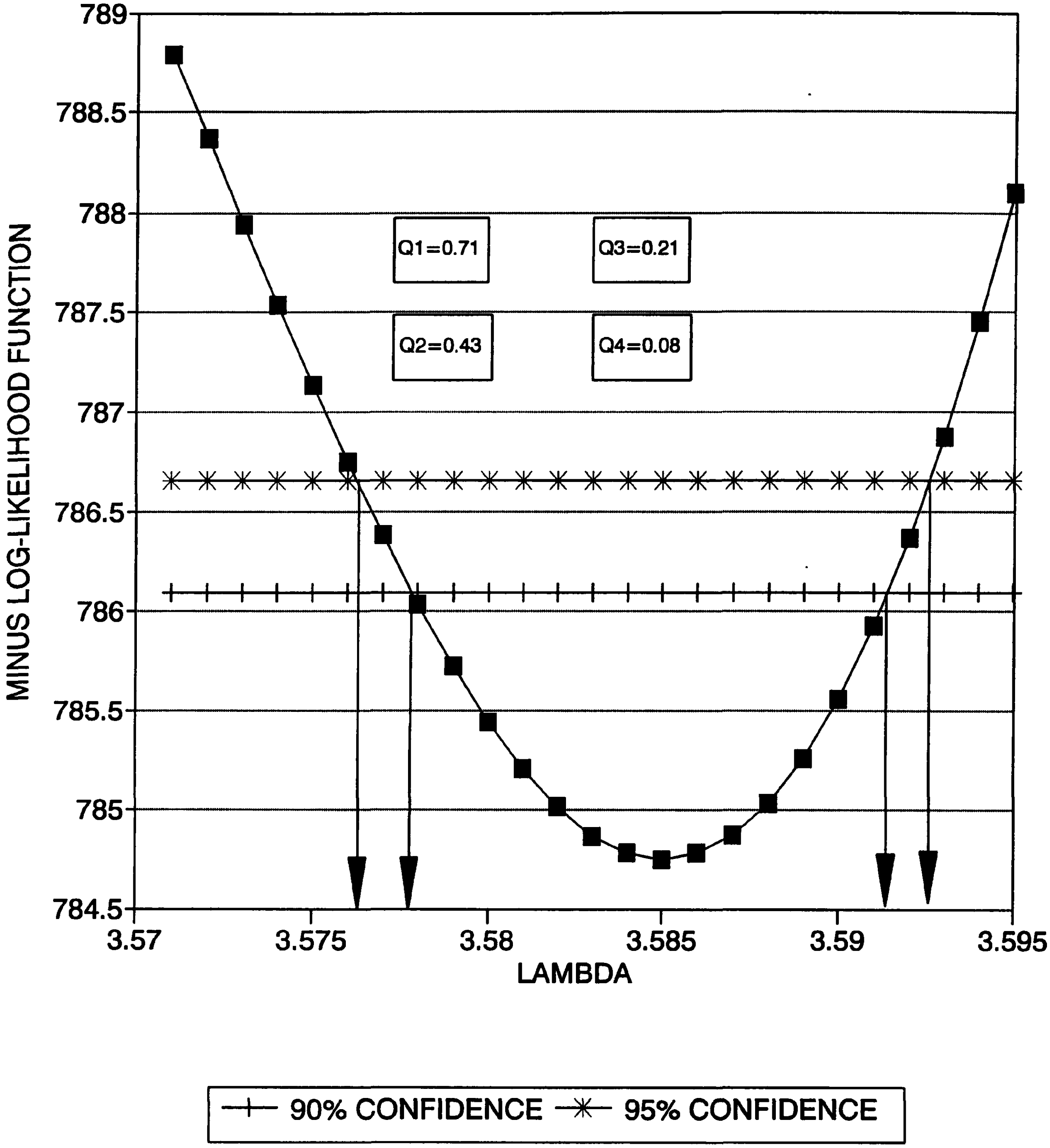


Figure (5.15) Profile Likelihood Function For Allocation (53)

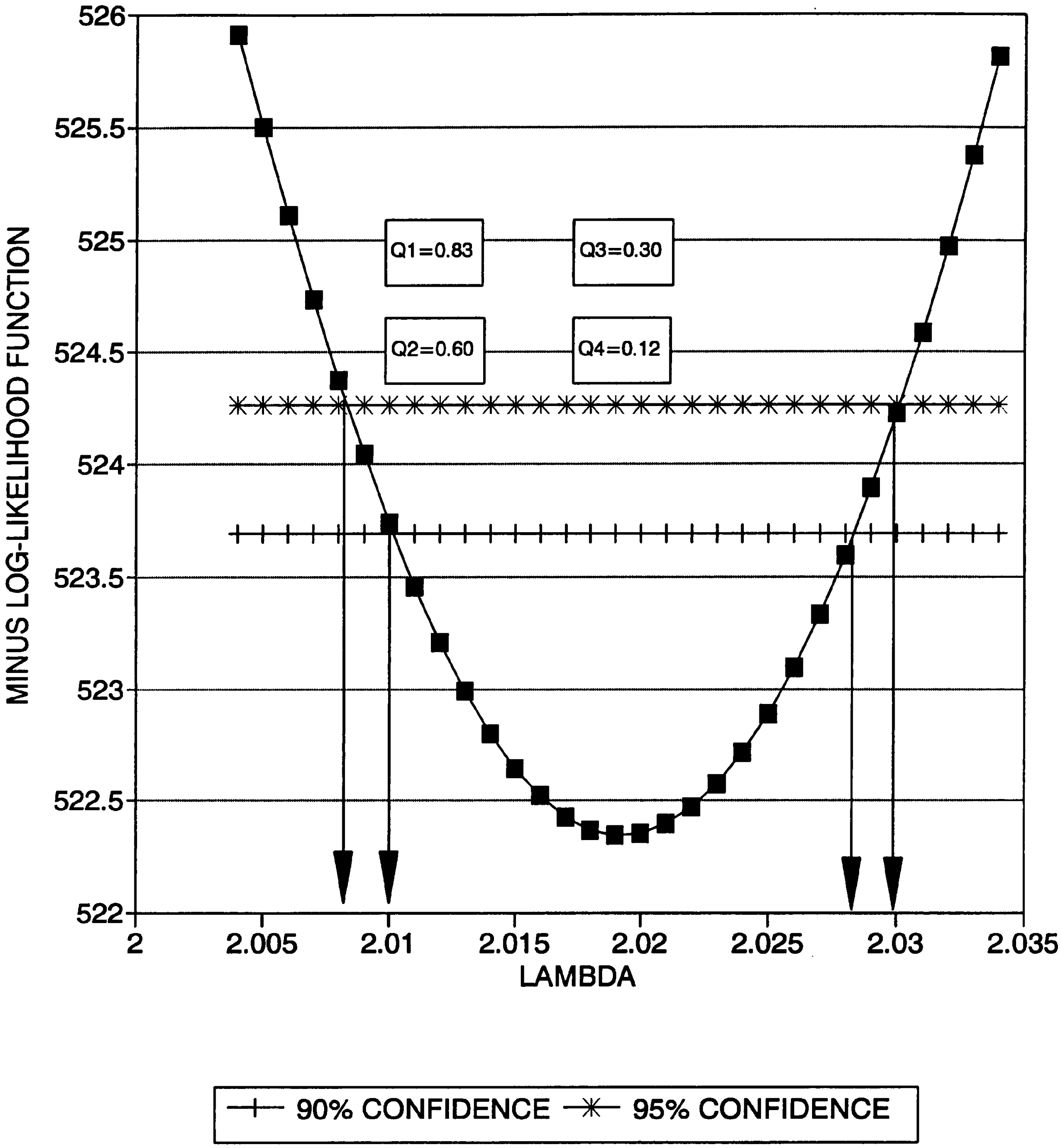


Figure (5.16) Profile Likelihood Function For Allocation (59)

CHAPTER 6

DISCUSSION

In chapter one we described the HF radio wave data set which has been modelled in this thesis and we described previous models developed by other workers for these data, typically at a higher level of aggregation than the "disjoint interval counts" or "binary data" analysed in this thesis.

Chapter two was concerned with a brief introduction to the underlying statistical theory for our modellng techniques.

Chapter three contained the main new theoretical development of this thesis. A previously suggested model for binary counts data which take the form

$$\Pr(Y_k) = \Delta_k^{-1} \exp(y_k^T \theta_k + W_k^T \lambda_k + C_k(Y_k))$$

where $W_k^T = (y_{k1}y_{k2}, y_{k1}y_{k3}, \ldots\ldots, y_{k2}y_{k3}, \ldots\ldots)$, $\theta_k^T = (\theta_{k1}, \theta_{k2}, \ldots\ldots, \theta_{km_k})$ and $\lambda_k^T = (\lambda_{k12}, \lambda_{k13}, \ldots\ldots, \lambda_{k23}, \ldots\ldots)$ are "canonical" parameters, and $\Delta_k = \Delta_k(\theta_k, \lambda_k)$ is the normalisation constant defined by $\Delta_k = \sum \exp(Y_k^T \theta_k + W_k^T + C_k(Y_k))$.

was successfully extended so as to cope with correlated sequences containing considerably more than the twelve, at most correlated binary counts, which this earlier model was competent to deal with. The HF radio data generates sequences of several hundred correlated binary values and hence an adaptation and extension was necessary to this previously suggested multivariate exponential family model.

Several steps of some complexity were required to achieve this successful extension from a few to several hundred observations. The main theoretical problem concerned calculation of the normalisation constant for this multivarite distribution, since this was necessary to evaluation of the likelihood for the data under computing parameter values. Model simplification was the first, and necessary, step, followed by some functional analysis and then numerical analysis of detailed steps in the computing algorithm.

The model then took the form

$$\Pr(y) = \frac{e^{\theta\sum_{i=1}^{n} y_i + \lambda\sum_{i=1}^{n-1} y_i y_{i+1}}}{\sum_{m=0}^{n}\binom{n}{m} e^{\theta m + (m-1)\frac{m}{n}}}$$

and was then successfully demonstrated on the available data. The practical results also discussed in chapter five were to suggest that for this particular set of data (which however could only be a very small fraction of the potentially large amount of binary data collected over the years by this radio detection equipment stet and developed at UMIST) serial correlation was not detectable as measured by statistical significance tests.

This is in contrast to the inferences from the other models as described elsewhere in this thesis. However, from the radio engineer's point of view the conclusion, overall, seems to be that the serial correlation is either non-existent or so small as to make little difference to most of the predictive uses for which the data is put.

However, for small scale local frequency predications, in those propagation regions where serial correlation is (possibly in future analyses) found to be significant, the multivariate model as developed in this chapter could be used to improve local conditioned predictions.

In chapter four multinomial models were fitted to a dissaggregated version of the main "occupancy over threshold" data sequence. The disjoint intervals used here in place of the overlapping intervals as used for previous binomial models had a strong theoretical advantage, since the standard assumption of independence for the data stream of summary counts was thereby made plausible, as opposed to being literally impossible because of the overlapping counts for the binomial model method.

However, although this method was technically superior, it required a large increase in the complexity of the model and on the fitting criterion of most interest to the radio engineer ("how well did it fit the data as collected") it failed to perform as well as the simpler binomial model, let alone produce an improvament.

However, the multinomial model fit was close enough to the binomial fit for most practical purposes and has the major advantage that it could be sensibly and easily extropolated to situations where different thresholds and bands were used.

In chapter five, Markov Chain models and the multivariate exponential models were fitted to selected sequences of binary counts (binary, as opposed to the more usual aggregated occupancy counts collected by the radio engineer). After some development of the Markov theory, its application to a particular set of data from winter 1991 suggested that a second order Markov Chain was a suitable model for these data, although the dependence was weak. This was in contrast to the results from fitting the multivariate exponential model (described in chapter three), and as described earlier in this section.

Then we showed that for a given signal either above or below threshold, what is the probability of the next one. This was very important result since in the past work they relied on this assumption for modelling and predicting the occupancy within bands. In this work we were predicting the signals at one threshold for a signal frequency, rather than for complete bnad, and this can be used to improve conditional one or two steps ahead predication.

APPINDEX (A)
DATA EXAMPLES
MULT-COUNTS OCCUPANCY DATA

Table 1a Multi-counts values for summer day 1982

	TIME	USER	BAND WIDTH	−∞,-107	-107,-97	-97,-87	-87,-77	-77,∞
1	S82D	FM	204	198	6	0	0	0
2	S82D	AM	40	39	1	0	0	0
3	S82D	FM	195	190	4	1	0	0
4	S82D	FM	255	252	3	0	0	0
5	S82D	FMB	200	198	1	1	0	0
6	S82D	FM	350	339	7	4	0	0
7	S82D	AE	305	303	2	0	0	0
8	S82D	FM	45	44	1	0	0	0
9	S82D	FMB	200	199	1	0	0	0
10	S82D	AE	100	98	2	0	0	0
11	S82D	FMA	300	282	16	2	0	0
12	S82D	FM	100	93	7	0	0	0
13	S82D	AE	50	49	1	0	0	0
14	S82D	FB	50	50	0	0	0	0
15	S82D	MM	438	406	21	8	3	0
16	S82D	FM	212	191	12	6	3	0
17	S82D	AE	100	95	3	2	0	0
18	S82D	FMB	250	233	15	2	0	0
19	S82D	FM	480	431	29	12	7	1
20	S82D	AE	250	243	4	3	0	0
21	S82D	FM	220	197	14	6	3	0
22	S82D	B	250	148	44	35	18	5
23	S82D	MM	325	258	25	21	14	7
24	S82D	AE	240	227	10	3	0	0
25	S82D	FM	235	200	21	11	3	0
26	S82D	AM	100	79	13	7	1	0
27	S82D	B	200	146	34	13	5	2
28	S82D	FM	500	452	32	13	3	0
29	S82D	FM	395	324	39	18	14	0
30	S82D	MM	305	270	20	11	2	2
31	S82D	MM	315	232	38	23	13	9
32	S82D	AE	225	213	11	1	0	0
33	S82D	F	460	379	40	26	10	5
34	S82D	B	400	229	73	55	28	15
35	S82D	F	100	92	3	5	0	0
36	S82D	AE	100	81	11	4	3	1
37	S82D	FA	50	27	10	8	3	2
38	S82D	FM	450	301	73	33	27	16
39	S82D	FM	575	421	85	33	20	16
40	S82D	AE	225	199	14	6	3	3
41	S82D	F	250	173	37	19	16	5
42	S82D	B	400	143	74	70	59	55
43	S82D	F	180	89	47	24	11	9
44	S82D	MM	500	364	68	39	23	6
45	S82D	MM	470	311	67	49	29	14
46	S82D	AE	160	138	18	3	1	0
47	S82D	FM	240	150	38	25	18	9
48	S82D	B	200	135	23	23	16	3

Table 1b Multi-counts values for summer day 1982

	TIME	USER	BAND WIDTH	$-\infty$,-107	-107,-97	-97,-87	-87,-77	-77,∞
49	S82D	FM	200	140	26	21	9	4
50	S82D	AM	350	252	72	19	6	1
51	S82D	FM	650	501	78	48	19	4
52	S82D	AE	100	69	21	6	3	1
53	S82D	B	500	122	112	101	84	82
54	S82D	F	400	270	54	41	27	8
55	S82D	F	360	239	55	37	25	4
56	S82D	MM	500	443	41	12	2	2
57	S82D	MM	550	436	59	36	17	2
58	S82D	F	140	110	19	4	6	1
59	S82D	B	350	105	85	63	53	45
60	S82D	AE	130	118	7	3	1	1
61	S82D	F	38	34	2	2	0	0
62	S82D	AM	100	84	9	7	0	0
63	S82D	F	612	500	63	24	18	7
64	S82D	MM	120	92	15	10	2	1
65	S82D	F	400	371	25	3	1	0
66	S82D	F	380	356	17	3	3	1
67	S82D	MM	120	112	6	2	0	0
68	S82D	F	200	185	9	3	2	1
69	S82D	FM	500	472	15	6	5	2
70	S82D	FM	500	481	15	3	1	0
71	S82D	AM	450	440	6	4	0	0
72	S82D	B	420	218	82	52	37	31
73	S82D	AE	130	127	2	1	0	0
74	S82D	MM	400	372	22	5	1	0
75	S82D	MM	455	426	20	8	1	0
76	S82D	F	145	139	4	1	1	0
77	S82D	FM	200	196	4	0	0	0
78	S82D	AE	150	149	1	0	0	0
79	S82D	FM	650	646	3	1	0	0
80	S82D	FM	500	500	0	0	0	0
81	S82D	FM	390	386	3	1	0	0
82	S82D	AM	110	110	0	0	0	0
83	S82D	MM	210	209	1	0	0	0
84	S82D	FM	340	340	0	0	0	0
85	S82D	RA	120	119	1	0	0	0
86	S82D	B	430	421	7	2	0	0
87	S82D	MM	75	75	0	0	0	0
88	S82D	FM	325	- 322	3	0	0	0
89	S82D	FM	500	490	10	0	0	0
90	S82D	FM	500	470	28	2	0	0
91	S82D	FMM	500	486	12	1	1	0
92	S82D	AM	500	500	0	0	0	0
93	S82D	AM	500	500	0	0	0	0
94	S82D	AM	700	700	0	0	0	0
95	S82D	AM	300	300	0	0	0	0

Table 1a Congestion values for summer day 1982

	TIME	USER	BAND WIDTH	-117	-107	-97	-87	-77
1	S82D	FM	204	17	3	0	0	0
2	S82D	AM	40	12	2	0	0	0
3	S82D	FM	195	5	3	1	0	0
4	S82D	FM	255	7	1	0	0	0
5	S82D	FMB	200	5	1	0	0	0
6	S82D	FM	350	10	3	1	0	0
7	S82D	AE	305	4	1	0	0	0
8	S82D	FM	45	9	2	0	0	0
9	S82D	FMB	200	5	0	0	0	0
10	S82D	AE	100	3	1	0	0	0
11	S82D	FMA	300	16	6	1	0	0
12	S82D	FM	100	17	7	0	0	0
13	S82D	AE	50	10	2	0	0	0
14	S82D	FB	50	2	0	0	0	0
15	S82D	MM	438	15	7	3	1	0
16	S82D	FM	212	18	10	4	1	0
17	S82D	AE	100	15	5	2	0	0
18	S82D	FMB	250	14	7	1	0	0
19	S82D	FM	480	22	10	4	2	0
20	S82D	AE	250	8	3	1	0	0
21	S82D	FM	220	19	10	8	1	0
22	S82D	B	250	63	41	23	9	2
23	S82D	MM	325	30	21	13	6	2
24	S82D	AE	240	19	5	1	0	0
25	S82D	FM	235	28	15	6	1	0
26	S82D	AM	100	47	21	8	1	0
27	S82D	B	200	43	27	10	3	1
28	S82D	FM	500	20	10	3	1	0
29	S82D	FM	395	31	18	8	4	0
30	S82D	MM	305	24	11	5	1	1
31	S82D	MM	315	47	26	14	7	3
32	S82D	AE	225	20	5	0	0	0
33	S82D	F	460	39	18	9	3	1
34	S82D	B	400	67	43	24	11	4
35	S82D	F	100	16	8	5	0	0
36	S82D	AE	100	53	19	8	4	1
37	S82D	FA	50	80	45	25	10	6
38	S82D	FM	450	70	33	17	10	4
39	S82D	FM	575	67	27	12	6	3
40	S82D	AE	225	50	12	5	3	1
41	S82D	F	250	70	31	16	8	2
42	S82D	B	400	90	64	47	28	14
43	S82D	F	180	82	50	24	11	5
44	S82D	MM	500	57	27	14	6	1
45	S82D	MM	470	68	34	20	9	3
46	S82D	AE	160	47	14	2	1	0
47	S82D	FM	240	66	37	22	11	4
48	S82D	B	200	66	32	21	9	1

Table 1b Congestion values for summer day 1982

TIME	USER	BAND WIDTH		-117	-107	-97	-87	-77
49	S82D	FM	200	60	32	17	9	1
50	S82D	AM	350	55	28	7	2	0
51	S82D	FM	650	49	23	11	4	1
52	S82D	AE	100	57	31	10	4	1
53	S82D	B	500	96	80	55	33	16
54	S82D	F	400	54	32	19	9	2
55	S82D	F	360	55	34	18	8	1
56	S82D	MM	500	28	11	3	1	0
57	S82D	MM	550	47	21	10	3	0
58	S82D	F	140	44	21	8	5	1
59	S82D	B	350	88	70	40	28	13
60	S82D	AE	130	25	9	4	2	1
61	S82D	F	38	28	10	5	5	0
62	S82D	AM	100	28	16	7	0	0
63	S82D	F	612	34	18	8	4	1
64	S82D	MM	120	37	23	11	2	1
65	S82D	F	400	18	7	1	0	0
66	S82D	F	380	23	8	2	1	0
67	S82D	MM	120	16	7	2	0	0
68	S82D	F	200	21	7	3	2	1
69	S82D	FM	500	13	6	3	1	0
70	S82D	FM	500	10	4	1	0	0
71	S82D	AM	450	14	2	1	0	0
72	S82D	B	420	71	48	29	10	7
73	S82D	AE	130	11	2	2	0	0
74	S82D	MM	400	17	7	1	0	0
75	S82D	MM	455	15	6	2	0	0
76	S82D	F	145	9	4	1	1	0
77	S82D	FM	200	5	2	0	0	0
78	S82D	AE	150	1	1	0	0	0
79	S82D	FM	650	2	1	0	0	0
80	S82D	FM	500	1	0	0	0	0
81	S82D	FM	390	2	1	0	0	0
82	S82D	AM	110	0	0	0	0	0
83	S82D	MM	210	2	0	0	0	0
84	S82D	FM	340	0	0	0	0	0
85	S82D	RA	120	2	1	0	0	0
86	S82D	B	430	3	2	0	0	0
87	S82D	MM	75	3	0	0	0	0
88	S82D	FM	325	3	1	0	0	0
89	S82D	FM	500	9	2	0	0	0
90	S82D	FM	500	22	6	0	0	0
91	S82D	FMM	500	19	3	0	0	0
92	S82D	AM	500	0	0	0	0	0
93	S82D	AM	500	0	0	0	0	0
94	S82D	AM	700	1	0	0	0	0
95	S82D	AM	300	0	0	0	0	0

APPINDEX B
BINARY DATA EXAMPLES

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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0

0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0:

ALLOCATION 58

T=-107

0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 0 1 0 0 1 1 1 1 1 1
1 0 0 0 0 0 0 1 1 1 1 0 1 0 0 1 1 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 1 1 1 1
1 1 1 1 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 1
1 0

T=-97

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 1 0 1 1 0 0
1 0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 1
0 0 0 1 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0
0 0

T=-87

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 0
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0

T=-77

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0

ALLOCATION 59

T=-107

1 1 1 0 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 1 1 0 0 1 1 1 1 0 1 1 1 0 0 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0

0 0 0 0 0 0 0 1 0 0 0 0 1 1 1 1 1 1 1 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 0 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 1 1 1 1 1 1 1 1

1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 :

T=-97

0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 1 0 0 0 0 1 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1

1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 0 1 1 1

1 1 1 1 1 1 0 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 0 0 0 1 0 0

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 1 0 0 1 1 1 1 0 1 1 1 0 0 1 1 0 0 0 0 1 0 0 0

0 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0

1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 0 1 1 0 1 1 1 1 1 0 1 1 1 1 1 0 1 0 0 0 0 0 0

1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 :

T=-87

0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0

1 1 1 1 1 1 1 0 0 1 0 0 0 0 1 0 0 0 0 1 0 1 0 0 1 1 0 1 1 1 1 0 1 0 0 0 0 0 0 0 1 0 0 0 1 1

1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0

0 1 0 0 0 0 1 0 1 0 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0

1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 0 0 1 0 0 0 1 1
1 0 0 0 1 0 0 0 1 1 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 1 1 1 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0
0 0 0 1 1 0 0 1 1 1 0 1 0 0 0 0 0 0 1 1 1 0 1 1 1 0 0 0 :

T=-77

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0
0 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1
1 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 :

APPENDIX C
GENSTAT PROGRAM EXAMPLE
SUMMER DAY PROGRAM
PROGRAM RUN FOR ALL USERS AND MULTI-COUNT $C_5 = (-77, \infty)$

```
SCALAR    [VALUE = 760]                                          N
VARIATE    [VALUES = 1...N]                                  INDEX
UNITS

"Xset DEFINES THE TERMS FOR THE MODEL FITTING"
TEXT [VALUE=' UBand + Sunspot + FSun + F2Sun                  Xset
"C DEFINE THE Q MULTINOMIAL VALUES TO BE USED "
" C5=(-77,INFINITY) "
TEXT [VALUE = 'C5']                                              C
TEXT [VALUE= ' Index,Vlevs,Sunspot,Bandno,C,FC,CErr']         Pset
TEXT [VALUES=' 10,10,10,10,10,10,10 ']                         Fset

" RESULTS WILL BE RESTRICTED TO THESE USERS "
VARIATE    [VALUES= 1...12]
Usrset

" THESE ARE SUMMER SUNSPOT NUMBERS"
VARIATE    [VALUES=95(115,65,46,30.8,14,31,104,189)]     Sunspot
"THESE ARE WINTER SUNSPOT NUMBERS"
" VARIATE   [VALUES=190(137,93,60,16.5,14,18,58,14)]     Sunspot"
CALCULATE Sunspot2=Sunspot*Sunspot

TEXT
[VALUES=FM,AM,FMB,AE,FMA,FB,MM,B,F,FA,RA,FMM]ALLOCS

FACTOR[LABELS=Allocs;  VALUES=((1,2,1,1,3,1,4,1,3,4,5,1,4,6,7,1,4,3,\
1,4,1,8,7,4,1,2,8,1,1,7,7,4,9,8,9,4,10,1,1,4,9,8,9,7,7,4,1,8,1,2,1,4,8,\
9,9,7,7,9,8,4,9,2,9,7,9,9,7,9,1,1,2,8,4,7,7,9,1,4,1,1,1,2,7,1,11,8,7,1,1,\
1,12,2,2,2,1))8]                                               User
FACTOR[ LEVELS=95;   VALUES=((1...95)8)]                     Bandno

VARITE[NVALUES=95;VALUES=204,40,195,255,200,350,305,45,200,100,\
300,100,50,50,438,212,100,250,480,250,220,250,325,240,235,100,\
```

```
200,500,395,305,315,225,460,400,100,100,50,450,575,225,250,400,\
180,500,470,160,240,200,200,350,650,100,500,400,360,500,550,140,\
350,130,38,100,612,120,400,380,120,200,500,500,450,420,130,400,\
455,145,200,150,650,500,390,110,210,340,120,430,75,325,500,500,\
500,500,500,700,300]                                    Bandwidth
VARIATE   [NVALUES=95]                                       Freq
CALCULATE    Freq = 1606 + CUM(Bandwidth)  -  (Bandwidth/2)
VARIATE         VFreq      ;   VALUES  = !(( Freq)8)
VARIATE         VBands     ;   VALUES  =  !((  Bandwidth)8)

VARIATE    Vuser     ;   VALUES =  !(( 1,2,1,1,3,1,4,1,3,4,5,1,4,6,7,1,4,3,\
1,4,1,8,7,4,1,2,8,1,1,7,7,4,9,8,9,4,10,1,1,4,9,8,9,7,7,4,1,8,1,2,1,4,8,\
9,9,7,7,9,8,4,9,2,9,7,9,9,7,9,1,1,2,8,4,7,7,9,1,4,1,1,1,2,7,1,11,8,7,1,1,\
1,12,2,2,2,1)8)

CALCULATE      FBand = Bandno  +  0 / (Vuser. IN .Usrset)



SORT    [INDEX=FBand ; GROUPS=UBand]
CALCULATE    Sunspot = Sunspot / 1000
&          FSun = VFreq*Sunspot
&          F2Sun = VFreq*FSun
READ            [CHANNEL=5]                      C1,C2,C3,C4,C5
```

Identifier	Minimum	Mean	Maximum	Values	Missing	
C1	23.0	248.2	700.0	760	0	
C2	0.00	23.98	143.00	760	0	Skew
C3	0.00	14.01	128.00	760	0	Skew
C4	0.000	8.116	129.000	760	0	Skew
C5	0.000	4.628	126.000	760	0	Skew

```
RESTRICT          C;   CONDITION = (Vuser . IN. Usrset)
```

```
" HELP environment , * "
MODEL [DISTRIBUTION = binomial; LINK = logit ; DISPERSION=*] ||
C;\
NBINPMIAL = VBands; FITTEDVALUES=FC
TERMS \\ Xset
FIT [CONSTANT=e] \\Xset
```

******** Regression Analysis ********

Response variate : C5

Binomial totals : VBands

Distribution : Binomial

Link function : Logit

Fitted terms : Constant + UBand + Sunspot + FSun + F2Sun

*** Summary of analysis ***

	d.f	deviancemean	deviance
Regression	97	9338.9	96.277
Residual	662	961.2	1.452
Total	759	10300.1	13.571
Change	-97	-9338.9	96.571

* MESSAGE : The following units have large residual :

134	-3.33
344	-3.28
402	3.44
595	3.47
615	4.64
623	3.54
718	-3.61

* MESSAGE : The following units have high leverage :

737	0.58

750	0.44
751	0.46
758	0.61

*** Estimates of regression coefficients ***

	estimate	s. e	t
Constant	-14.3	19.9	-0.72
UBand 2	1.6	28.3	0.06
UBand 3	0.1	28.2	0.00
UBand 4	-0.2	28.1	-0.01
UBand 5	0.1	28.2	0.00
UBand 6	-0.5	28.2	-0.02
UBand 7	-0.3	28.2	-0.01
UBand 8	1.6	28.3	0.06
UBand 9	8.4	19.9	0.42
UBand 10	0.8	28.2	0.03
UBand 11	-0.3	28.2	-0.01
UBand 12	0.8	28.2	0.03
UBand 13	1.5	28.3	0.05
UBand 14	1.5	28.3	0.05
UBand 15	8.5	19.9	0.43
UBand 16	9.4	19.9	0.47
UBand 17	0.9	28.3	0.03
UBand 18	8.2	19.9	0.41
UBand 19	8.3	19.9	0.42
Uband 20	0.0	28.2	0.00
UBand 21	8.2	19.9	0.41
UBand 22	11.5	19.9	0.58
UBand 23	10.8	19.9	0.54
UBand 24	7.8	19.9	0.39
UBand 25	8.8	19.9	0.44
UBand 26	8.3	19.9	0.42

UBand 27	10.6	19.9	0.53
UBand 28	9.3	19.9	0.47
UBand 29	9.6	19.9	0.48
UBand 30	10.1	19.9	0.51
UBand 31	11.5	19.9	0.58
UBand 32	0.2	28.2	0.01
UBand 33	9.7	19.9	0.49
UBand 34	11.6	19.9	0.58
UBand 35	8.7	19.9	0.44
UBand 36	9.3	19.9	0.47
UBand 37	10.4	19.9	0.52
UBand 38	10.4	19.9	0.52
UBand 39	9.9	19.9	0.50
UBand 40	8.8	19.9	0.44
UBand 41	10.1	19.9	0.51
UBand 42	12.6	19.9	0.63
UBand 43	10.8	19.9	0.54
UBand 44	10.0	19.9	0.50
UBand 45	11.1	19.9	0.56
UBand 46	8.2	19.9	0.41
UBand 47	10.7	19.9	0.54
UBand 48	11.3	19.9	0.57
UBand 49	10.0	19.9	0.50
UBand 50	7.7	19.9	0.39
UBand 51	9.6	19.9	0.48
UBand 52	10.4	19.9	0.52
UBand 53	12.7	19.9	0.64
UBand 54	10.2	19.9	0.51
UBand 55	10.2	19.9	0.51
UBand 56	9.1	19.9	0.46
UBand 57	10.1	19.9	0.51
UBand 58	9.5	19.9	0.48

UBand 91	-2.3	27.0	-0.09
UBand 92	-2.5	26.9	-0.09
UBand 93	3.8	19.9	0.19
UBand 94	-3.1	26.5	-0.12
UBand 95	-2.4	26.4	-0.09
Sunspot	5.52	3.15	1.75
Fsun	-0.001168	0.000475	-2.46
F2sun	0.58E-07	0.17E-07	3.42

```
66    RESTRICT  FC, QErr ;    CONDITION = RESTRICT(#C)
67    CALCULATE      [PRINT=summary]        CErr = 100*(#C-FC)/VBands
   Identifier      Minimum       Mean      Maximum     Values   Missing
      CErr          -6.203        0.00       7.125        760         0
68      &        CErr = ABS(CErr)
   Identifier    Minimum      Mean      Maximum    Values    Missing
      CErr       0.0000       0.5153     7.1252       760        0     skew
69
70    "PRINT      \\ Pset;   FIELDWIDTH = \\ Fset"
71    CALCULATE      [PRINT + summary]          Q=100*\\ C/VBands
   Identifier     Minimum     Mean     Maximum     values    missing
       Q          0.000       1.292     25.200       760        0      skew
72    VARIATE    [VALUES = 1,2...40]   CBounds
73    HISTOGRAM    [LIMITS = CBounds ; SCALE = 20]              Q
Histogram of Q grouped by CBounds
      -  1.000         568  * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      1.000-2.000      73   * * * *
      2.000-3.000      35   * *
      3.000-4.000      20   *
      4.000-5.000      11   *
      5.000-6.000      8
      6.000-7.000      9
      7.000-8.000      4
```

8.000-9.000	3
9.000-10.000	5
10.000-11.000	2
11.000-12.000	1
12.000-13.000	5
13.000-14.000	4
14.000-15.000	1
15.000-16.000	1
16.000- 17.000	3
17.000-18.000	2
18.000-19.000	0
19.000-20.000	2
20.000-21.000	1
21.000-22.000	0
22.000-23.000	1
23.000-24.000	0
24.000-25.000	0
25.000-26.000	1
26.000-27.000	0
27.000-28.000	0
28.000-29.000	0
29.000-30.000	0
30.000-31.000	0
31.000-32.000	0
32.000-33.000	0
33.000-34.000	0
34.000-35.000	0
35.000-36.000	0
36.000-37.000	0
37.000-38.000	0
38.000-39.000	0
39.000-40.000	0

40.000-	0	

Scale: 1 asterisk represents 20 units.

74 VARIATE [VALUES = 1...50] PBounds

75 HISTOGRAM [LIMITS = PBounds ; SCALE = 20] CErr

Histogram of CErr groubed by PBounds

- 1.000	642	******************************
1.000 - 2.000	68	***
2.000 - 3.000	21	*
3.000 - 4.000	14	*
4.000 - 5.000	9	
5.000 - 6.000	3	
6.000 - 7.000	2	
7.000 - 8.000	1	
8.000 - 9.000	0	
9.000 - 10.000	0	
10.000 - 11.000	0	
11.000 - 12.000	0	
12.000 - 13.000	0	
13.000 - 14.000	0	
14.000 - 15.000	0	
15.000 - 16.000	0	
16.000 - 17.000	0	
17.000 - 18.000	0	
18.000 - 19.000	0	
19.000 - 20.000	0	
20.000 - 21.000	0	
21.000 - 22.000	0	
22.000 - 23.000	0	
23.000 - 24.000	0	
24.000 - 25.000	0	

```
      PROGRAM
LINE  ************************************
 1    SET M=3800;
 2    SMPL 1,M;
 3    READ(FORMAT=FREE,FILE='GLAM5S DATA')
 3    QCAT,INDEX,YR,BAND,N,T;
 4    READ (FORMAT=FREE,FILE=' METTS3 DAT ') F1;
 5    READ (FORMAT=FREE,FILE=' SUNSPOTS DATA ') SUN;
 6    READ (FORMAT=FREE,FILE=' BWIDTH DATA ') W;
 7    F2=F1*F1;
 8    F3=F2*F1;
 9    FSUN=F1*SUN;
10    F2SUN=F2*SUN;
11    SELECT T=4;
12    MSD QCAT YR BAND N F1 F2 F3 FSUN F2SUN;
13    DUMMY BAND;
14    DUMMY QCAT;
15    FRML EQ1 LOGL=N*[QCAT2*XQ2 + QCAT3*XQ3 + QCAT4*XQ4 +
15    QCAT5*XQ4-LOG[1+EXP(XQ2)+EXP(XQ3)+EXP(XQ4)+EXP(XQ5)]];
16    FRML EXPT EXPECTED=W*EXP(QCAT2*XQ2+QCAT3*XQ3+QCAT4*
16    XQ4+QCAT5*XQ5)/(1+EXP(XQ2)+EXP(XQ3)+EXP(XQ4)+EXP(XQ5));
17    FRML EQXQ2 XQ2=B21*F1+B22*F2+B23*F3+B24*SUN+B25*FSUN+
17    B26*F2SUN;
18    FRML EQXQ3 XQ3=B31*F1+B32*F2+B33*F3+B34*SUN+B35*FSUN+
18    B36*F2SUN;
19    FRML EQXQ4 XQ4=B41*F1+B42*F2+B43*F3+B44*SUN+B45*FSUN+
19    B46*F2SUN;
20    FRML EQXQ5 XQ5=B51*F1+B52*F2+B53*F3+B54*SUN+B55*FSUN+
20    B56*F2SUN;
21    EQSUB EXPT EQXQ2 EQXQ3 EQXQ4 EQXQ5;
22    EQSUB (NAME=LOGIT3) EQ1 EQXQ2 EQXQ3 EQXQ4 EQXQ5;
```

```
PARAM  B21-B26   B31-B36   B41-B46   B51-B56;
ML   (HITER=N,HCOV=NBW,MAXIT=100)  LOGIT3;
GENR  EXPT;
DIFF=ABS(N-EXPECTED);
MSD  EXPECTED  DIFF;
HIST   (MIN=0,MAX=100,NBIN=10)DIFF;
END;
```

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS : 560

	MEAN	STD DEV	MIN	MAX	SUM	VAR
QCAT	3.00000	1.41548	1	5	1680.000	2.00358
YR	4.5	2.29334	1	8	2520.000	5.25939
BAND	46.35714	31.76119	1	95	51920	1008.773
N	32.35714	67.1298	0	305	18120	4506.416
F1	10.64982	6.56715	3.0025	8.27499	5963.899	43.127
F2	156.469	170.3303	9.015	581.251	87622.66	29012.42
F3	2755.58	3954.29	27.068	1268.645	1543124	1.564D+7
FSUN	791.814	841.643	42.035	438.7266	443415.8	708362.8
F2SUN	11633.47	17913.6	126.21	1023.000	6514743	3.2089D8

MAXIMUM LIKELIHOOD ESTIMATION

EQUATION: LOGIT3

Working space used: 26565

STARTING VALUES

	B51	B52	B53	B54	B55	B56
VALUE	0.000	0.000	0.000	0.000	0.000	0.000
	B41	B42	B43	B44	B45	B46
VALUE	0.000	0.000	0.000	0.000	0.000	0.000
	B31	B32	B33	B34	B35	B36
VALUE	0.000	0.000	0.000	0.000	0.000	0.000

	B21	B22	B23	B24	B25	B26
VALUE	0.000	0.000	0.000	0.000	0.000	0.000

F=9163. FNEW= 589.6 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=59119

F=6589.6 FNEW=5936.1 ISQZ=1 STEPSIZE=0.5000 CRITERIORION=2093.5

F=5936.1 FNEW=5839.9 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=161.25

F=5839.9 FNEW=5812.0 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=44.467

F=5812.0 FNEW=5802.2 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=15.938

F=5802.2 FNEW=5799.8 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=4.0882

F=5799.8 FNEW=5799.6 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=0.3905

F=5799.6 FNEW=5799.6 ISQZ=0 STEPSIZE=1.0000 CRITERIORION=0.4E-02

CONVERGENCE ACHIEVED AFTER 8 ITERATIONS

17 FUNCTION EVALUTIONS.

LOG OF LIKELIHOOD FUNCTION = -5799.6

NUMBER OF OBSERVATIONS = 560

PARAMETER	ESTIMATE	STANDARD ERROR	T-STATISTIC
B51	-3.215163	0.3611698	-8.902083
B52	0.3505505	0.49008E-01	7.15292
B53	-0.102687E-01	0.16628E-02	-6.175389
B54	-0.242236E-01	0.18895E-01	-1.281981
B55	0.708722E-02	0.32278E-02	2.195658
B56	-0.362479E-03	0.14511E-03	-2.497965
B41	-2.167767	0.1656282	-13.008815
B42	0.2414497	0.2503614E-01	9.644047
B43	-0.756435E-02	0.9493676E-03	-7.967780
B44	-0.274003E-01	0.1235463E-01	-2.217818
B45	0.368531E-02	0.2197146E-02	1.677316
B46	-0.104807E-03	0.9691849E-04	-1.081389
B31	-1.482506	0.6289108E-01	-23.57260
B32	0.1546792	0.8834038E-02	17.50945

B33	-0.4443248E-02	0.3095046E-03	-14.356
B34	-0.2272254E-01	0.4481996E-02	-5.069737
B35	0.3901287E-02	0.8540896E-03	4.567772
B36	-0.1595019E-03	0.3937604E-04	-4.050736
B21	-1.127314	0.3842442E-01	-29.33849
B22	0.1146356	0.5169533E-02	22.17523
B23	-0.3230564E-02	0.1711181E-03	-18.87915
B24	-0.2200017E-02	0.2811201E-02	-7.825899
B25	0.3684900E-02	0.4999541E-03	7.370478
B26	-0.1313949E-03	0.2143541E-04	-6.129803

STANDARD ERRORS COMPUTED FROM ANALYTIC SECOND DERIVATIVES (NEWTON).

PARAMETER	ESTIMATE	STANDARD ERROR	T-STATISTIC
B51	-3.215163	0.3885539	-8.274691
B52	0.3505505	0.5282603E-01	6.635943
B53	-0.1026872E-01	0.1793650E-02	-5.725041
B54	-0.2422355E-01	0.2738607E-01	-0.8845209
B55	0.7087223E-02	0.4207269E-02	1.684519
B56	-0.3624796E-03	0.1658688E-03	-2.185339
B41	-2.167767	0.1198476	-18.08769
B42	0.2414497	0.1822602E-01	13.24753
B43	-0.7564352E-02	0.6949467E-03	-10.88479
B44	-0.2740031E-01	0.9462624E-02	-2.895636
B45	0.3685308E-02	0.1562203E-02	2.359045
B46	-0.1048065E-03	0.6464116E-04	-1.621359
B31	-1.482506	0.2727166E-01	-54.36068
B32	0.1546792	0.3959954E-02	39.06085
B33	-0.4443248E-02	0.1449826E-03	-30.64677
B34	-0.2272254E-01	0.2208962E-02	-10.28652
B35	0.3901287E-02	0.4130235E-03	9.445678

B36	-0.1595019E-03	0.1853864E-04	-8.603757
B21	-1.127314	0.1140544E-01	-98.84009
B22	0.1146356	0.1555627E-02	73.69091
B23	-0.3230564E-02	0.5291072E-04	-61.05688
B24	-0.2200017E-01	0.9384048E-03	-23.44422
B25	0.3684900E-02	0.1554689E-03	23.70185
B26	-0.1313949E-03	0.6244278E-05	-21.04244

STANDARD ERROR COMPUTED FROM COVARIANCE OF ANALYTIC FIRST DERIVATIVES (BHHH).

PARAMETER	ESTIMATE	STANDARD ERROR	T-STATISTIC
B51	-3.21563	0.4542794	-7.077503
B52	0.3505505	0.6449236E-01	5.435536
B53	-0.1026872E-01	0.2269041E-02	-4.525576
B54	-0.2422355E-01	0.1675936E-01	-1.445374
B55	0.7087223E-02	0.3979133E-02	1.781097
B56	-0.3624796E-03	0.2052578E-03	-1.765972
B41	-2.167767	0.2660041	-8.149372
B42	0.2414497	0.4072235E-01	5.929169
B43	-0.7564352E-02	0.1551821E-02	-4.874501
B44	-0.2740031E-01	0.1858786E-01	-1.474097
B45	0.3685308E-02	0.3835004E-02	0.9609660
B46	-0.1048065E-03	0.1824213E-03	-0.5745302
B31	-1.482506	0.1773598	-8.358751
B32	0.1546792	0.2471988E-01	6.257278
B33	-0.4443248E-02	0.8339258E-03	-5.328110
B34	-0.2272254E-01	0.1106335E-01	-2.053858
B35	0.3901287E-02	0.2267256E-02	1.720708
B36	-0.1595019E-03	0.1072944E-03	-1.486581
B21	-1.127314	0.1544759	-7.297673
B22	0.1146356	0.2066464E-01	5.547427

B23	-0.3230564E-02	0.6649648E-03	-4.858248
B24	-0.2200017E-01	0.1006938E-01	-2.184859
B25	0.3684900E-02	0.2090251E-02	1.762898
B26	-0.1313949E-03	0.9732304E-04	-1.350090

STANDARD ERRORS COMPUTED FROM ANALYTIC FIRST AND SECOND DERIVATIVES (EICKER-WHITE)

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS : 560

	MEAN	STD DEV	MIN.	MAX.	SUM	VAR.
EXPECT.	32.35714	66.34075	2.25D-09	300.9192	18119.99	4401.095
DIFF.	2.73600	3.91628	2.25D-09	10.22681	1532.159	15.33722

HISTOGRAM OF DIFF

```
MIN.                                                                  MAX
0.0                                                                   526
-------------------------------------------------------------------------
0  ******************************************************************  526
   ******************************************************************
10 *********************                                               31
   *********************
20 *                                                                   2
   *
30 *                                                                   1
   *
40 *                                                                   0
   *
50 *                                                                   0
   *
```

60 *	0
*	
70 *	0
*	
80 *	0
*	
90 *	0

T.S.P PROGRAM EXAMPLE
SUMMER DAY
PROGRAM RUN FOR ALL MULTI-COUNT AND USER FIXED MOBILE USING C.D.F. OF NORMAL DISTRIBUTION

```
        PROGRAM
LINE
*******************************************************************

 1      SET  M=3800;
 2      SMPL  1,M;
 3      READ(FORMAT=FREE,FILE='GLAM5S
 3      DATA')QCAT,INDEX,YR,BAND,N,T;
 4      READ   (FORMAT=FREE,FILE=' METTS3 DAT ') F1;
 5      READ   (FORMAT=FREE,FILE=' SUNSPOTS DATA ') SUN;
 6      READ   (FORMAT=FREE,FILE=' BWIDTH DATA ') W;
 7      F2=F1*F1;
 8      F3=F2*F1;
 9      FSUN=F1*SUN;
10      F2SUN=F2*SUN;
11      SELECT  T=1;
12      MSD  QCAT  YR  BAND  N  F1  F2  F3  FSUN  F2SUN;
13      DUMMY   BAND;
14      DUMMY   QCAT;
15      SET   B21=0.005;
16      FRML   EQ1   LOGL=N*[QCAT1*LOG[CNORM(XQ2)-0]+QCAT2*LOG
16      [CNORM(XQ3)-CNORM(XQ2)]+QCAT3*LOG[CNORM(XQ4)-
16      CNORM(XQ3)]
16      +QCAT4*LOG[CNORM(XQ5)-CNORM(XQ4)]+QCAT5*LOG[1-
16      CNORM(XQ5)]];
17      FRML  EXPT  EXPECTED =W*EXP[[QCAT1*LOG[ CNORM(XQ2)-0]+
17      QCAT2[CNORM(XQ3)-CNORM(XQ2)]+QCAT3*LOG[CNORM(XQ4)-
17      CNORM(XQ3)]+QCAT4*LOG[CNORM(XQ5)-
CNORM(XQ4)]+QCAT5*LOG
17      [ 1-CNORM(XQ50]]];
18      FRML  EQXQ2   XQ2=B21*(-107)+B22*SUN+B23*FSUN+B24*F2SUN+
18      B25*F1;
19      FRML  EQXQ3   XQ3=B21*(-97)+B22*SUN+B23*FSUN+B24*F2SUN+
19      B25*F1;
```

```
20   FRML EQXQ4 XQ4=B21*(-87)+B22*SUN+B23*FSUN+B24*F2SUN+
20   B25*F1;
21   FRML EQXQ5 XQ5=B21*(-87)+B22*SUN+B23*FSUN+B24*F2SUN+
21   B25*F1;
22   EQSUB EXPT EQXQ2 EQXQ2 EQXQ3 EQXQ4 EQXQ5;
23   EQSUB (NAME=LOGIT5) EQ1 EQXQ2 EQXQ3 EQXQ4 EQXQ5;
24   PARAM B21-B25  B31-B34  B41-B44  B51-B54;
25   ML  (HITER=N,HCOV=NBW,MAXIT=100) LOGIT5;
26   GENR EXPT;
27   DIFF=ABS(N-EXPECTED);
28   MSD EXPECTED  DIFF;
29   HIST (MIN=0 , MAX=100, NBIN=10)DIFF;
30   END;
```

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS : 1120

	MEAN	STD DEV	MIN	MAX	SUM	VAR
QCAT	3.00000	1.41548	1	5	3360.000	2.00179
YR	4.5	2.29334	1	8	5040.000	5.25469
BAND	46.35714	31.76119	1	95	51920	1008.773
N	71.50714	139.9917	0	649.000	80088.00	19597.67
F1	13.89962	9.49204	1.70800	29.84999	15567.57	90.0987
F2	283.2177	288.7442	2.91726	891.0217	317203.8	83373.22
F3	6617.284	8003.361	4.98268	26506.99	7411358	6.41D+07
FSUN	1033.437	1161.012	23.91199	5641.645	1157448	1347948
F2SUN	21057.23	30847.21	40.84164	168403.1	2.36D+07	9.52D+08

MAXIMUM LIKELIHOOD ESTIMATION

EQUATION: LOGIT3

Working space used: 26565

STARTING VALUES

	B21	B22	B23	B24	B25
VALUE	0.005	0.000	0.000	0.000	0.000

F=0.12E+6 FNEW=55157 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=0.12E6

F=55157 FNEW=48625 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=11088

F=48625 FNEW=47726 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=1601.3

F=47726 FNEW=47683 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=82.266

F=47683 FNEW=47683 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=0.3987

CONVERGENCE ACHIEVED AFTER 5 ITERATIONS
10 FUNCTION EVALUATIONS.

LOG OF LIKELIHOOD FUNCTION = -47682.5

NUMBER OF OBSERVATIONS= 1120

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.103561E-03	113.3522
B22	0.381808E-01	0.363799E-03	104.9501
B23	-0.416143E-02	0.512938E-04	-81.12938
B24	0.976854E-04	0.160984E-05	60.68027
B25	0.1553404	0.106783E-02	145.4736

STANDARD ERRORS COMPUTED FROM ANALYTIC SCOND DERIVATIVES (NEWTON).

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.122160E-04	960.9411
B22	0.381808E-01	0.398566E-04	957.9529
B23	-0.416143E-02	0.533449E-05	-780.0994
B24	0.976854E-04	0.171240E-06	570.4583
B25	0.1553404	0.126054E-03	1232.331

STANDARD ERRORS COMPUTED FROM CONVERIANCE OF ANALYTIC FIRST DERIVATIVES (BHHH).

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.962655E-03	12.19426
B22	0.381808E-01	0.341521E-02	11.17963
B23	-0.416143E-02	0.502525E-03	-8.281036
B24	0.976854E-04	0.154465E-04	6.324121
B25	0.1553404	0.962682E-02	16.13622

STANDARD ERRORS COMPUTED FROM ANALYTIC FIRST AND SECOND DERIVATIVES (EICKER-WHITE).

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS: 1120

	MEAN	STD DEV	MIN.	MAX.	SUM	VAR.
EXPECT.	71.50713	138.9511	0.00001	643.7893	80087.98	19307.40
DIFF.	19.34163	30.92411	0.00001	27.03441	21662.62	956.3007

HISTOGRAM OF DIFF

NUMBER OF OBSERVATIONS: 1081

```
MIN                                                                  MAX
8.00                                                                 664
------------------------------------------------------------------------
-
0  ************************************************************      664
   ************************************************************
10 ********************                                               150
   ********************
20 ***********                                                        82
   ***********
30 ******                                                             48
   ******
40 *****                                                              38
   *****
50 ****                                                               29
   ****
60 ****                                                               30
   ****
70 **                                                                 18
   **
80 *                                                                  8
   *
90 **                                                                 14
   **
------------------------------------------------------------------------
8.000                                                                644.0
MIN.
      MAX.
END  OF  OUTPUT
```

T.S.P PROGRAM EXAMPLE

SUMMER DAY

PROGRAM RUN FOR ALL MULTI-COUNTS AND USER

FIXED MOBILE USING C.D.F. OF NORMAL DISTRIBUTION

(FULL MULTINOMIAL MODEL)

```
      PROGRAM
LINE
*************************************************************
1     SET  M=3800;
2     SMPL  1,M;
3     READ(FORMAT=FREE,FILE='GLAM5SDATA')
3     QCAT,INDEX,YR,BAND,N,T;
4     READ  (FORMAT=FREE,FILE=' METTS3 DAT ') F1;
5     READ  (FORMAT=FREE,FILE=' SUNSPOTS DATA ') SUN;
6     READ  (FORMAT=FREE,FILE=' BWIDTH DATA ') W;
7     F2=F1*F1;
8     F3=F2*F1;
9     FSUN=F1*SUN;
10    F2SUN=F2*SUN;
11    SELECT  T=1;
12    MSD  QCAT  YR  BAND  N  F1  F2  F3  FSUN  F2SUN;
13    DUMMY   BAND;
14    DUMMY   QCAT;
15    SET   B21=0.011738;
16    SET   B22=3.25;
17    SET   B23=-0.354502;
18    SET   B24=0.0083216;
19    SET   B25=13.23307;
20    FRML  EQ1  LOGL=N*[QCAT1*LOG[CNORM(XQ2)-0]+QCAT2*LOG
20    [CNORM(XQ3)-CNORM(XQ2)]+QCAT3*
20    LOG[CNORM(XQ4)-CNORM(XQ3)]
20    +QCAT4*LOG[CNORM(XQ5)-CNORM(XQ4)]+QCAT5*LOG[1-
20    CNORM(XQ5)]];
21    FRML  EXPT  EXPECTED =W*EXP[[QCAT1*LOG[ CNORM(XQ2)-0]+
21    CAT2[CNORM(XQ3)-CNORM(XQ2)]+QCAT3*LOG[CNORM(XQ4)-
21    NORM(XQ3)]+QCAT4*LOG[CNORM(XQ5)-CNORM(XQ4)]+QCAT5*LOG
21     1-CNORM(XQ50]]];
```

```
22      RML  EQXQ2  XQ2=B21*((-107)+B22*SUN+B23*FSUN+B24*F2SUN
22      B25*F1);
23      RML  EQXQ3  XQ3=B21*((-97)+B22*SUN+B23*FSUN+B24*F2SUN
23      B25*F1);
24      RML  EQXQ4  XQ4=B21*((-87)+B22*SUN+B23*FSUN+B24*F2SUN
24      B25*F1);
25      RML  EQXQ5  XQ5=B21*((-77)+B22*SUN+B23*FSUN+B24*F2SUN
25      B25*F1);
26      EQSUB EXPT EQXQ2   EQXQ2  EQXQ3  EQXQ4  EQXQ5;
27      QSUB (NAME=LOGIT5) EQ1  EQXQ2  EQXQ3  EQXQ4  EQXQ5;
28      ARAM B21-B25   B31-B34   B41-B44   B51-B54;
29      ML  (HITER=N,HCOV=NBW,MAXIT=100)  LOGIT5;
30      GENR EXPT;
31      DIFF=ABS(N-EXPECTED);
32      MSD  EXPECTED  DIFF;
33      HIST  (MIN=0,MAX=100,NBIN=10)DIFF;
34      END;
```

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS : 1120

	MEAN	STD DEV	MIN	MAX	SUM	VAR
QCAT	3.00000	1.41548	1	5	3360.000	2.00179
YR	4.5	2.29334	1	8	5040.000	5.25469
BAND	46.35714	31.76119	1	95	51920	1008.773
N	71.50714	139.9917	0	649.000	80088.00	19597.67
F1	13.89962	9.49204	1.70800	29.84999	15567.57	90.0987
F2	283.2177	288.7442	2.91726	891.0217	317203.8	83373.22
F3	6617.284	8003.361	4.98268	26506.99	7411358	6.41D+07
FSUN	1033.437	1161.012	23.91199	5641.645	1157448	1347948
F2SUN	21057.23	30847.21	40.84164	168403.1	2.36D+07	9.52D+08

MAXIMUM LIKELIHOOD ESTIMATION

EQUATION: LOGIT5

Working space used: 27567

STARTING VALUES

	B21	B22	B23	B24	B25
VALUE	0.01174	3.2500	-0.35450	0.0083	

213.23307

F=47683 FNEW=47683 ISQZ=0 STEPSIZE=1.0000
CRITERIORION=0.17763

CONVERGENCE ACHIEVED AFTER 1 ITERATIONS

10 FUNCTION EVALUATIONS.

LOG OF LIKELIHOOD FUNCTION = -47682.5

NUMBER OF OBSERVATIONS= 1120

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.103561E-03	113.3522
B22	3.252518	0.314177E-01	103.5251
B23	-0.3545047	0.455355E-02	-77.85247
B24	0.832179E-02	0.144106E-03	57.74759
B25	13.23291	0.886515E-01	149.2689

STANDARD ERRORS COMPUTED FROM ANALYTIC SECOND DERIVATIVES (NEWTON).

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.122159E-04	960.9582
B22	3.252518	0.307612E-02	1057.344
B23	-0.3545047	0.462890E-03	-765.8504

B24	0.832179E-02	0.156074E-04	533.1967
B25	13.23291	0.820141E-02	1613.492

STANDARD ERRORS COMPUTED FROM COVARIANCE OF ANALYTIC FIRST DERIVATI VES (BHHH).

PARAMETER STATISTIC	ESTIMATE	STANDARD ERROR	T-
B21	0.117389E-01	0.962665E-03	12.19431
B22	3.252518	0.3344272	9.725640
B23	-0.3545047	0.473045E-01	-7.494105
B24	0.832179E-02	0.140921E-02	5.905300
B25	13.23291	0.9969773	13.27303

STANDARD ERRORS COMPUTED FROM ANALYTIC FIRST AND SECOND DERIVATIVES (EICKER-WHITE).

RESULTS OF COVARIANCE PROCEDURE

NUMBER OF OBSERVATIONS = 1120

	MEAN	STD DEV	MIN.	MAX.	SUM	VAR.
EXPECT.	71.50713	138.9512	0.00001	643.7896	80087.98	19307.42
DIFF.	19.34163	30.92411	0.00001	27.03572	21662.52	956.3063

HISTOGRAM OF DIFF

NUMBER OF OBSERVATIONS : 1081

MIN MAX

8.000 644

-0 *** 664

****

10 ******************** 150

****

20 *********** 82

****

30 ****** 48

******

40 ***** 38

*****

50 **** 29

****

60 **** 30

****

70 ** 18

**

80 * 8

*

90 ** 14

**

8.000 644.0

MIN

MAX.

END OF OUTPUT

APPENDIX E

FORTRAN PROGRAMS

```
      PROGRAM MAXIMUM
C
C     ***************************************************************
C     * THE AIM OF THIS PROGRAM IS TO FIND THE ESTIMATION
C     * OF THE PARAMETERS USING MAXIMUM LIKELIHOOD METHOD
C     ***************************************************************
     INTEGER N, LH, LIW, LW
     PARAMETER(N1=200,NS=4)
      PARAMETER(N=4,LH=N*(N-1)/2,LIW=2,LW=7*N+N*(N-1)/2)
      INTEGER     NOUT
      PARAMETER   (NOUT=6)
C        LOCAL SCALARS
      DOUBLE PRECISION   DELTA, ETA, F, STEPMX, XTOL
      INTEGER      IBOUND, IFAIL, IPRINT, J, MAXCAL
C        LOCAL ARRAYS
      DOUBLE PRECISION  BL(N), BU(N), G(N), HESD(N)
      DOUBLE PRECISION  HESL(LH), W(LW), X(N)
      INTEGER      ISTATE(N), IW(LIW)
C       EXTERNAL SUBROUTINES

      EXTERNAL       E04HCF, E04KDF, FUNCT, MONIT
      INTEGER Y(N1,NS)
      COMMON S,T
      DOUBLE PRECISION S(2,NS),T(NS)
      OPEN(11,FILE='AL68-107.DAT')
```

```
CALL E04HCF (N, FUNCT, X, F, G, IW, LIW, W, LW, IFAIL)

IPRINT = 1

MAXCAL =50*N

ETA= 0.5E+0

XTOL = 1.0E-12

DELTA = 0.0E+0

STEPMX = 4.0E0

IBOUND = 0

BL(1)= -20.0E+0

BU(1)= 14.0E+0

BL(2)= -3.0E+0

BU(2)= 3.0E0

BL(3)= -1.0E-2

BU(3)= 18.0

BL(4)=-4.0

BU(4)=5.0

C     SET UP STARTING POINT


X(1)= -1.0E-4

X(2)=-1.0E-4

X(3)= 1.0E-4

x(4)=-1.0E-4

IFAIL = 1

CALL
E04KDF(N,FUNCT,MONIT,IPRINT,MAXCAL,ETA,XTOL,DELTA,STEPMX,
```

```
    *       IBOUND,BL,BU,X,HESL,LH,HESD,ISTATE,F,G,IW,LIW,W,LW,
    *       IFAIL)
     IF(IFAIL.NE.0) WRITE (15,FMT=99998) IFAIL
     IF(IFAIL.NE.1) THEN
        WRITE (15,FMT=99997) F
        WRITE (15,FMT=99996) (X(J),J=1,N)
        WRITE (15,FMT=99995) (G(J),J=1,N)
        WRITE (15,FMT=99994) (ISTATE(J),J=1,N)
        WRITE (15,FMT=99993) (HESD(J),J=1,N)
        WRITE (15,FMT=99992) XTOL
        END IF
        STOP
C
99999 FORMAT (' E04KDF PROGRAM RESULTS',/1X)
99998 FORMAT (///' ERROR EXIT TYPE', I3,'- SEE ROUTINE DOCUMEN')
99997 FORMAT (///'FUNCTION VALUE ON EXIT IS', F16.8)
99996 FORMAT (' AT THE POINT', 3F9.4)
99995 FORMAT ('THE CORRESPONDING(MACHINE DEPENDENT GRADIENT
IS',/23X,
    *     1P,3E12.3)
99994 FORMAT ('ISTATE CONTAINS',3I5)
99993 FORMAT ('AND HESD CONTAINS',3E12.4)
99992 FORMAT ('AND XTOL CONTAINS',D12.4)
     END
```

```
      GC(1)=GC(1)+LAAM1(N1,THETA,LAMBDA)/LAAM(N1,THETA,LAMBDA)
     *-S(1,J)
 25   CONTINUE
      GC(2)=0.0
      DO 15 J=1,NS
        THETA=XC(1)+T(J)*XC(2)
        LAMBDA=XC(3)+T(J)*XC(4)
      GC(2)=GC(2)+T(J)*LAAM1(N1,THETA,LAMBDA)/LAAM(N1,THETA,LAMBDA)
     * -T(J)*S(1,J)
 15   CONTINUE
      GC(3)=0.0
      do 35 j=1,ns
        THETA=XC(1)+T(J)*XC(2)
        LAMBDA=XC(3)+T(J)*XC(4)
        GC(3)=GC(3)+LAAM2(N1,THETA,LAMBDA)/LAAM(N1,THETA,LAMBDA)
     *  -S(2,J)
 35   CONTINUE
      GC(4)=0.0
      DO 45 J=1,NS
        THETA=XC(1)+T(J)*XC(2)
        LAMBDA=XC(3)+T(J)*XC(4)
        GC(4)=GC(4)+T(J)*LAAM2(N1,THETA,LAMBDA)/
     *  LAAM(N1,THETA,LAMBDA)-T(J)*S(2,J)
 45   CONTINUE
```

```
      RETURN
      END
      SUBROUTINE
MONIT(N,XC,FC,GC,ISTATE,GPJNRM,COND,POSDEF,NITER
     *,NF,IW,LIW,W,LW)
      INTEGER        NOUT
      PARAMETER      (NOUT=6)
      DOUBLE PRECISION  COND, FC, GPJNRM
      INTEGER        LIW,LW,N,NF,NITER
      LOGICAL        POSDEF
      DOUBLE PRECISION  GC(N), W(LW),XC(N)
      INTEGER        ISTATE(N), IW(LIW)
      INTEGER        ISJ, J
      WRITE (15,FMT=99999) NITER, NF, FC, GPJNRM
      WRITE (15,FMT=99998)
      DO 20 J=1,N
         ISJ=ISTATE(J)
         IF(ISJ.GE.0)THEN
           WRITE(15,FMT=99997) J, XC(J), GC(J)
         ELSE IF (ISJ.EQ.-1) THEN
           WRITE(15,FMT=99996) J, XC(J), GC(J)
         ELSE IF (ISJ.EQ.-2) THEN
           WRITE(15,FMT=99994) J, XC(J), GC(J)
         ELSE IF (ISJ.EQ.-3) THEN
           WRITE(15,FMT=99994) J, XC(J), GC(J)
```

```
        END IF
 20   CONTINUE
      IF(COND.NE.0.0E0) THEN
        IF(COND.GT.1.0E6) THEN
          WRITE(15,FMT=99993)
      ELSE
          WRITE(15,FMT=99992) COND
      END IF
      IF( .NOT. POSDEF) WRITE (15,FMT=99991)
      END IF
      RETURN
99999 FORMAT(//'ITNS     FN EVALS          FN VALUE     NO',
    *     'RM OF PROJ GRADIENT',/1X,I3,6X,I5,2(6X,1P,E20.4))
99998 FORMAT(/'J          X(J)              G(J)       STA',
    *      'TUS')
99997 FORMAT(' ',I2,1X,1P,2E20.4,'     FREE')
99996 FORMAT(' ',I2,1X,1P,2E20.4,'     UPPER BOUND')
99995 FORMAT(' ',I2,1X,1P,2E20.4,'     LOWER BOUND')
99994 FORMAT(' ',I2,1X,1P,2E20.4,'     CONSTANT')
99993 FORMAT(/'ESTIMATED CONDITION NUMBER OF PROJECTED HESSIAN IS MO',
    *     'RE THAN 1.0E+6')
99992 FORMAT (/'ESTIMATED CONDITION NUMBER OF PROJECTED HESSIAN =',1P,
    *      E10.2)
99991 FORMAT(/'PROJECTED HESSIAN MATRIX IS NOT POSTIVE DEFINITE')
```

```
      END
C
C=====================================================================
=========
C     CALCULATING THE NORMALIZING CONSTANT
C=====================================================================
=========
      FUNCTION LAAM(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM
      DOUBLE PRECISION  E,NR,NM,T,PSI,LAM
      INTEGER  M,N,R,T1
      T=FLOAT(N)*EXP(THETA)
      LAM=T+1
      DO 20 M=1,N-1
      PSI=(FLOAT(N-M))/(FLOAT(M+1))*EXP(THETA)
     **EXP((2*LAMBDA*M)/N)
         T=PSI*T
         LAM=LAM+T
 20   CONTINUE
      LAAM=LAM
      RETURN
      END
      FUNCTION LAAM1(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM1
      DOUBLE PRECISION  E,NR,NM,T,PSI,LAM
      INTEGER  M,N,R
```

```
      T=FLOAT(N)*EXP(THETA)
      LAM=T
      DO 20 M=1,N-1
      PSI=(FLOAT(N-M))/(FLOAT(M))*EXP(THETA)
     **EXP((2*LAMBDA*M)/N)
         T=PSI*T
         LAM=LAM+T
 20   CONTINUE
      LAAM1=LAM
      RETURN
      END
      FUNCTION LAAM2(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM2
      DOUBLE PRECISION  E,NR,NM,T,PSI,LAM
      INTEGER  M,N,R
      T=(FLOAT(N-1))*EXP(2*THETA)*EXP((2*LAMBDA)/(FLOAT(N)))
      LAM=T
      DO 20 M=2,N-1
      PSI=(FLOAT(N-M)/FLOAT(M-1))*EXP(THETA)
     **EXP((2*LAMBDA*M)/N)
         T=PSI*T
         LAM=LAM+T
 20   CONTINUE
      LAAM2=LAM
      RETURN
```

```
PROGRAM FIS
C ======================================================================
C THE AIM OF THIS PROGRAM IS CALCUKATING HESSIAN MATRIX
C ======================================================================
*    .. Parameters ..
   INTEGER        N, LHES, LWORK
 PARAMETER   (N=4,LHES=N,LWORK=N*N+N)
   INTEGER        NOUT
   PARAMETER      (NOUT=6)
*    .. Local Scalars ..
   DOUBLE PRECISION EPSRF, OBJF
   INTEGER        I, IFAIL, IMODE, IWARN, J, MODE, MSGLVL
*    .. Local Arrays ..
   DOUBLE   PRECISION   HCNTRL(N),   HESIAN(LHES,N),   HFORW(N),
OBJGRD(N),
  +             USER(1), WORK(LWORK), X(N)
   INTEGER        INFO(N), IUSER(1)
*    .. External Subroutines ..
   EXTERNAL       E04XAF, OBJFUN
*    .. Executable Statements ..
   PARAMETER  (N1=200,NS=4)
   INTEGER Y(N1,NS)
   COMMON S,T
   DOUBLE PRECISION S(2,NS),T(NS)
```

```
      T(1)=-107
      T(2)=-97
      T(3)=-87
      T(4)=-77
      OPEN(15,FILE='ALI1.RES')
      OPEN(11,FILE='AL68-107.DAT')
      OPEN(12,FILE='AL68-97D.DAT')
      OPEN(13,FILE='AL68-87D.DAT')
      OPEN(14,FILE='AL68-77D.DAT')
      READ(11,101)(Y(I,1),I=1,N1)
      READ(12,101)(Y(I,2),I=1,N1)
      READ(13,101)(Y(I,3),I=1,N1)
      READ(14,101)(Y(I,4),I=1,N1)
  101 FORMAT(79I1)
      DO 15 J=1,NS
        S(1,J)=0.0
        DO 10 I=1,N1
          S(1,J)=S(1,J)+Y(I,J)
 10   CONTINUE
        S(2,J)=0.0
        DO 21 I=1,N1-1
          S(2,J)=S(2,J)+Y(I,J)*Y(I+1,J)
 21     CONTINUE
 15   CONTINUE
      WRITE (15,*) 'FIS PROGRAM RESULTS'
```

```
      MSGLVL = 0
*     Set the point at which the derivatives are to be estimated.
      X(1)=-5.7429
      X(2)=-0.0177
      X(3)=3.8605
      X(4)=0.0559
*     Take default value of EPSRF.
      EPSRF = -1.0D0
*     Illustrate the different values of MODE.
      DO 40 IMODE = 0, 2
        MODE = IMODE
        WRITE (15,*)
        IF (MODE.EQ.0) THEN
          WRITE (15,*)
     +      'Find gradients and Hessian diagonals given function only'
          WRITE (15,*) '( i.e. MODE = 0 ).'
        ELSE IF (MODE.EQ.1) THEN
          WRITE (15,*)
     +      'Find Hessian matrix given function and gradients'
          WRITE (15,*) '( i.e. MODE = 1 ).'
        ELSE IF (MODE.EQ.2) THEN
          WRITE (15,*)
     +      'Find gradients and Hessian matrix given function only'
          WRITE (15,*) '( i.e. MODE = 2 ).'
        END IF
```

```
*     Set HFORW(I) = -1.0 so that E04XAF computes the initial trial
*     interval.
      DO 20 I = 1, N
        HFORW(I) = -1.0D0
   20 CONTINUE
      IFAIL = 1
*
      CALL E04XAF(MSGLVL,N,EPSRF,X,MODE,OBJFUN,LHES,HFORW,OBJF,
     +            OBJGRD,HCNTRL,HESIAN,IWARN,WORK,IUSER,USER,INFO,
     +            IFAIL)
*
      IF (IFAIL.EQ.0 .OR. IFAIL.EQ.2) THEN
        WRITE (15,99999) 'Function value is ', OBJF
        IF (MODE.EQ.1) THEN
          WRITE (15,*) 'Gradient vector is'
          WRITE (15,99998) (OBJGRD(I),I=1,N)
        ELSE
          WRITE (15,*) 'Estimated gradient vector is'
          WRITE (15,99998) (OBJGRD(I),I=1,N)
        END IF
        IF (MODE.EQ.0) THEN
          WRITE (15,*) 'Estimated Hessian matrix diagonal is'
          WRITE (15,99998) (HESIAN(I,1),I=1,N)
        ELSE
          WRITE (15,*)
```

```
     +          'Estimated Hessian matrix (machine dependent) is'
              WRITE (15,99998) ((HESIAN(I,J),J=1,N),I=1,N)
            END IF
          ELSE
            WRITE (15,*)
            WRITE (15,99997) 'On exit from E04XAF IFAIL = ', IFAIL
          END IF
   40 CONTINUE
      STOP
*
99999 FORMAT (1X,A,1P,D12.4)
99998 FORMAT (3(1X,1P,D12.4))
99997 FORMAT (1X,A,I2)
      END
*
      SUBROUTINE OBJFUN(MODE,N,X,OBJF,OBJGRD,NSTATE,IUSER,USER)
*     .. Scalar Arguments ..
      DOUBLE PRECISION  OBJF
      INTEGER           MODE, N, NSTATE
*     .. Array Arguments ..
      DOUBLE PRECISION  OBJGRD(N), USER(*), X(N),T1
      INTEGER           IUSER(*)
*     .. Local Scalars ..
      PARAMETER(N1=200,NS=4)
```

```
      DOUBLE                                                    PRECISION
     S(2,NS),THETA,LAMBDA,LAAM,LAAM1,LAAM2,T(NS)
      COMMON S,T
*     .. Executable Statements ..
      OBJF=0.0
      DO 10 J=1,NS
      THETA=X(1)+T(J)*X(2)
      LAMBDA=X(3)+T(J)*X(4)
      OBJF = OBJF+ LOG(LAAM(N1,THETA,LAMBDA))-THETA*S(1,J)
     +       -LAMBDA*S(2,J)
 10   CONTINUE
      IF (MODE.EQ.1) THEN
      OBJGRD(1)=0.0
      DO 25 J=1,NS
      THETA=X(1)+T(J)*X(2)
      LAMBDA=X(3)+T(J)*X(4)
        OBJGRD(1) =OBJGRD(1)+ LAAM1(N1,THETA,LAMBDA)/
     +           LAAM(N1,THETA,LAMBDA)-S(1,J)
 25   CONTINUE
      OBJGRD(2)=0.0
      DO 15 J=1,NS
      THETA=X(1)+T(J)*X(2)
      LAMBDA=X(3)+T(J)*X(4)
      OBJGRD(2) =OBJGRD(2)+ T(J)*LAAM1(N1,THETA,LAMBDA)/
     +        LAAM(N1,THETA,LAMBDA)-T(J)*S(1,J)
```

```
 15  CONTINUE
     OBJGRD(3)=0.0
     DO 35 J=1,NS
     THETA=X(1)+T(J)*X(2)
     LAMBDA=X(3)+T(J)*X(4)
     OBJGRD(3)=OBJGRD(3)+LAAM2(N1,THETA,LAMBDA)/
    +        LAAM(N1,THETA,LAMBDA)-S(2,J)
 35  CONTINUE
     OBJGRD(4)=0.0
     DO 45 J=1,NS
     THETA=X(1)+T(J)*X(2)
     LAMBDA=X(3)+T(J)*X(4)
     OBJGRD(4)=OBJGRD(4)+T(J)*LAAM2(N1,THETA,LAMBDA)/
    +  LAAM(N1,THETA,LAMBDA)-T(J)*S(2,J)
 45  CONTINUE
     END IF
     RETURN
     END
```

```
C
C==========================================================================
C
      FUNCTION LAAM(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM
      DOUBLE PRECISION  T,PSI,LAM,T1
      INTEGER    M,N
      T=FLOAT(N)*EXP(THETA)
       LAM=T+1
      DO 20 M=1,N-1
      PSI=(FLOAT(N-M))/(FLOAT(M+1))*EXP(THETA)
     +    *EXP((2*LAMBDA*M)/N)
        T=PSI*T
        LAM=LAM+T
 20   CONTINUE
        LAAM=LAM
      RETURN
      END

      FUNCTION LAAM1(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM1
      DOUBLE PRECISION  T,PSI,LAM
      INTEGER    M,N
      T=FLOAT(N)*EXP(THETA)
       LAM=T
```

```
      DO 20 M=1,N-1
      PSI=(FLOAT(N-M))/(FLOAT(M))*EXP(THETA)
     +   *EXP((2*LAMBDA*M)/N)
        T=PSI*T
        LAM=LAM+T
 20   CONTINUE
        LAAM1=LAM
      RETURN
      END

      FUNCTION LAAM2(N,THETA,LAMBDA)
      DOUBLE PRECISION  THETA,LAMBDA,LAAM2
      DOUBLE PRECISION  T,PSI,LAM
      INTEGER   M,N
      T=(FLOAT(N-1))*EXP(2*THETA)*EXP((2*LAMBDA)/(FLOAT(N)))
       LAM=T
      DO 20 M=2,N-1
      PSI=(FLOAT(N-M))/(FLOAT(M-1))*EXP(THETA)
     +   *EXP((2*LAMBDA*M)/N)
        T=PSI*T
        LAM=LAM+T
 20   CONTINUE
        LAAM2=LAM
      RETURN
      END
```

```
      PROGRAM CORR
C
C =============================================================
C     THE AIM OF THIS PROGRAM IS TO CALCULATE THE CORRELATION
C     COEFFICIENT OF YI AND YI+1
C =============================================================
      INTEGER   N,I,J,K,L,M
      PARAMETER (I=5,J=4,K=495,L=500,M=0,N=494)
      REAL      LAMBDA,THETA,FC,LAAM
        THETA=-1.9521
        LAMBDA=1.96
       FC=(EXP(theta+lambda)*LAAM(J,THETA,LAMBDA)*
     * LAAM(N,THETA,LAMBDA))*(LAAM(m,theta,lambda)
     *  -(LAAM(k,theta,lambda)*LAAM(I,THETA,LAMBDA)/
     *   LAAM(L,THETA,LAMBDA)))/(sqrt(LAAM(J,THETA,LAMBDA)*
     *   LAAM(k,THETA,LAMBDA)*(1-(EXP(THETA+LAMBDA)*
     *   LAAM(J,THETA,LAMBDA)*LAAM(K,THETA,LAMBDA))/
     *   LAAM(L,THETA,LAMBDA)))
     *    *sqrt(LAAM(I,theta,lambda)*LAAM(n,theta,lambda)
     *   *(1-(EXP(theta+lambda)*LAAM(I,theta,lambda)*
     *    LAAM(n,theta,lambda))/LAAM(L,THETA,LAMBDA))))
      WRITE(*,*) 'FC='
      WRITE(*,*) FC
      END
```

```
C
C===========================================================
C
      FUNCTION  LAAM(N,THETA,LAMBDA)
      REAL      THETA,LAMBDA,LaAM
      REAL      E,NR,NM,T,PSI,LAM
      INTEGER   M,N,R
        T=FLOAT(N)*EXP(THETA)
        LAM=T+1
        DO 20 M=1,N-1
          PSI=(FLOAT(N-M))/(FLOAT(M+1))*EXP(THETA)
     *       *EXP((2.0*LAMBDA*M)/N)
          T=PSI*T
          LAM=LAM+T
20      CONTINUE
        LaAM=LAM
      RETURN
      END
```

```
      PROGRAM INVERS
C
C ==================================
C     THE AIM OF THIS PROGRAM IS TO FIND
C     THE INVERS OF THE MATRIX
C ==================================
*   .. Parameters ..
    INTEGER        NIN, NOUT
    PARAMETER        (NIN=5,NOUT=6)
    INTEGER        NMAX, LDA, LWORK
    PARAMETER        (NMAX=8,LDA=NMAX,LWORK=64*NMAX)
*   .. Local Scalars ..
    INTEGER        I, IFAIL, INFO, J, N
*   .. Local Arrays ..
    DOUBLE PRECISION A(LDA,NMAX), WORK(LWORK)
    INTEGER        IPIV(NMAX)
*   .. External Subroutines ..
    EXTERNAL        DGETRF, DGETRI, X04CAF
*   .. Executable Statements ..
      WRITE (NOUT,*) 'Program Results'
*   Skip heading in data file
    READ (NIN,*)
    OPEN(5,FILE='RAB.DAT')
    READ (NIN,102) N
102   FORMAT(I4)
```

```
      IF (N.LE.NMAX) THEN
*
*      Read A from data file
*
       OPEN(33,FILE='MAT1.DAT')
       READ (33,103) ((A(I,J),J=1,N),I=1,N)
 103   FORMAT(3D12.4)
*
*      Factorize A
*
       CALL DGETRF(N,N,A,LDA,IPIV,INFO)
*
       WRITE (NOUT,*)
       IF (INFO.EQ.0) THEN
*
*         Compute inverse of A
*
          CALL DGETRI(N,A,LDA,IPIV,WORK,LWORK,INFO)
*
*         Print inverse
*
          IFAIL = 0
          CALL X04CAF('General',' ',N,N,A,LDA,'Inverse',IFAIL)
       ELSE
          WRITE (NOUT,*) 'The factor U is singular'
```

```
  END IF
END IF
STOP
END
```

```
      PROGRAM NORM1
C
C ==========================================
C     THE AIM OF THIS PROGRAM TO FIND THE INVERS
C     OF THE NORMAL DISTRIBUTION FUNCTION
C ==========================================

      INTEGER    NOUT
      PARAMETER  (NOUT=6)
      DOUBLE PRECISION   X
      INTEGER    I, IFAIL
      DOUBLE PRECISION   P(8)
      DOUBLE PRECISION   G01CEF
      EXTERNAL          G01CEF
      DATA      P/0.001E0, 0.01E0, 0.1E0, 0.2E0, 0.8E0, 0.9E0,
     *          0.99E0, 0.999/
      WRITE (NOUT,*) 'G01CEF EXAMPLE PROGRAM RESULTS'
      WRITE (NOUT,*)
      WRITE (NOUT,*) ' PROB. DEVIATE'
      WRITE (NOUT,*)
      DO 20 I=1, 8
        IFAIL = 1
        X = G01CEF(P(I),IFAIL)
        IF(IFAIL.NE.0) THEN
```

```
      WRITE (NOUT,99999) P(I),' FAILED IN G01CEF. IFAIL =',
   *     IFAIL
     ELSE
       WRITE (NOUT,99998) P(I), X
     END IF
  20 CONTINUE
    STOP
99999 FORMAT (1X,F7.3,A,I2)
99998 FORMAT (1X,F7.3,F11.4)
    END
```

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