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RUPTURE PROPERTIES OF TWISTED CONTINUOUS FILAMENT YARNS

Presented

by

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A C K N O W L E D G E M E N T

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A U T H O R

The author was awarded the degree of B.Text. by the University of Bombay, India, in 1952. He joined the Khatau Makanji Spinning and Weaving Mills Ltd., as a departmental assistant in the spinning section of the Mills.

In 1955, he was admitted as a graduate student in the Textile Engineering Department of the Lowell Technological Institute, Lowell, Mass., U. S. A. There he completed one semester's advance study in textile technology and engineering. In 1956, he was admitted as a research student in the Department of Textile Industries of the Manchester College of Science and Technology, Manchester, U.K., preparing for the M.Sc.Tech. degree. After completion of one year's training in research under the guidance of Dr. J. W. S. Hearle, he was admitted to the Ph.D. course in October 1957.

ABSTRACT

Rupture properties of twisted continuous filament yarns are investigated over a twist factor range up to 100 tex$^{\frac{1}{2}}$ turns/cm. The rupture properties studied are mechanism of yarn failure, yarn breaking extension, tenacity and work of rupture. The effect of twisting tension and the method of twisting are also studied.

Chapter I is exclusively devoted to the review of earlier work on the theoretical and the experimental aspects of the subject. The gaps in the knowledge and the chosen field of work have also been discussed.

The results of the main experimental work, carried out by the methods decribed in chapter II, are presented in chapter III. The experimental data are mainly concerned with tensile test results on the Instron tester, Uster automatic strength tester and IP2 Scott tester, and the results of the determination of yarn structural parameters. The yarns investigated, viz. viscose, acetate, Tenasco, nylon and Terylene, were twisted on both an uptwister and a ring doubler. Effect of twisting tensions on the rupture properties and twist contraction factor is also studied.

Subsidiary experiments were conducted to learn more about actual yarn structure, actual mechanism of yarn failure and the influence of twist on the rupture properties of single filaments. Attention was particularly focussed on the explanation of the yarn breaking extension

behaviour with twist. The experimental procedure, the results obtained and the discussion are included in chapter IV.

In chapter V, the theory of the relations between yarn extension and tenacity values and filament properties is developed by considering some of the important neglected factors like yarn lateral contraction ratio, large extension values and the influence of both tensile and compressive forces when Hooke's law relation ceases to hold.

In chapter VI, the reasons for the variation of stress-strain curves, yarn breaking extension and tenacity are discussed and the results compared with the theoretical predictions. The explanations of all the results assume the idealised helical model of the yarn structure. The influence of the twisting conditions to produce buckled yarn structure is considered in the explanation of the breaking extension behaviour with twist. The nature of yarn failure is observed to be more complicated than so far postulated.

With regard to the tenacity values, the limitations of the assumptions, viz. Hooke's law or linear form of relation, are discussed. The simple relation ($\cos^2\alpha$) is found to give less satisfactory agreement with the experimental results.

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LIST OF SYMBOLS

A	Constant.
$a_1, a_2 a_3$	Constants.
a	The number of units in the core structure; constant.
B	Constant.
b	Breaking stress of fibre; constant.
c	Cosine of the surface helix angle.
C_y	Yarn contraction factor.
D	Yarn diameter.
d	Fibre diameter.
dG	Small increment in G
dh	Small increment in h
dl	Small increment in l
dr	Small increment in r
dx	Small increment in x
$d\theta$	Small increment in θ
E	Initial modulus.
f	Filament stress below rupture.
f_r	Fibre specific stress function.
$f_1(c, a_1, a_2, a_3, G_y)$	Function of c, a_1, a_2, a_3 and G_y
G	Tangential compressive stress.
h_o	Length per turn of twist before extension.
h_b	Length per turn of twist at yarn break.
k	Helix constant.

K	Factor depending upon material density.
L	Filament length in the surface helix.
L_o	Untwisted length of yarn.
L_c	Twisted length of yarn
L_E	Length to which 'L_o' is extended in helical path.
ℓ	Length of filament in the yarn structure.
ℓ_o	Length of filament before extension
ℓ_b	Length of filament at break.
m	Constant.
m_o	Number of filaments crossing unit area of cross-section.
N	The number of pseudo cores in yarn.
n	Constant.
P	Breaking load of yarn.
P_o	Breaking load of yarn at zero twist.
P_t	Breaking load of yarn at 't' twist.
R_b	Yarn radius at break.
R_o	Yarn radius before extension.
R_y	Yarn retraction.
r	Helix radius.
r_s	Radius of Sth pseudo core.
S	Total number of structural units.
T	Count of yarn in tex.
t	Turns per unit length.

t_o	Turns per unit length before extension.
t_b	Turns per unit length at break.
V	Specific volume of yarn
X	Axial specific stress.
X_f	Specific stress in fibre under the same extension as yarn.
x	Measure of position of fibre in yarn $= \frac{l}{L}$
Y	Yarn stress
α	Surface helix angle.
α_o	Surface helix angle before extension.
α_b	Surface helix angle at break.
δ	Common difference for summation.
ϵ	Extension.
ϵ_b	Filament extension at break.
ϵ_f	Filament extension.
ϵ_y	Yarn extension at break.
η_r	Number of units in r^{th} pseudo core.
η_s	Number of units in s^{th} pseudo core.
θ	Helix angle
	~~Helix angle of fibres in helix radius~~
θ_b	Helix angle at break
θ_o	Helix angle before extension.
θ_r	Helix angle of fibres in helix radius
ν	Number of units per area of yarn cross-section.

σ_1, σ_f	Poisson's ratio of fibre.
σ_y	Yarn contraction ratio.
γ	The number of pseudo cores in the structure.

CONTENTS

CHAPTER VI DISCUSSIONS Page

Appendixes

CHAPTER I

INTRODUCTION

1.1 GENERAL INTRODUCTION

The basic materials available to a textile technologist are textile fibres, but many properties of yarns and fabrics depend upon how the inherent fibre properties are modified by processing techniques and the geometry of the structures. Recent trends in textile technology are in the direction of replacing hit and miss techniques of designing by the precision of engineering design. F. T. Peirce[1] in his classical paper "Geometry of Cloth Structure" has pointed the way. All fibres have to be converted into yarns before they are woven into cloth, or used as yarns alone. Therefore, knowledge about the properties of yarns, and the way in which they are related to yarn geometry and inherent fibre properties is of considerable technical importance.

Among the yarn properties which can be related to fibre properties and yarn geometry, the mechanical properties of twisted yarns are of great importance. The natural and man-made staple fibres are almost invariably converted into yarns by twisting them together; and almost all continuous filament yarns have some twist, while some are highly twisted. Voile and crepe yarns, some bulked yarns, and tyre cords are examples of commercially employed twisted continuous filament yarn structures. Some yarns are twisted for better strength realization, while others are twisted for appearance and handle.

The simplest of the mechanical properties of twisted yarns which can be investigated is the behaviour under an applied load or extension along the yarn axis i.e. the tensile load-extension relation up to rupture. Knowledge about the behaviour under low extension or load will be of importance in establishing the best tensions and techniques to be employed in converting yarns into cloth or other composite structures. The rupture behaviour will be a useful guide in predicting the serviceability of the final industrial and commercial products.

The present investigation is mainly devoted to experimental studies of the rupture properties of twisted continuous filament yarns. Materials with different inherent fibre properties - nylon, Terylene, viscose rayon, Tenasco and acetate have been studied. The effect of twisting conditions has also been investigated. The rupture properties studied are breaking extension, breaking load and work of rupture. Theoretical aspects of the subject have also been developed, and their limitations are discussed.

1.2 IMPORTANCE OF TWISTED CONTINUOUS FILAMENT YARN STUDIES.

The constituent fibre elements in yarns may be discontinuous elements as in man-made staple fibre yarns and natural fibre yarns like cotton, wool, etc. They may also be continuous elements as in natural silk fibre or man-made fibre yarns.

In the staple yarns the mechanism of load transmission and yarn failure, are more complicated than those of continuous filament yarns. The staple elements introduce the factor of fibre slippage in the prediction of yarn extension behaviour at any load; and the variability in the number of fibres in the cross-section introduces limitations to the mathematical analysis of the problem.

Most of the commercially manufactured yarns and fabrics are made from discontinuous fibre elements. Therefore, the study of the behaviour of yarns spun from discontinuous elements will no doubt be of considerable importance. But for progress in the study of any problem, one has to make certain assumptions to simplify the methematical treatment.

The validity of these hypotheses may be experimentally confirmed, provided the test sample is a reasonable approximation to the idealised structure. The study of continuous filament yarns may well serve as a stepping stone towards an understanding of staple yarns, because of the two main simplifications in the theoretical developments - firstly, the reduction of variability in yarn structural parameters, and secondly, the elimination of the dominant influence of frictional forces as observed in staple yarn behaviour.

The study is also of considerable technical importance in itself, in view of the wide use of continuous filament man-made fibre yarns. Their satisfactory processing and use depend on their mechanical properties.

1.3 REVIEW OF LITERATURE.

In the systematic study of a technological problem, it is very important to collect and understand relevant theoretical and experimental findings discussed and developed by various workers in the past. In the light of such a critical examination, the line of further approach, to advance the present knowledge, can then be effectively planned.

In the study of the rupture properties of continuous filament yarns as influenced by twist, the present technological developments can be summarised under the following headings:-

1) The yarn structure - Theoretical and Experimental Aspects.

2) The Rupture Properties i.e. Tenacity, Breaking Extension and Work of Rupture - Theoretical and Experimental Aspects.

3) The way in which the structural properties of twisted yarn influence their rupture properties, together with the closely related topic of twist contraction.

4) The gaps in the present knowledge, and the chosen field of work.

1.31 Yarn Structure - Theoretical Aspects

The commonly defined elements of yarn structure in practical use are twist and mass per unit length (tex). The twist factor or twist multiplier, which takes into account both these elements of yarn structure is a valuable measure for use in comparing the properties of different yarn sizes. The methods available for the determination of these important elements were of the destructive type. There was a need to define the twisted yarn structure so as to calculate the relations between various geometrical parameters and explain the dependence on twist of various yarn properties like construction factor, tenacity, breaking extension etc.

Schwarz[2] and Woods[3] made valuable contributions by defining the twist per unit length. A useful geometrical relation is given as

$$t = \frac{\tan\alpha}{\pi D} \qquad \text{.......... (1.31a)}$$

where,

t = twist per unit length

and D = yarn diameter.

Schwarz[2,4] corrected equation (1.31a) by introducing the concept of helix constant 'K'. This correction factor is of practical use in computing twist per unit length from the optical measurements of yarn diameter 'D' and surface helix angle 'α'. The corrected expression is

$$t = \frac{\tan\alpha}{\pi KD} \qquad \text{........... (1.31a)}$$

where,

$$K = \frac{D - d}{D}$$

d = fibre diameter

For yarn structure with infinite number of filaments as assumed in theoretical concept, the value of helix constant 'K' is unity.

This relation assumes a circular yarn cross section and a helical fibre path. To simplify the mathematical approach most of the workers have used idealised yarn structures. Fig. 1.31A shows such a widely accepted idealised yarn structure. The main assumptions are

1) The yarn is uniform and circular in cross-section. So also are the elements of yarn structure.

2) The yarn is built up of a number of superimposed concentric circles, the circumferences of these circles are the loci of the centres of the individual circular elements which fall into a rotationally symmetric array in cross-sectional view.

3) Yarn diameter is very large as compared to that of the structural units.

4) The centre line of each structural unit lies in a perfect helix, with the centre of helix located at the centre of yarn cross-section.

5) The packing of these structural units, in the yarn cross-section, is such as to keep their number per unit area, at a direction normal to their axis, constant.

It is generally recognised that such a structure will not produce a coherent staple yarn. Peirce[5] postulated that, superimposed on such a structure there must be some degree of random tangle.

As regards the density of fibre packing, the alternative assumptions have been considered by some workers[6,7]. They assumed that the number of fibres, per unit area at a direction normal to the yarn axis is constant. In other words, fibre packing density is

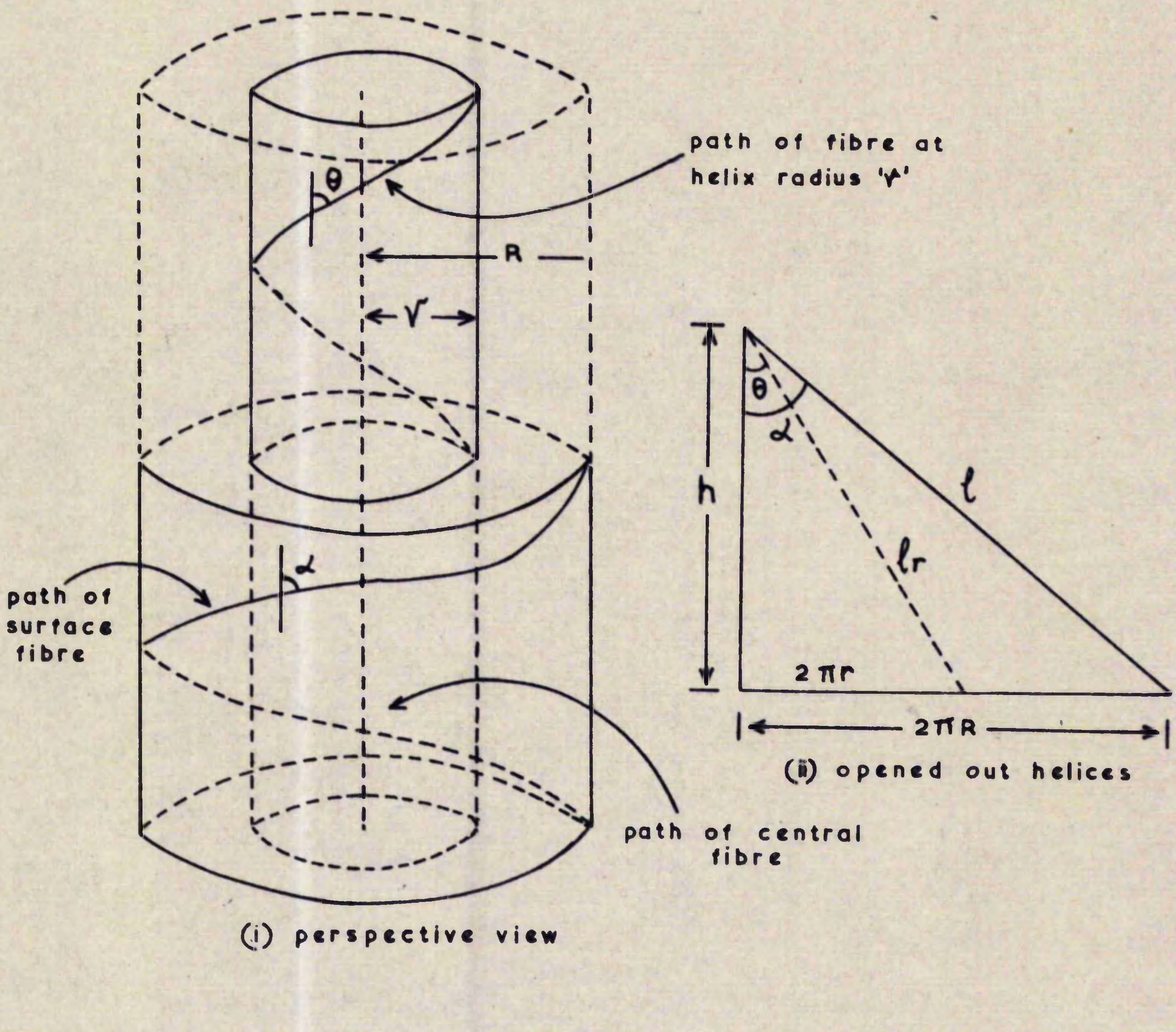


FIG.(1.31A) IDEALISED HELICAL YARN STRUCTURE minimum at the yarn centre and maximum at the surface.

The geometrical packing of units into yarn structure has been discussed by Schwarz[8]. He assumed the yarn structure to be built up of a core and pseudo-cores. The core itself may be composed of one to six units. Whatever may be the core, a circle may be drawn to circumscribe it. Fibres with their centres on another circle concentric with this will constitute the first pseudo core. The radius of this pseudo-core will be equal to the circumscribing radius of the core plus the radius of the structural units. The number of units which can be packed in this pseudo-core without any distortion can be given by a mathematical expression.

$$\eta_s = \frac{2\pi r_s}{d} \qquad \text{.......(1.31b)}$$

where η_s = The number of units in s^{th} pseudo-core.

r_s = The radius of the s^{th} pseudo-core.

d = The diameter of the structural unit.

Equation (1.31b) assumes close packing and allows the counting of fractions of the structural units in a given pseudo-core. Schwarz[8] has further discussed the open packing structure. The total number of units in such an open packing may be given as

$$S = \frac{N}{2}\left[\frac{2a}{N} + (N+1)\delta\right] \qquad \text{...... (1.31c)}$$

where S = Total number of structural units

N = The number of pseudo-cores in the yarn

δ = The common difference $\delta = 6$ when $a = 1$

a = The number of units in the core structure.

Fig. 1.31B represents the idealised geometry of the cross-section of a zero twist yarn. The cross-section of the units are also circular. When twist is introduced these units of cross section will appear as ellipses whose major axes lie circumferentially and whose minor axes lie radially. The increase in the ellipticity is a function of the sine of the helix angle θ . Fig. 1.31B represents the geometry of cross-section of such a twisted yarn structure.

Hamburger et al[9] have shown that the maximum number of units possible in such a twisted structure without distortion can be given by an expression

$$\eta_r = 2\pi \; N \cos\theta_r \qquad \ldots\ldots\ldots (1.31d)$$

where N = The number of pseudo-cores

η_r = The number of units in r^{th} pseudo-core

θ_r = The helix angle for the elements in r^{th} pseudo-core

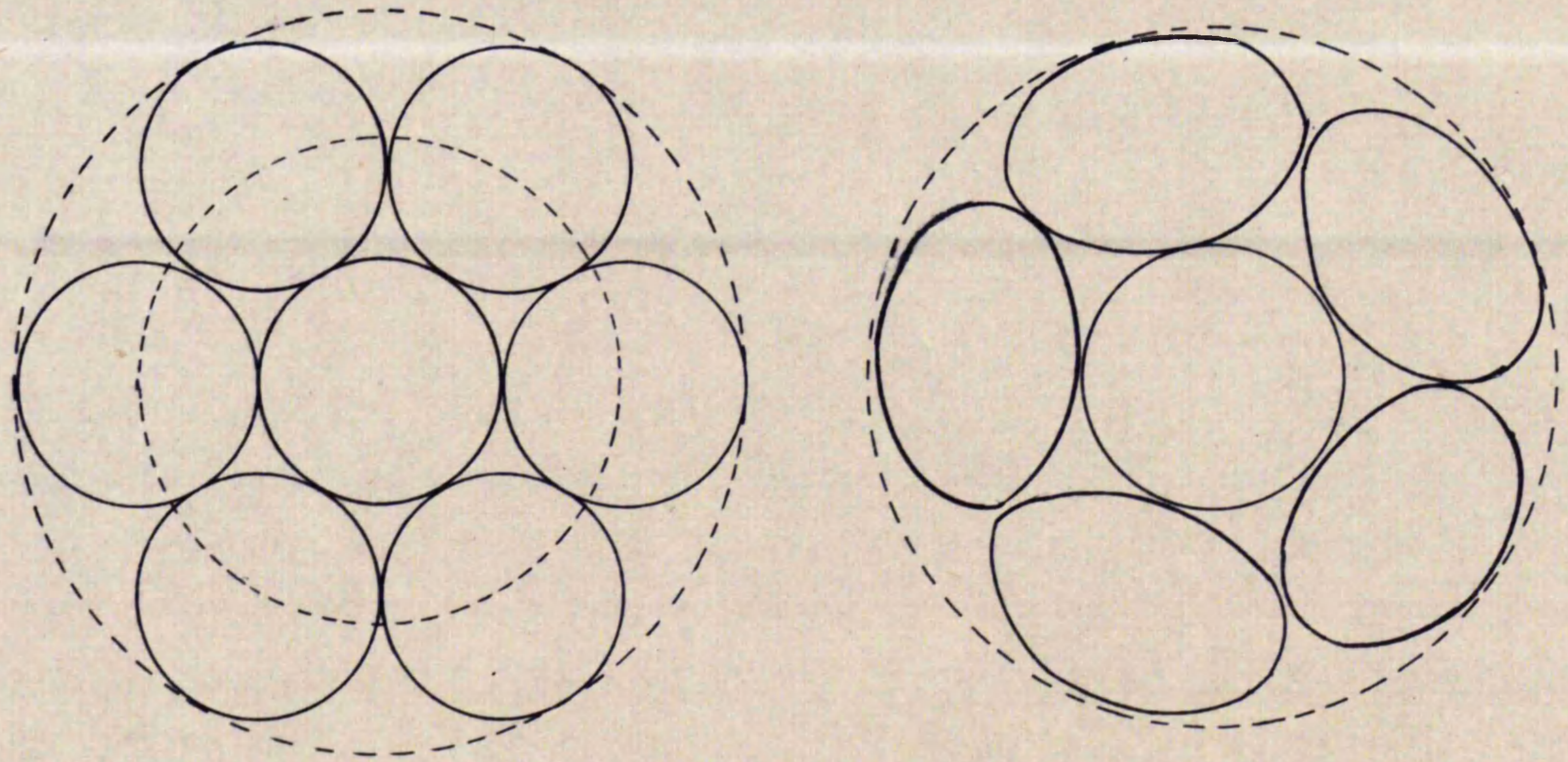
If packing factor is defined as the ratio of the area of the yarn cross-section to that of the structural units in the cross-section, the theoretical close-packed yarn structure as defined above results in the packing factor of 0.75. This can be expressed mathematically as

$$\text{packing factor} = \frac{1 + 3\gamma + 3\gamma^2}{1 + 4\gamma + 4\gamma^2} \qquad \ldots\ldots\ldots (1.31e)$$

where γ = Number of pseudo-cores in a structure having single core unit.

In practice packing factors of the order of 0.6 have been reported[9] in the commercially twisted yarns.

This theoretical geometry of yarn structure has not been widely



(i) ZERO TWIST YARN

(ii) 't' TWIST YARN

FIG. I.31B — IDEALISED GEOMETRY OF YARN CROSS-SECTION.

used in the application to the theoretical developments of the yarn properties. This may be mainly attributed to the difficulties in the mathematical computations. All the workers have assumed the idealised yarn structure built up of idealised structural units having infinitesimal dimensions. Thus the number of units in an annular ring of radius r and thickness dr, in the yarn cross-section, can be given by an equation

$$N = 2\pi r dr \cdot \nu \qquad \ldots\ldots (1.31f)$$

where N = Total number of structural units in the annular ring

ν = The number of units per unit area of yarn cross-section.

1.32 YARN STRUCTURE - EXPERIMENTAL ASPECTS.

Very little work has been reported to check the assumptions made in defining the theoretical yarn structure. The deviation from such an idealised yarn structure may be due to

(a) Migratory behaviour of fibres during twisting

(b) Buckled inner fibre layers to allow for the recovery of strain developed during twisting. The tensile strain due to twisting will be at a maximum in the fibres forming outer yarn structure and the least in those forming the core structure.

No work has been reported to prove or disprove this buckled structure of twisted yarn. As regards migratory behaviour of fibres some workers have shown experimentally the evidence of the radii of the helices followed by given fibres changing along the length of twisted yarn. This migration behaviour has been reported and studied in both continuous filament and staple yarns.

Morton and Yen[10] reported the arrangement of fibres in Fibro yarns. They employed a tracer fibre technique for the study of this migration behaviour. The method involves optical observation of the path followed by dyed tracer fibres, when all other fibres in the samples are rendered transparent by immersion in a liquid of suitable refractive index.

The technique has certain limitations in studying the continuous filament yarn structure due to difficulties in the introduction of tracer fibre.

Morton[11] studied the coefficient of migration in Fibro yarns. The fibre migration principles may be summarised as

1) Migration must take place because of the difference in the tensions developed among fibres during twisting.

2) In a blend of two fibres differing in the initial modulus, the one having the higher initial modulus would tend to occupy inner zones of the structure.

3) The greater the number of fibres in a given yarn cross-section, the greater the obstruction which each fibre has to overcome in migrating.

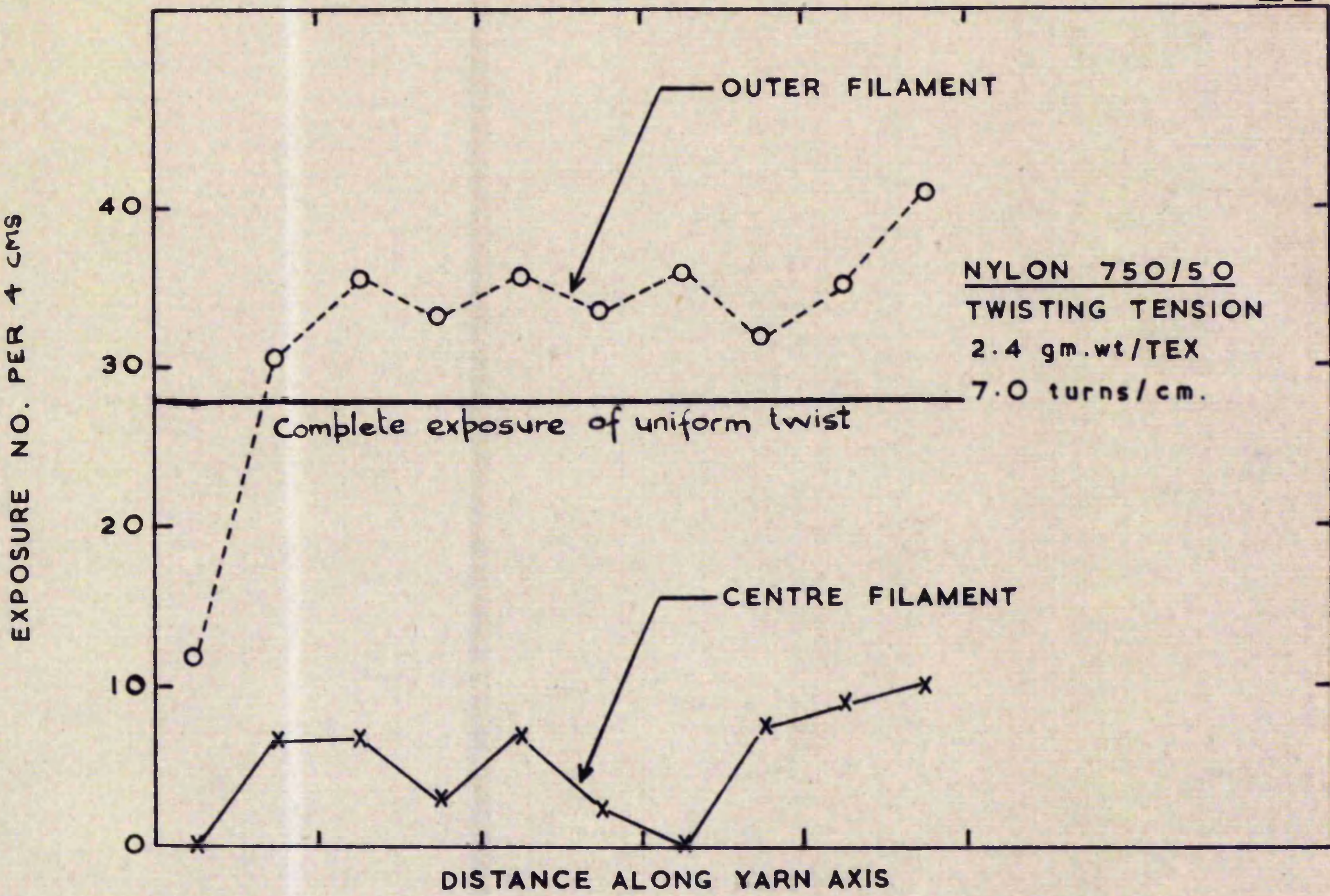
4) Higher tension differences must be developed among the fibres in the coarse yarns in order to produce the same effect.

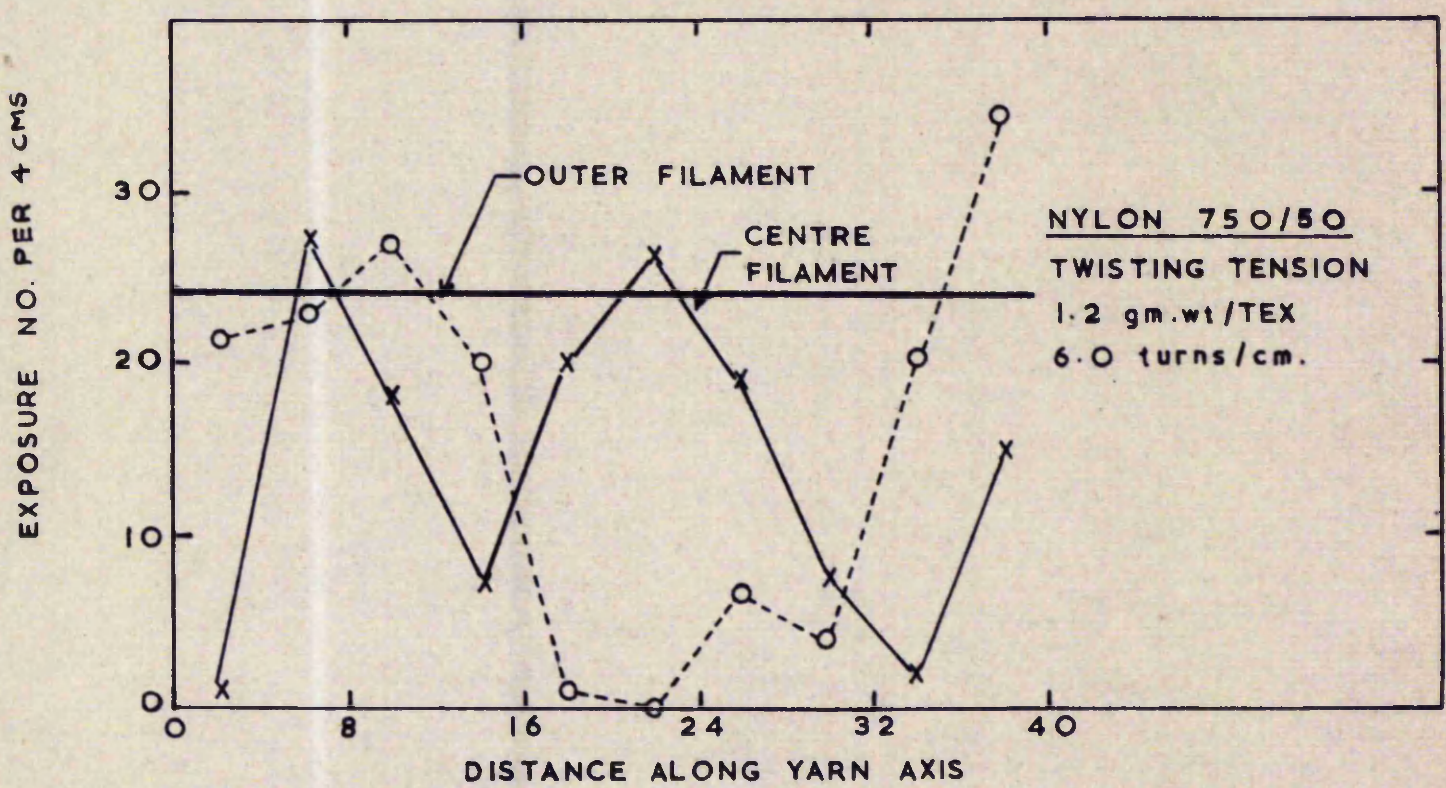
Kakiage[12] studied the effect of twisting tension on migration pattern for nylon 750-50 yarns. He obtained a bundle of fifty 15 denier monofilaments having one coloured filament in the centre and one in the outer layer. He twisted the bundle to 5, 6 and 7 turns/cm. using twisting tensions of 0.13, 0.20 and 0.25 gms./denier. In all the cases the bundle was twisted by a static device. He counted the number of times on which a particular dyed filament was exposed on the surface of the yarn. In other words, if a particular dyed filament was observed to be all the time on the surface, the exposure number per centimeter would be equal to the turns per centimeter in that region. The exposure number was counted for each successive region of 4 cm. in 40 cm. twisted yarn. He observed that at higher twisting tensions, the surface filament is exposed more frequently than the central filament as shown in Fig. (1.32A).

1.33 Yarn rupture properties - Theoretical aspects

1.33 A - Tenacity

Many attempts have been made to predict the properties of twisted yarn structures from the yarn geometry and inherent properties of the structural units. From Fig. (1.33A) it can be seen that the stress-strain relations of commercially employed textile materials, follow Hooke's law only in the initial portion of the curve. In the region of breakage the curves may be regarded as linear or parabolic in shape. Mathematical expressions relating the initial modulus, the breaking





FILAMENT EXPOSURE NUMBER FOR TWISTED NYLON YARN.

FIG.(1.32 A)

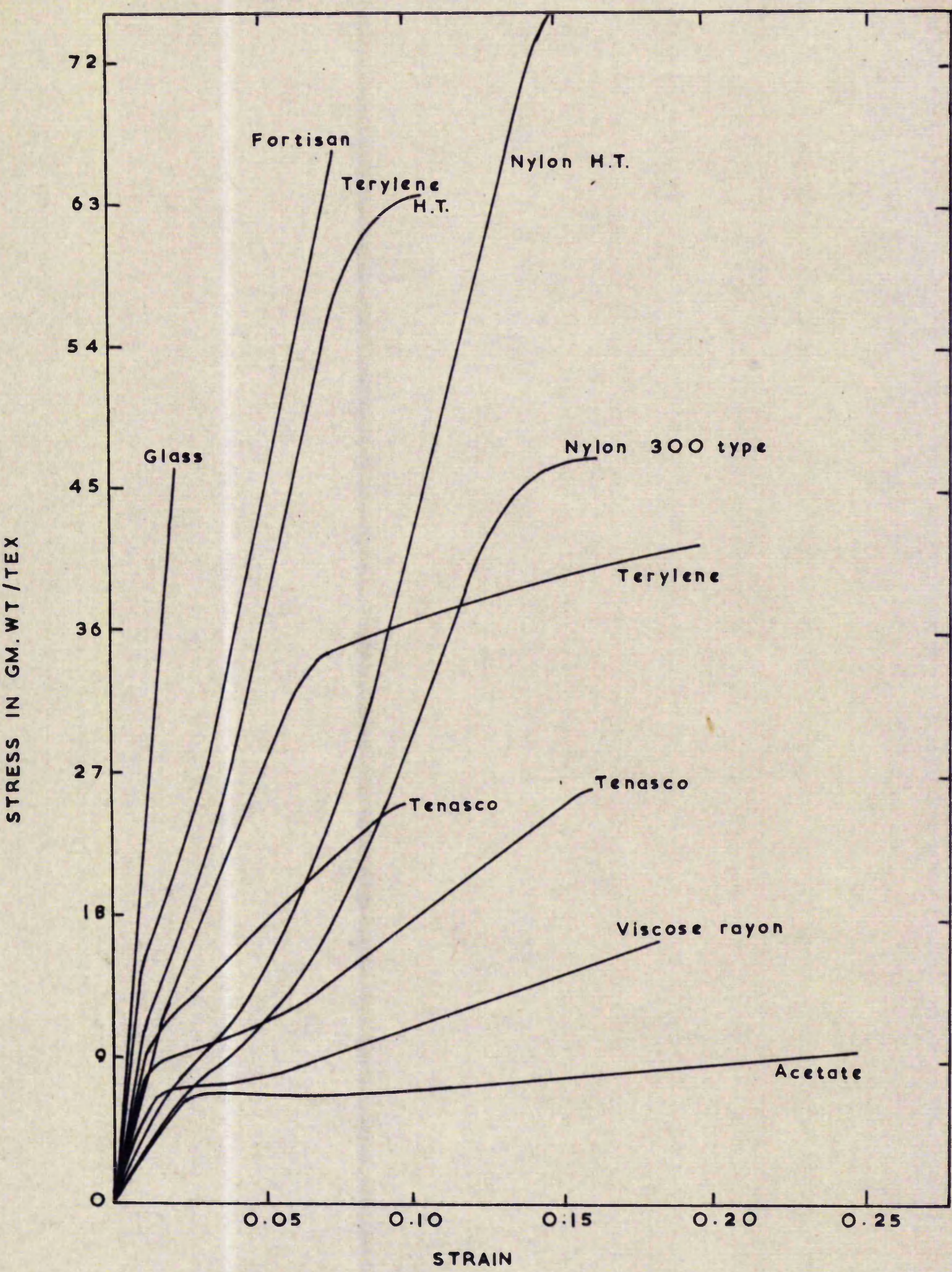


FIG 1.33A STRESS-STRAIN CURVES FOR TYPICAL TEXTILE FIBRES.

extension, tenacity and stress-strain characteristics of twisted multifilament yarns to the inherent filament properties have been discussed by many workers.

Gegauff[13] in 1907 obtained an expression for breaking strength of twisted yarns. In his theoretical approach, he made certain assumptions which have not been further modified. His expression was

$$\text{Breaking load} = 2\pi b \int_0^{R_b} \frac{r\,dr}{(1 + 4\pi^2 t^2 r^2)^2} \qquad \ldots.(1.33a)$$

where b = The breaking stress of fibres

R_b = The radius of yarn at break

r = The helix radius

t = Turns per unit length at break.

This equation assumes

1) Idealised yarn structure as explained in para. 1.31.
2) Only the axial tensile forces contributing to the breaking load of twisted yarn.
3) The structural units follow Hooke's law and are all identical.
4) Any changes in yarn diameter during extension are negligible.
5) The strain values are low so as to make changes in the surface helix angle negligible.
6) The yarn extension at break equals the breaking extension of the constituent filaments.

From the assumptions 1, 4 and 5, the filament extension 'ϵ_f' can be related to yarn extension 'ϵ_y' by the expression

$$\epsilon_f = \epsilon_y \cos^2\theta_o \quad \text{.........} \quad (1.33\ b)$$

where θ_o = The helix angle of the filament.

Platt[14] obtained more accurate geometrical relations and analytically showed that the errors are negligible, thus justifying the use of equation number (1.33 b).

The relation of yarn strain to filament strain can be given as

$$\epsilon_f = (1 + \epsilon_y) \frac{\sec\theta}{\sec\theta_o} - 1 \quad \text{.......} \quad (1.33\ c)$$

where θ_o and θ are the filament helix angles before and after yarn extension.

Expression (1.33 c) can be written in terms of initial or at break values for yarn diameter and twist per unit length.

Platt[14] has reported a more accurate relation by substituting initial values of yarn diameter ($2R_o$) and twist per unit length to namely

$$\epsilon_f = \left[\frac{(1 + \epsilon_y)^3 + 4\pi^2 t_o^2 r_o^2}{(1 + \epsilon_y)(1 + 4\pi^2 t_o^2 r_o^2)}\right]^{\frac{1}{2}} - 1 \quad \text{.....} \quad (1.33\ d)$$

and this is equivalent to

$$\epsilon_f = \frac{\epsilon_y}{2}\left[3\cos^2\theta - 1\right] \quad \text{.......} \quad (1.33\ e)$$

It should be noted that equation (1.33 e) assumes the change in yarn diameter to follow constant volume deformation. It seems more useful to express equation (1.33c) in terms of both yarn diameter and twist per unit length values at break.

Platt et al[15], have developed a theoretical approach when Hooke's law ceases to hold (Assumption No. 3). They introduced 'f_r', a general stress function of fibre strain, but otherwise made similar assumptions. They showed that

$$\text{Breaking load of yarn} \quad 2\pi \int_0^{R_b} \frac{f_r \cdot r \cdot dr}{1 + 4\pi^2 t_b^2 r^2} \qquad \ldots.(1.33f)$$

where t_b = Turns per unit length at break

R_b = The yarn radius at break

r = The helix radius

f_r = The stress function depending upon the properties of material.

The fibre specific stress function 'f_r' has a form which depends on the material properties. It may be linear as in viscose, acetate, or it may be parabolic as in nylon. When it is linear

$$f_r = A + B\epsilon_f \qquad \ldots\ldots.. (1.33g)$$

where, A and B are fibre constants.

As shown in figure (1.33B) A is the intercept on the stress axis and B is the slope of the fibre stress-strain curve.

From equation (1.33f) he obtained the following expression by making some mathematical approximations and omitting to correct for the changes in the linear density of yarn which occurs on twisting.

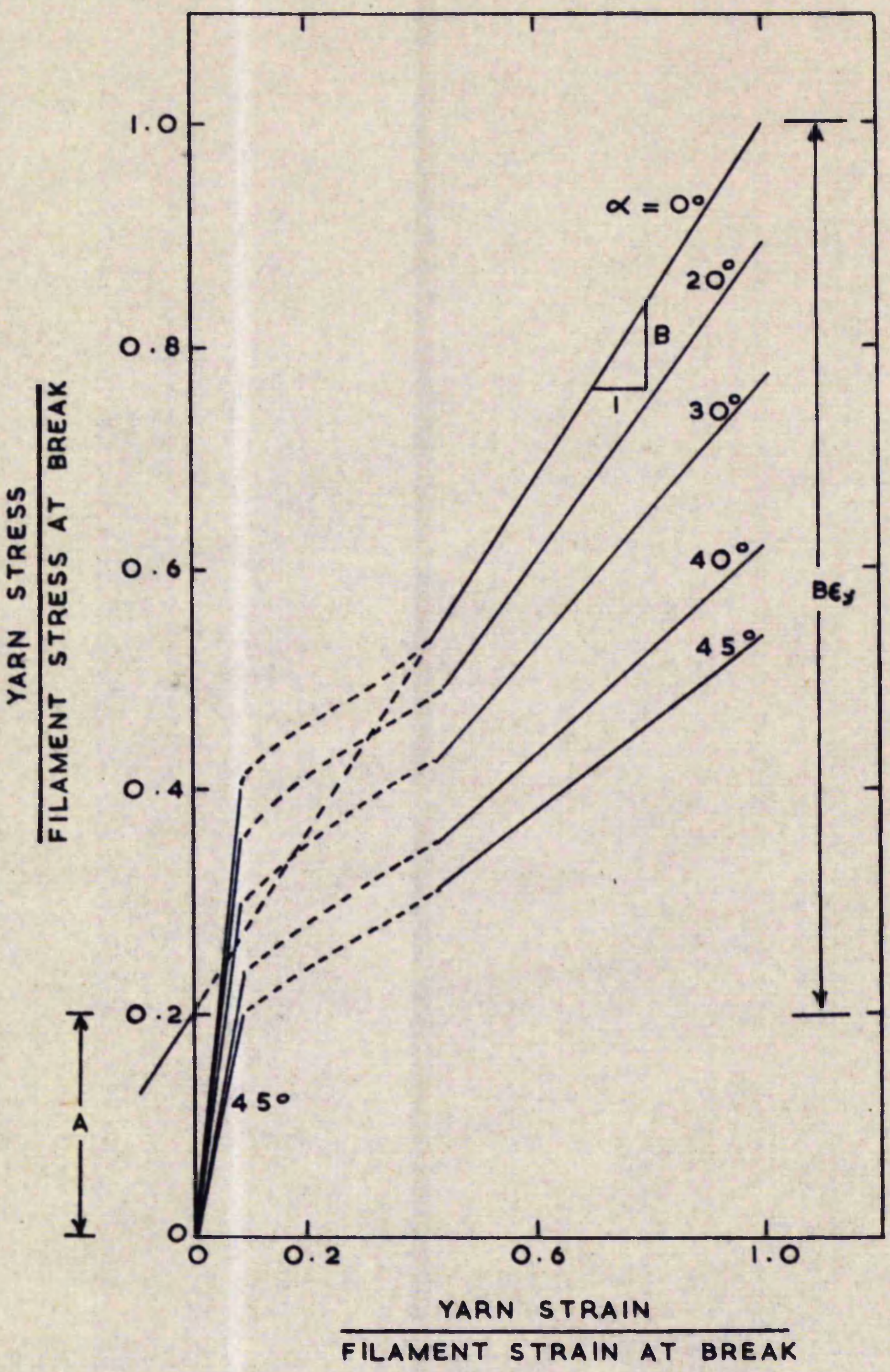


FIG.(1.33B) HYPOTHETICAL LINEAR STRESS-STRAIN RELATIONS OF YARNS WITH SUCCESSIVELY HIGHER TWISTS.

$$P_t = P_o - \tan^2\alpha_b(\frac{A}{2} + B\epsilon_b) \text{ .... } (1.33\ h)$$

where P_t = Breaking load of yarn with twist 't' per unit length

P_o = Breaking load of zero twist yarn.

α_b = The helix angle of the outer filaments in yarn structure at break.

ϵ_b = The breaking extension of constituting material.

Equation (1.33 f) can be integrated and simplified after correcting for twist contraction to obtain a mathematical relation between yarn tenacity, twist angle and fibre parameters.

$$\text{Yarn Tenacity} = 2\ A \cot^2\alpha_b \log_e \sec\alpha_b + B\epsilon_b \cos^2\alpha_b \text{ .....} (1.33\ I)$$

Shorter[16] pointed out that the expression (1.33 h) will hold good on condition that as the twist is increased the number of filaments in the yarn cross section must be reduced.

Neal Truslow[17] has corrected equation (1.33 h) for twist contraction. Thus

$$P_t = \left\{2A\cot^2\alpha_b \log_e \sec\alpha_b + B\epsilon_b \cos^2\alpha_b\right\}\frac{P_o C_y}{A + B\epsilon_b} \text{..} (1.33\ j)$$

where Cy = The yarn contraction factor twist.

The application of an external tensile load, along the axis of the yarn results in forces being applied to the various fibres. In general, the stresses which can act on the structural units of the twisted yarn are

1) Tensile forces - direction along fibre axis and normal to the fibre cross-section.

2) Compressive forces - direction radial and tangential to the element of yarn cross-section.

3) The bending moment.

4) The shear forces.

5) The torsional moment.

It is generally assumed that the fibres are perfectly flexible members, having an extremely low modulus of elasticity and an extremely large ratio of fibre length to fibre diameter. Platt[15] considered that under such assumptions it is justifiable to consider the influence of tensile forces only, and neglect the effect of others stresses.

In a recent paper on mechanics of twisted yarns, Hearle[18] has considered the influence of tensile and compressive forces, but neglected the influence of other forces mentioned above. Assuming the idealised theoretical yarn structure he obtained a condition for the radial equilibrium of the stresses acting on the small yarn element shown in figure 1.33k. His expression for yarn tenacity, where the fibre extensions follow Hooke's law, may be written as mean specific

$$\text{stress} = \frac{\text{yarn tension}}{\pi R^2/v}$$

$$= \frac{v}{\pi R^2}\int_c^1 \frac{L^2}{2\pi v}\left[X \times \frac{c^2}{x^2} - G\left(1 - \frac{c^2}{x^2}\right)\right] x\,dx \qquad \ldots\ldots (1.33\ k)$$

where c = cosine of the surface helix angle

X = axial stress

G = tangential stress

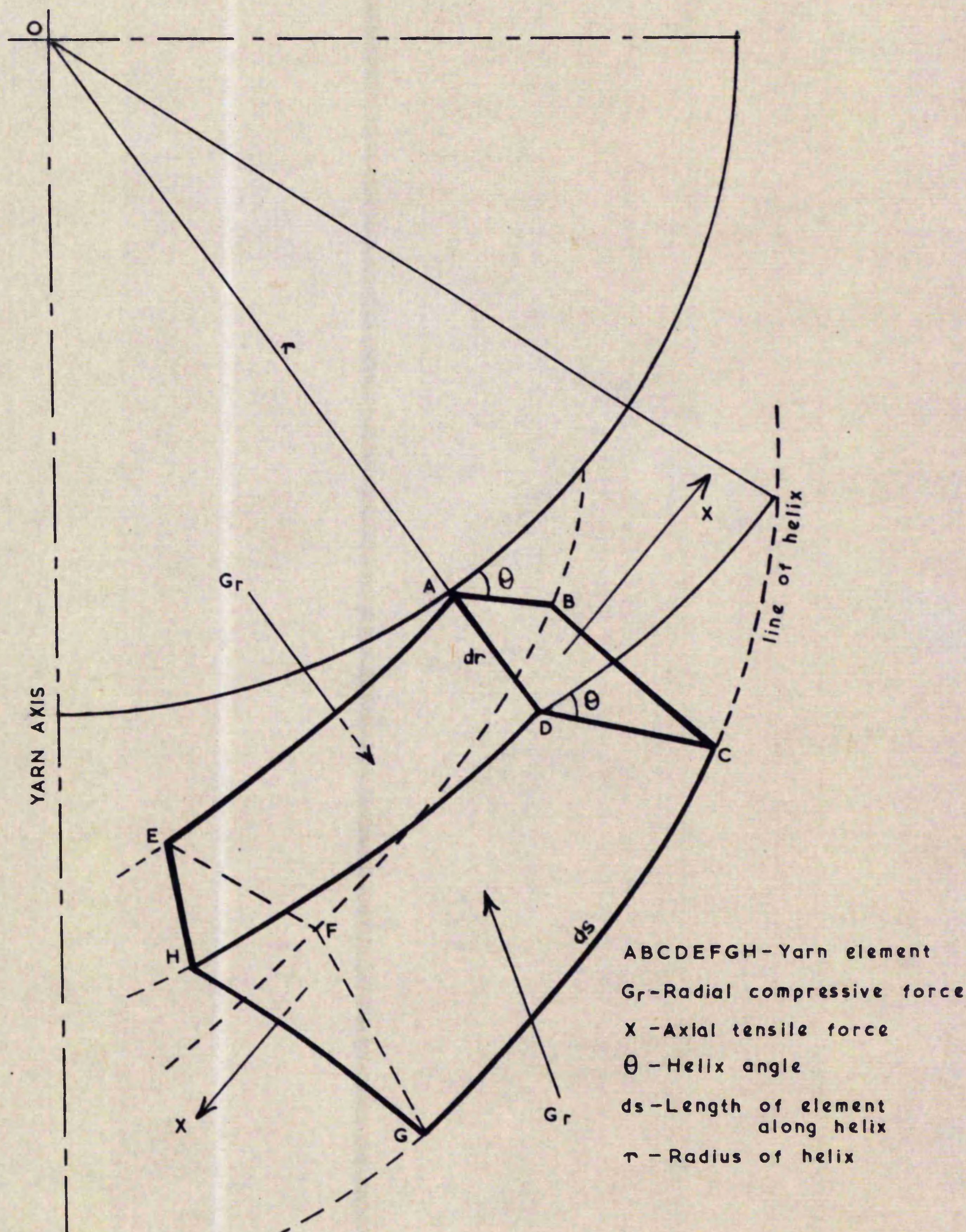


FIG. 1.33E FORCES ACTING ON THE YARN ELEMENT

x = The measure of the position of fibre in yarn

v = The specific volume of yarn.

On substituting the values for X and G from the equation governing the radial equilibrium of the yarn, he obtained an expression giving the yarn stress, Y, in terms of fibre stress Xf, axial poisson ratio σ_1, and the parameter, C, depending on the yarn twist.

$$Y = \frac{X_f \cdot C^2}{(1+\sigma_1)(1-C^2)}\left[\frac{4+3\sigma_1}{2(1+\sigma_1)} - \frac{1+\sigma_1}{2\sigma_1}\cdot C^2 - \frac{2\sigma_1^2+2\sigma_1-1}{2\sigma_1(1+\sigma_1)}C^{2(1+\sigma_1)} + \log_e C\right]$$

$$= X_f \cdot F(c, \sigma_1) \qquad \text{........... (1.331)}$$

where X_f = Tensile specific stress in a fibre under the same extension as the yarn.

In his radial equilibrium condition he assumes the magnitude of both the tangential and the radial compressive forces to be equal. Values calculated on this theory would be expected to differ from the experimental tenacity values, as Hooke's law ceases to hold near breakage.

Hearle further discusses what happens when the central filament is no longer able to contribute to the yarn tension. Figure (1.33D) represents the stress-strain curve of twisted yarns after the breakage has started with the stress expressed as a fraction of fibre tenacity,

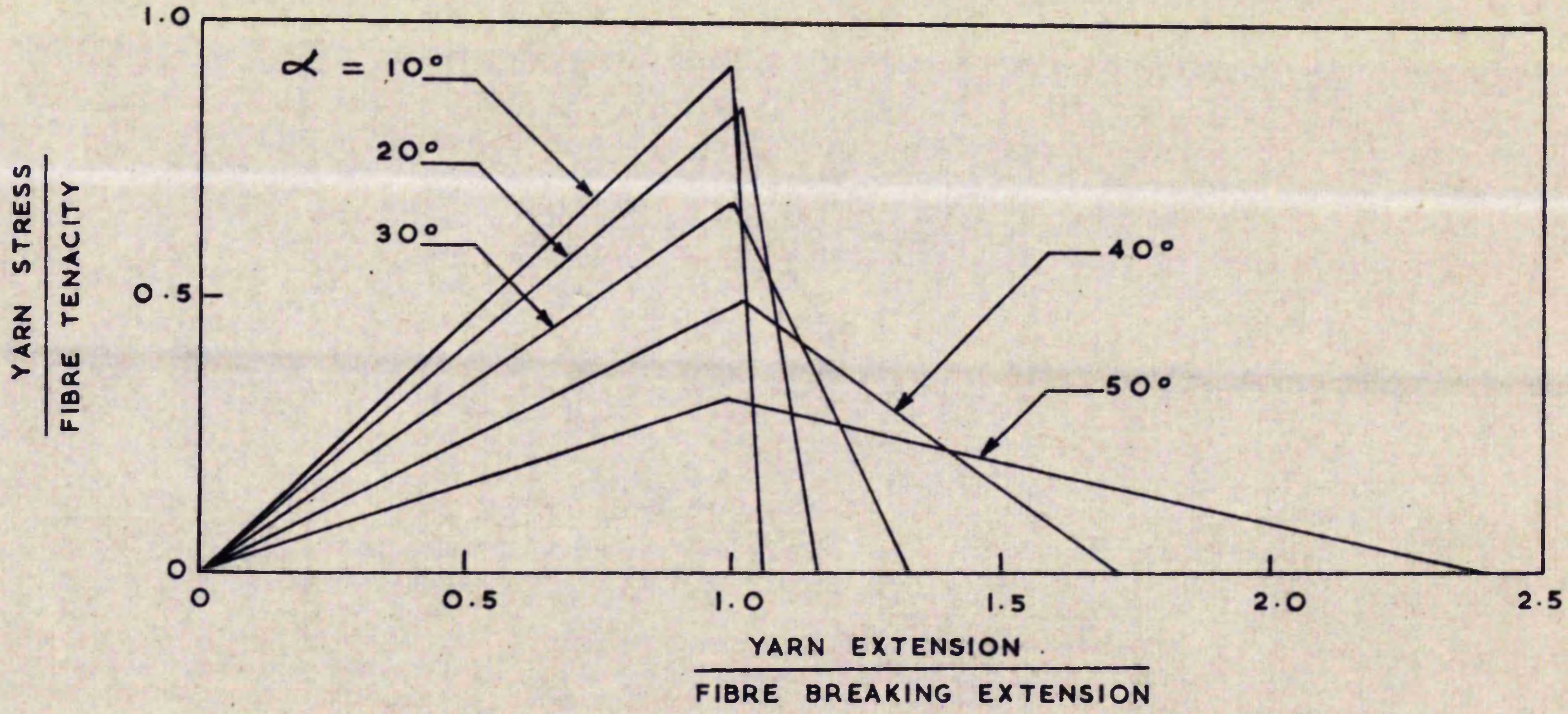


FIG.(1.33D) THEORETICAL STRESS-STRAIN CURVES ASSUMING HOOKE'S LAW AND THE INFLUENCE OF COMPRESSIVE FORCES.

and extension as its ratio to the fibre breaking extension.

1.33B BREAKING EXTENSION

Simple theory, on the basis of idealised yarn structure, predicts that the yarn elongation to rupture should be independent of twist. Since, the fibres close to the yarn axes are the most highly strained, with an extension equal to that of the yarn, they will be the first to fail, and the yarn elongationto rupture should be equal to that of the fibre material. Thus where twisting does not influence the tensile properties of fibres, yarn breaking extension will be independent of the yarn linear density (Tex) or yarn twist per unit length.

Platt[15] and Hamburger[19] have used the above simple theory to explain their experimental results. They also pointed out the possibility of the mathematical prediction of the stress-strain relations of yarns with successively higher twists (Fig. 1.33B). At any load below rupture the elongation of higher twist yarns exceeds that of lower twist yarns. In other words, for a given stress 'f' below yield point the strain 'ϵ_t' of the twisted yarn may be related to the strain ϵ_o of zero twist yarn by a mathematical expression as

$$\epsilon_t = \epsilon_o \cdot \operatorname{Sec}^2\alpha \qquad \text{........ (1.33m)}$$

Platt[7] corrected this expression for twist contraction, which seems to be irrelevant.

Hearle[18] has pointed out that the fibres close to the yarn axis in an idealised yarn structure are under the influence of very high compressive forces. Also textile fibres are known to undergo lateral dimensional change (Poisson's ratio effect) whenever they are axially extended or radially compressed. In such conditions, the filament breaking extension may be different from that obtained in filament tensile tests. No experimental work has been reported to study the effect of the compressive forces on breaking extensions and tenacities of the textile fibres.

1.34 Yarn rupture properties - Experimental aspects

There is a good deal of experimental work reported on breaking extension and tenacity of continuous multifilament yarns. The continuous multifilament yarns can be twisted in various ways: for example, static twisting, twisting on a ring doubler and twisting on an uptwister. The tension during twisting can be varied over a wide range in all the three types of twisting. Uptwisting is the most common in commercial practice. Very little work has been reported on the influence of twisting tensions on the rupture properties of twisted yarns.

Platt[15], Taylor et al[21], Alexander and Sturley[22], and others[24,25] have studied the rupture properties of twisted continuous filament yarns. Their experimental procedure, yarns tested and results are summarised in Tables ($1.34A_1$ & A_2).

TABLE 1.34 A (1)

Summary of experimental procedure.

Reference	Experimental procedure (a) Twisting; (b) Testing
Taylor et al[21]	(a) Twisted on Atwood universal model 110 uptwister with a twisting tension 3.0 g.wt./tex. Yarns were found to shear at twist factor 105 $tex^{\frac{1}{2}}$ turns/cm. (b) Tensile tests on IP2 tester. Twenty tests for each yarn were taken by using 10 inches specimen length.
Grover et al[20]	(a) & (b) Twisting and testing was carried out simultaneously. Predetermined length of zero twist yarn that would give a contracted length of 12 ins. was twisted on the IP2 Scott Tester. Turns per inch, yarn dia. in cm. and surface helix angle was measured. Ten inches length from the twisted twelveinches of yarn was then used for tensile tests.
Platt et al[15]	Uptwisted.
Shrinagabhushan[23]	(a) Twisted on a silk doubler of the uptwister type. (b) Tested on the Goodbrand single thread tester with 12 inches rate of traverse per minute. The average value was obtained by testing 25 samples of 10 ins specimen lengths on the Goodbrand single thread tester

TABLE 1.34 A (2)

Summary of the results obtained.

Reference	Nominal den./fil.no.	(initial) Twist factor $Tex^{\frac{1}{2}}$ turns/cm (maximum)	RESULTS (a) Tenacity; (b) Breaking extension
A) VISCOSE RAYON			
Taylor et al[21]	150/40 150/40 250/60 300/44 300/120	95.5 98.4 104.2 90.3 90.7	(a) The maximum tenacity occurs at twist factor 12 units and then decreases continuously. Relative tenacities when plotted against twist factor at break show a common curve for all yarns. However, wide scatter is seen at higher twist factors. (b) For all yarns, the breaking extension initially increases to a maximum and then remains fairly constant at high twist factors.
Grover et al[20]	50/20 75/30 100/40 100/60 150/40 150/90 300/44	67.6 71.3 87.6 83.1 90.3 85.4 90.3	(a) The maximum tenacity occurs at twist factor 15 units and then decreases continuously as the twist factor is increased. Relative tenacities when plotted against twist factor at break exhibits a common curve for all yarns except viscose 50/20. (b) The breaking extension initially increases to a maximum and then decreases continuously as the twist factor is increased. The rate of decrease is higher in viscose 50/20 yarn.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	Twist factor $Tex^{\frac{1}{2}}$ turns/cm. (initial) (maximum)	R E S U L T S (a) Tenacity; (b) Breaking extension
Platt et al[15]	150/40	44.2	(a) The tenacity initially remains fairly constant and then decreases continuously. (b) The breaking extension remains constant over the entire range of twist factors (35 $tex^{\frac{1}{2}}$ turns/cm.). No initial increase is observed.
Shrinagabhushan[23]	150/30 60/45	105 101	(a) The maximum tenacity occurs at the twist factor of 15 $tex^{\frac{1}{2}}$ turns/cm. and then decreases continuously. The relative tenacity values when plotted against twist factor at break do not lie on a common curve. For a given twist factor, the higher the yarn count in tex the lower is the relative tenacity. (b) The breaking extension initially increases to a maximum at twist factor of 20 units. At higher twist factors the rate of decrease in breaking extension is higher for finer yarns.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	(initial) Twist factor $Tex^{\frac{1}{2}}$ turns/cm. (maximum)	R E S U L T S (a) Tenacity; (b) Breaking extension
B) <u>ACETATE</u>			
Grover et al[20]	55/14 75/20 100/28 120/32 150/41 300/80	74.6 89.1 88.2 98.9 90.9 72.0	(a) The maximum tenacity occurs at 30 $tex^{\frac{1}{2}}$ turns/cm. and then decreases continuously. The relative tenacities when plotted against twist factor at break show a common curve for all yarns except 300/80. All values lie above those reported by Platt et al[15]. (b) The breaking extension initially increases to a maximum, remains constant up to twist factor of 65 units, and then decreases continuously.
Platt et al[15]	300/104 600/208	56.7 67.4	(a) The tenacity initially remains constant and then decreases continuously. Relative tenacities when plotted against twist factor at break show a common curve for both yarns but the relative values lie below those reported by Grover et al[20]. (b) The breaking extension remains constant over the entire range of twists. No initial increase or later decrease is observed.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	(initial) Twist factor $Tex^{\frac{1}{2}}$ turns/cm. (maximum)	R E S U L T S (a) Tenacity; (b) Breaking extension
C) <u>NYLON</u>			
Taylor et al[21]	20/7 T 200 30/10 T 200 40/13 " 70/34 T 100 70/34 T 200 70/34 T 300 100/34 " 210/34 " 260/17 "	60.9 74.5 88.9 81.5 84.2 80.6 64.4 74.5 85.4	(a) The maximum tenacity occurs at twist factor 10 units at break and then decreases continuously. Relative tenacities when plotted against twist factor at break show a wide scatter. (b) The breaking extension initially remains constant and then increases more or less rapidly for all the yarns except 20/7. The relaxation time has a direct effect on the extension behaviour of nylon yarns.
Alexander et al[22]	15/1 30/10 45/15 60/20	36.9 56.0 71.3 85.8	(a) The tenacity values when plotted against twist factor lie on a common curve. The maximum tenacity occurs at 18 $tex^{\frac{1}{2}}$ turns/cm. and then decreases continuously. The tenacity is fairly constant for 15 denier monofilament. (b) The breaking extension initially decreases to a minimum and then increases for all yarns except 15/1 where it is fairly constant at all twists.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	Twist factor (initial) $Tex^{\frac{1}{2}}$ turns/cm. (maximum)	RESULTS (a) Tenacity; (b) Breaking extension
Grover et al[20]	15/1 30/10 40/13 50/17 70/34 100/34	36.4 52.5 61.4 70.4 85.9 86.9	(a) The maximum tenacity occurs at twist factor 10 units (at break) for 100-34 and 15/1 while the tenacity is initially constant for all other yarns. On further insertion of twist it decreases continuously. The relative tenacity values when plotted against twist factor at break show a wide scatter at high twist factors. (b) The breaking extension initially decreases to a minimum and then increases fairly rapidly for all yarns except 100/34 and 15/1 yarns where it increases continuously. The effect of twisting tension is to reduce both the breaking load and the breaking extension.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	(initial) Twist factor $Tex^{\frac{1}{2}}$ turns/cm. (maximum)	R E S U L T S (a) Tenacity; (b) Breaking extension
D) DACRON Taylor et al[21]	40/34 70/34 210/34	84.6 80.2 104.2	(a) The tenacity values for all yarns decrease continuously as the twist factor is increased. No initial rise in tenacity is observed. (b) The breaking extension increases continuously as the twist is increased for all yarns except 40/34 where it initially decreases to a minimum and then increases continuously.
Grover et al[20]	40/34 70/34	51.8 70.0	(a) The tenacity values for both yarns initially increases to a maximum and then decreases continuously as the twist factor is increased. (b) The breaking extension increases continuously as the twist is increased.

TABLE 1.34 A (2) cont'd

Reference	Nominal den./fil.no.	(initial) Twist factor $Tex^{\frac{1}{2}}$ turns/cm. (maximum)	RESULTS (a) Tenacity; (b) Breaking extension
E) <u>ORLON</u>			
Taylor et al[21]	75-30 100-40 200-80 400-160	85.0 63.9 69.7 63.4	(a) Tenacity initially increases to a maximum and then decreases continuously. (b) Breaking extension is fairly constant at all twist factors for all yarns except 75/30 where a sharp decrease is observed at 85 $tex^{\frac{1}{2}}$ turns/cm.
Grover et al[20]	75-30 100-40 150-80 200-40 200-80	73.0 70.0 78.0 59.0 59.0	(a) Tenacity initially increases to a maximum and then decreases continuously. Relative tenacity values are lower than those reported by Taylor et al[21]. (b) Breaking extension initially increases to a maximum and then decreases continuously at all twist factors higher than 40 $tex^{\frac{1}{2}}$ turns/cm.

1.34A TENACITY

All the experimental data was replotted to show corrected tenacities (tenacity in gms.wt./tex at break). Most of the curves show a rise in tenacity as the twist factor rises to a twist factor of 10 to 20 $tex^{\frac{1}{2}}$ turns/cm., and then a fall for higher twist factors. The initial rise is usually explained as being due to the increased mutual support of the filaments as the frictional forces are brought into play. The fall in tenacity is in qualitative agreement with the theory as expressed by the equations 1.33(i) or 1.33(l).

For comparative purposes, it is more convenient to plot relative values of the tenacity against twist factor. This has been done in figures ($1.34A_1$ - A_8). It would seem logical to use these ratios relative to the values at zero twist. But the tenacities of zero twist yarns are very much influenced by the variability of filaments in the composite specimen. As the twist is increased, the influence of this effect is eliminated and the yarn tenacity reaches its maximum. In order to eliminate the influence of this factor, the ratios relative to the tenacity values at twist factors of 20 $tex^{\frac{1}{2}}$ turns/cm. for Acetate and 10 $tex^{\frac{1}{2}}$ turns/cm. for other yarns have been plotted. These points are given in Table (1.34B). They correspond to the maximum in the tenacity curves for the respective yarns.

1.34B BREAKING EXTENSIONS

As regards breaking extension, most of the results depart appreciably from the simple prediction that the breaking extension should be constant

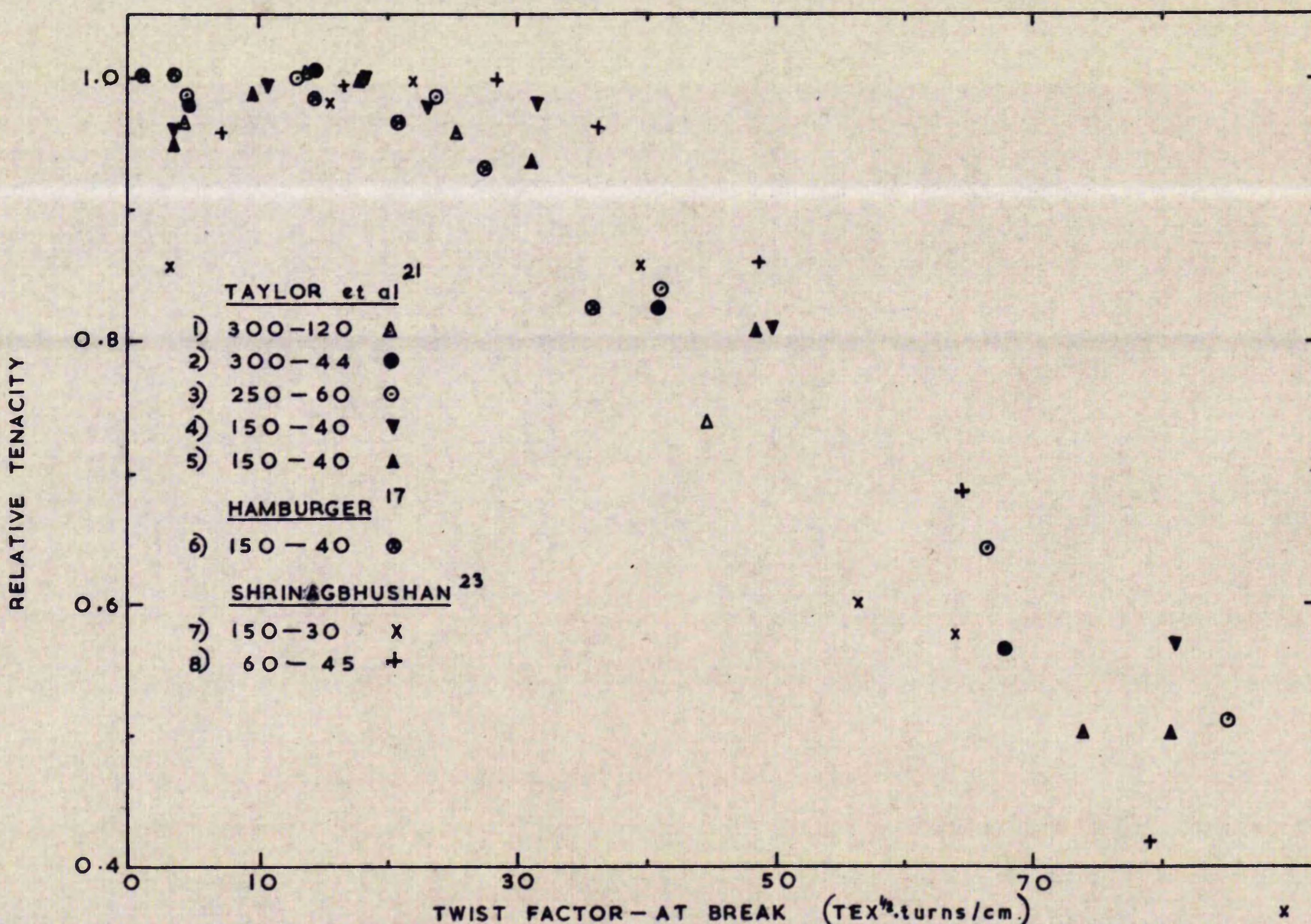


FIG. 1.34 A_1 RELATIVE TENACITIES OF SOME VISCOSE YARNS.

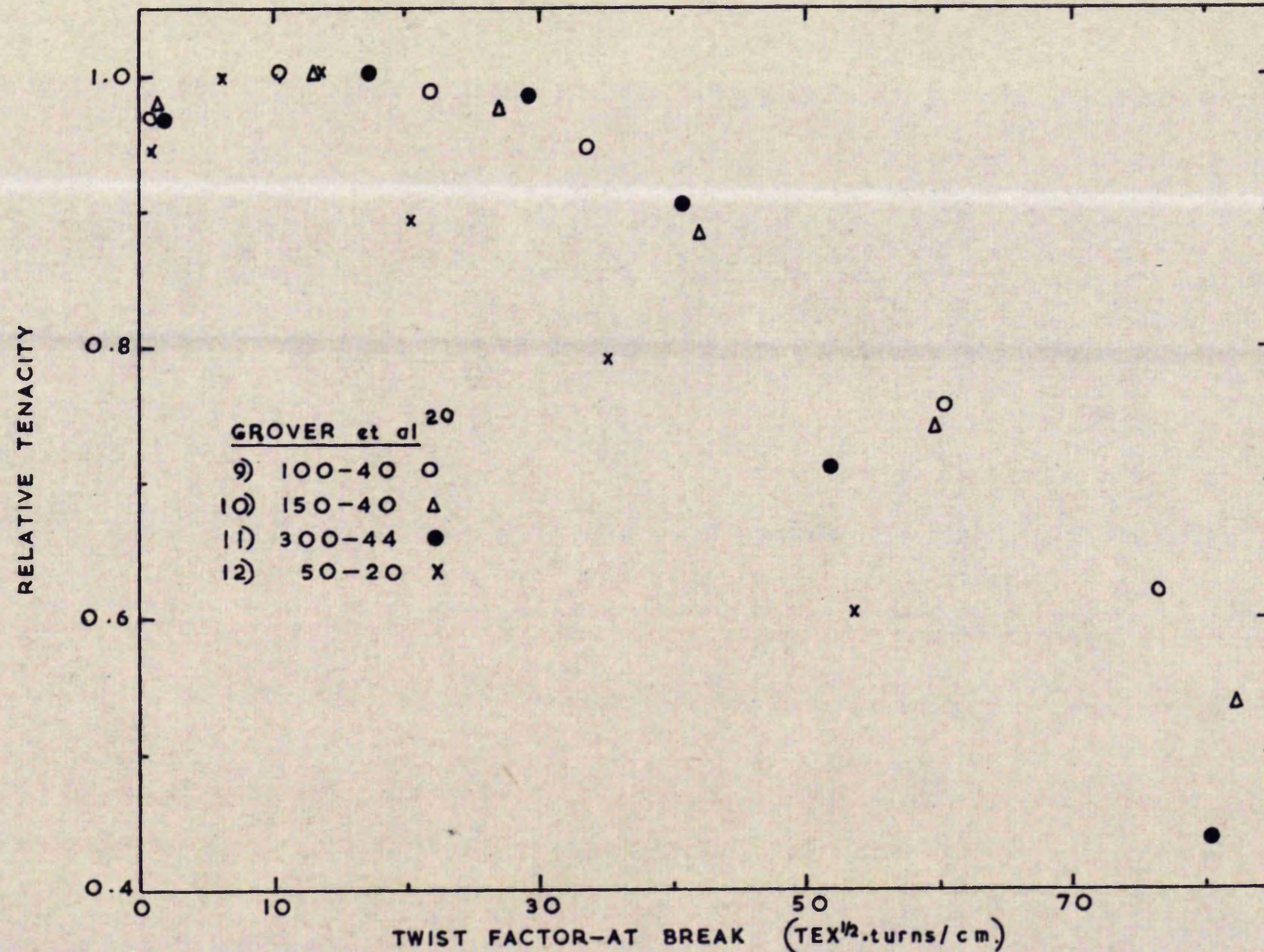


FIG. 1.34 A_2 RELATIVE TENACITIES OF SOME VISCOSE YARNS

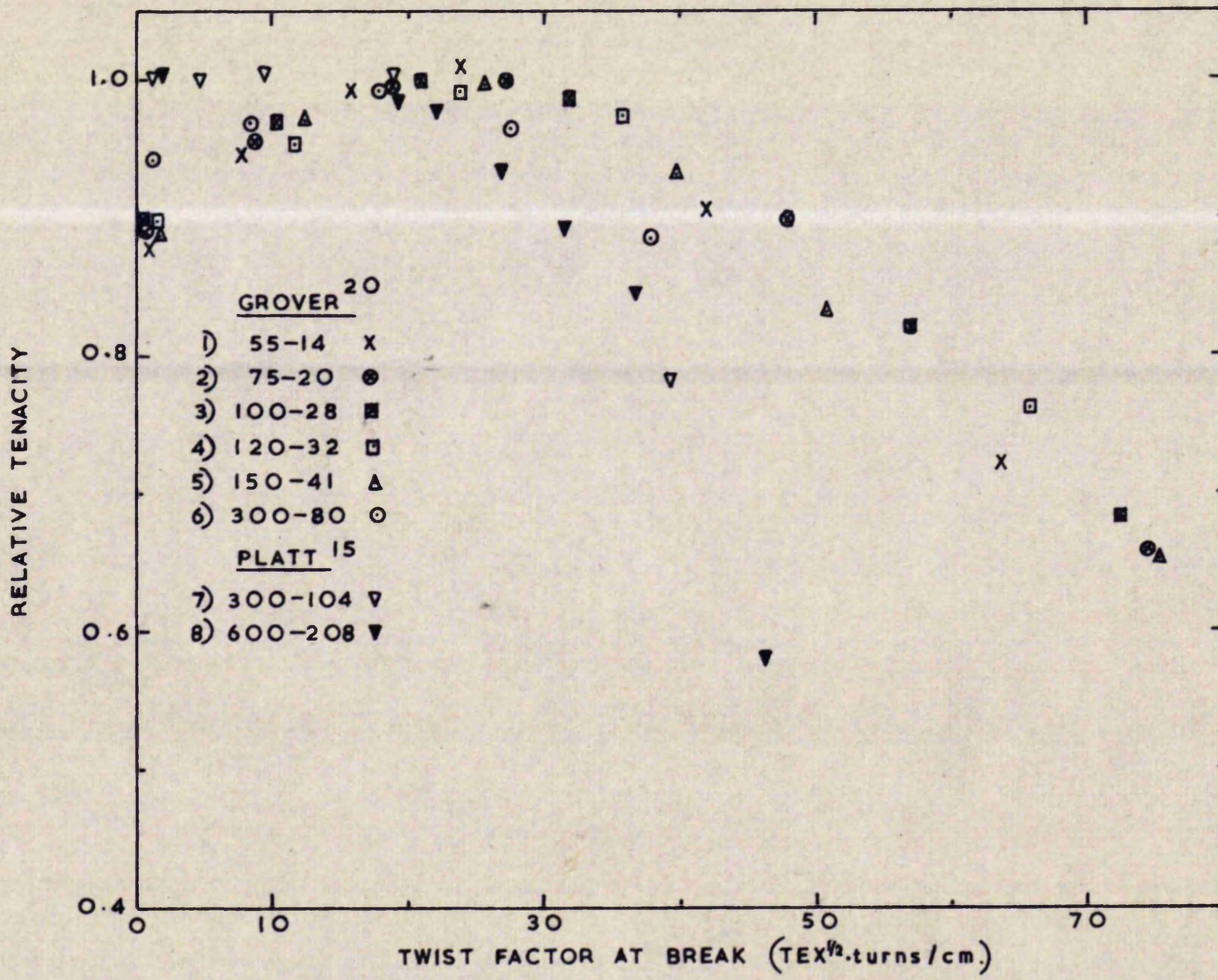


FIG. 1.3 4 A_3 RELATIVE TENACITIES OF SOME ACETATE YARNS.

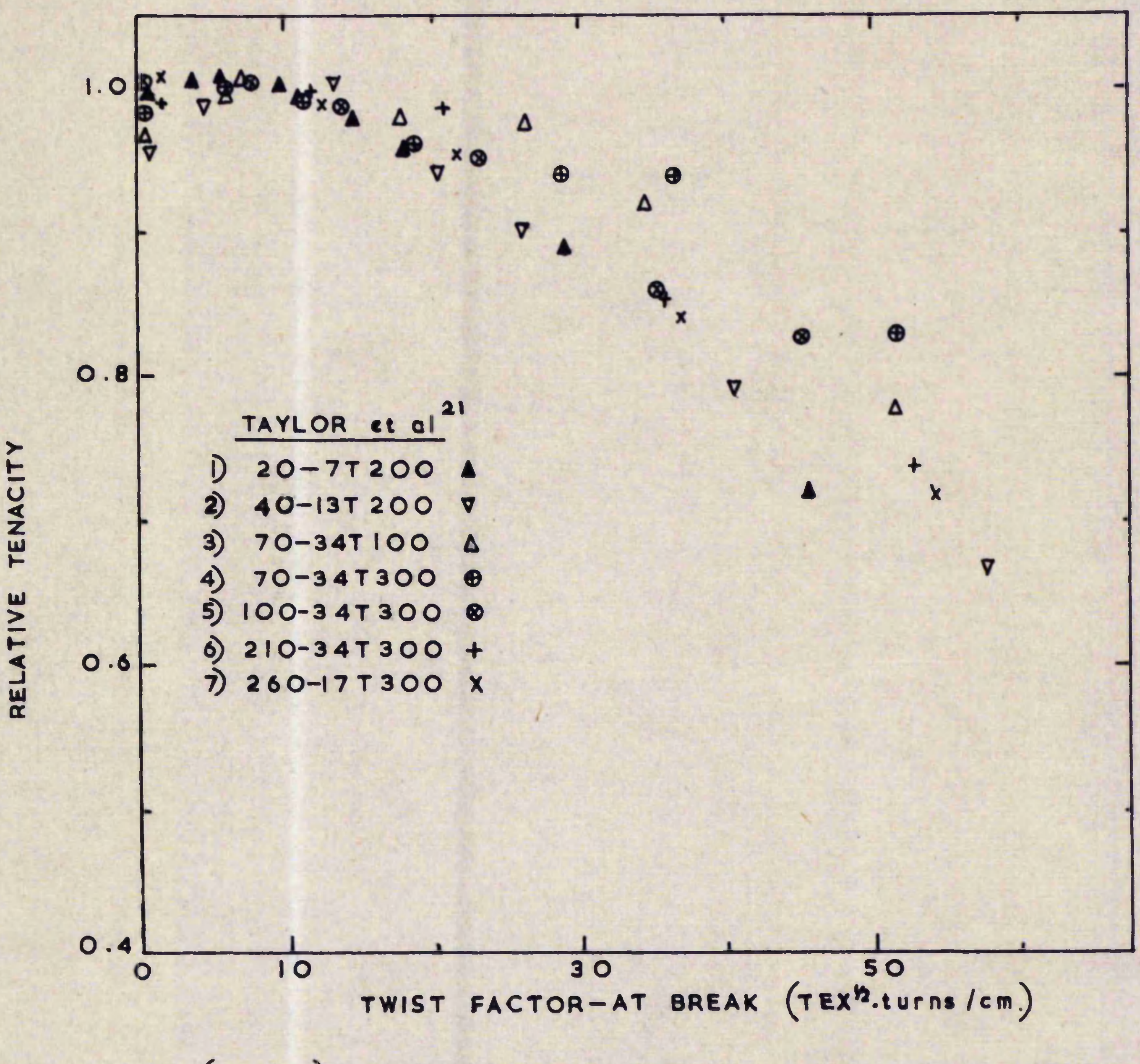


FIG.(1.34 A_4) RELATIVE TENACITIES OF SOME NYLON YARNS.

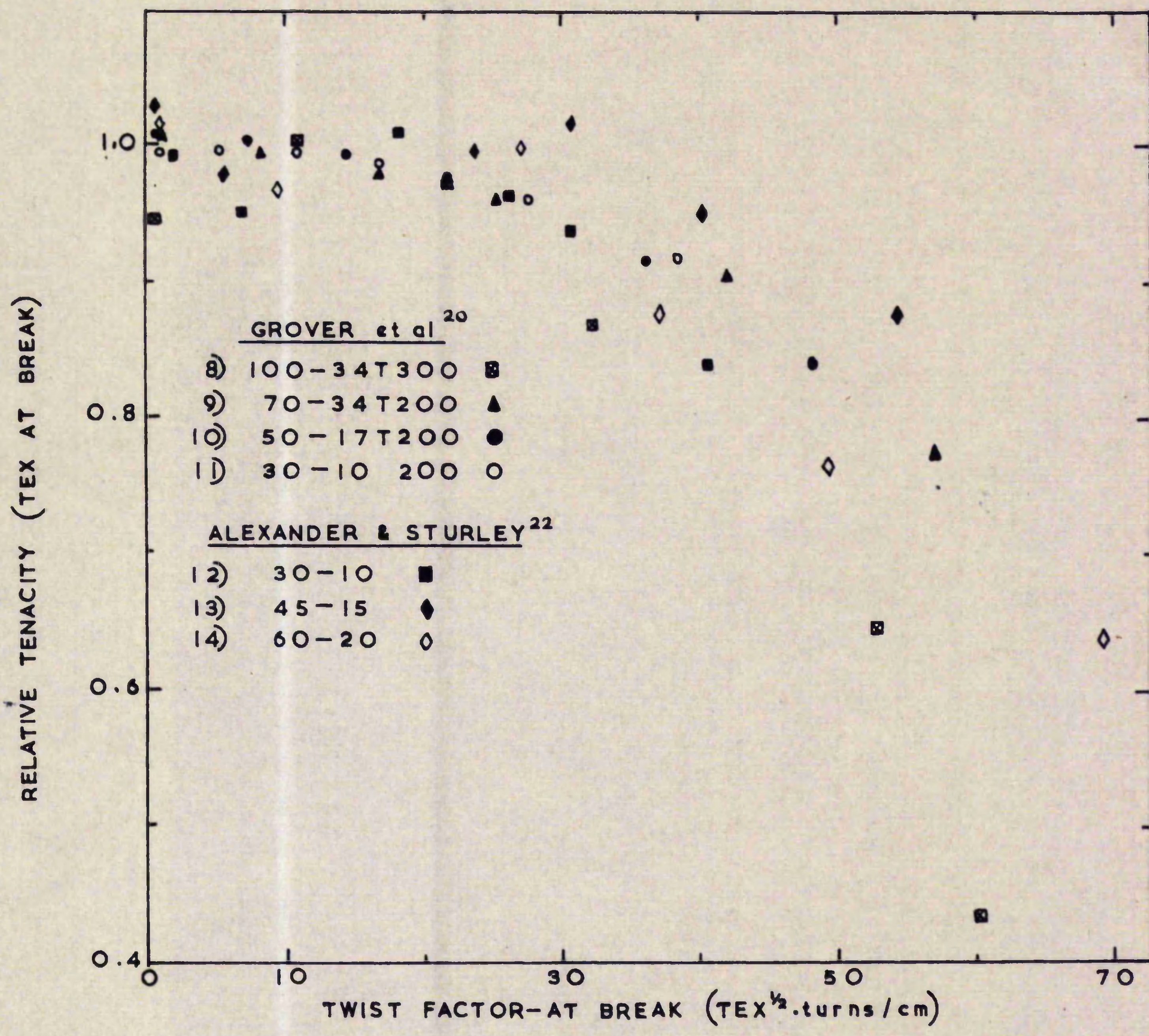


FIG.(1.34 A5) RELATIVE TENACITIES OF SOME NYLON YARNS.

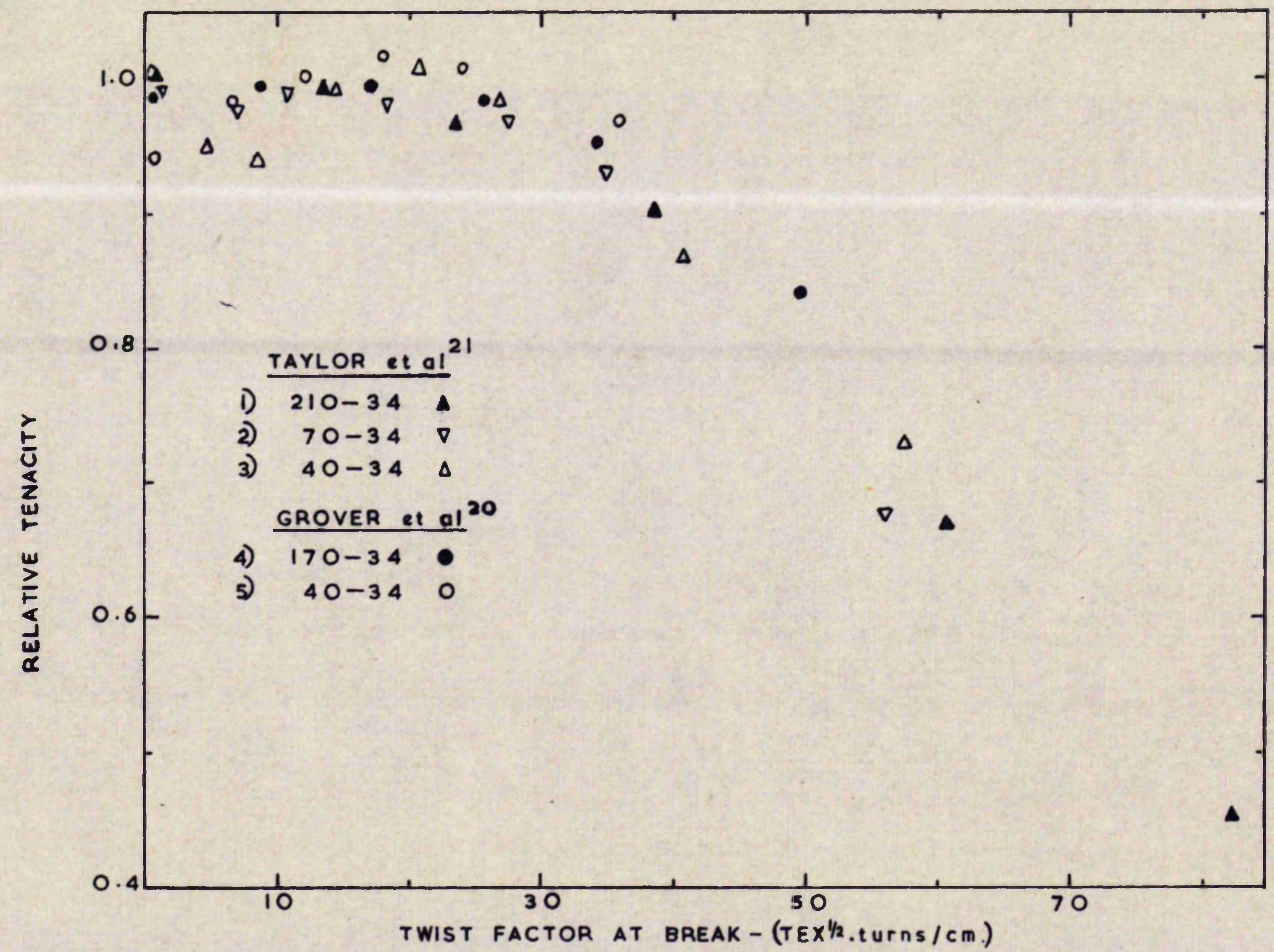


FIG.(1.34A_6) RELATIVE TENACITIES OF SOME DACRON YARNS.

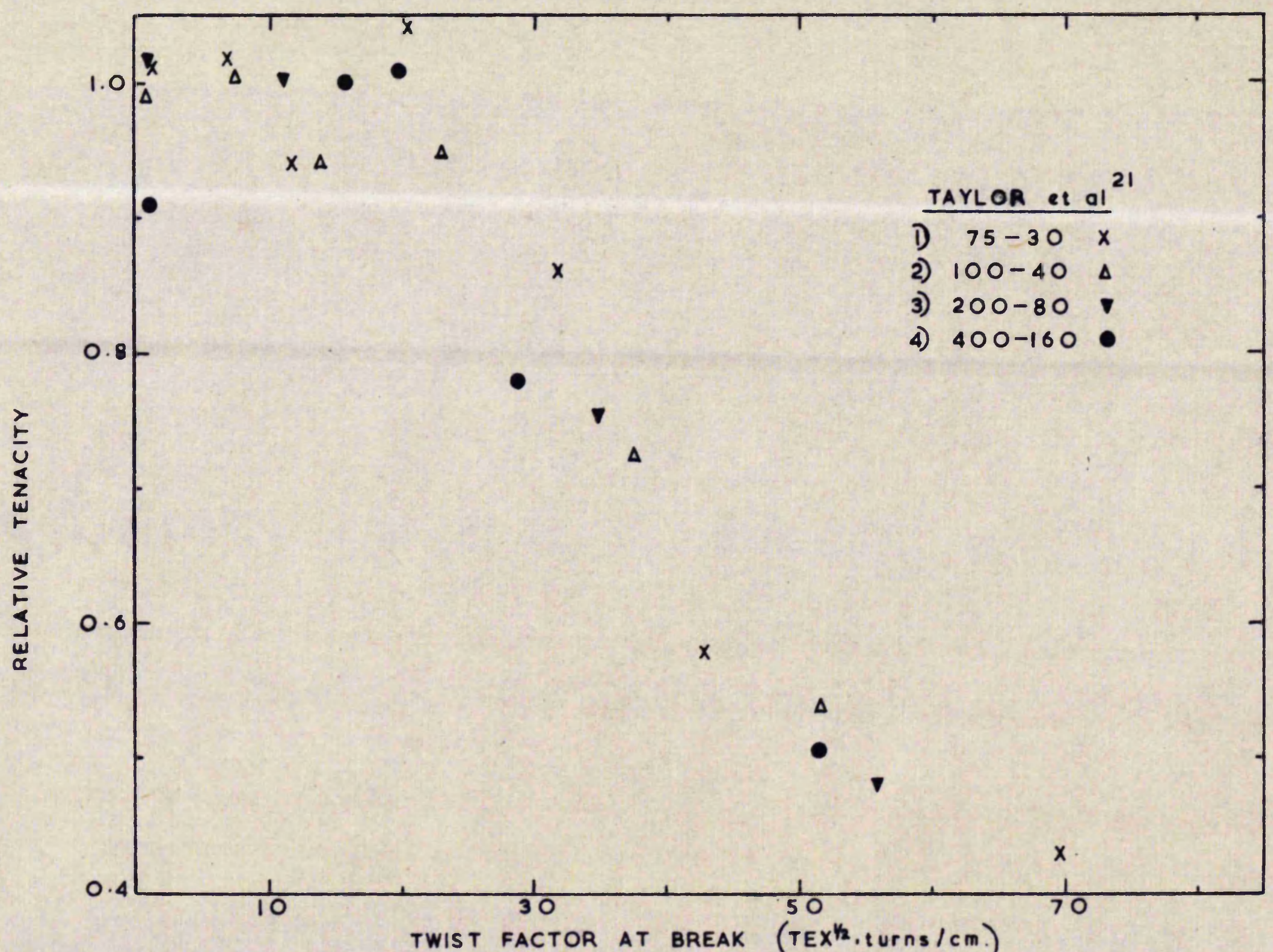


FIG.(1.34 A_7) RELATIVE TENACITIES OF SOME ORLON YARNS.

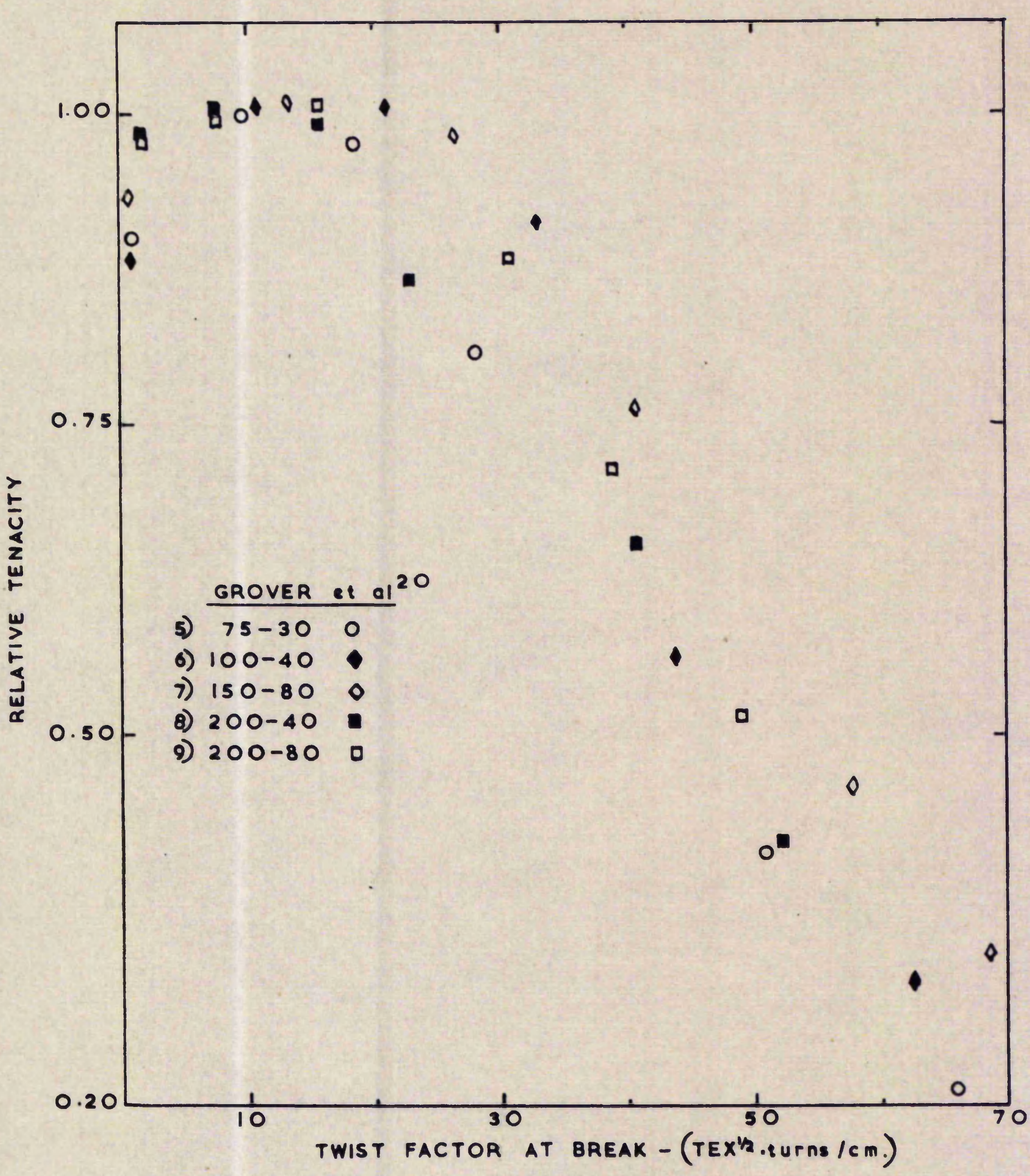


FIG.(1.34A8) RELATIVE TENACITIES OF SOME ORLON YARNS.

TABLE 1.34B

The tenacity values used as standard tenacities in calculating the relative tenacity values in figures $1.34A_1 - A_8$.

Reference	Denier Fil. No.	Standard Tenacity (corrected) g.wt./tex.	Reference	Denier Fil. No.	Standard Tenacity (corrected) g.wt./tex.
		A) VISCOSE RAYON			
(1) Taylor et al[21]	150-40 H.T.	28.1	(2) Grover et al[20]	50-20	27.0
	150-40 AVISCO	27.2		100-40	23.0
	250-60 "	22.2		100-60	22.3
	300-44 "	26.0		300-44	22.3
	300-120 "	28.8			
(3) Platt et al[17,15]	150-40	21.8	(4) Shrinaga-bhushan[23]	60-45	16.0
				150-30	16.2
		B) ACETATE			
(1) Grover et al[20]	55-14	16.4	(2) Platt[15]	300-104	17.2
	75-20	16.4		600-208	17.2
	100-28	16.4			
	120-32	15.4			
	150-41	15.4			
	300-80	15.5			

TABLE 1.34B - cont'd

Reference	Denier Fil. No.	Standard Tenacity (corrected) g.wt./tex.	Reference	Denier Fil. No.	Standard Tenacity (corrected) g.wt./tex.
		C) NYLON			
(1) Taylor et al[21]	20-7 T200	75.8	(2) Grover et al[20]	30-10 T200	63.2
	40-13 T200	62.8		50-17 "	58.8
	70-34 "	64.9		70-34 "	65.3
	70-34 T300	78.4		100-34 T300	69.0
	100-34 "	78.9			
	210-34 "	86.4			
	260-17 "	72.0			
(3) Alexander et al[22]	30-10	52.6			
	45-15	50.6			
	60-20	51.6			
		D) DACRON			
(1) Taylor et al[21]	40-34	59.4	(2) Grover et al[20]	40-34	60.4
	70-34	61.8		70-34	62.4
	210-34 H.T.	70.8			
		E) ORLON			
(1) Taylor et al[21]	75-30	49.6	(2) Grover et al[20]	75-30	49.6
	100-40	53.0		100-40	42.1
	200-80	51.6		150-80	40.1
	400-160	50.0		200-40	40.5
				200-80	42.0

and independent of twist. From figures ($1.34B_1$ - B_5), it can be seen that the only exception to this rule are Platt's values for viscose rayon and acetate yarns, and some of the other results for viscose rayon and orlon yarns. Most of the curves for acetate and viscose rayon show an initial increase in breaking extension and then a decrease as the twist is increased. On the other hand, most of the nylon and Dacron yarns show an initial drop followed by a rise in breaking extension as twist increases.

1.34C WORK OF RUPTURE AND WORK FACTOR.

The work of rupture is defined as the area under the load extension curve expressed as gm.wt.-cm. per tex for a centimetre length. The values of work of rupture for viscose and nylon yarns reported by Taylor et al[21] have been re-calculated and plotted in figure ($1.34C_1$ - C_2) as relative values of work of rupture.

Meredith[33] has defined the work factor as the ratio of work of rupture to the product of breaking extension and tenacity. This work factor is a measure of deviation from Hooke's law. This work factor has been also plotted in figure ($1.34C_3$ - C_4).

The work factor remains more or less constant in all viscose rayon yarns, while the work of rupture initially increases to a maximum and then decreases continuously, in the same way as the tenacity values. No initial rise in work of rupture is observed in nylon yarns. For all nylon yarns the work of rupture decreases continuously except for nylon 70-34

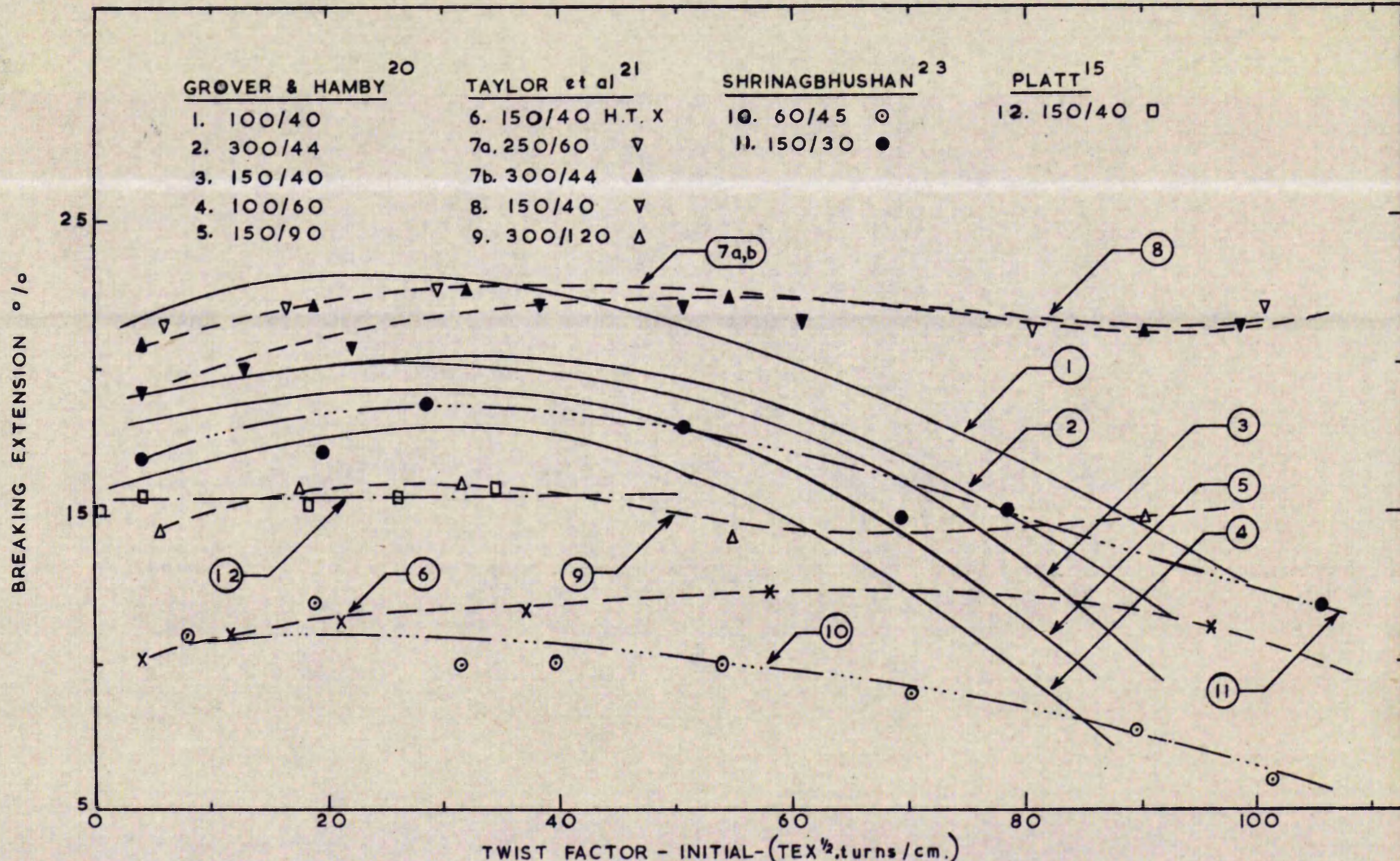


FIG. 1.34B, BREAKING EXTENSION OF SOME VISCOSE YARNS

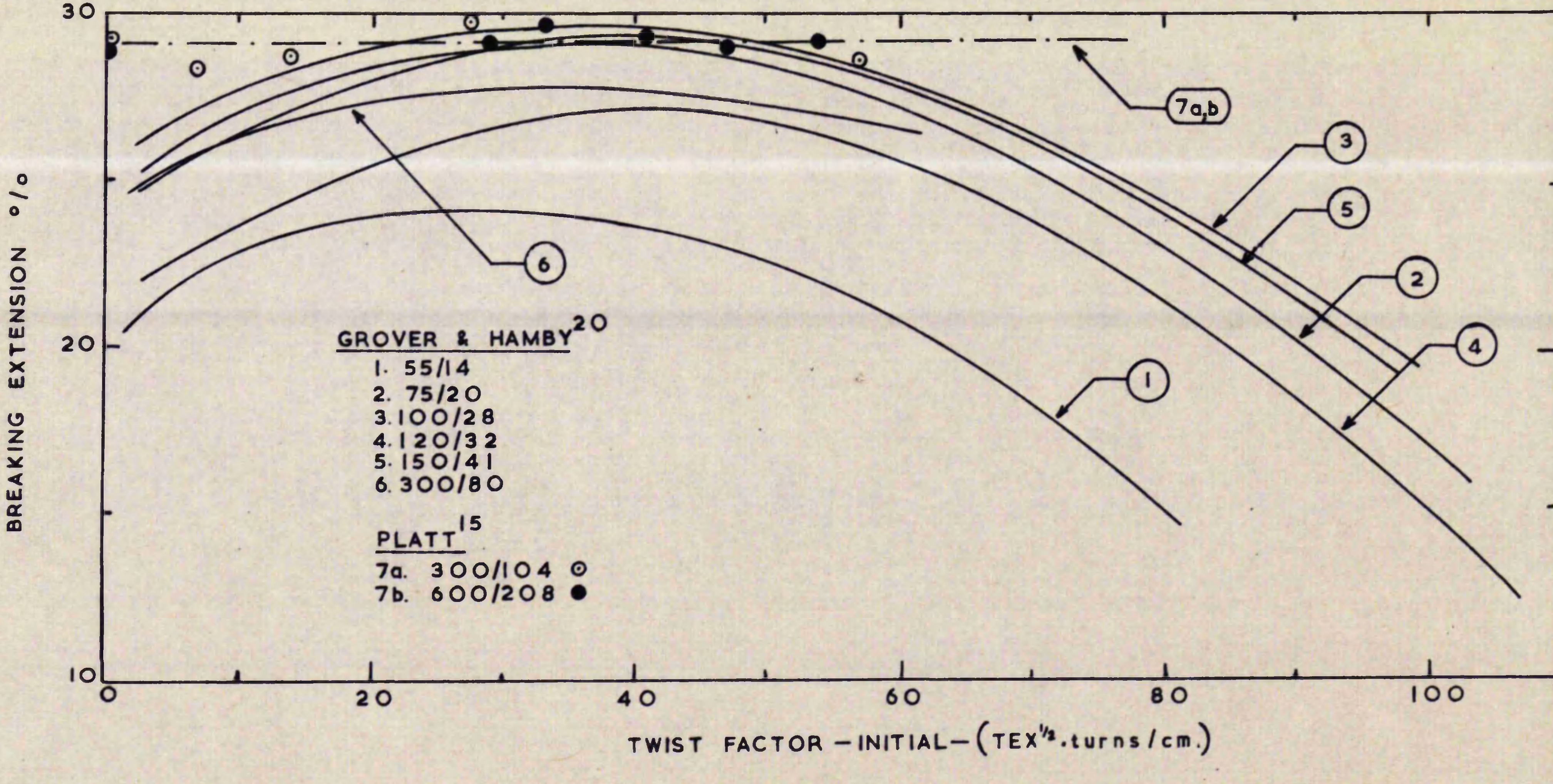


FIG. 1.34 B$_2$ BREAKING EXTENSION OF SOME ACETATE YARNS.

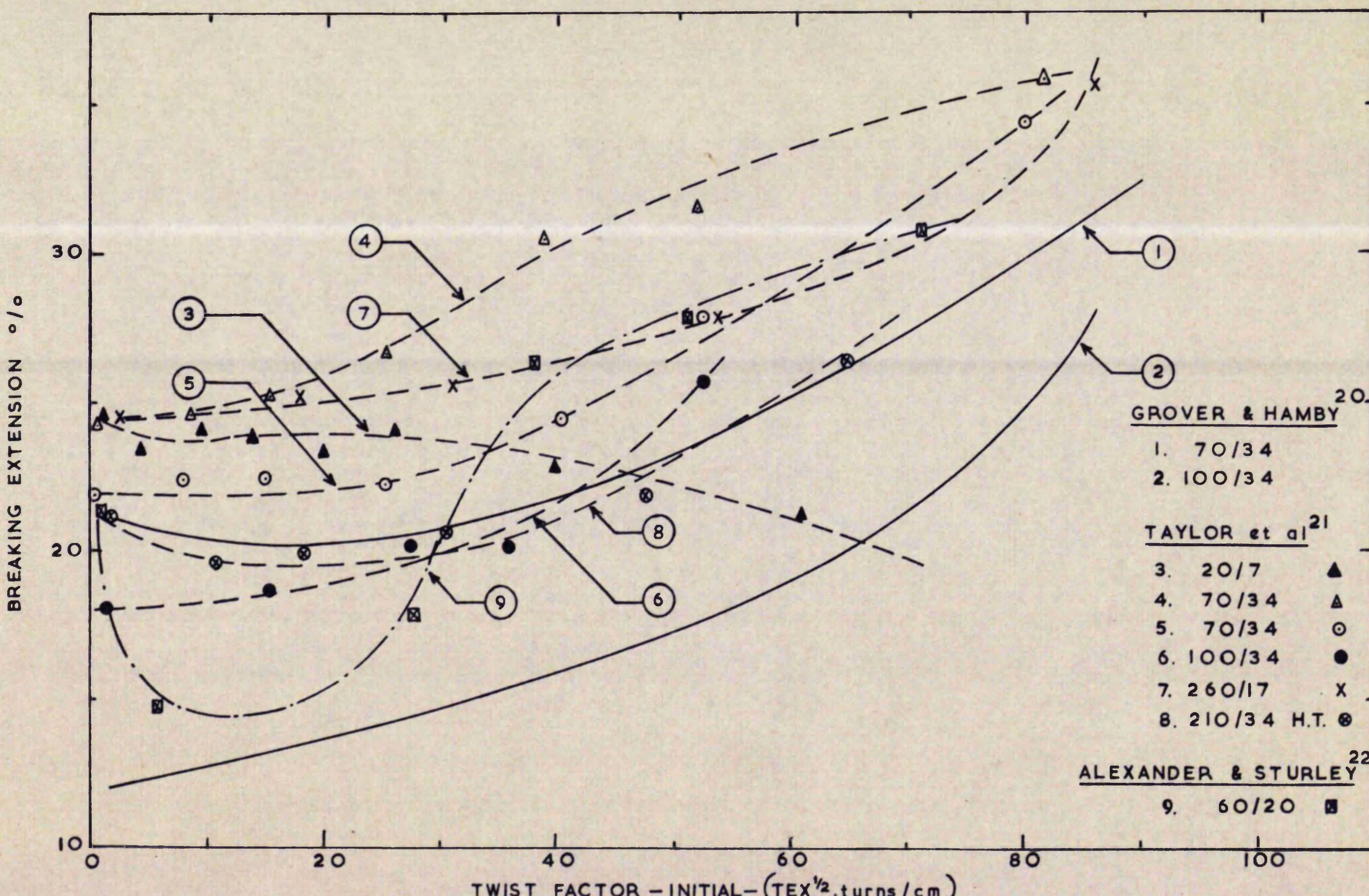


FIG. 1.34B$_3$ BREAKING EXTENSION OF SOME NYLON YARNS.

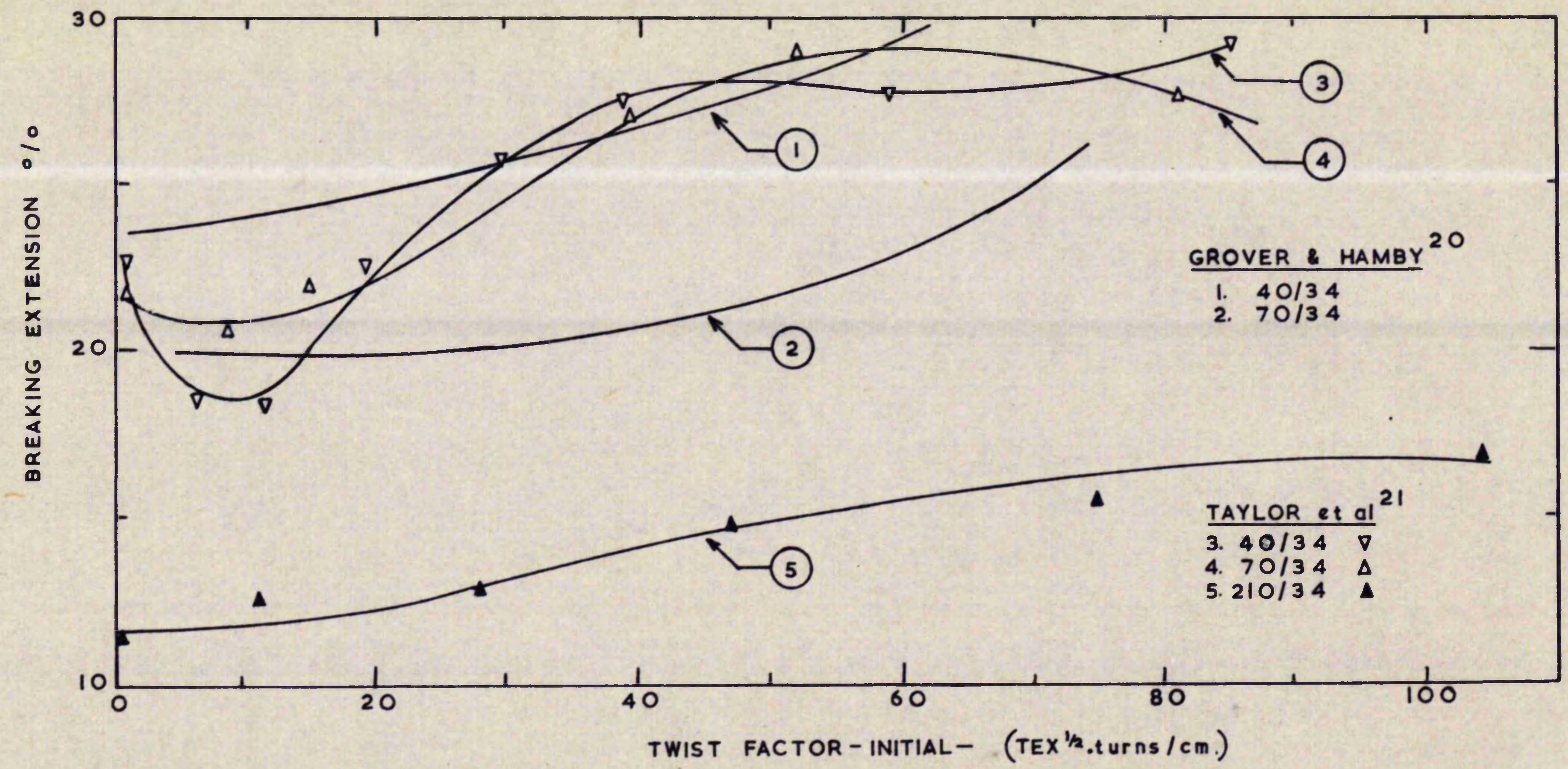


FIG. 1·34B$_4$ BREAKING EXTENSION OF SOME DACRON YARNS.

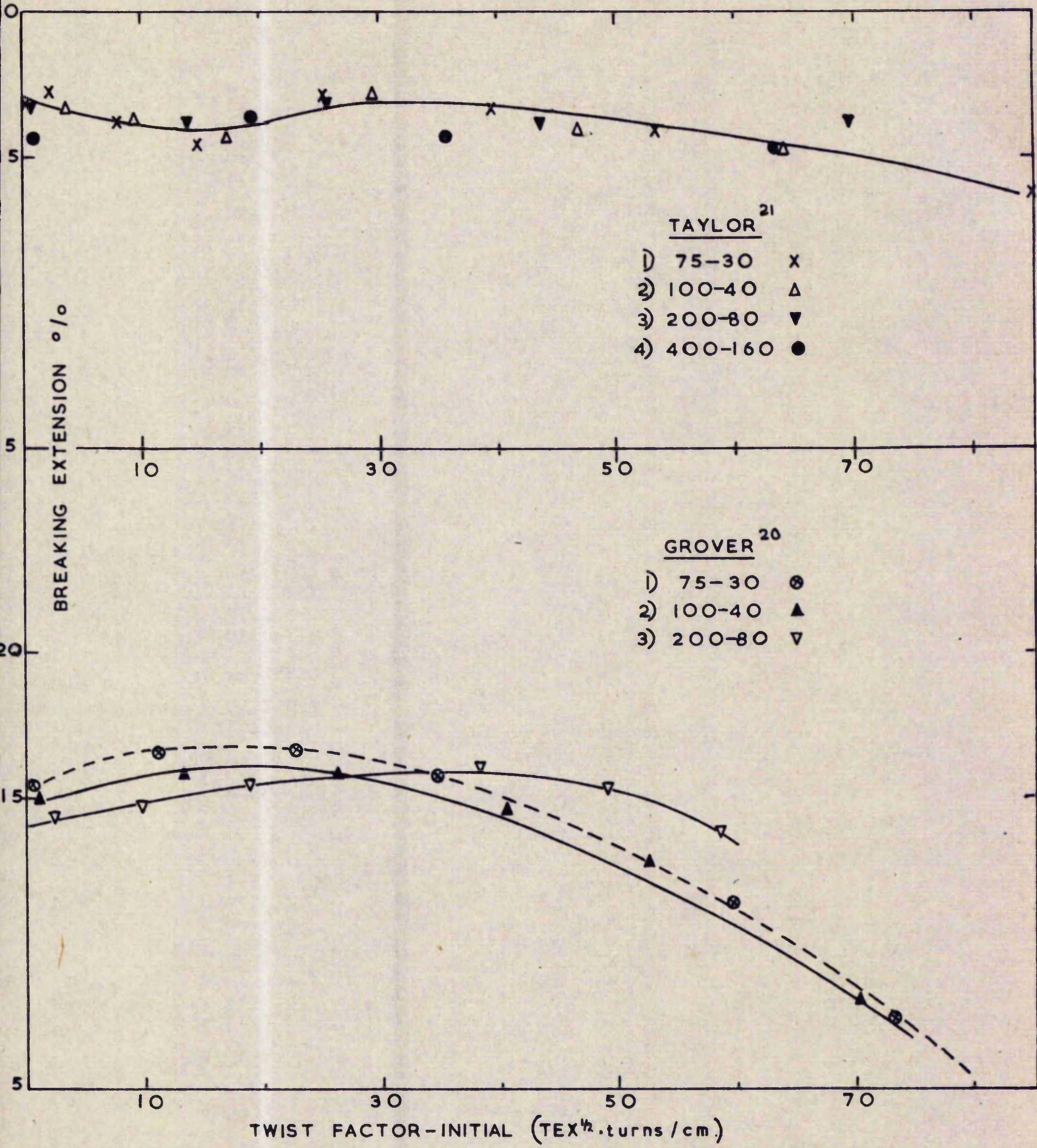


FIG. (1.34 B_5) THE BREAKING EXTENSION OF SOME ORLON YARNS.

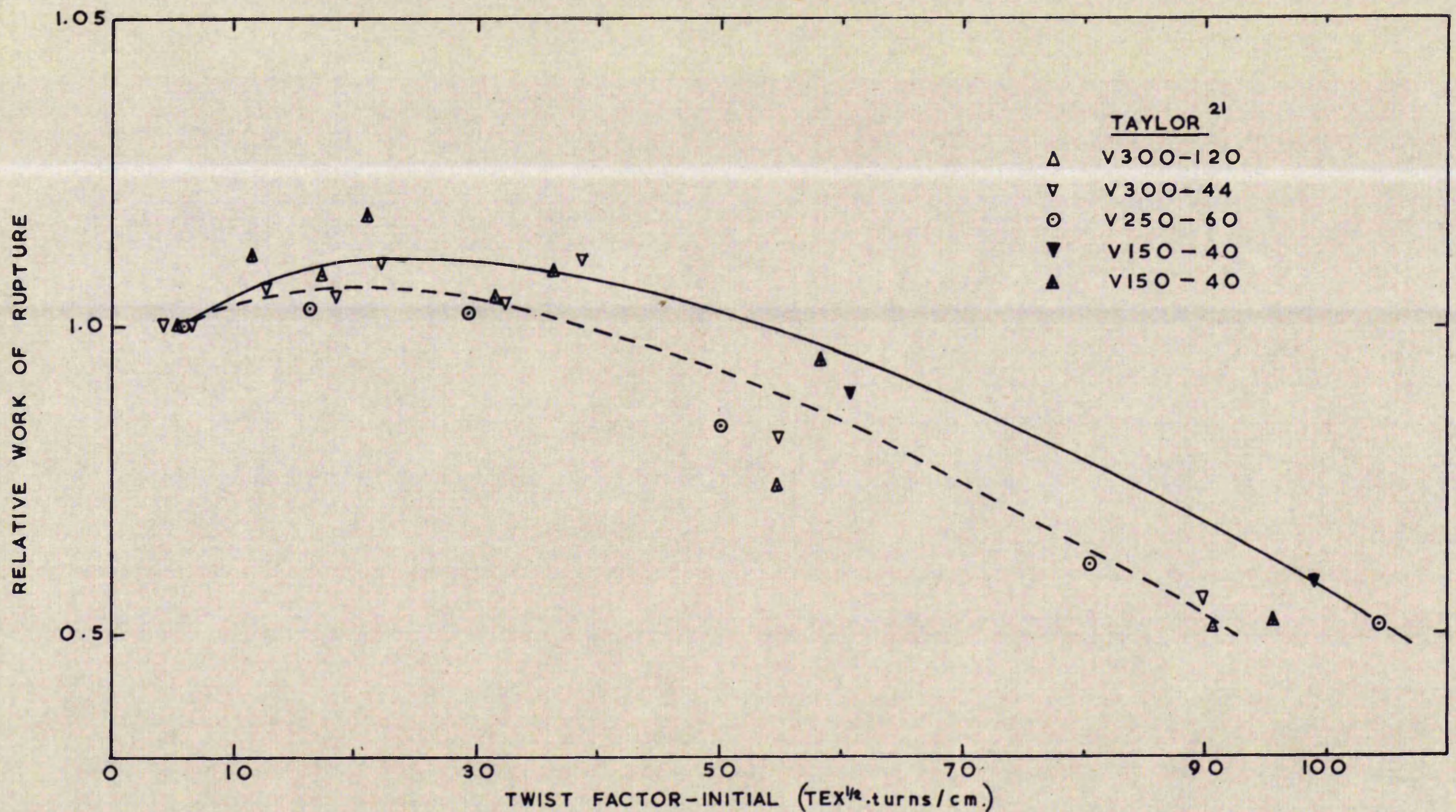


FIG. (1.34 C$_1$) RELATIVE WORK OF RUPTURE FOR SOME VISCOSE YARNS.

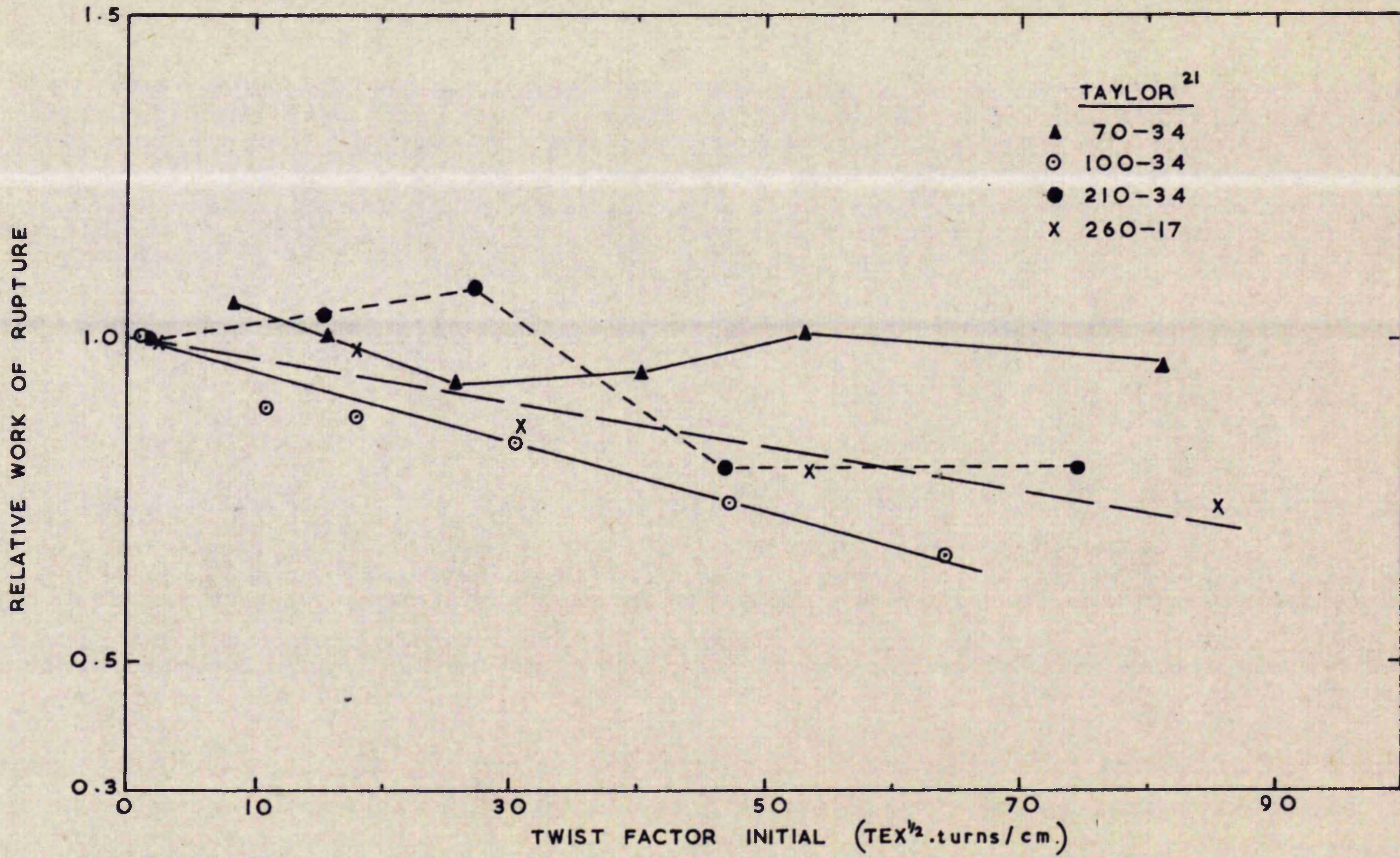


FIG. (1.34 C2) RELATIVE WORK OF RUPTURE FOR SOME NYLON YARNS.

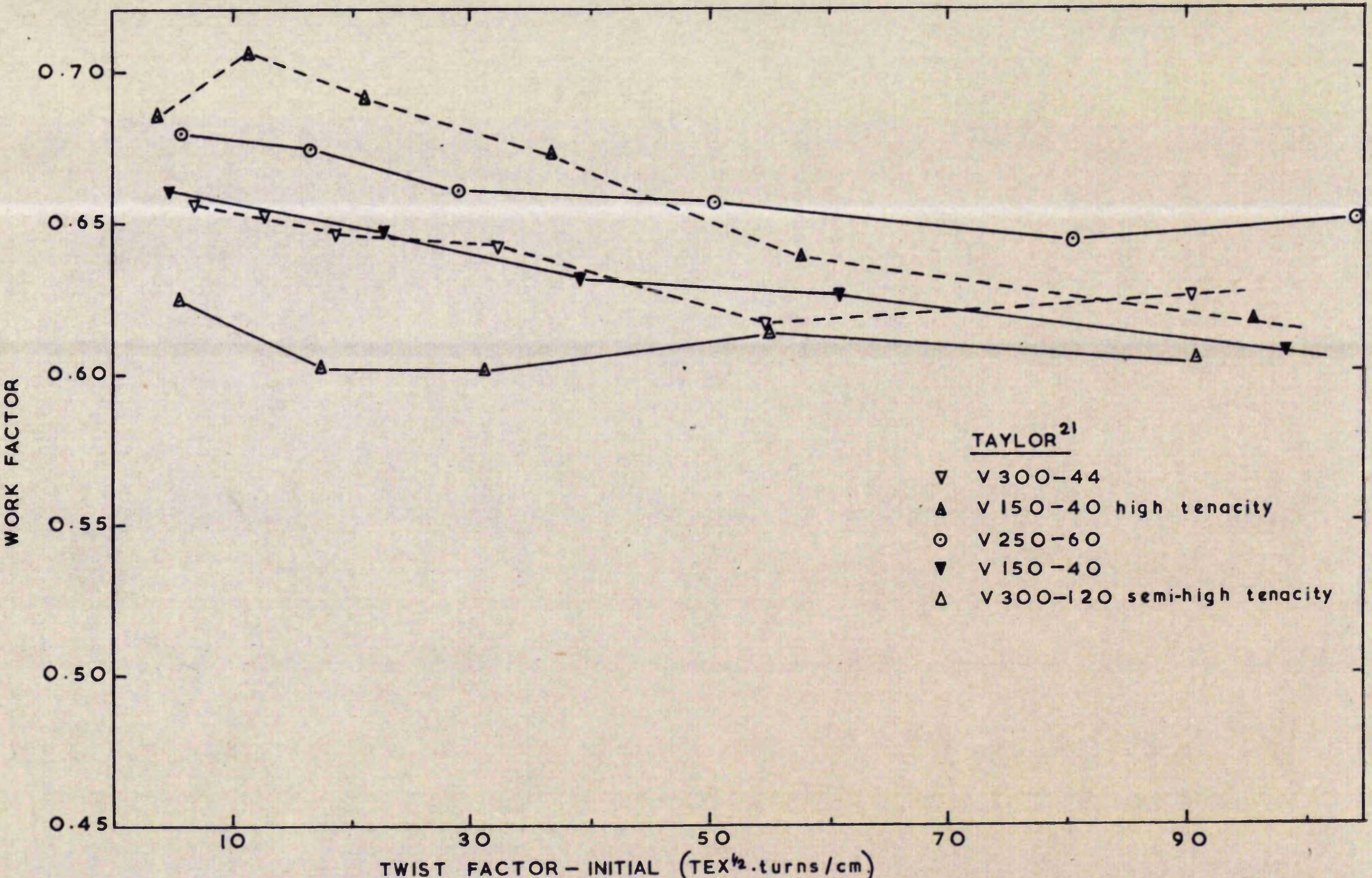


FIG. (1.34 C_3) WORK FACTOR FOR SOME VISCOSE YARNS.

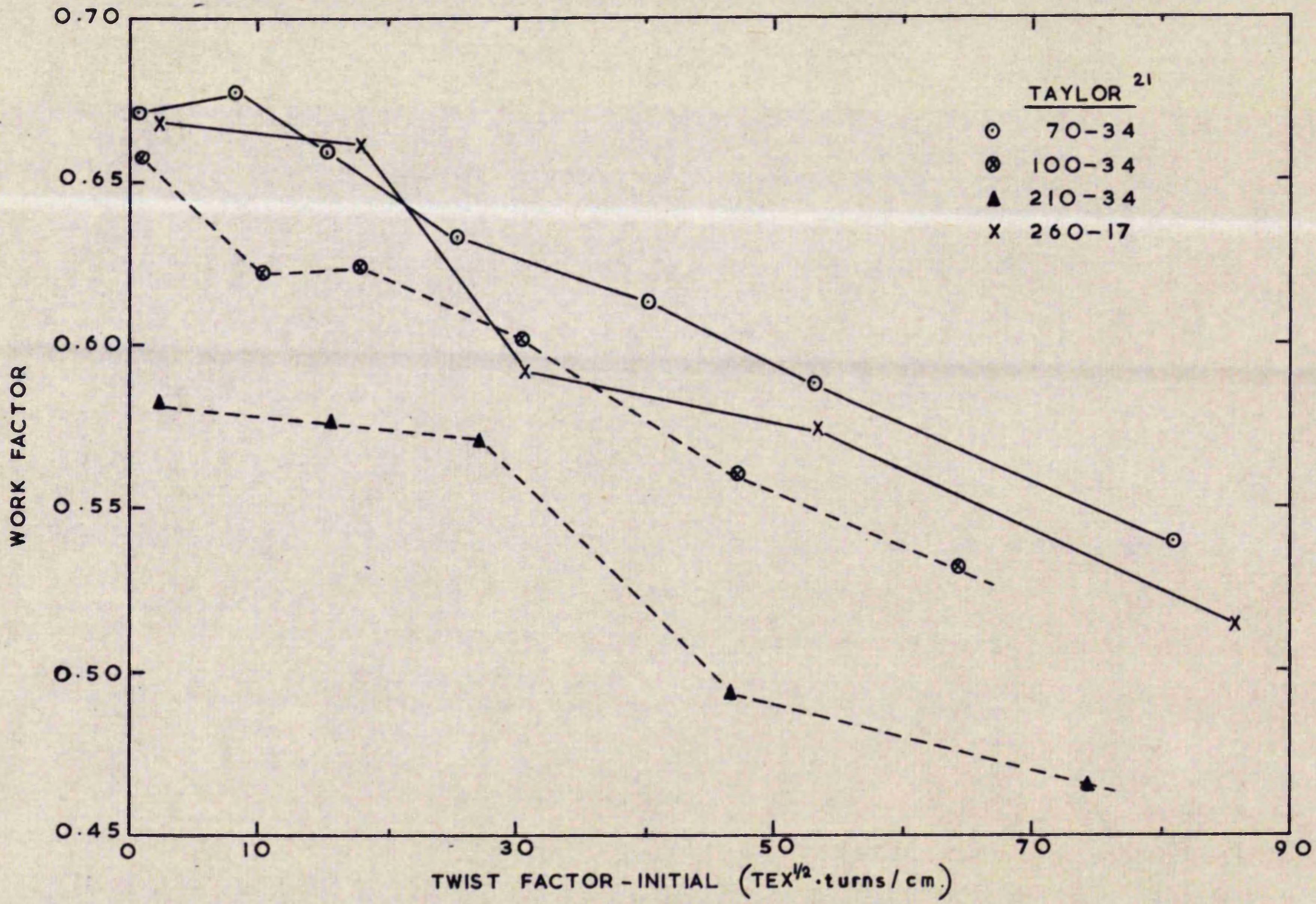


FIG. ($1.34C_4$) WORK FACTOR FOR SOME NYLON YARNS.

where it remains constant. The work factor is found to decrease continuously for all nylon yarns. No explanations of such behaviour have been reported.

1.34D EFFECT OF TWISTING AND TESTING CONDITIONS ON THE RUPTURE PROPERTIES.

Taylor has reported that for nylon yarns the elongation results are erratic in many cases. The relaxation time has a direct effect on the elongation, particularly of lower denier yarns. Figure ($1.34D_1$ - D_2) shows such a behaviour for nylon 70-34 and 40-13 yarns. Elongation of higher denier yarn is affected less.

Grover and Hamby have reported the effect of twisting tension on the tenacity and breaking extension of nylon 100-34 and 30-10 yarns. At high twist factors, the effect of higher twisting tension is to decrease the tenacity and the breaking extension as shown in figure ($1.34D_3$ and D_4).

1.35 TWIST CONTRACTION

1.35A Theoretical Aspects

One of the assumptions made in the mathematical approach to mechanical properties of twisted yarns is that the filaments do not change their length in twisting. The validity of this assumption, can only be established by comparing the theoretical predictions with actual observed contraction results.

Braschlor[26], Platt[7] and Treloar[6] have reported theoretical expressions relating surface helix angle and twist contraction factor or yarn retraction.

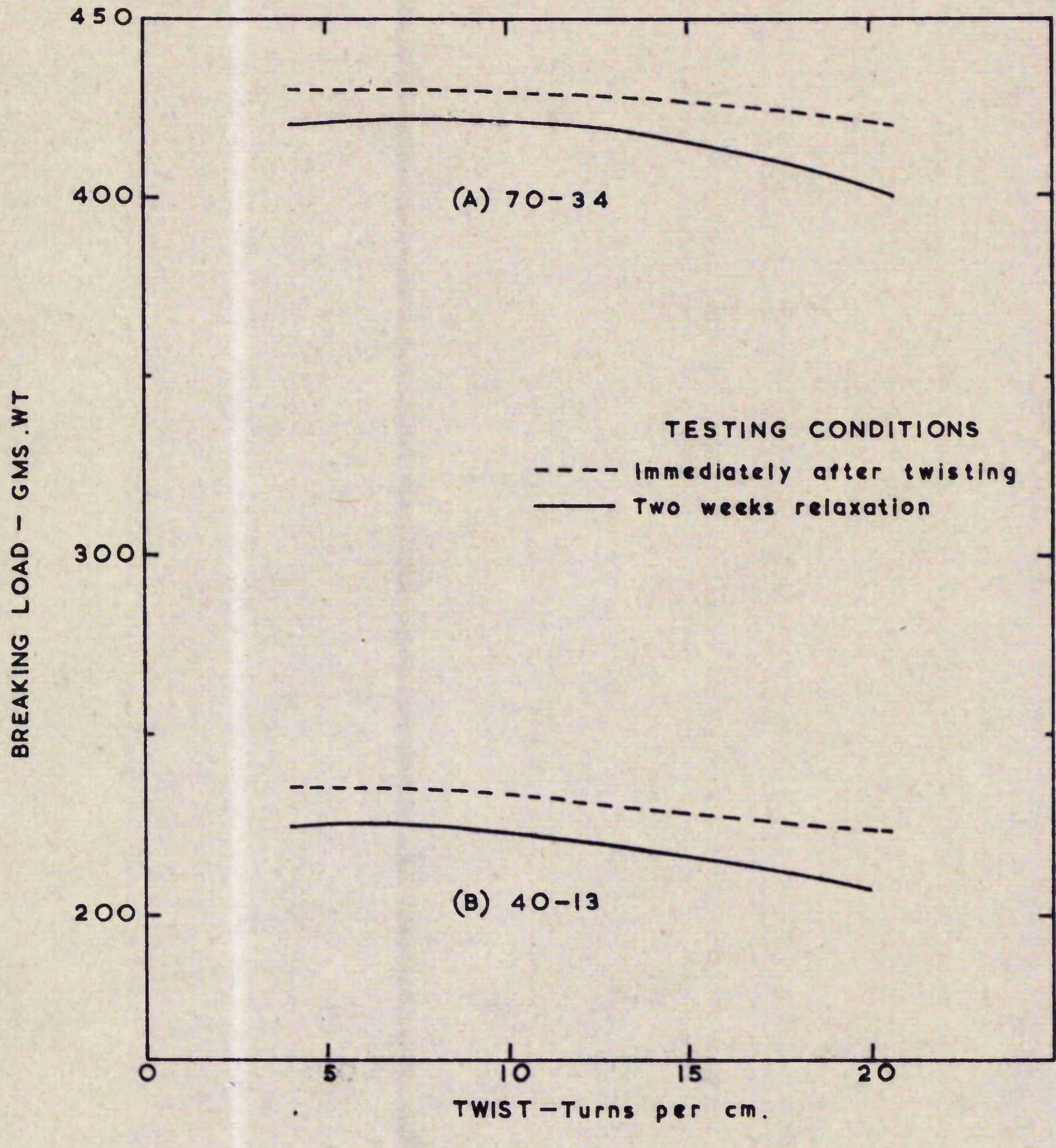


FIG. (1.34 D_1) EFFECT OF RELAXATION PERIOD ON BREAKING EXTENSION OF NYLON YARNS — TAYLOR et al.[21]

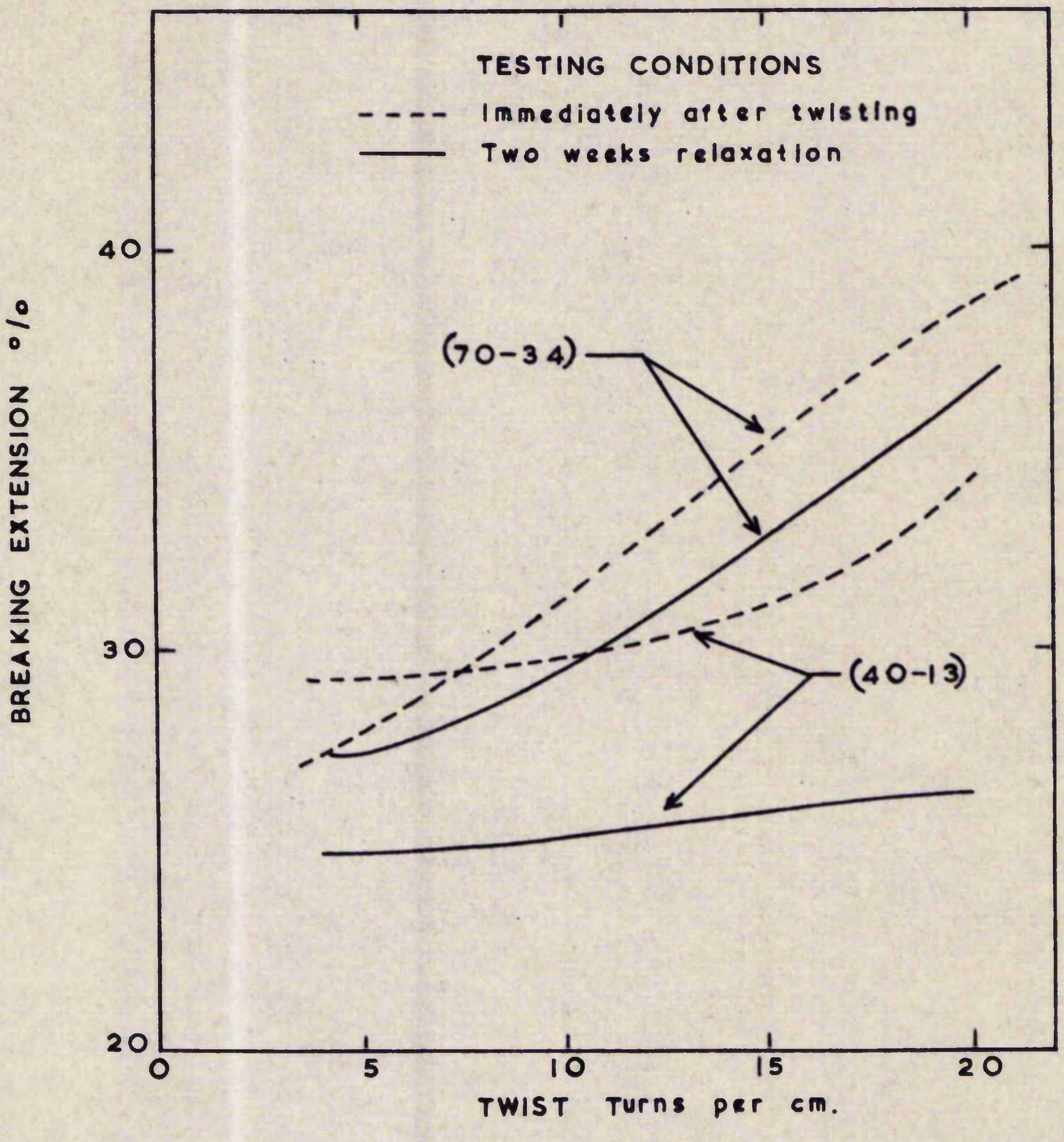


FIG. (1.34D$_2$) EFFECT OF RELAXATION PERIOD ON BREAKING EXTENSION OF NYLON YARNS.— TAYLOR et al.[21]

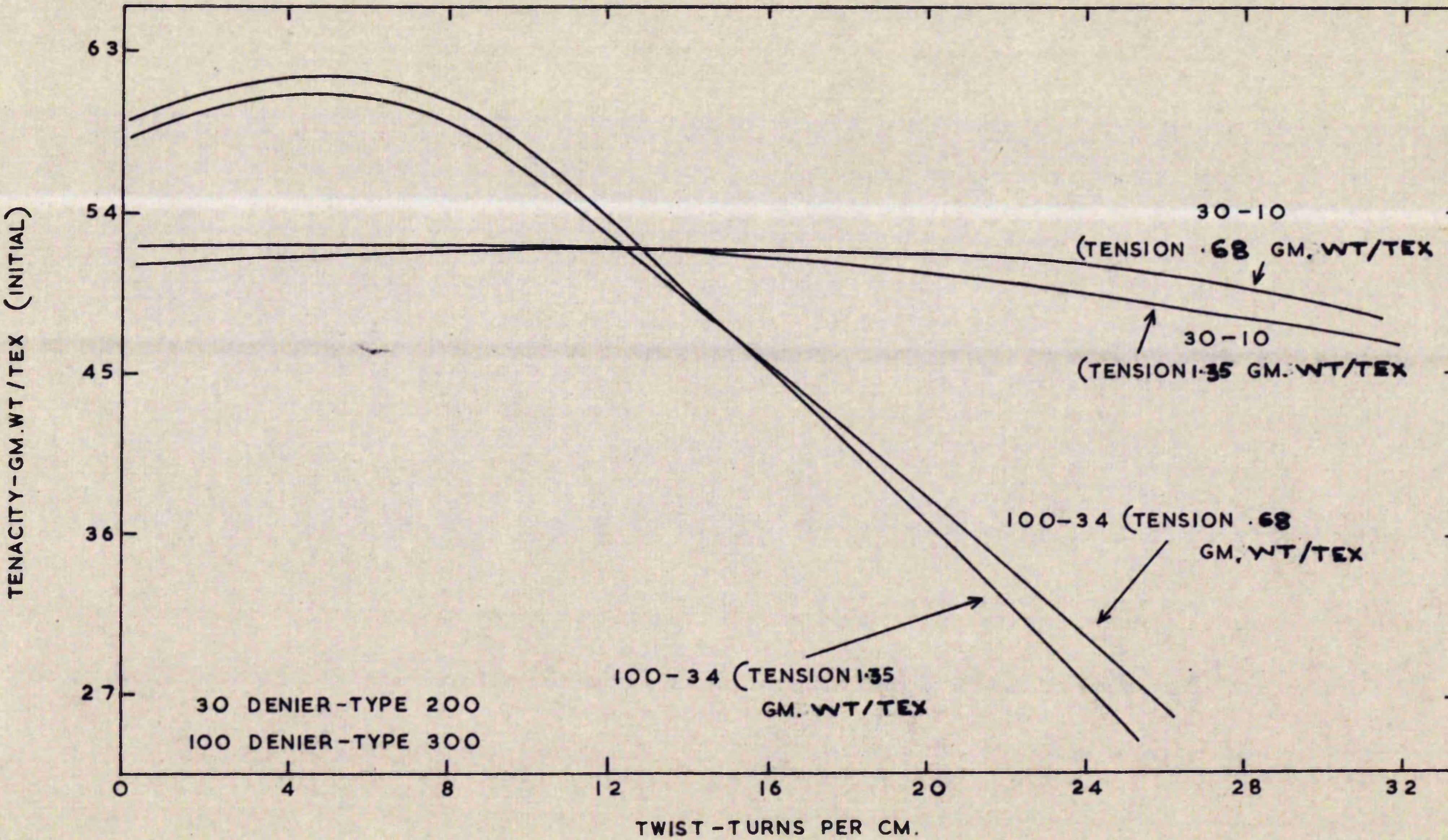


FIG. 1.34D_3 EFFECT OF TENSION ON TENACITY OF 30 AND 100 DENIER CONTINUOUS FILAMENT NYLON YARNS — GROVER et al.[20]

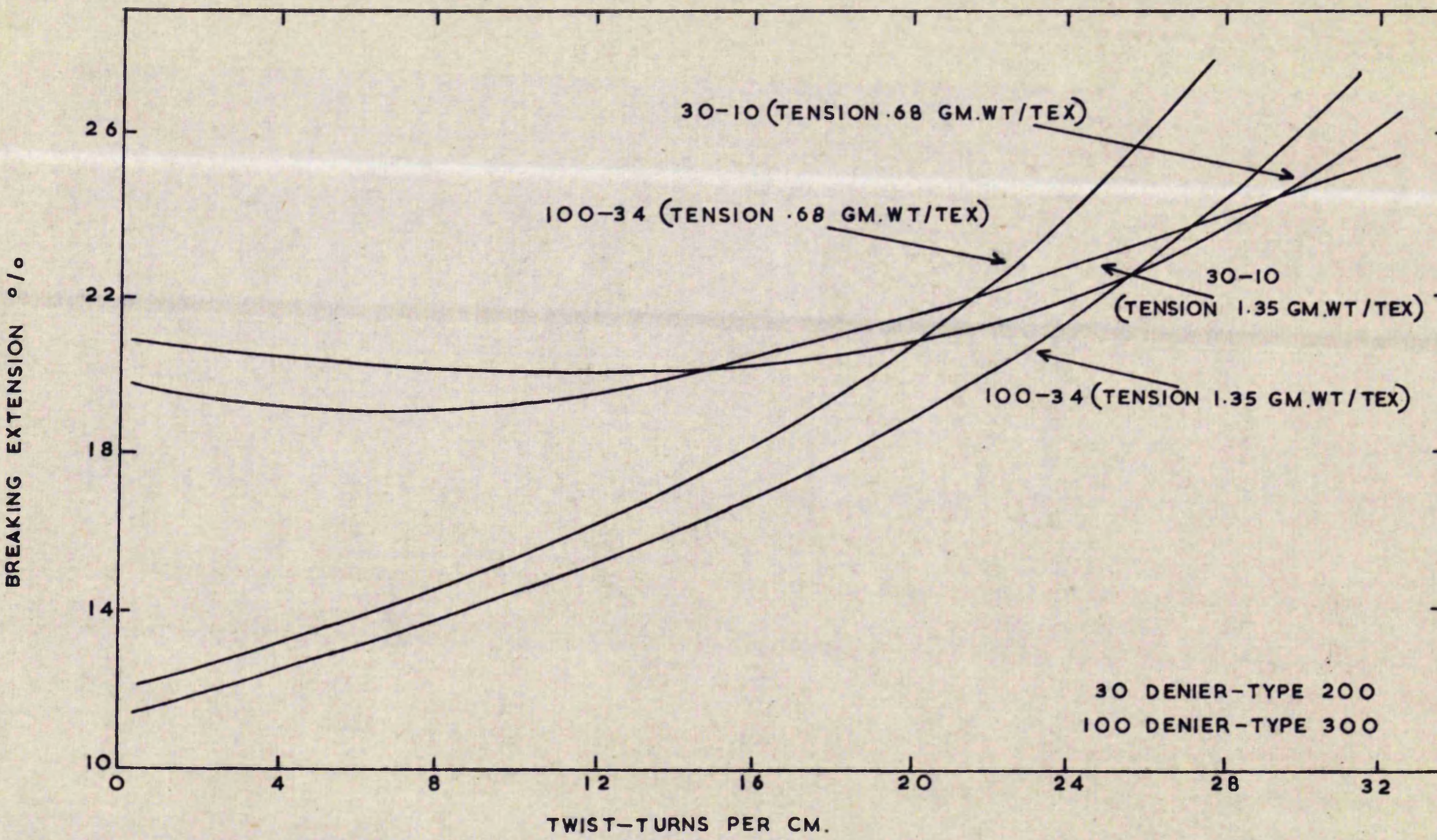


FIG. 1·34 D_4 EFFECT OF TENSION ON BREAKING EXTENSION OF 30 AND 100 DENIER CONTINUOUS FILAMENT NYLON YARNS — GROVER et al[20]

Consider a parallel strand of filaments of length ' L_o' each. when twisted to surface helix angle 'α' such a strand will contract to an axial length 'L_c'. Then by definition

$$\text{Yarn Retraction } 'R_y' = \frac{Lo - Lc}{Lo} \qquad \ldots\ldots\ldots (1.35a_1)$$

and

$$\text{Yarn Contraction } 'Cy' = \frac{Lo - Lc}{L_c} \qquad \ldots\ldots\ldots (1.35a_2)$$

The two terms yarn retraction and yarn contraction can be mathematically related as

$$Ry = \frac{Cy - 1}{1\,Cy\,Cy} \qquad \ldots\ldots\ldots (1.35a)$$

Considering an annular ring in a yarn, obtained by twisting unit length of initially untwisted yarn, Treloar[6] worked out his expression for yarn retraction as

$$Ry = 1 - \text{Mean axial length per unit filament length on twisting.}$$

$$= 1 - \frac{\int_0^R 2\pi r \cdot dr \cdot m_o \cos\theta_r}{\int_0^R 2\pi r \cdot dr \cdot m_o}$$

$$= \tan^2 \frac{\alpha}{2} \qquad \ldots\ldots\ldots (1.35b)$$

where

R = The yarn radius

r = The radius of annular ring

dr = The thickness of ring

m_o = The number of filaments per unit area of cross-section

θ_r = The filament helix angle at helix radius 'r'

α = The filament surface helix angle at helix radius R

In deriving this expression Treloar[6] assumed a constant number of filaments crossing unit area normal to the yarn axis throughout the cross-section.

He also derived an alternative expression, assuming a constant number of filaments crossing unit area normal to the filament axes. This expression was

$$Ry = 1 - \frac{\int_0^R 2\pi r \cdot dr \cdot m_o \cos\theta_r \cdot \cos\theta_r}{\int_0^R 2\pi r \cdot dr \cdot m_o \cos\theta_r}$$

$$= 1 - \frac{\log_e \sec^2\alpha}{2(\sec\alpha - 1)} \qquad \text{..... (1.35c)}$$

From equation (1.35b) and (1.35a) it can also be shown that

$$\text{Contraction factor } Cy = \frac{1 + \sec\alpha}{2} \qquad \text{...... (1.35d)}$$

This is an alternative form of the expression derived by Braschler[26] and referred to by Gregory[27] namely:

$$Cy = \frac{1 + \cos\alpha}{2\cos\alpha} \qquad \text{......... (1.35e)}$$

In a paper on " Influence of yarn twist on modulus of elasticity" Platt[7] has corrected for denier increase by an expression

$$Cy = \frac{2}{3}\ \frac{\sec^3\alpha - 1}{\tan^2\alpha} \qquad \text{.......... (1.35f)}$$

This equation assumes that the number of filaments per unit area at a direction normal to the yarn axis is given by an expression $m_o \sec\theta$

Hearle and Morton[28] have pointed out the difference between the two methods of calculating retraction or contraction factor.

Treloar[6] and Platt[7] consider the twisting of initially untwisted yarn having unit length. The other method, preferred by Hearle and Morton, considers the untwisting of unit length of twisted yarn, and, if the number of filaments per unit area normal to the yarn is given by $m_o \cos\theta_r$, gives an expression which is identical with equation (1.35b):-

$$Ry = 1 - \frac{\int_0^R 2\pi r \cdot dr \cdot m_o \cos\theta_r}{\int_0^R 2\pi r \cdot dr \cdot m_o \cos\theta_r \cdot \sec\theta_r} \qquad \ldots\ldots (1.35h)$$

$$\tan^2 \frac{\alpha}{2} \qquad \ldots\ldots (1.35i)$$

Table 1.35 (I) shows the magnitude of the errors involved in such alternative assumptions.

1.35B Twist Contraction - Experimental

Experimental results have been reported by Treloar[6], Alexander and Sturley[22], Grover and Hamby[20], Sparke[29] and Tattersall[30]. Treloar[6] has reported unpublished data supplied by Courtaulds Limited, and the Dunlop Rubber Co., Ltd., and has shown that the degree of agreement with the theory is very good. In a recent paper, Tattersall[30] has compared the experimental results and concluded that Tenasco 1650/750

TABLE 1.35(I)

Comparison of different theoretical equations predicting the contraction factor and retraction

Surface helix angle	CONTRACTION FACTORS Equ.1.35(c)	Equ.1.35(b) & 1.35(d)	Equ.1.35(f)
10^o	1.0078	1.0077	1.0064
20^o	1.0320	1.0321	1.0328
30^o	1.0774	1.0773	1.0792
40^o	1.1477	1.1527	1.1594
50^o	1.2576	1.2778	1.2979

yarn uptwisted to about 12 T.P.I. with tensions of 0.15 gms/denier was found to give a good agreement with theory. Alexander and Sturley[22] obtained a common curve for all nylon yarns (30/10, 60/20, 45/15). When percentage contraction was plotted against twist factor, Grover and Hamby[31] have shown that the contraction and twist data reveal an excellent fit to the algebraic form

$$C_y = at^b \qquad \text{........ (1.35i)}$$

where

C_y = The percentage contraction

t = Turns per inch

a,b = Least square constants.

Sparke[29] studied the effect of twisting tension on contraction and breaking twist of some multifilament viscose rayon yarns. He reported that the effect of increasing the tension is to reduce both the twist contraction and the twist necessary to produce rupture. He derived a theoretical relation to obtain the filament extension during twisting. If yarn as a whole contracts but individual filaments are stretched

$$L_E^2 = L_C^2 + \pi t^2 D^2 \qquad \text{..... (1.35j)}$$

where L_E = The length to which L_o is extended in helical path

L_c = The contracted length i.e. (1 - Retraction) original length.

t = The number of turns

D = The diameter of twisted yarn.

If the diameter of twisted yarn is not sensibly different from that of a monofil of equivalent denier and density then $\pi^2 D^2 = K^2 \dfrac{T\,Lo}{Lc}$

where K is a factor depending upon the density of material

T = the count of yarn in tex

Lo = the original length of yarn (untwisted).

By twisting statically at constant length, Sparke[29] reported that the experimental value of 'K' is very close to the theoretical value although it is slightly higher. In such a case

$$K = \left[\frac{(1 + \text{Breaking strain})^2 - 1}{\text{Turns to produce rupture x count in tex}} \right]^{\frac{1}{2}} \qquad \text{....(1.35k)}$$

Hence

$$L_E^{\,2} = Lc^2 + K^2 \frac{T\,Lo}{Lc} \qquad \text{........ (1.351)}$$

Thus knowing L_o, L_c and K the corresponding value of the filament extension at a given twist factor can be obtained. (Fig.2) But the assumption that the diameter of yarn at a given twist is approximately the same as that of a monofil of equivalent density and count is doubtful.

Experimental data for turns required to rupture nylon, viscose rayon, Tenasco and Fortisan yarns have been reported by Marathe[32]. The results confirm the theoretical approach based on the assumptions made by Sparke[29].

Tattersall[30] observed that the degree of departure from the theoretical predictions depends upon the tension during twisting and the method of twisting. The lower the denier, the less is the effect of twisting tension. For Tenasco 1650/750 yarn, as the twisting tension is increased from 0.54 to 5.4 gms. per tex, the experimental curve was found to shift from one side of the theoretical curve to the other as shown in figure ($1.35B_6$).

All the available experimental data has been collected and replotted against (twist factor -initial).[2] Figures [$1.35B_1$–B_5] show the contraction behaviour of viscose rayon, acetate and nylon multi-filament yarns. It could be noticed that contraction factor values reported by Taylor[21] are higher for viscose rayon than those reported by Grover[20] and Nagabhushan[23], who observed that breaking extension decreases continuously at higher twists. As regards acetate yarns, the contraction factor is low at high T.F. for all yarns showing decrease in breaking extension. The contraction factor of nylon yarns is higher when twisted yarns are heat set[22], but the data reported by Taylor[21] and Grover[20] fall on a single curve showing that the results are very little affected by twisting tensions used or the methods of twisting.

1.4 SCOPE FOR STUDIES

From the foregoing study of the literature it may be realised that there are wide gaps in our knowledge of the rupture behaviour of twisted continuous filament yarns. The influence of some factors have

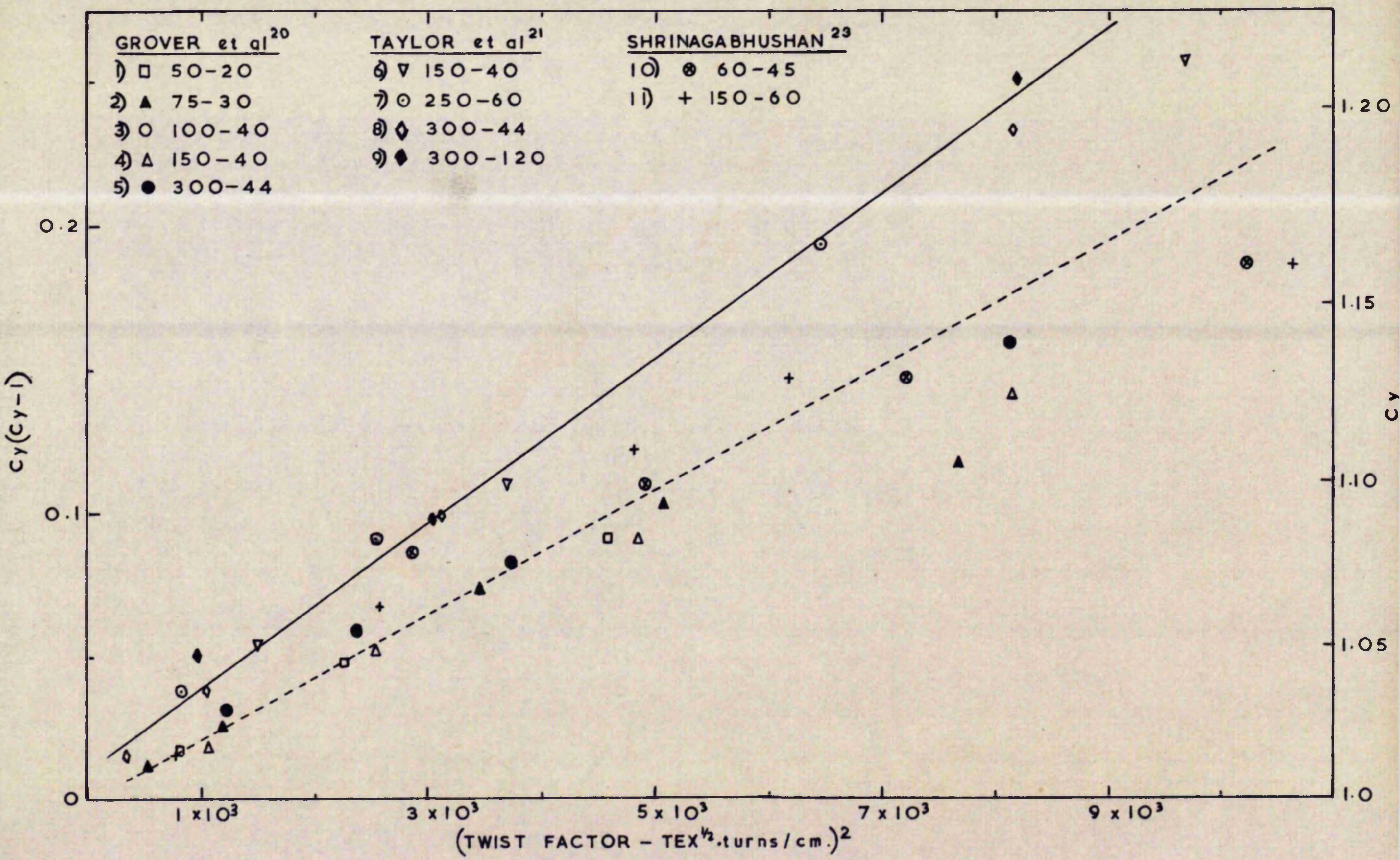


FIG. (1.35B1) CONTRACTION FACTOR OF SOME VISCOSE YARNS.

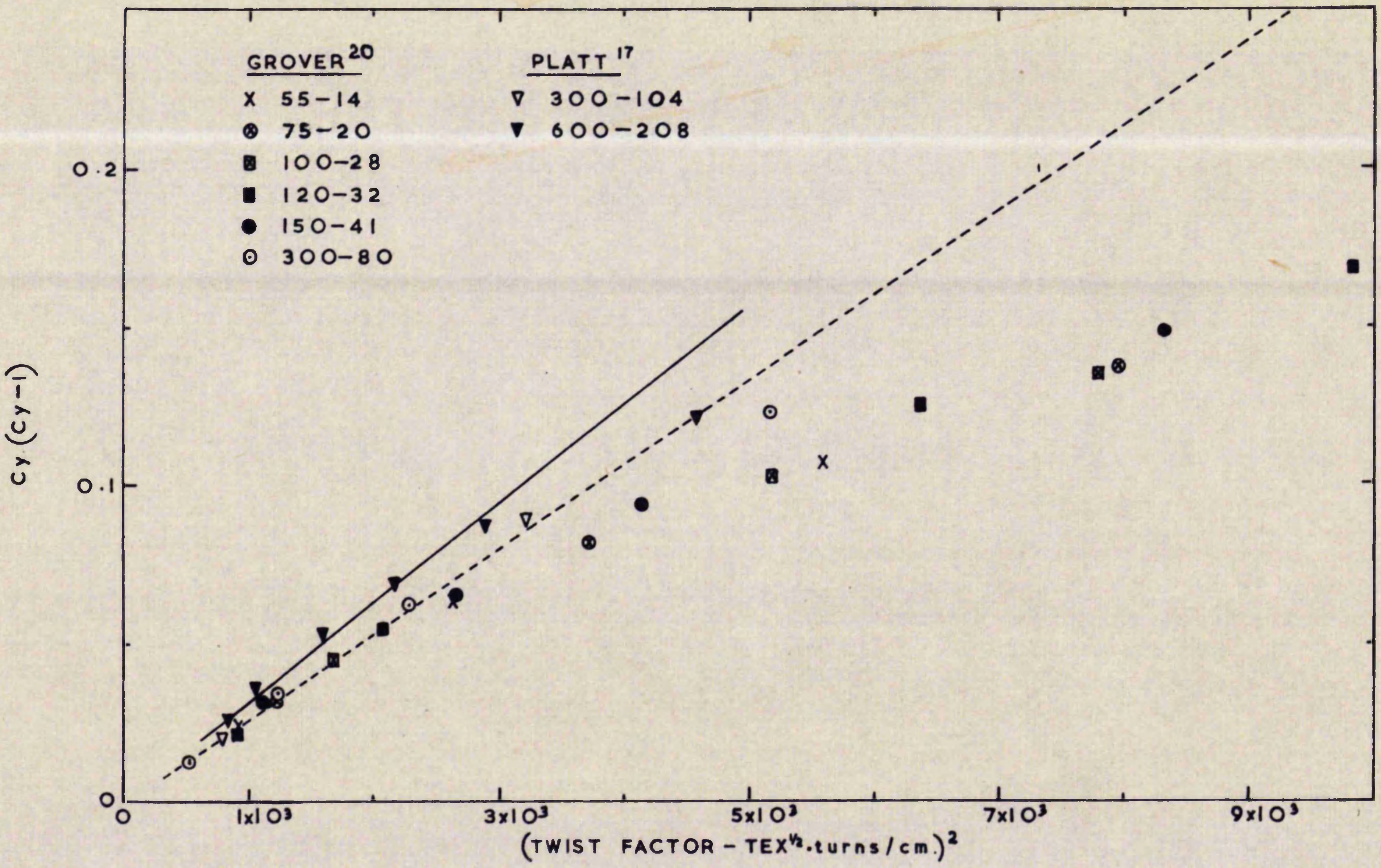


FIG.(1.35B$_2$) CONTRACTION FACTOR OF SOME ACETATE YARNS.

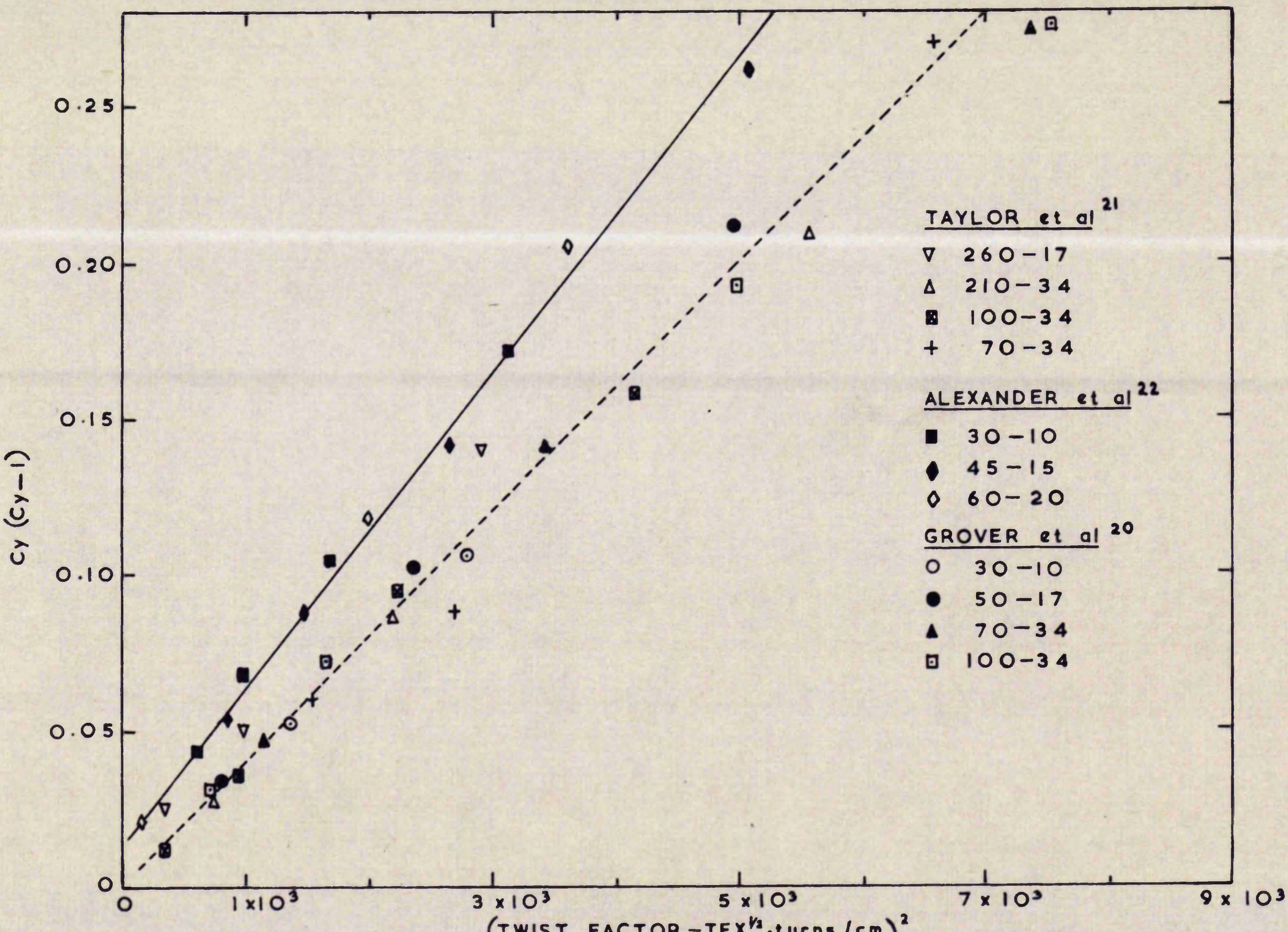


FIG.(1.35B$_3$) <u>CONTRACTION FACTOR OF SOME NYLON YARNS.</u>

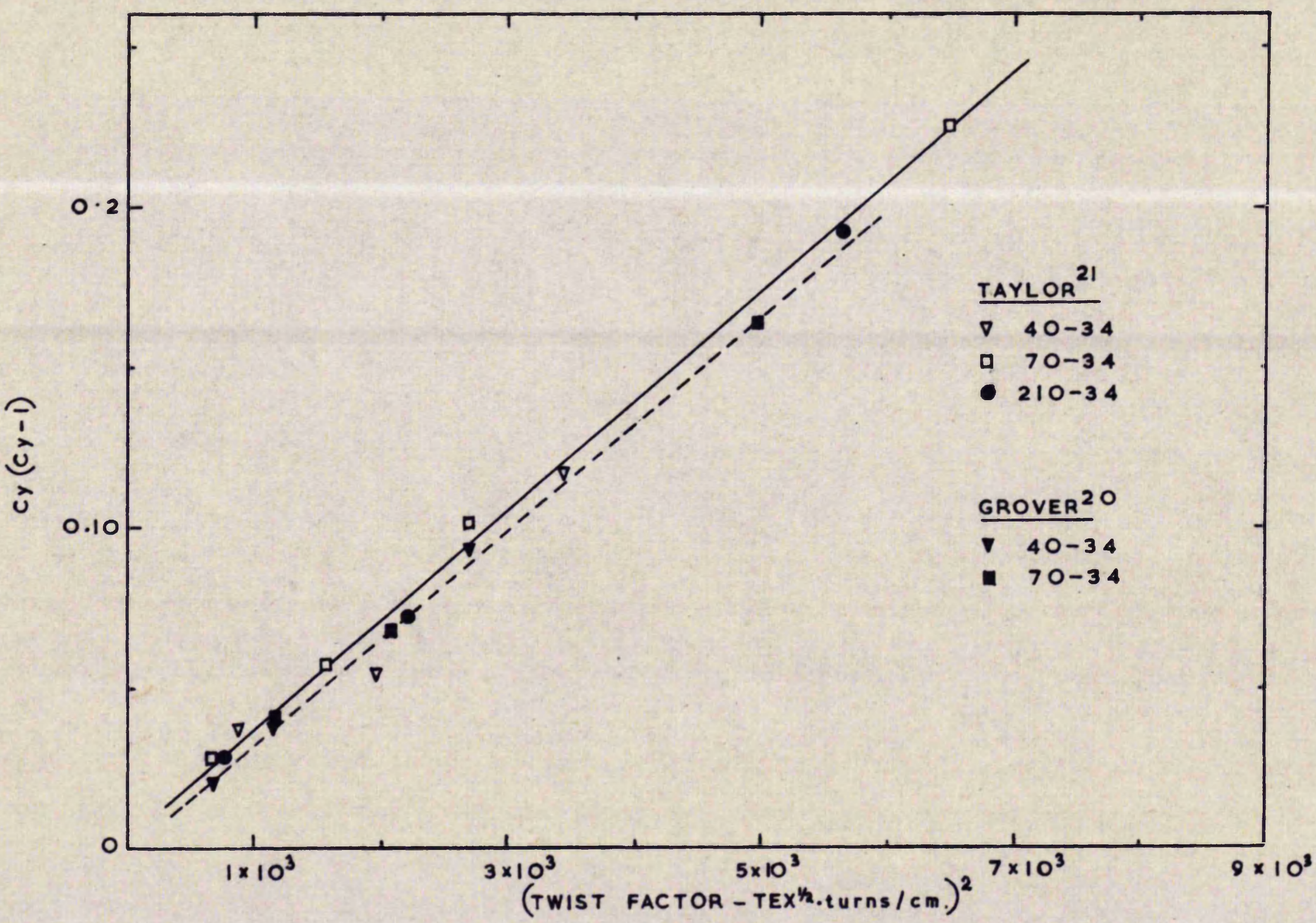


FIG (1.35 B_4) CONTRACTION FACTOR OF SOME DACRON YARNS.

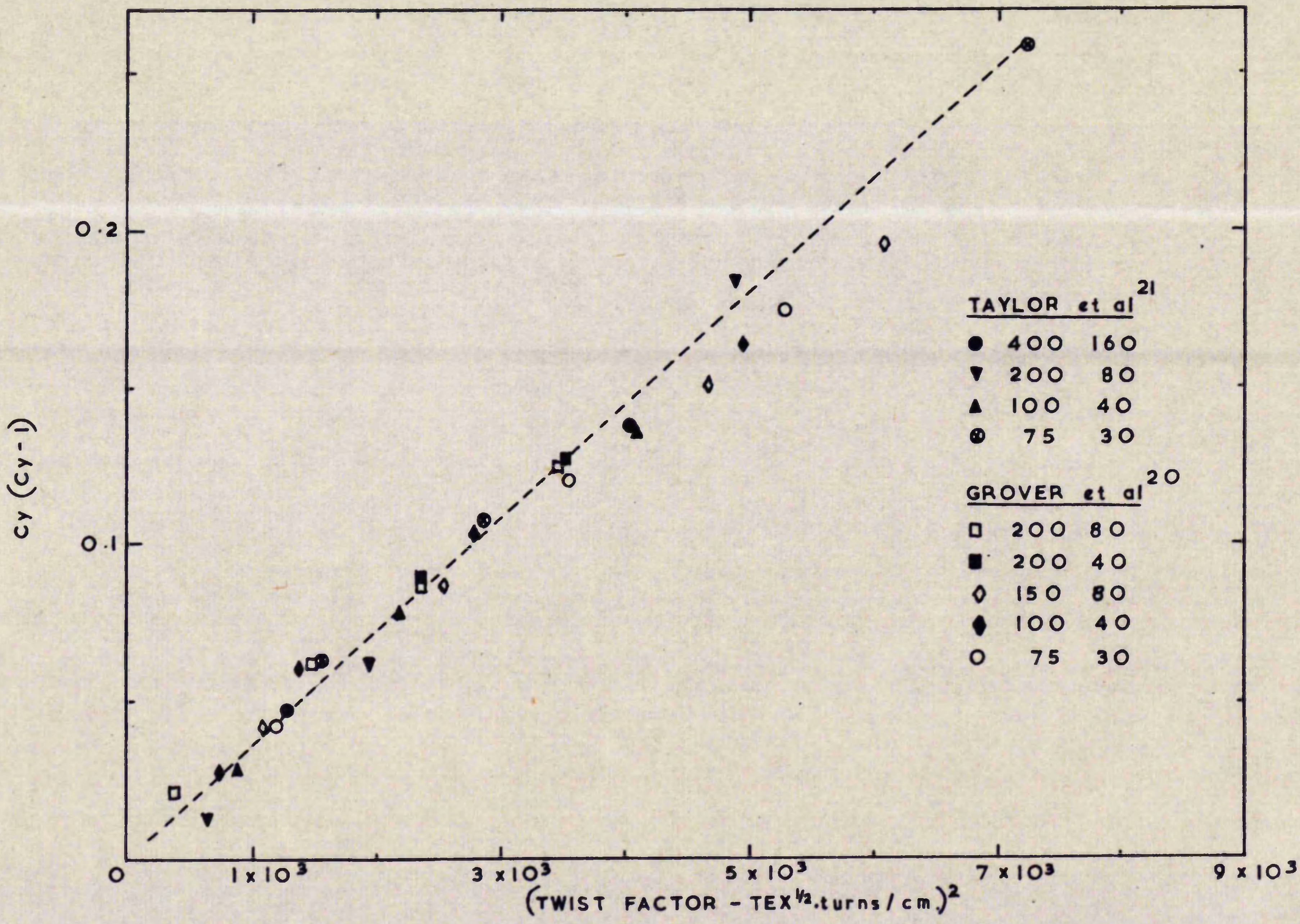


FIG ($1 \cdot 35\ B_5$) CONTRACTION FACTOR OF SOME ORLON YARNS.

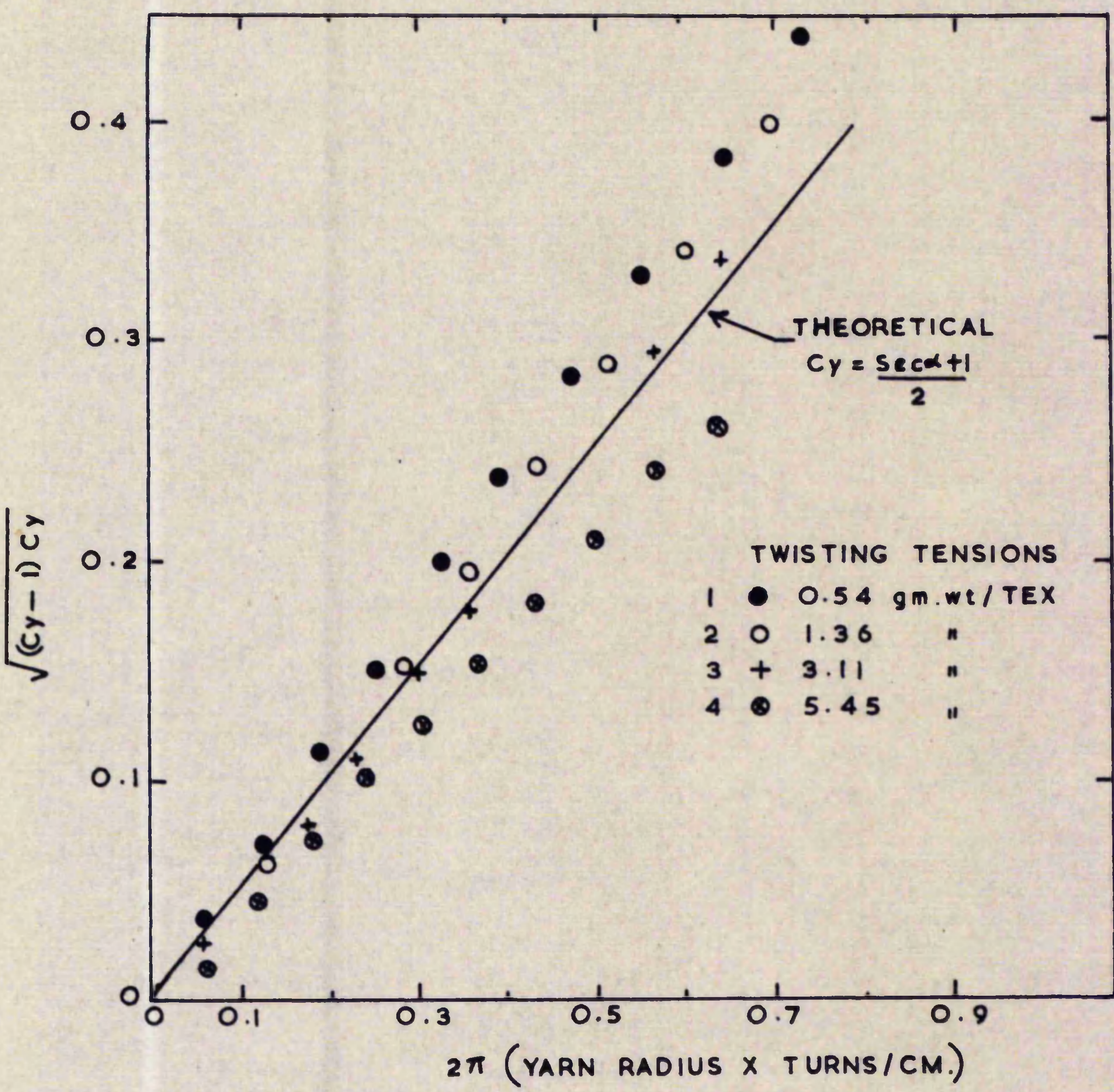


FIG. [1.35 B_6] EXPERIMENTAL RETRACTION DATA FOR TENASCO 1650-750 YARN - TATTERSALL[30] been neglected in order to simplify the theoretical approach, without experimental studies being made to test the validity of such assumptions.

1.41 Gaps in the Knowledge

These may be summarised as follows:

(a) Basic Experimental Data:

The rupture properties of twisted continuous filament yarns are influenced by twisting conditions. There is a basic need to obtain some more experimental data to study the effect of such factors as twisting methods, twisting tensions, etc.

(b) The yarn diameter changes during extension are important in obtaining an accurate expression for the relation of yarn to filament strain at break. Also, an important yarn parameter - Twist Angle at break which relates theory and experimental results, is affected by this behaviour. Some workers assume it to follow a constant volume relation while others neglect the influence of this factor. Experimental work should be carried out to test such assumptions.

Very little is known about the actual mechanism of yarn breakage which may be more complex than is usually assumed. The mechanism of yarn breakage is important in explaining the breaking extension behaviour.

(c) It has been assumed that the twisting process does not alter the properties of the constituent filaments. Experiments can be conducted to test the validity of such assumptions.

The compressive forces may influence the filament rupture properties. The contribution by such an experimental study will help in explaining rupture properties of twisted continuous filament yarns.

(d) The yarn structure has been known to depart from the idealised one to one in which the constituent filaments migrate indefinitely. Some experimental work on model yarn can be carried out to determine the magnitude of errors.

(e) As more and more information is obtained about the actual structure and the validity of these assumptions, the theory will have to be modified to agree with the experimental findings. The theoretical developments will have to consider many factors such as

(i) The influence of tensile and compressive forces on the yarn rupture properties when Hooke's law ceases to hold.

(ii) Consideration of the yarn lateral contraction factor in the derivation of yarn tenacity expressions.

(iii) Consideration of the large strain values in place of the instantaneous strain relations.

(iv) Influence of some other factors so far neglected.

(v) New theoretical approach on the basis of the precise knowledge about the actual yarn structure and the mechanism of breakage.

1.42 The chosen field of work

The explanation of the breaking extension behaviour of twisted yarns will be a most valuable advance towards a better understanding of the rupture properties of continuous filament yarns. Attempts are made to conduct the experiments so as

1) To obtain some more information about the stress strain behaviour for some nylon, viscose rayon, Tenasco, Terylene and acetate yarns.

2) To study the influence of the twisting conditions and testing procedure commonly employed.

3) To study the actual yarn lateral contraction ratio, actual mechanism of breakage and the actual yarn structure.

4) To study the changes in the filament rupture properties under the influence of torsional [redacted] and tensile forces.

The experimental approach will increase our knowledge of yarn breakage. It will also help in explaining the contradictory results reported by various workers.

In the light of the above experimental findings, the theory can be modified to predict the tenacity values more accurately. The theoretical approach is developed to consider the influence of many so far neglected factors such as

1) Yarn lateral contraction factor;

2) The large strain values;

3) The influence of the compressive forces when Hooke's law ceases to hold.

TABLE (2.1a)

The yarn samples studied

Material	Nominal Denier & Fil.No.	Method of Twisting: Ring Doubler	Method of Twisting: Uptwister	Maximum Twist factor $tex^{\frac{1}{2}}$ turns/cm.	Tensile Test Methods: IP2	Tensile Test Methods: Uster	Tensile Test Methods: Instron
Viscose rayon	75-75	✓		88.7		✓	✓
	100-24		✓	104.3	✓	✓	✓
	100-40		✓	102.1	✓	✓	✓
	300-100	✓		94.3		✓	✓
Tenasco	400-180	✓		95.9		✓	✓
	1650-750	✓	✓	63.1			✓
Acetate	100-28		✓	101.9	✓	✓	✓
	100-48	✓	✓	101.2		✓	✓
	300-78	✓		78.0		✓	✓
Nylon	100-34		✓	97.5	✓	✓	✓
	840-136	✓		61.7			✓
Terylene	100-48	✓		113.9		✓	✓
	250-48	✓	✓	60.9		✓	✓

Samples from all these yarns were also heat set and then tested for rupture properties on the IP2 Scott tester.

CHAPTER II

EXPERIMENTAL

2.1 INTRODUCTION

It has already been pointed out that the experimental results do not altogether support the theories put forward. The discrepancies that exist may be due to the techniques used in the twisting or testing. It was therefore decided to obtain more experimental data and study the rupture properties more critically by employing available techniques of twisting and testing.

In selecting the material, twisting techniques and testing methods, attention was given to the results obtained by others and the commercial practice. Yarns were selected so as to have a wide range of linear densities (tex), filament number and fibre properties. Two sets of twisting methods and three sets of testing instruments were employed. Yarns were twisted up to the maximum twist factor of 90-100 units, which is commercially employed. Table (2.1a) shows the range of materials, yarn linear densities, twisting and testing methods, etc. The influence of twisting tensions on yarn rupture properties was also investigated.

2.2 TWISTING PROCEDURE

All yarns tested were twisted in 4 sets of experiments as shown in table (2.2I). Some measurements of twisting tension were carried out on both uptwister and ring doubler. In the ring doubler the tension was measured between the delivery roller and the thread guide as shown

TABLE (2.2.I)

Twisting Methods

SET NO.	YARNS material/denier/filament number		TWISTING METHOD	REMARKS
1	Viscose " Acetate Nylon	100-24 100-40 100-28 100-34	Uptwister 10,000 r.p.m.	Processed by British Rayon Research Association. Those with 10,20, T.P.I. without and 30,50,70, T.P.I. with the flyer American 2 Arm Type. 30 gauge. A sample from all these twisted yarns was heat set on cheeses.
2	Viscose " Tenasco " Acetate " Nylon Terylene	75-75 300-100 400-180 1650-750 100-48 300-78 840-136 100-48	Brooks & Doxey Ring Doubler English system of threading. Spindle speed - 6,000 r.p.m. Ring diameter - 2"	Processed in the departmental spinning laboratory. In twisting Tenasco 400-180 and Acetate 300-78, two twisting tensions were kept. While in all other yarns tensions were controlled to 0.2 to 0.5 g.wt./tex.

Table (2.2I) cont'd

SET NO.	YARNS material/denier/filament number		TWISTING METHOD	REMARKS
3	Viscose Tenasco Nylon Terylene	300-100 1650-750 840-136 250-48	Brooks & Doxey Ring Doubler English system of threading. Spindle speed - 6,000 r.p.m. Ring diameter - 2"	Processed in the departmental spinning laboratory. In twisting Terylene 250-48 and Viscose 300-100 three twisting tensions were controlled, while two for Tenasco 1650-750 and Nylon 840-138.
4	Acetate Tenasco Terylene	100-48 1650-750 250-48	Uptwister	Processed by the British Rayon Research Association. Acetate 100-48 and Terylene 250-48 were first wound on the Atwood redrawer, and then uptwisted on Platt's MU_1. Uptwister at 7000 r.p.m. and r.h. % 58/60 at 70°/72°F. While Tenasco 1650-750 was first rewound on to a suitable bobbin and then uptwisted on MU_3 type uptwister at spindle speed of 5500 r.p.m. and r.h. % 59/61 at 70°/72°F. In twisting these yarns, the twisting tension was found to be 0.9 g.wt./tex.

in figure (2.2A). It was observed that the twisting tension increases as the twist is increased, although the traveller number is kept the same. This may be due to the decreased coefficient of friction between the yarn and the traveller as the twist is increased. In the third set of experiments, the traveller number was adjusted to maintain the same twisting tensions for all the twist samples of a given yarn. The values of the traveller numbers and the corresponding tension measurements are given in Appendix I.

The filaments in some highly twisted acetate and Tenasco yarns were found to have ruptured in the process of twisting on a ring doubler.

2.3 TESTING PROCEDURE FOR YARN STRUCTURAL PARAMETERS

The most important structural parameters to be determined are twist factor (tex$^{\frac{1}{2}}$ turns/cm.) and surface helix angle at break. All the yarns were tested for the determination of twist per centimetre, tex and diameter in millimetres.

The twist per centimetre was determined from twenty tests on the twist tester using 25 cm. lengths at a tension of 0.45 g.wt. per tex. The specimen was cut and weighed on the torsion balance to determine its linear density (tex). The twist factor is then calculated from these results.

The yarn diameter was measured by observations of yarns with a microscope containing a scale in the eye piece. Except for the last two sets of twisted yarns, five observations were made on each of five 60 cm. lengths of yarn held at a tension of 0.9 g.wt/tex. For the last sets of twisted yarns, the diameter measurements were done on twenty

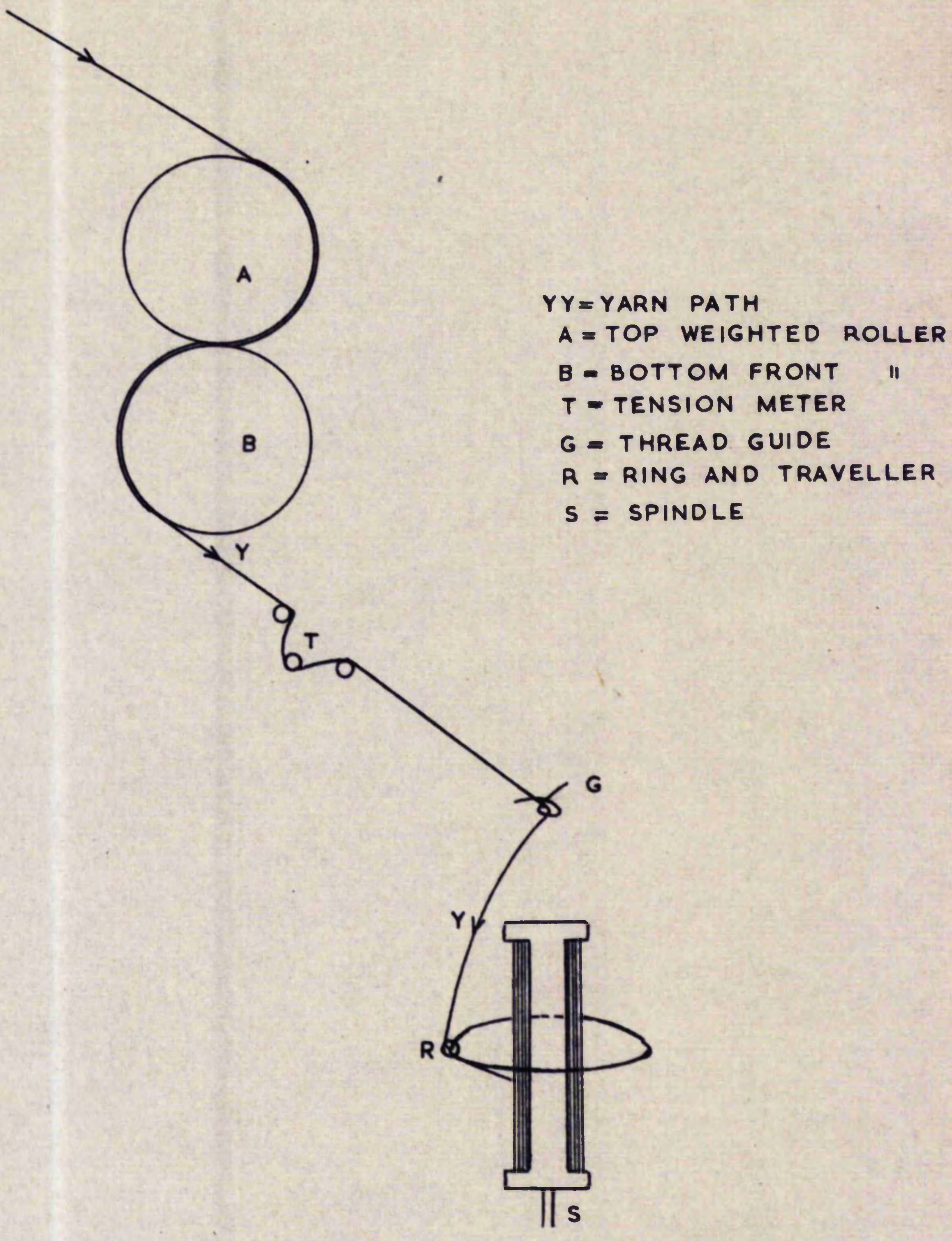


FIG. (2.2 A) TWISTING TENSION MEASUREMENTS ON A RING DOUBLER.

25 cm. lengths of yarn before the twist determination (Fig. 2.3A). The surface twist angle was calculated from these results using the relation

$$\mathrm{Tan}^{-1}\pi\, Dt \qquad \ldots\ldots\ldots\ldots\ (2.3a)$$

where,

D = Yarn diameter in cm.

α = The surface helix angle

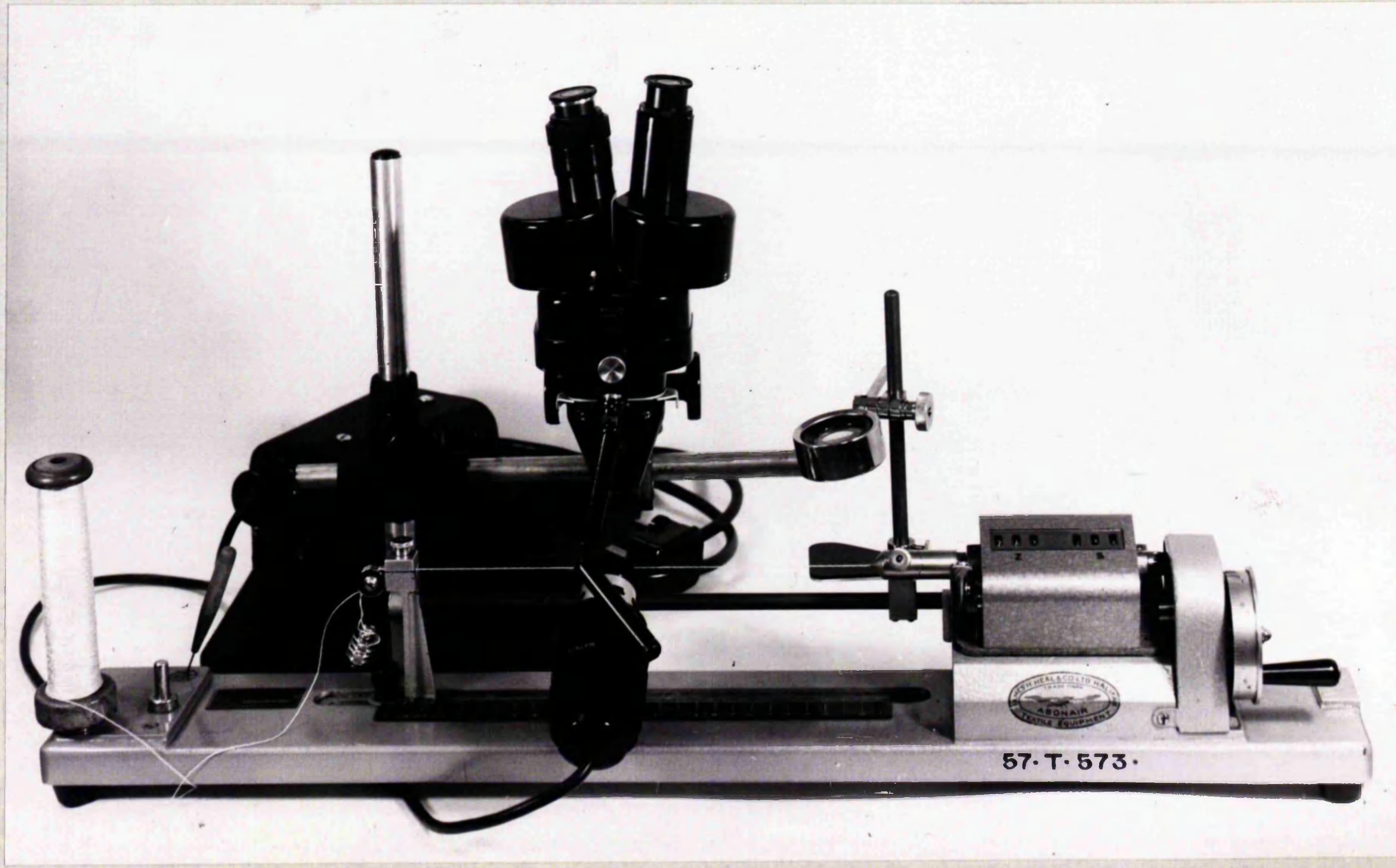
t = Turns per centimetre.

In chapter I, the significance of helix constant 'K' has been discussed. However, in deriving the theoretical prediction expressions one of the assumptions is to consider a yarn structure as composed of infinitisimal fibre elements. This condition assumes the value of helix constant 'K' as unity. Moreover, where the yarn structure is composed of sufficiently high number of filaments, the value of constant 'K' is negligible. Thus, it is justifiable to neglect this factor 'K' in computing twist angle by equation (2.3a).

Yarn diameter experimentally determined was corrected to give actual value at break by an appropriate factor $(1+ \epsilon_{by})^{-\frac{1}{2}}$ assuming constant volume deformation. This corrected yarn diameter value was used in calculating the twist angle at break by equation (2.3a).

Similarly, the twist factor at break was calculated by correcting turns per cm. and tex for breaking extension. In other words, the twist factor (initial) was corrected by an appropriate factor $(1 + \epsilon_{by})^{-3/2}$.

FIG. 2.3A DETERMINATION OF YARN STRUCTURAL PARAMETERS.



For all yarns investigated, the values of actual yarn count in tex, yarn diameter in centimetres and twist factor (initial) in $tex^{\frac{1}{2}}$ turns/cm. are given in Appendix I.

2.4 TENSILE TEST METHODS

Three sets of tests were carried out, one of them on the constant rate of elongation tester (table model Instron) and two of them on the constant rate of loading testers (IP2 Scott and Uster automatic yarn strength testers). The essential details of all these tests, all of which were carried out in a standard atmosphere of 65 $\pm$ 2% r.h. and 20 $\pm$ 2°C. are given in Table (2.4A).

The curves obtained in set B were used to obtain a representative load extension diagram, the constants for linear stress-strain relation near break and the work of rupture. The tenacity and the breaking extension values obtained from set A (earlier tests using the IP2 test) and set B (table model Instron) were used as a check to those obtained in set C (Uster automatic strength tester).

The problem of jaw breaks for nylon and Terylene yarns in particular, was solved by using suitable grips - cellophane tape and rubber packings of different hardness.

It was not possible to test low twist nylon 100-34 yarn by using the Uster automatic yarn strength tester. The within filament variability of the filament breaking extension was so high that the breakage detecting mechanism failed to function satisfactorily. This resulted in inaccurate records of the breaking load and the extension values.

The average values for breaking load in g.wt. and breaking extension in percentage for all yarns tested are given in Appendix I.

TABLE 2.4A

Tensile Test Methods

SET	INSTRUMENT	TYPE OF TEST	NO. OF TESTS	SPEC. LENGTH CM.	RATE	INITIAL TENSION G.WT.TEX.
A	Scott IP2	Whole load extension curve	50	50.8 cm.	Time of break = 21 $\pm$ 5 secs.	0.68
B	Instron	"	10	10 cm.	40% per min. except for acetate (100% per min.)	0.50
C	Uster	Break	40	50 cm.	Time of break = 20 $\pm$ 2 secs.	1.0

CHAPTER III

RESULTS

3.1 INTRODUCTION

In this chapter, the results of experiments carried out using the methods described in chapter II are presented. They show the influence of the magnitude of twist, twisting method and twisting tension on the rupture properties of continuous filament yarns.

Although, the unit for the magnitude of twist has been taken as turns per centimetre, it is more useful to plot the results against twist factor (tex$^{\frac{1}{2}}$ turns/cm), which takes into account both turns per unit length and the linear density of the yarns. The rupture properties when plotted against twist factor (initial), are of great practical importance, while when plotted against twist angle are useful in explaining the relevance of theoretical developments.

3.2 LOAD EXTENSION DIAGRAMS TO BREAK

3.21 General Studies

An example of the load extension diagrams as obtained on the Instron tester is shown in figure (3.21A). Similar load extension diagrams for other yarns are given in the Appendix. In order to obtain a representative load extension diagram, one curve from each set of 10 curves was chosen because its breaking load and breaking extension are close to the average values. Examples of these curves are plotted in figure (3.21 B).

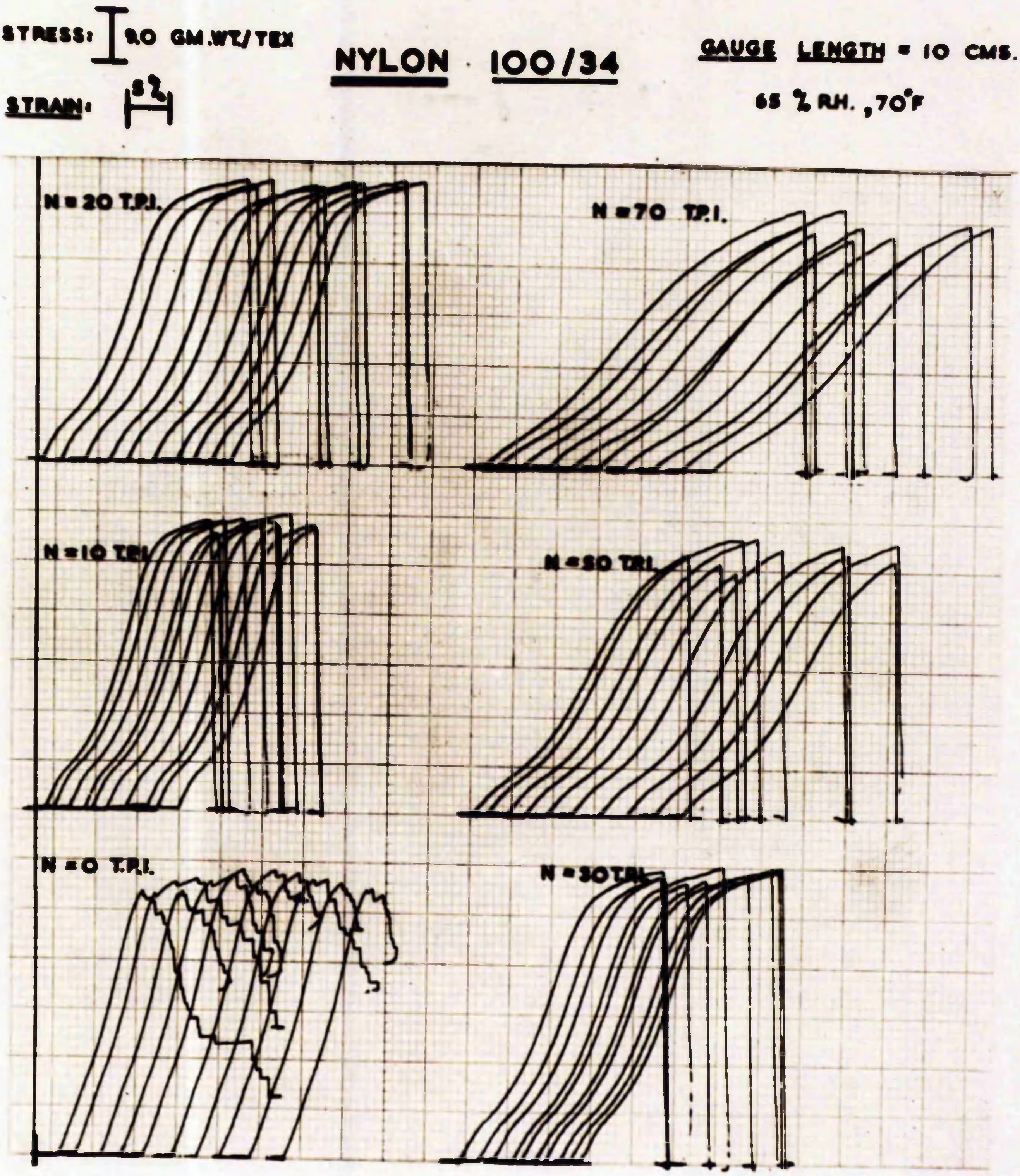


Fig. 3.21 A. Sample of load elongation curves in Instron test.

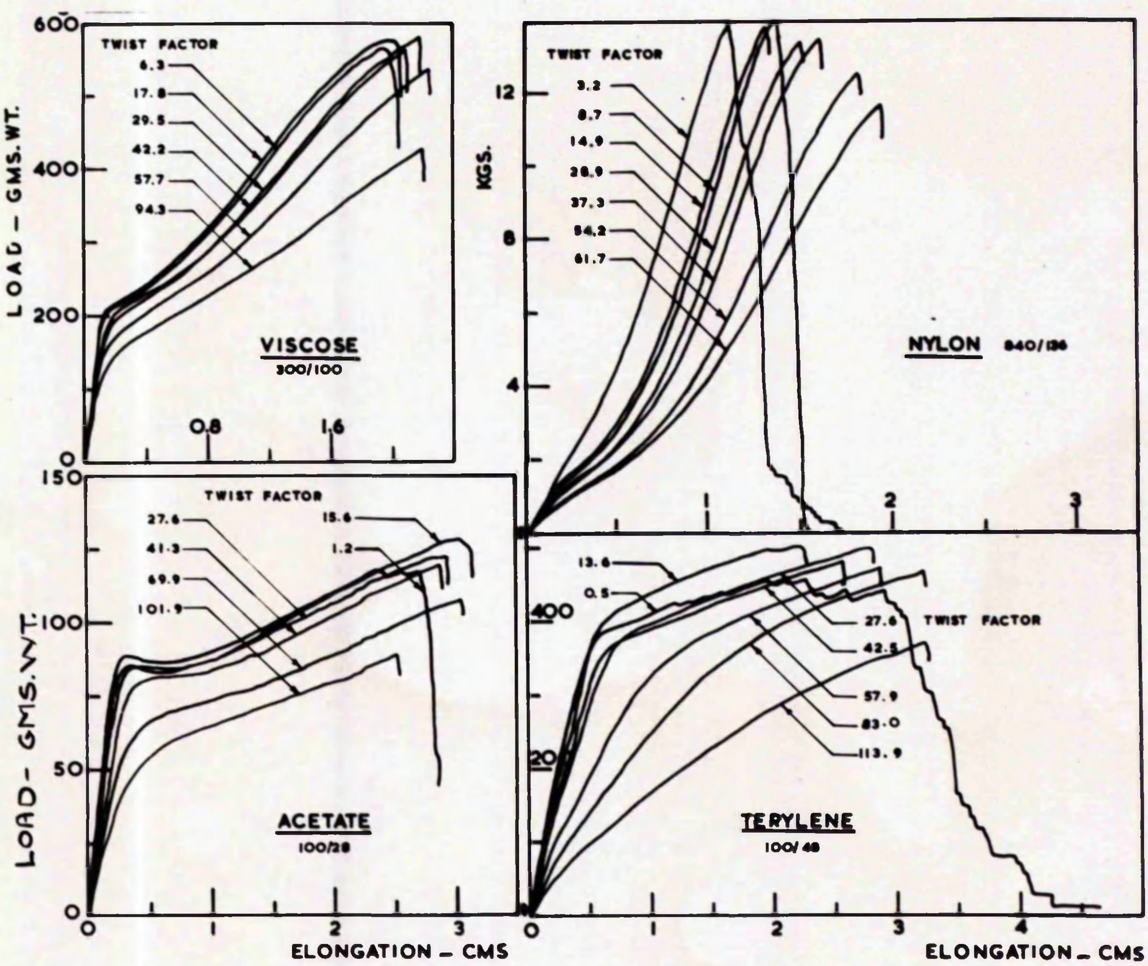


Fig. 3.21 B. Selected load elongation curves of yarns.

From these figures it can be seen that:

(1) At low twists, the load extension curves of all the materials end in a series of steps corresponding to the breakage of each filament or group of filaments, but at higher twist factors the break occurs almost instantaneously.

(2) For all yarns, at any load below rupture, the elongation of higher twist yarn exceeds that of lower twist yarn.

(3) For all twisted nylon yarns, the stress-strain curve shows an apparent yield point at about 2% elongation which is absent in that of zero twist yarn.

<u>3.22 Effect of twisting tension on load extension diagram to break</u>.

The results are shown in figure (3.22 A_1-A_5) It can be seen that

(1) In general, effect of higher twisting tension is to increase the initial modulus and the slope of the load extension diagram near break.

(2) This effect is more so, in yarns with higher twist factors, resulting in decreased breaking extension.

(3) The breaking load of all materials initially increases to a maximum and then decreases as the twisting tension is increased. The level of this optimum tension is different for different materials.

<u>3.23 Slope and intercept of linear stress-strain relations near break</u>.

It was pointed out in chapter I, that the stress-strain relation in the rupture region may be linear or parabolic in form. It is linear for viscose rayon and acetate yarns and parabolic for nylon.

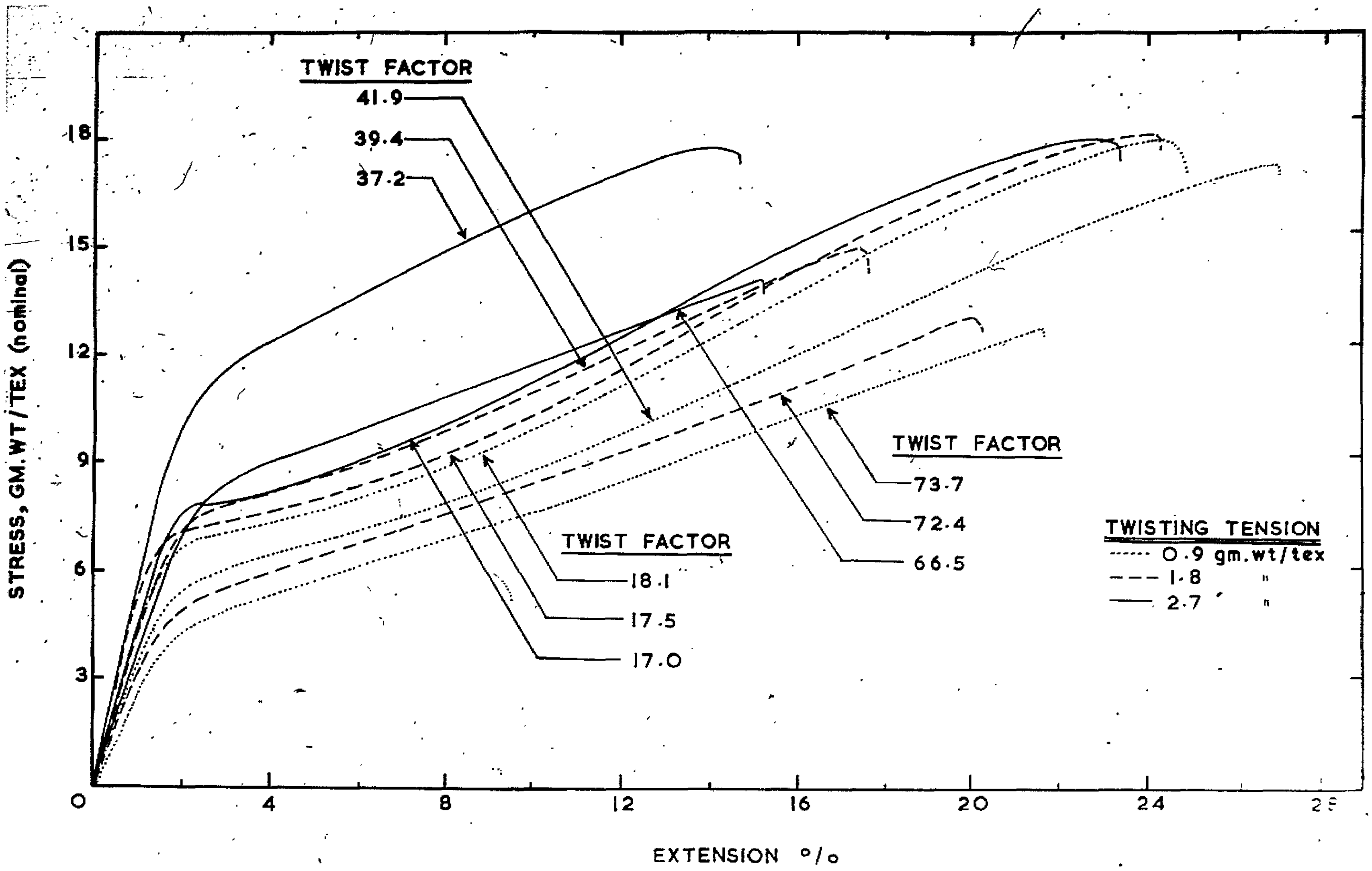


FIG. 3.22 A₁ EFFECT OF TWISTING TENSION ON LOAD-EXTENSION CURVES OF VISCOSE (300-100) YARNS

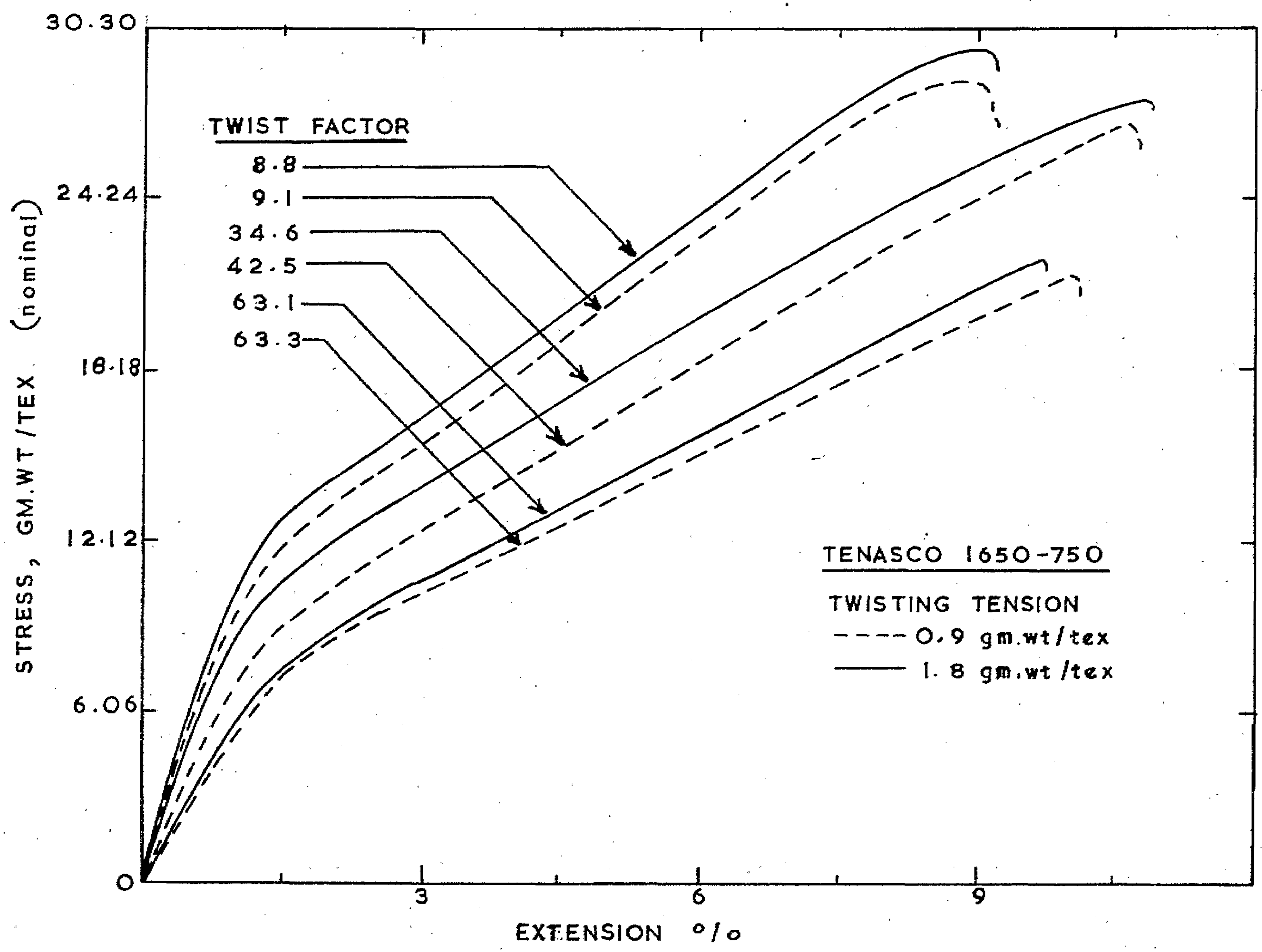


FIG. 3.22 A_2 EFFECT OF TWISTING TENSION ON LOAD-EXTENSION CURVES OF TENASCO YARNS.

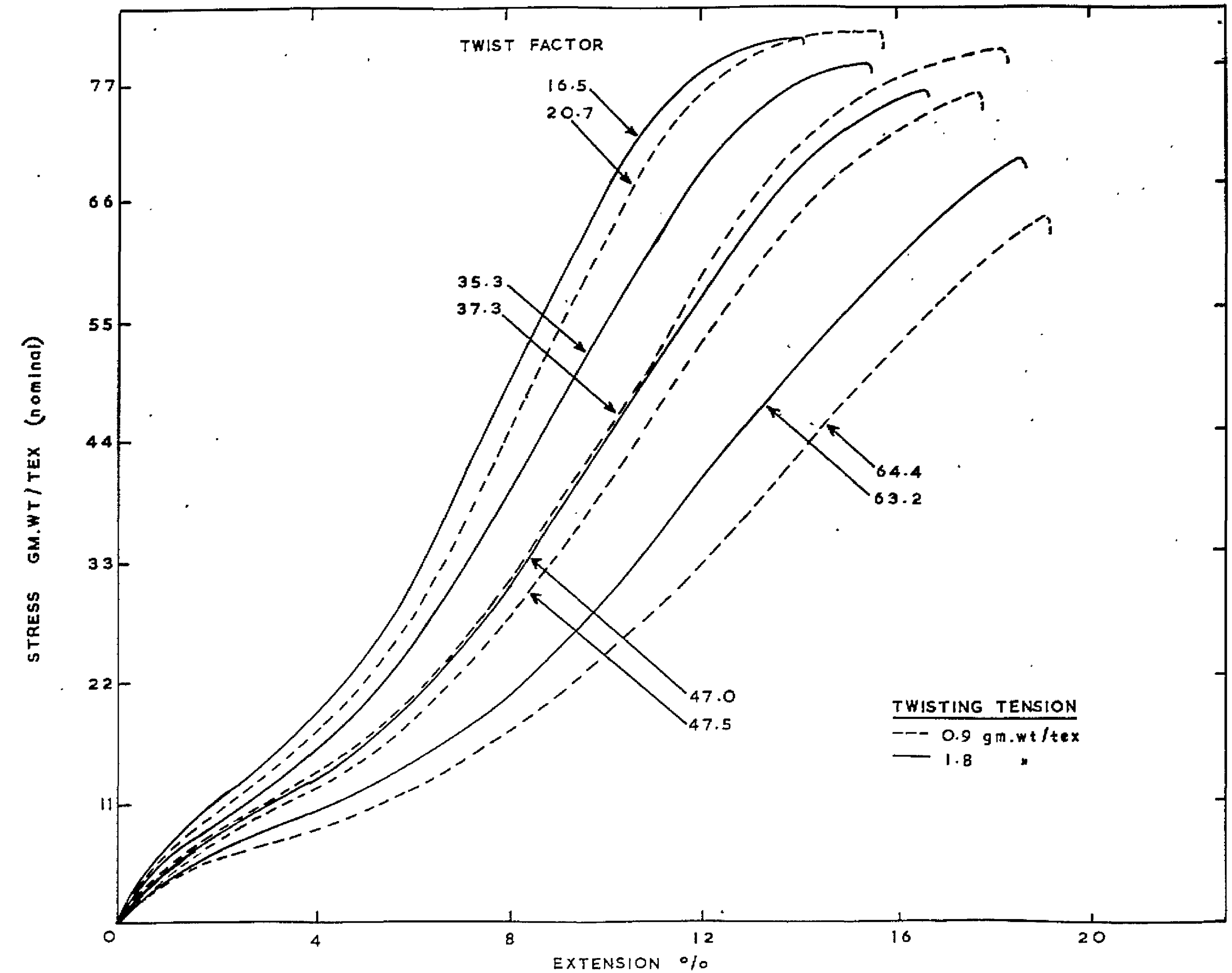


FIG. 3.22 A_3 EFFECT OF TWISTING TENSION ON LOAD.EXTENSION CURVES OF NYLON 840-136 YARN.

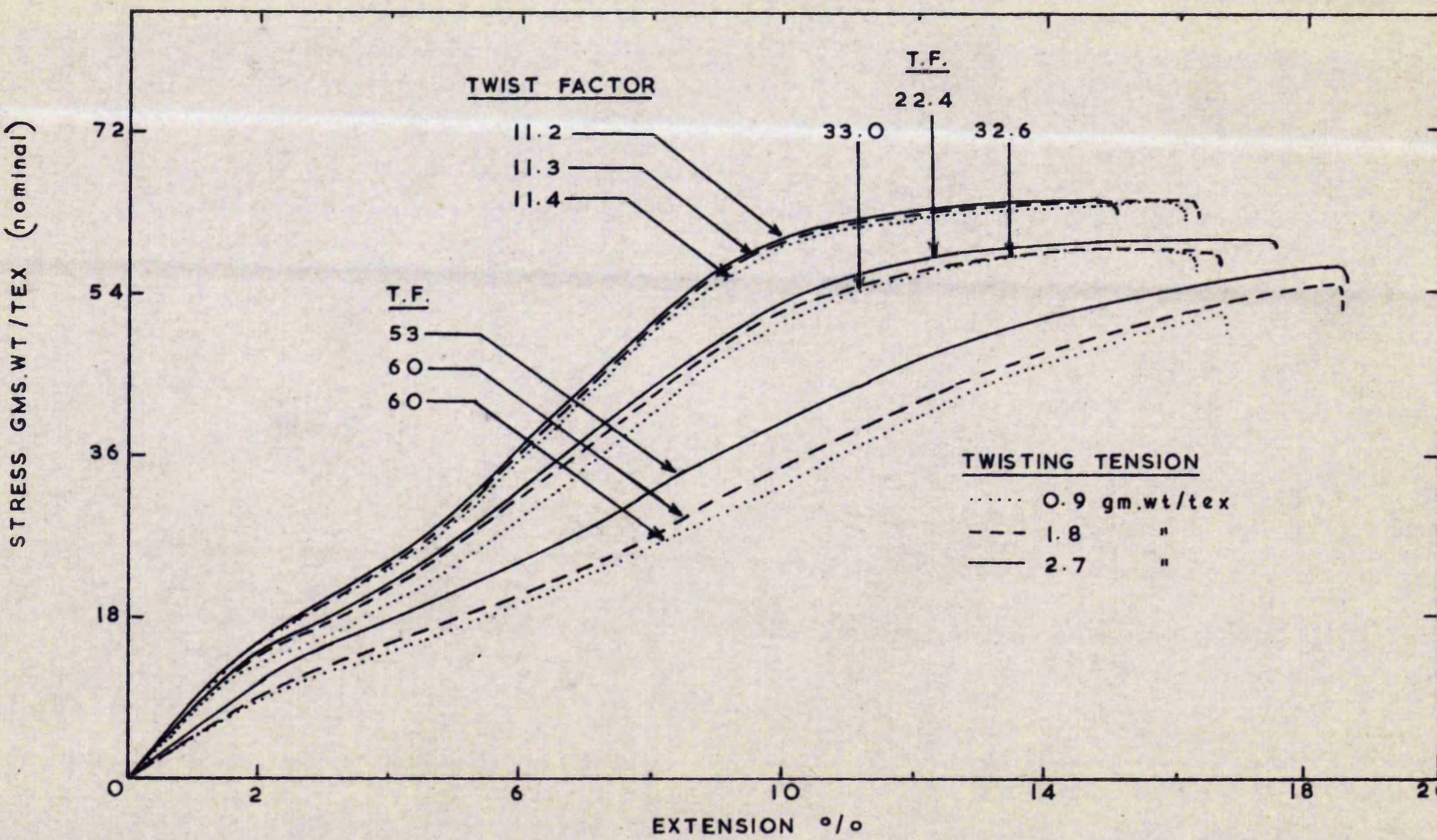


FIG. 3.22A_4 EFFECT OF TWISTING TENSION ON LOAD-EXTENSION CURVES OF TERYLENE 250-48

—INSTRON TESTS—

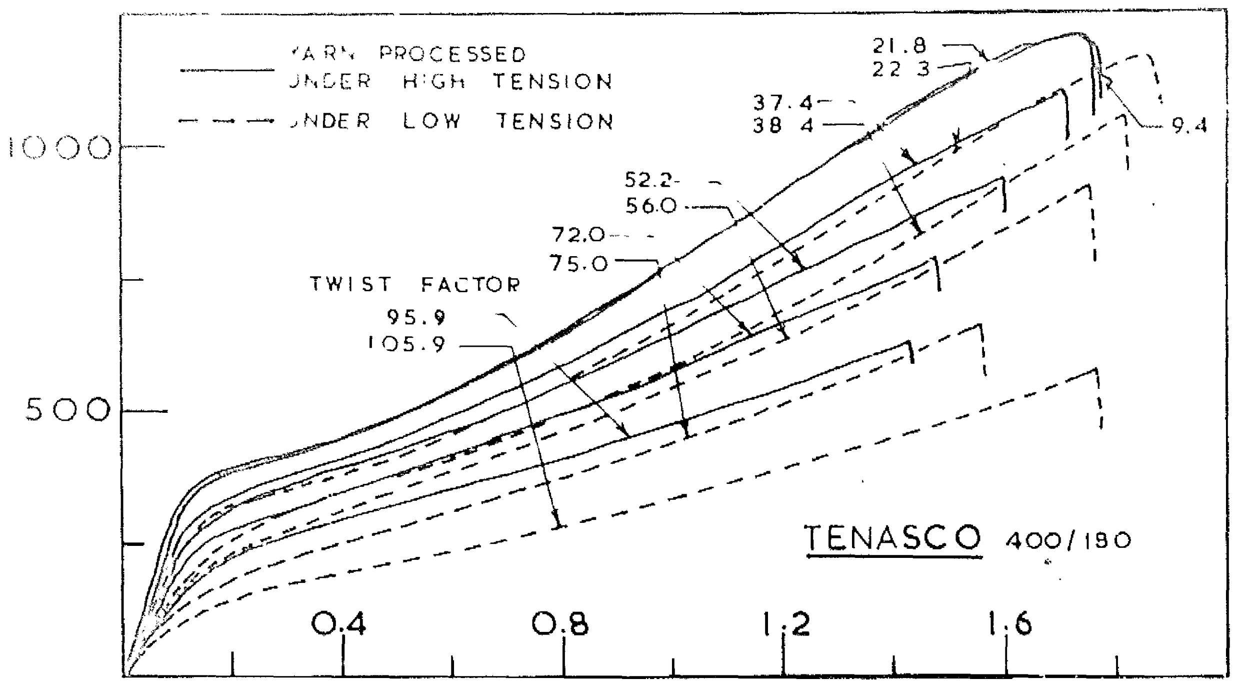


Fig. 3.22 A_5 Effect of Twisting Tension.

When the stress-strain relation is linear it can be expressed by an equation of the form

$$Y = A + B\epsilon \qquad \text{........ (3.13a)}$$

where Y = the stress in g.wt./tex.

B = the slope in g.wt./tex per unit strain.

A = the intercept in g.wt./tex.

ϵ = the strain.

In figure (3.23 A), let $\epsilon_1 Y_1$ and $\epsilon_2 Y_2$ be two points on a load extension diagram. It is necessary to correct for the reduction in yarn count (tex) due to extensions. Then by equation 3.13a, the slope B of linear stress-strain relation can be calculated

$$B = Y_2(1+\epsilon_2) - Y_1(1+\epsilon_1) \quad T[\epsilon_2 - \epsilon_1] \qquad \text{..........(3.13 b)}$$

where T = tex based on original count.

And the intercept A from the relation

$$A = \frac{Y_2(1+\epsilon_2)}{T} - B\epsilon_2 \qquad \text{........(3.13 c)}$$

These constants A and B have been calculated and plotted against twist factor at break in figures (3.23 B_1, B_2, C_1 & C_2)

It can be seen that

(i) In general, the slope B initially increases to a maximum and then decreases continuously as the twist factor is increased.

(ii) The effect of increasing the twisting tension is to increase both the intercept A and the slope B for a given twist factor.

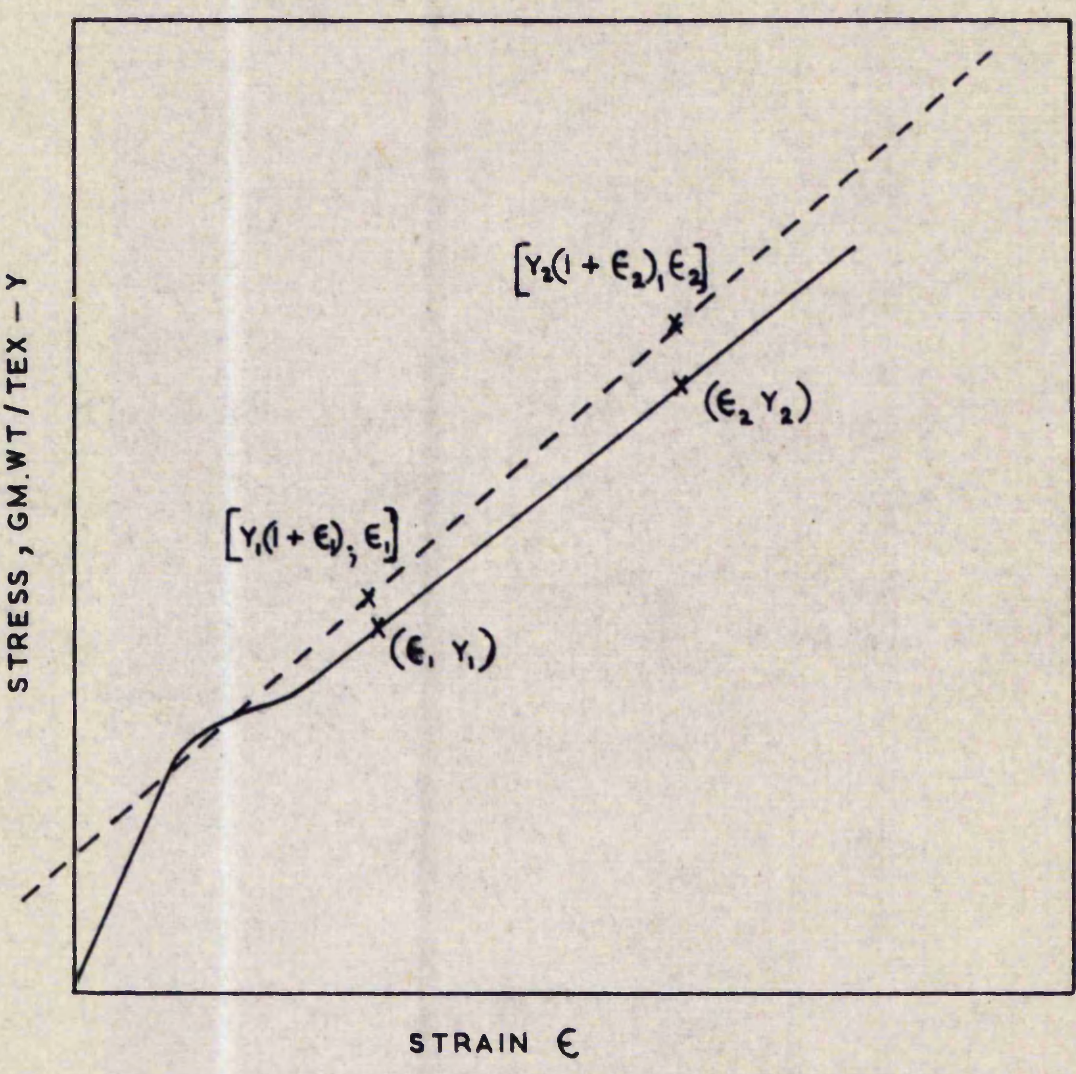


FIG. 3.23A METHOD OF CORRECTING INTERCEPT AND SLOPE OF STRESS-STRAIN RELATION NEAR BREAK.

FIG. 3.23B₁ THE EFFECT OF TWISTING TENSION ON SLOPE B OF THE STRESS-STRAIN CURVE FOR VISCOSE 300-100

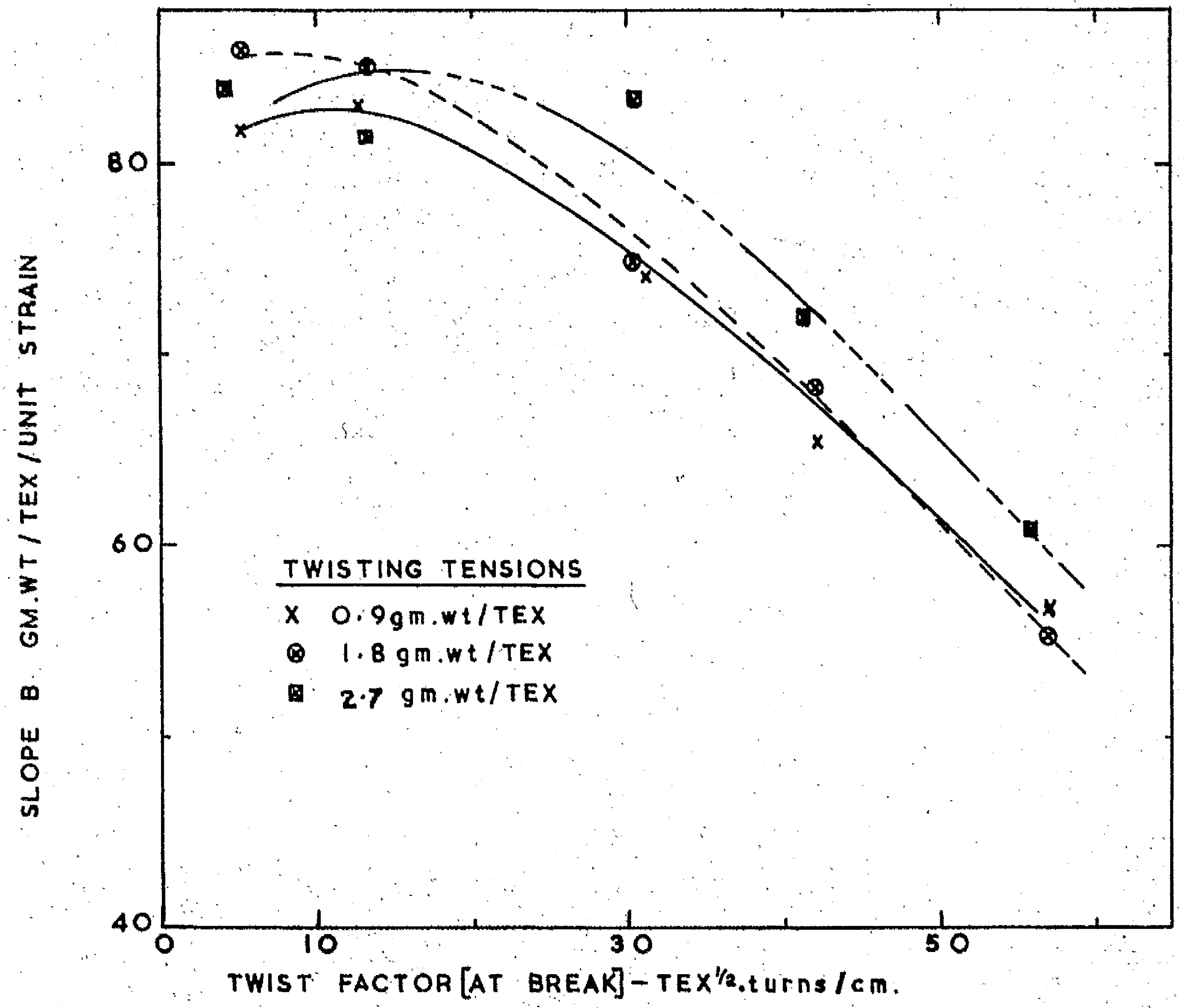
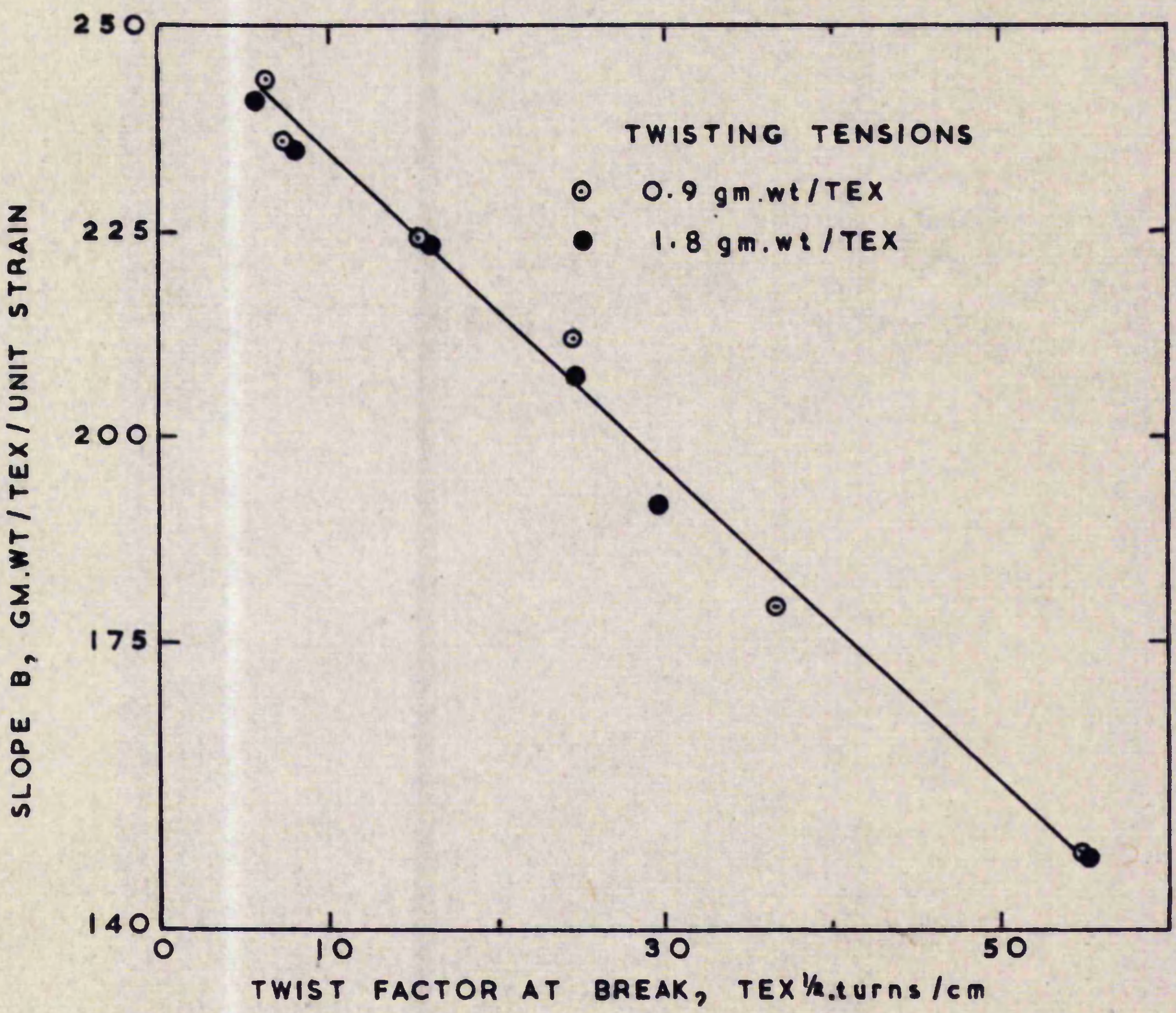


FIG. 323 B_2 THE EFFECT OF TWISTING TENSION ON SLOPE B OF STRESS-STRAIN CURVE FOR TENASCO 1650-750



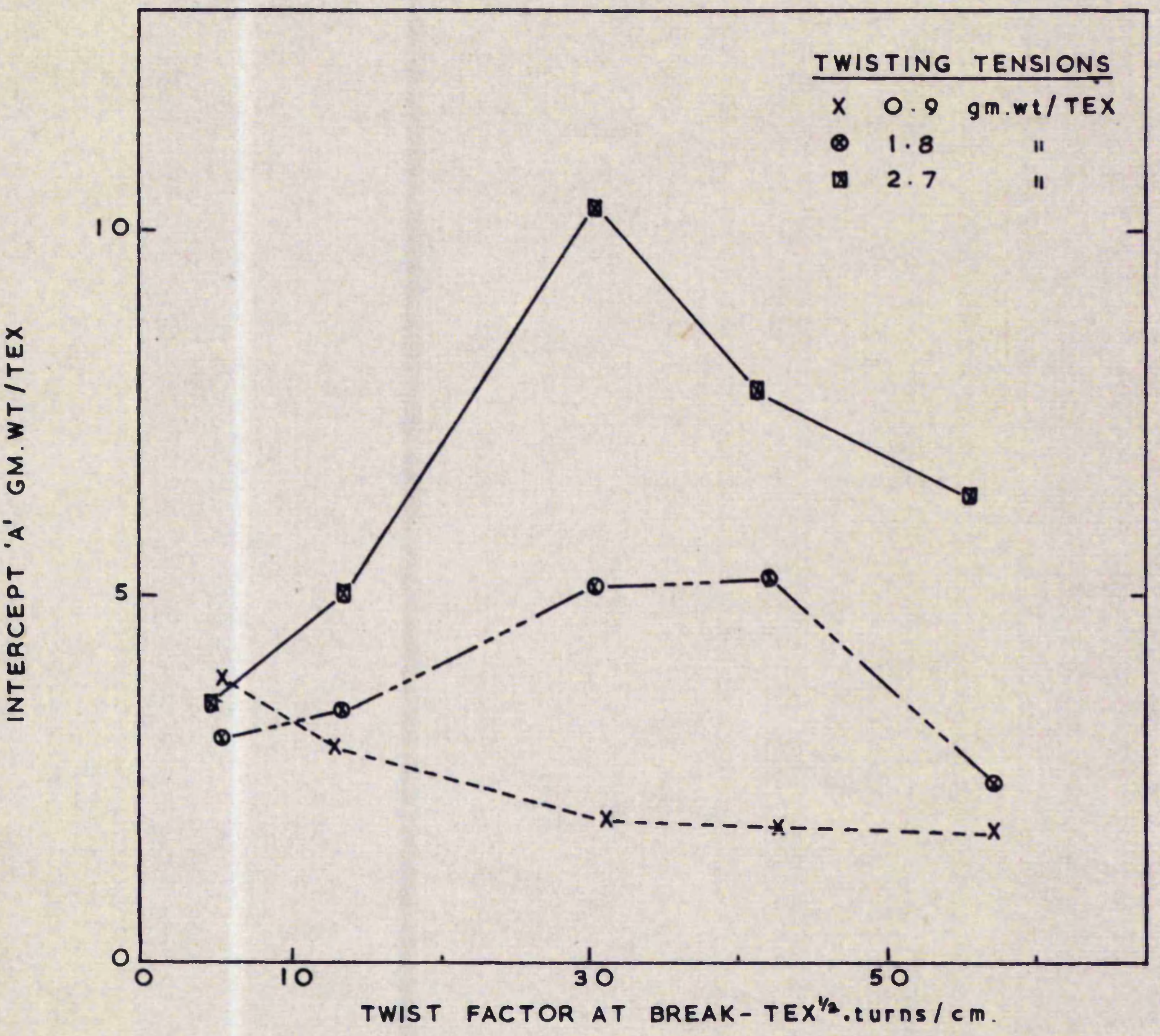


FIG. 3·23C$_1$ EFFECT OF TWISTING TENSION ON INTERCEPT 'A' OF STRESS-STRAIN RELATION FOR VISCOSE 300–100 YARNS.

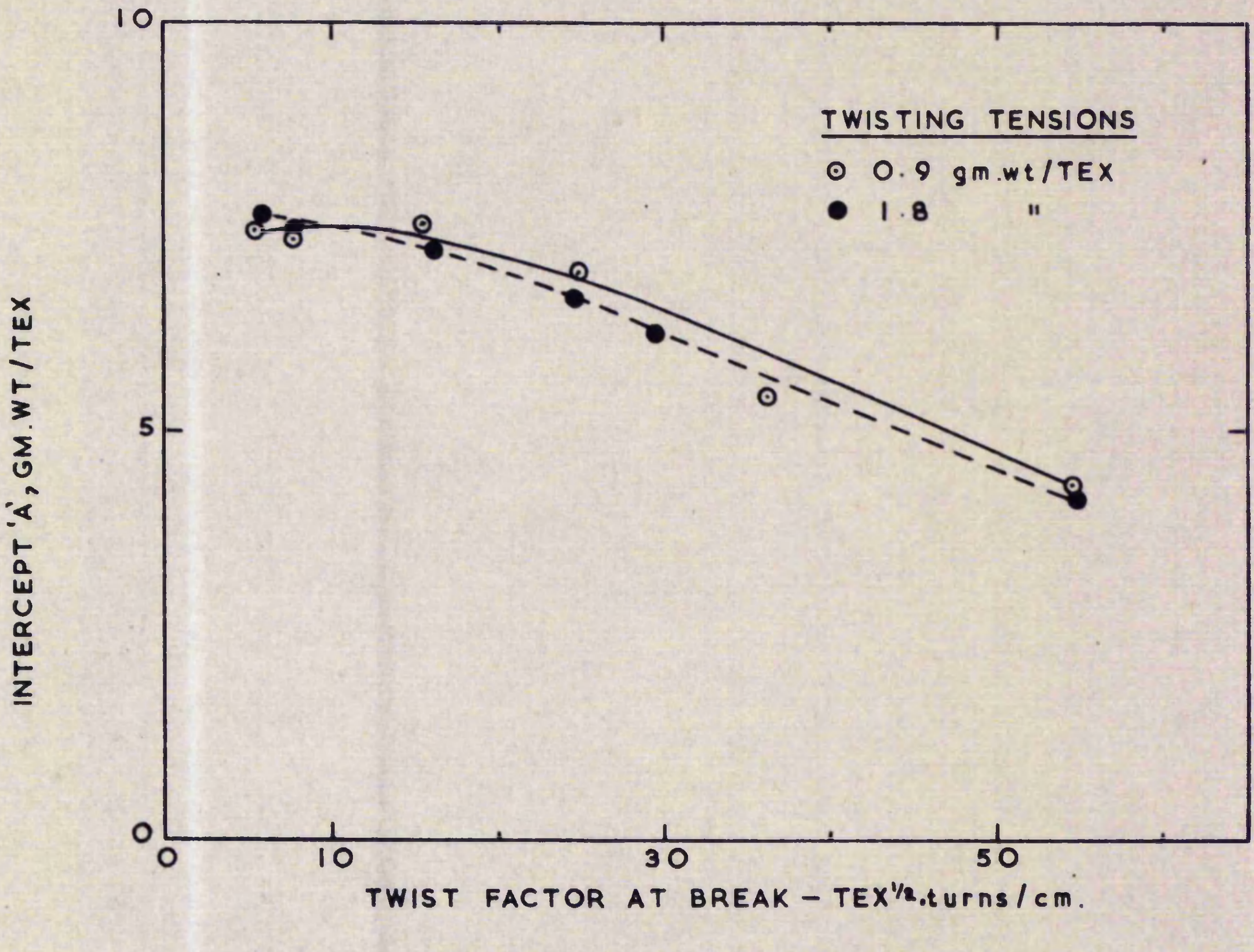


FIG. 3.23C$_2$ EFFECT OF TWISTING TENSION ON INTERCEPT 'A' OF STRESS-STRAIN RELATION FOR TENASCO 1650–750.

(iii) The behaviour of change in intercept A with twist is very erratic as the twisting tension is increased. This is observed more so in viscose 300-100 yarns.

3.3 EFFECT OF TWIST ON BREAKING EXTENSION

3.31 General studies

The variation of breaking extension with twist is shown in figures (3.31 A_1-A_5). Breaking extension values as low as 10% for Tenasco and as high as 35% for acetate have been obtained.

From these figures, it could be seen that:

(i) In general, the breaking extension values obtained by using three different types of tensile testers, show similar qualitative results. However, for a given yarn sample extension values from Instron tests are very low for zero twist yarns and are slightly higher for higher twist yarns, as compared to those obtained from the IP2 and Uster automatic strength tests: the explanation for this behaviour is given in chapter VI.

(ii) Viscose rayon: Figures (3.31 A_1, A_2 & A_3).

(a) For all yarns tested, the breaking extension initially increases and reaches a maximum at twist factors of 20 to 25 tex$^{\frac{1}{2}}$ turns/cm. On further insertion of twist, it remains more or less constant over the entire range of twists up to a twist factor of 100 units, except for viscose 75-75 where it rapidly decreases for yarns having twist factors higher than 70 units.

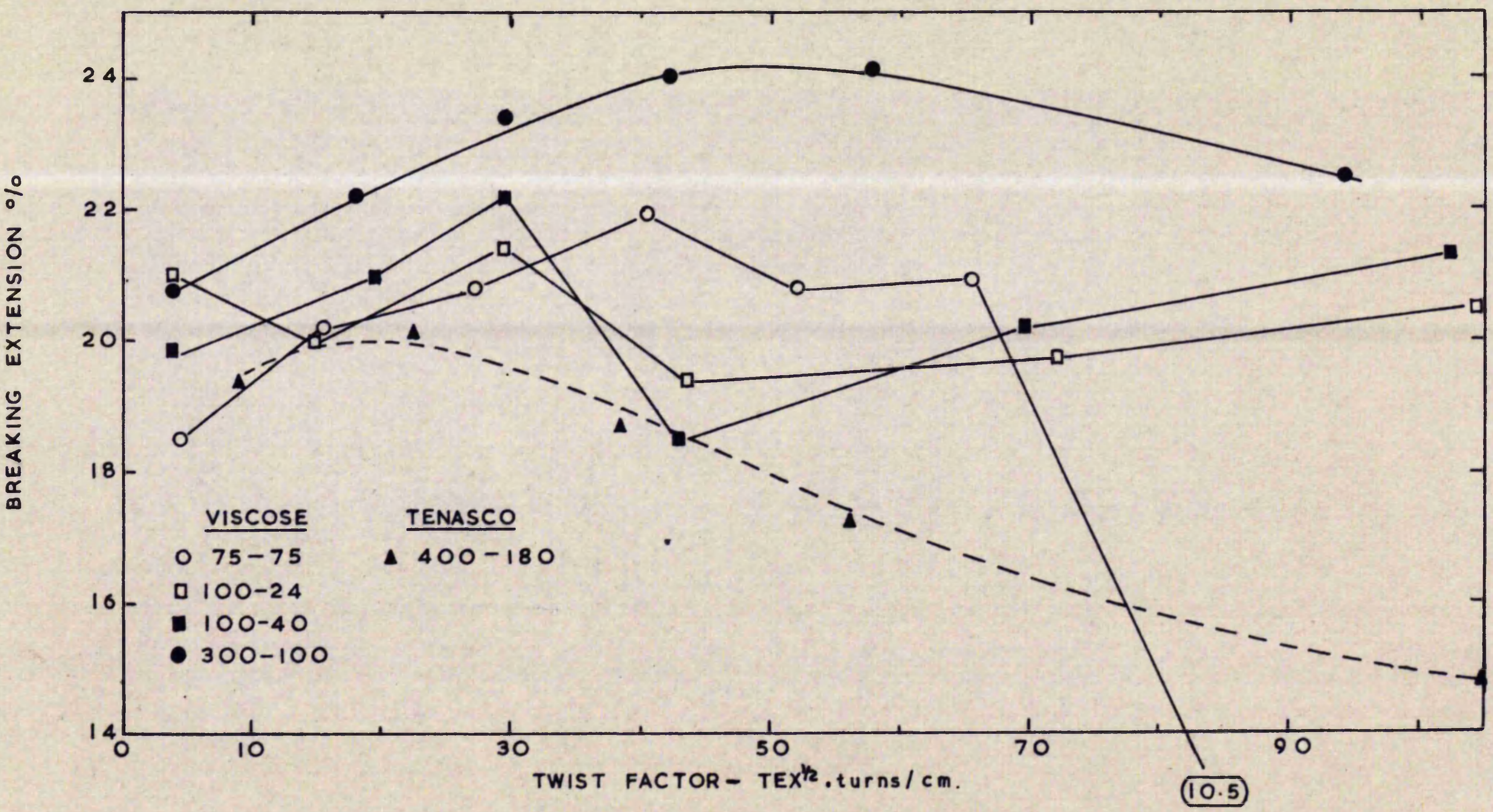


FIG. 3·31A, BREAKING EXTENSION OF VISCOSE AND TENASCO YARNS.

- USTER TESTS -

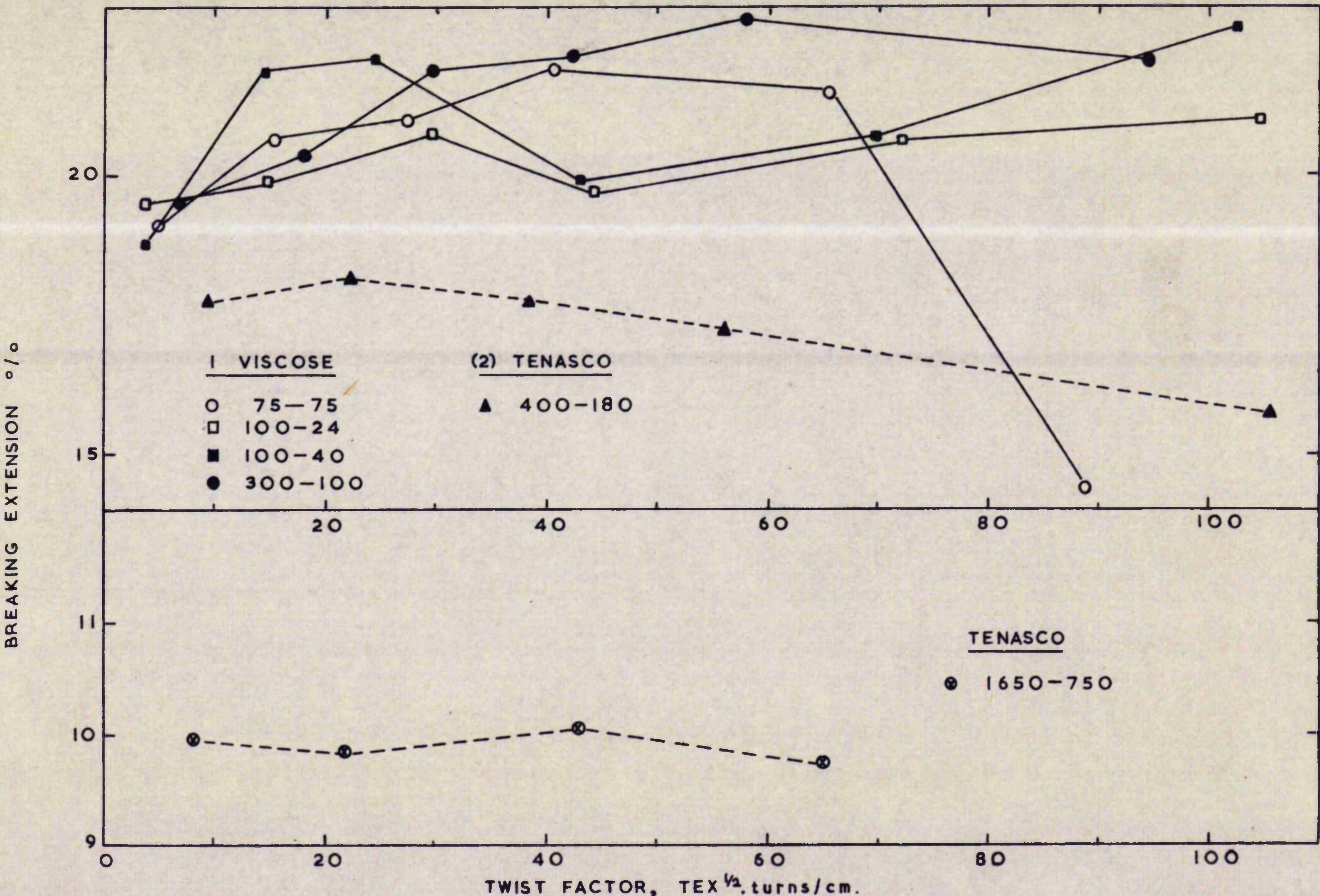


FIG. 3.31A$_2$ BREAKING EXTENSION OF VISCOSE AND TENASCO YARNS — INSTRON TESTS

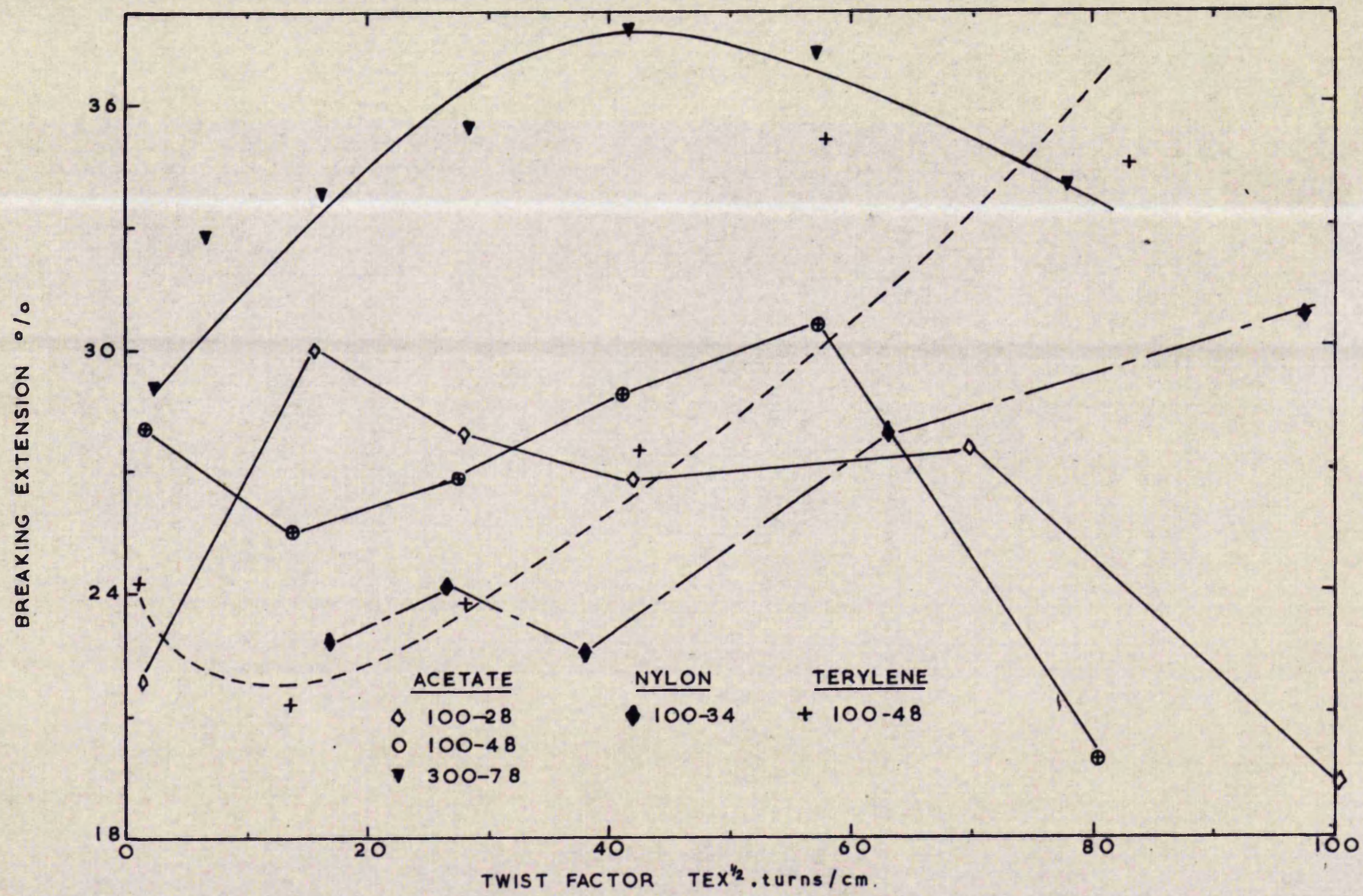


FIG. 3·31A₃ BREAKING EXTENSION OF ACETATE, NYLON & TERYLENE YARNS. –USTER TESTS–

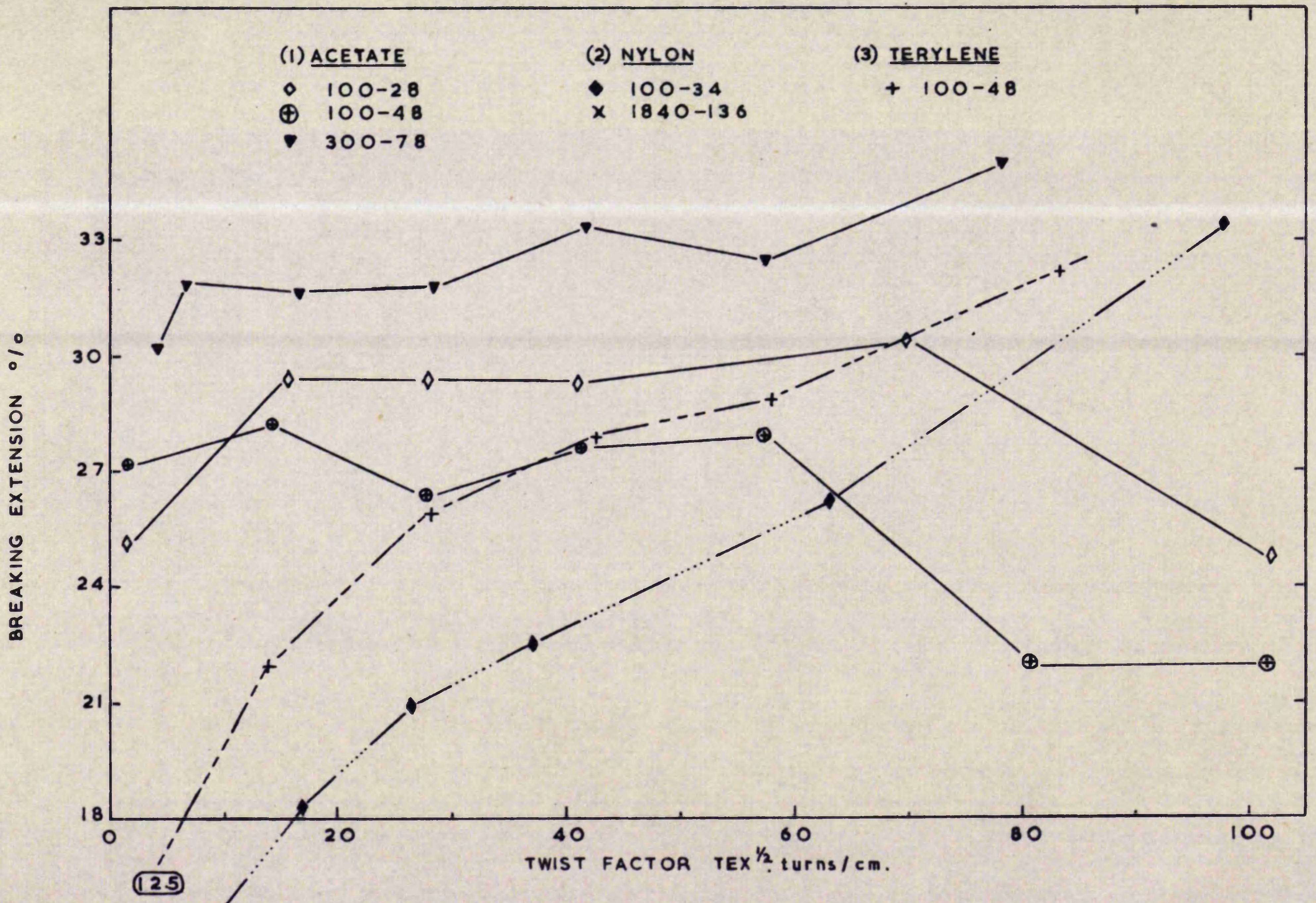


FIG. 3.31A$_4$ BREAKING EXTENSION OF ACETATE, NYLON AND TERYLENE YARNS — INSTRON TESTS

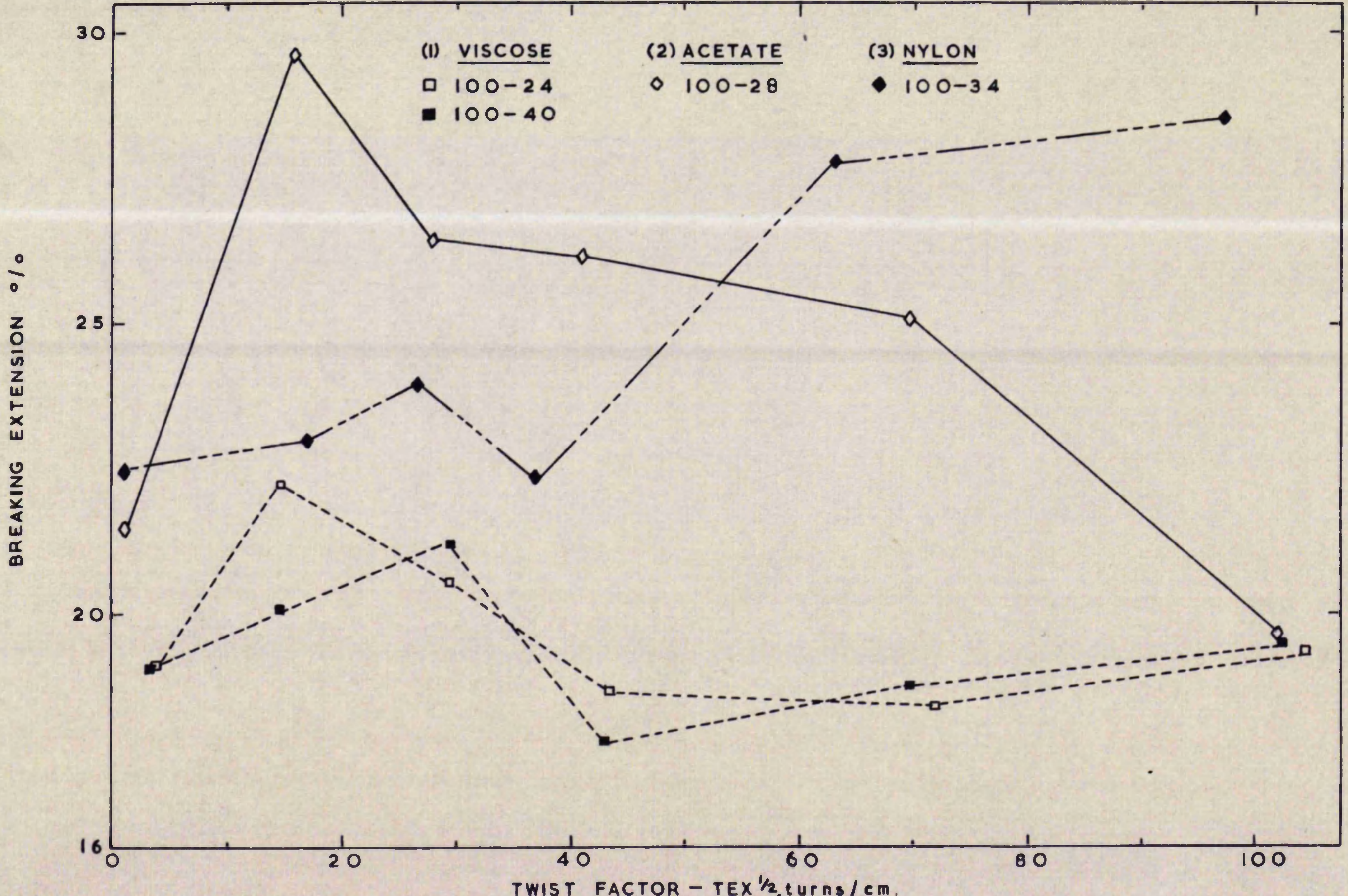


FIG. 3.31 A_5 BREAKING EXTENSION OF VISCOSE ACETATE AND NYLON YARNS — IP_2 TESTS

(b) The greater the number of filaments constituting the yarn structure, the greater is the initial increase in the breaking extension.

(iii) Tenasco : Figures ($3.31A_1$ & $3.31A_2$).

For Tenasco 1650-750, the breaking extension remains more or less constant over the whole twist factor range of 7 to 65 $tex^{\frac{1}{2}}$ turns/cm. However, for Tenasco 400-180 yarns the breaking extension does exhibit a slight initial rise reaching a maximum at a twist factor of 20 to 25 units, and shows a tendency to decrease for twist factors higher than 40 $tex^{\frac{1}{2}}$ turns/cm.

(iv) Acetate : Figures ($3.31A_3$, A_4 and A_5).

For all yarns tested the breaking extensions initially increase rapidly. A maximum value occurs at twist factors of 35-40 $tex^{\frac{1}{2}}$ turns/cm. Above twist factors of 60 to 70 units the breaking extension decreases rapidly.

(v) Nylon and Terylene : Figures ($3.31A_3$, A_4 and A_5).

In general, the breaking extension is an increasing function of twist for all yarns and over a wide range of twists up to a twist factor of 100 $tex^{\frac{1}{2}}$ turns/cm. The only exception is the Uster test results (Fig. $3.31A_4$) for Terylene 100-48, which shows an initial decrease followed by a continuous increase in breaking extension values: this initial increase is not observed in Instron test results (Fig. $3.31A_3$) on the same yarn.

3.32 Effect of twisting tension on breaking extension

Preliminary experiments were conducted by using two traveller numbers to control the tensions during twisting Tenasco 400-180 and acetate 300-78 yarns. It was then observed that the yarn tension at the twisting point increases as the twist is increased, although all other twisting conditions are kept the same (traveller number; spindle speed, r.h. etc.).

In later experiments on Tenasco 1650-750 viscose 300-100, nylon 840-136 and Terylene 250-48 yarns, the traveller number was adjusted for every change of twist wheel so as to maintain constant twisting tension. Three different tensions were used during twisting these yarns. The breaking extension results obtained in the tensile tests have been plotted in figures ($3.32A_1$-A_5).

The breaking extension values are found to vary from 10 to 35% for different materials tested. It was therefore found easier to compare the results when plotted as relative values rather than as absolute ones.

In the load extension diagrams obtained with the Instron tester, the breaking extensions for zero twist yarns can be read as the extension of either the first or the last filament to fail. It has been decided to define the relative breaking extension as the ratio of the actual breaking extension of a given twisted yarn (corresponding to the maximum load value in the load extension diagram) to that of the last filament in the zero twist yarn. Such relative breaking extension values have been plotted in figures ($3.32A_6$ and $3.32A_7$).

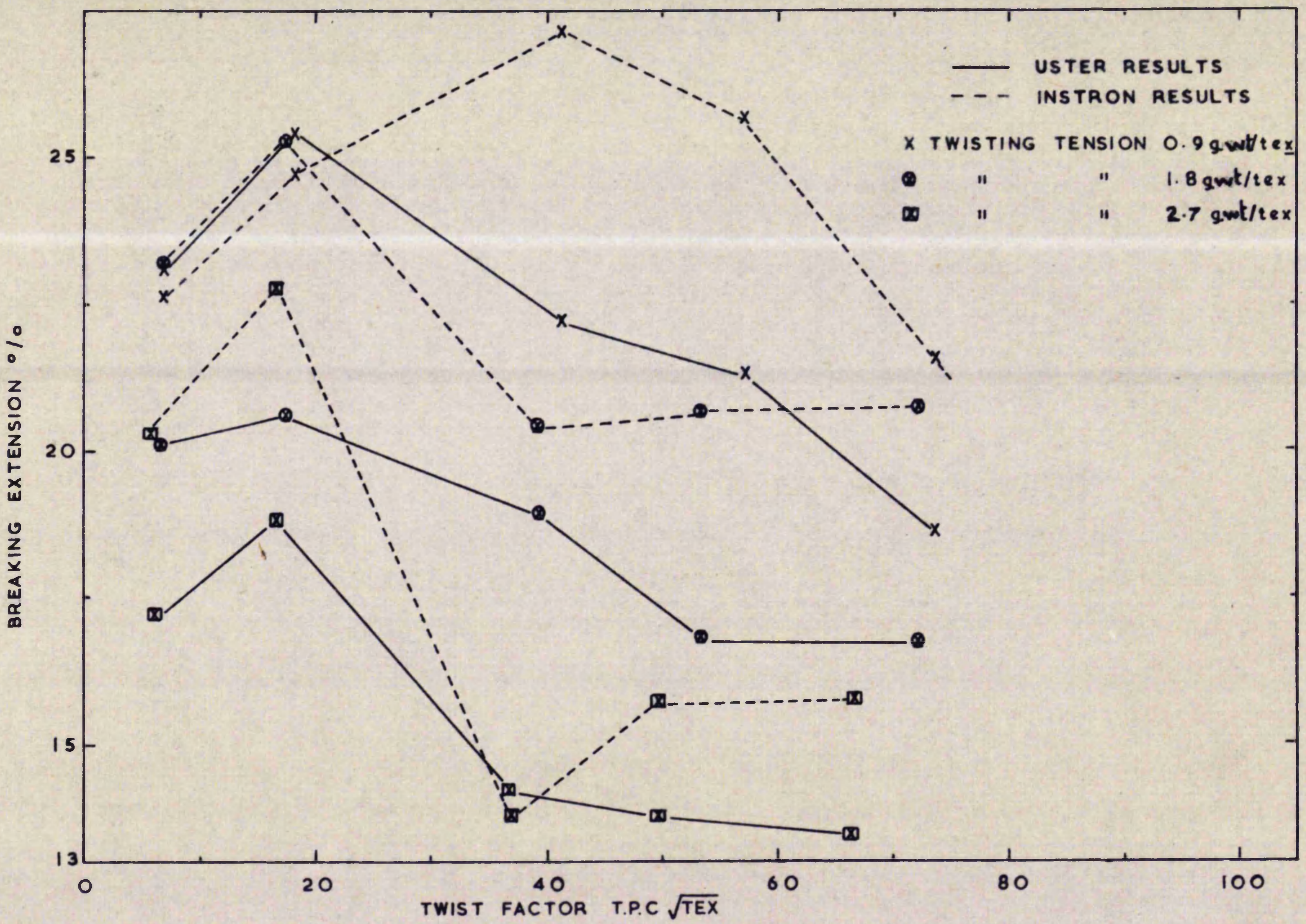


FIG. 3.32 A, THE EFFECT OF TWISTING TENSIONS ON THE BREAKING EXTENSIONS OF V300-100

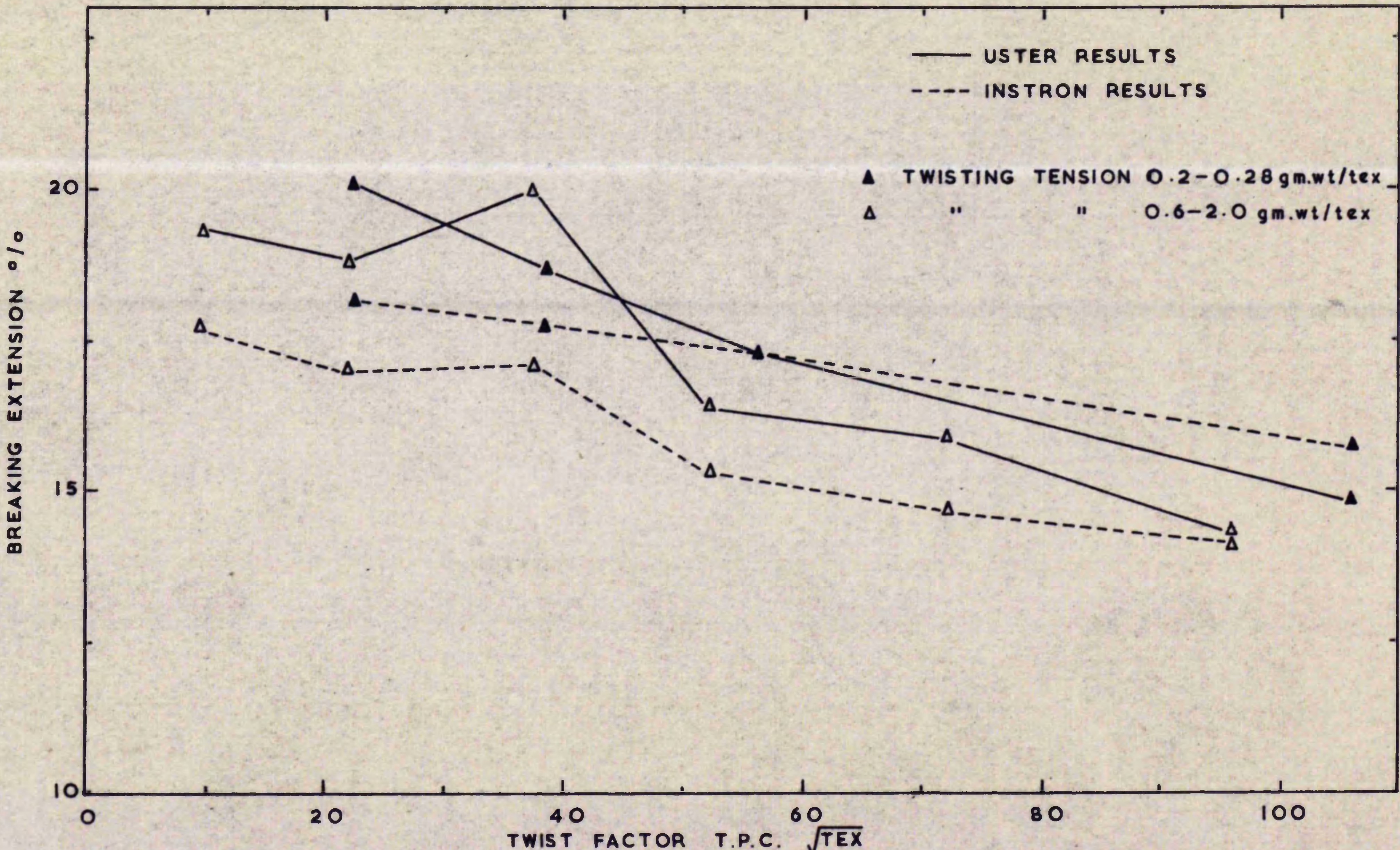


FIG. 3.32 A_2 THE EFFECT OF TWISTING TENSIONS ON THE BREAKING EXTENSION OF TENASCO 400-180 YARN

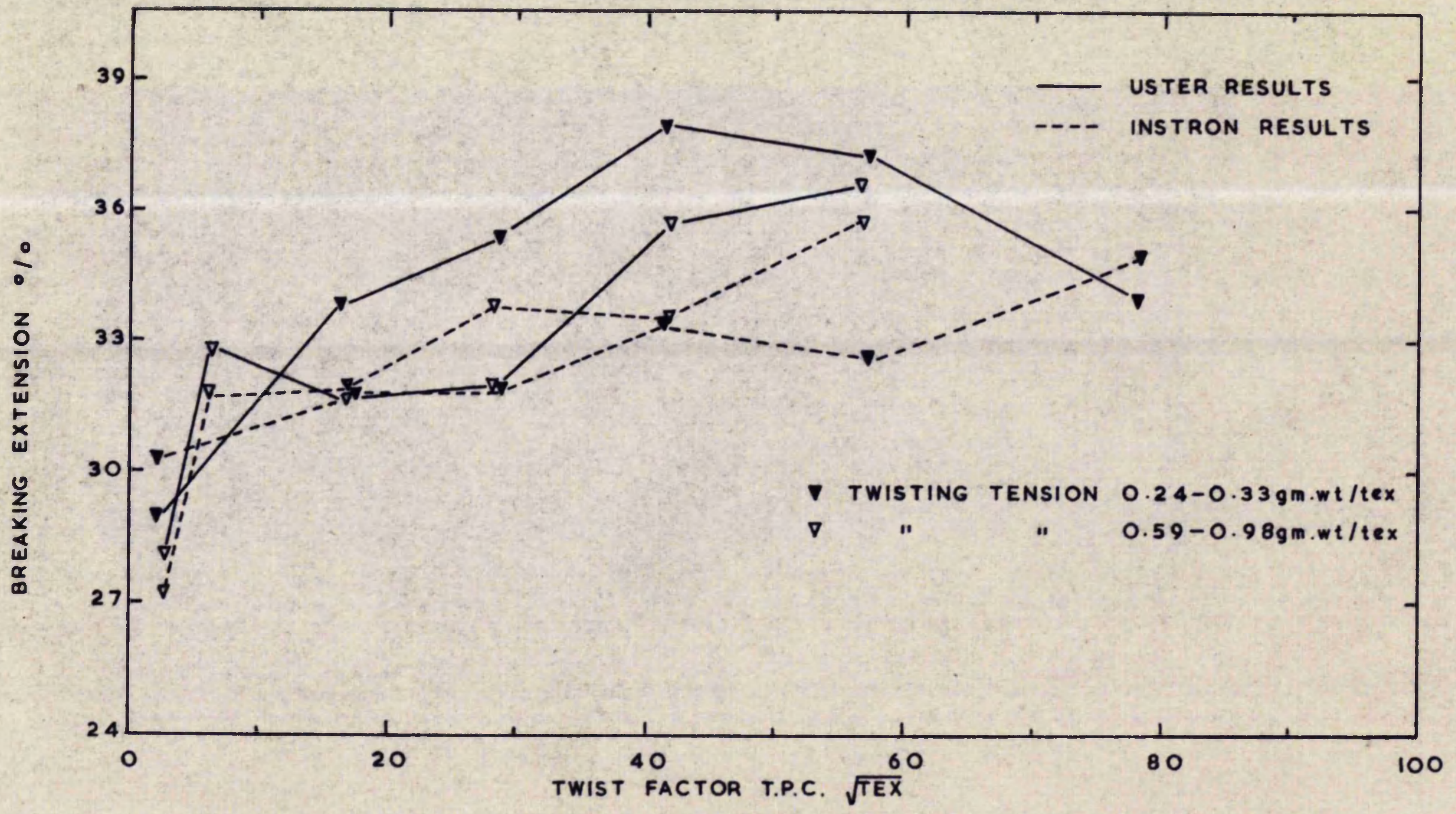
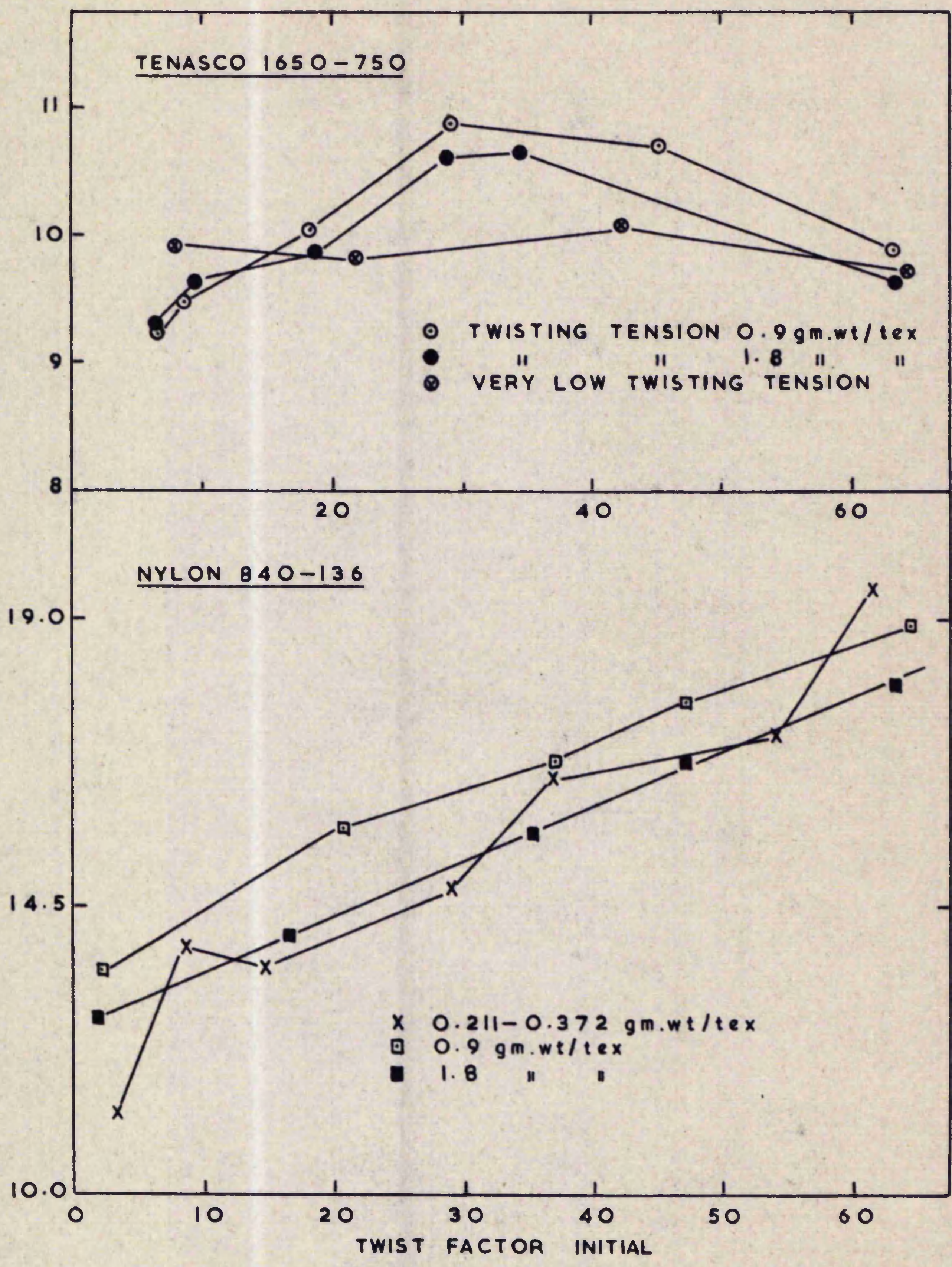


FIG. 3.32A, THE EFFECT OF TWISTING TENSION ON THE BREAKING EXTENSION OF ACETATE 300-78



3.32A4 THE EFFECT OF TWISTING TENSION ON THE BREAKING EXTENSIONS OF TENASCO AND NYLON (INSTRON TESTS)

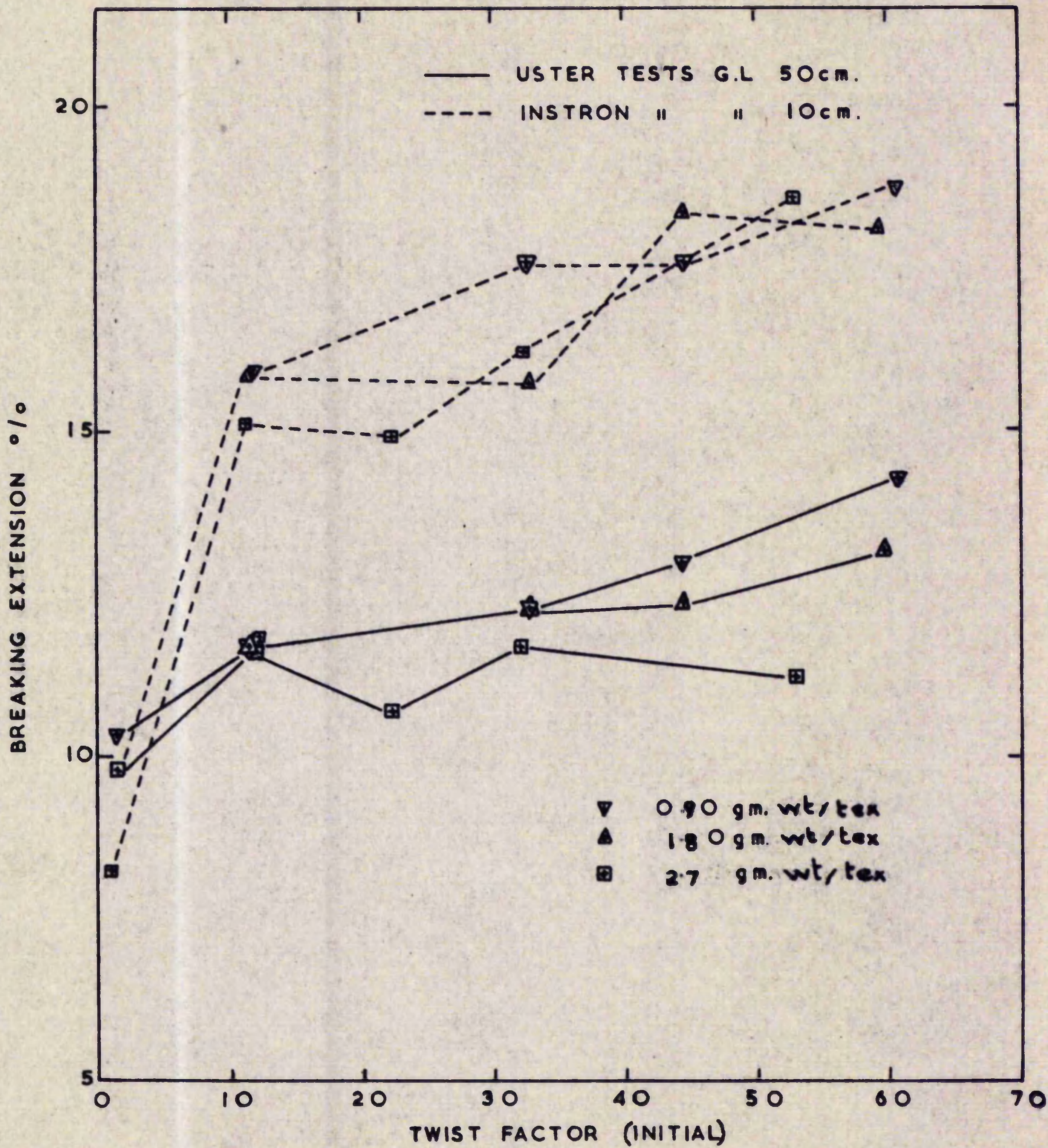


FIG. 3.32 A_5 THE EFFECT OF TWISTING TENSIONS ON THE BREAKING EXTENSION ON TERYLENE YARNS (250-48)

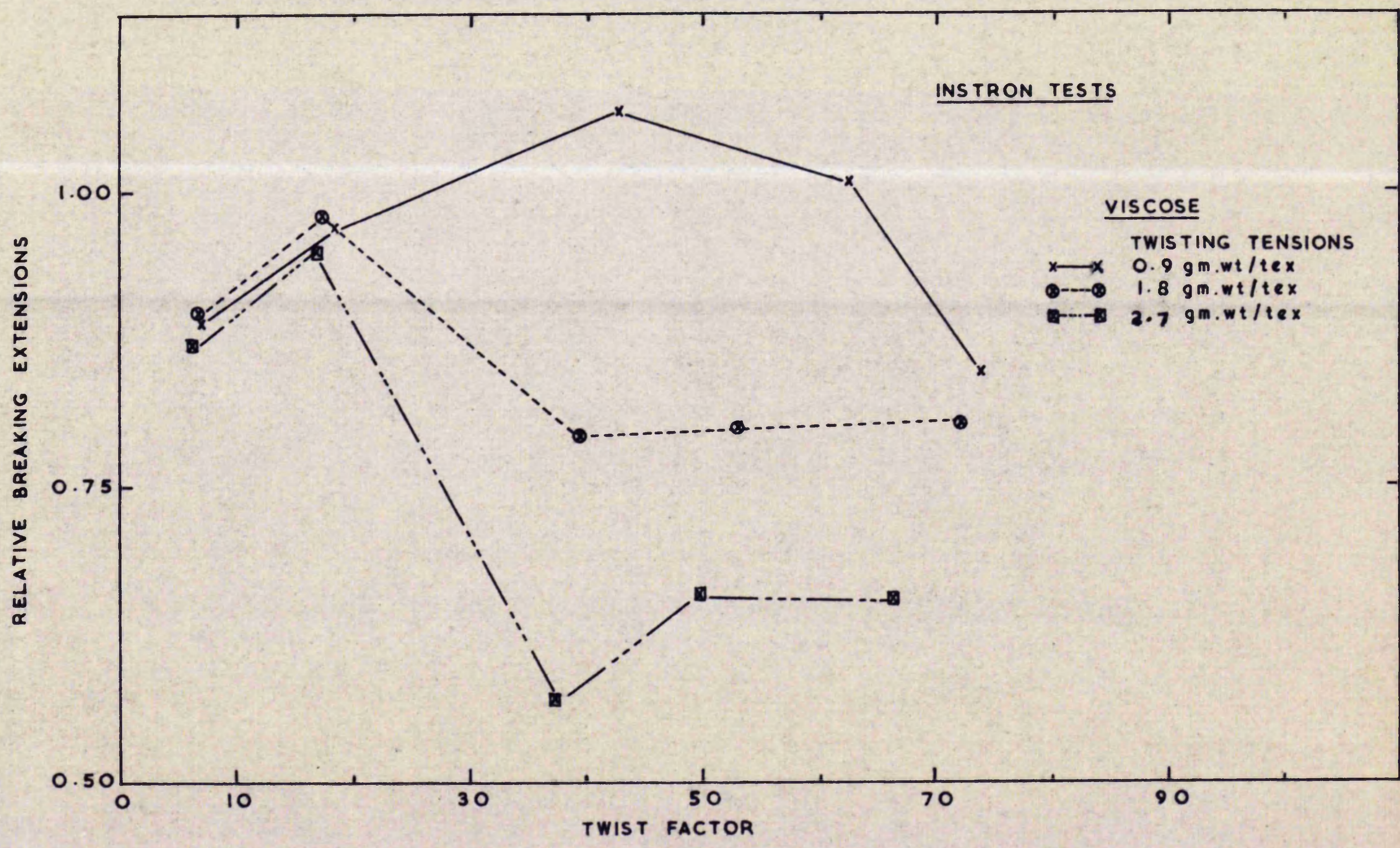


FIG. 3.32A6 THE EFFECT OF TWISTING TENSION ON THE BREAKING EXTENSION OF VISCOSE

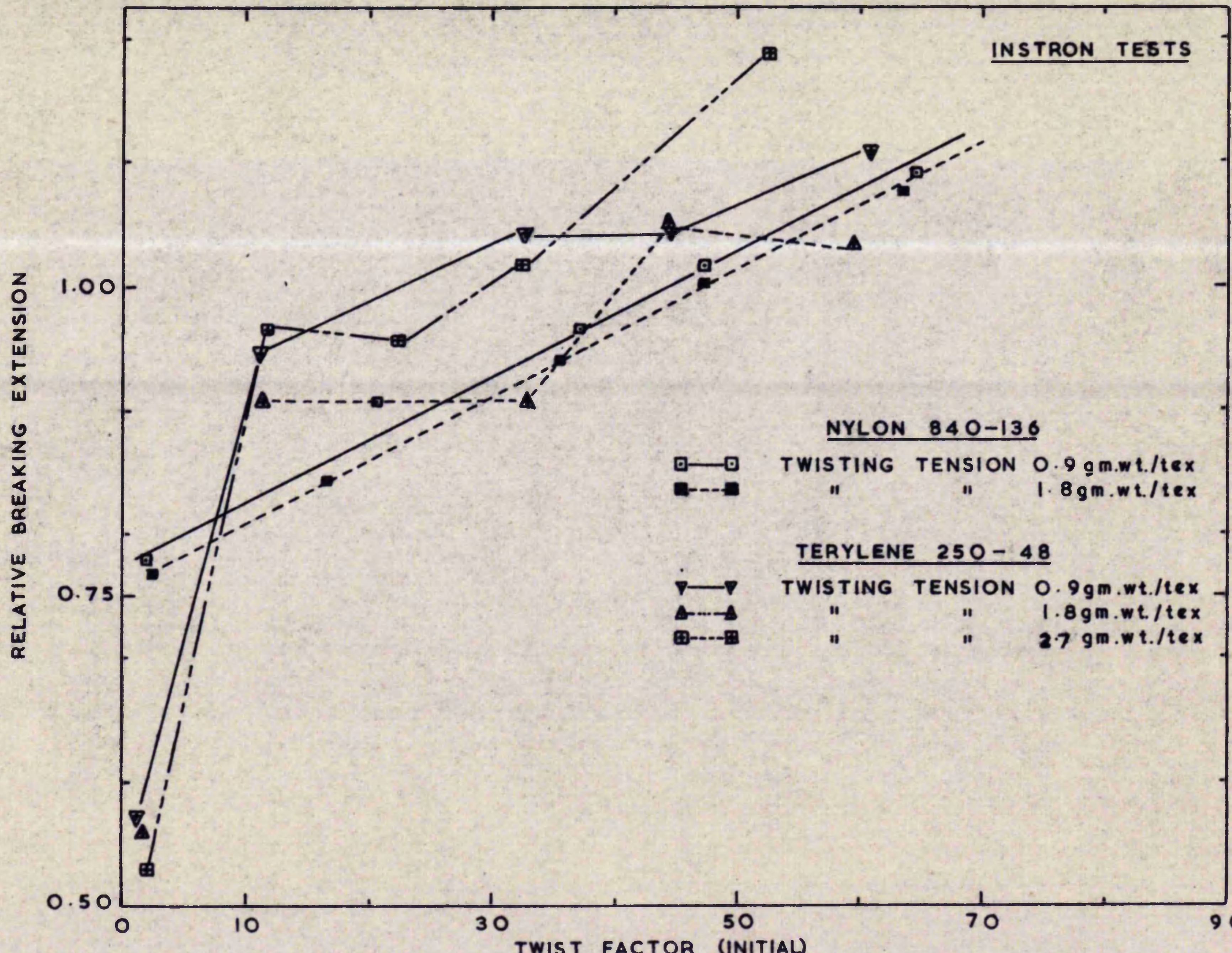


FIG. 3.32 A, THE EFFECT OF TWISTING TENSION ON THE BREAKING EXTENSION OF NYLON 840-136 AND TERYLENE 250-48

From these figures it can be concluded that

(i) In general, for a given twist factor, the higher the twisting tension, the lower is the breaking extension.

(ii) The effect of twisting tension in lowering the breaking extension is more pronounced in the behaviour of viscose 300-100 yarns than in nylon, Terylene and Tenasco yarns.

(iii) The breaking extension values for Tenasco 1650-750 yarns which were ring twisted under very low tensions are almost constant over the entire range of twist factors (7 to 65 tex$^{\frac{1}{2}}$turns/cm.). However, when moderate or higher twisting tensions were employed, the breaking extension values were found to show an initial increase reaching a maximum at a twist factor of 25 to 30 tex$^{\frac{1}{2}}$ turns/cm. and then a continuous decrease as further twist is inserted (fig. 3.32A_4).

3.33 The effect of method of twisting on breaking extension

These results are plotted in figures (3.33A_1 and A_2). The methods of twisting used were:- a ring doubler and an uptwister. A sample from the same spinning package was processed by these methods and tested on an Instron tester for tensile properties (Section 2.2).

From these figures, it can be seen that

(1) Method of twisting has very little effect on the breaking extension behaviour of twisted yarns, provided the twisting tensions are kept identical.

(2) Terylene and Tenasco yarns showed similar qualitative behaviour in both the methods of twisting, although the absolute values are somewhat different.

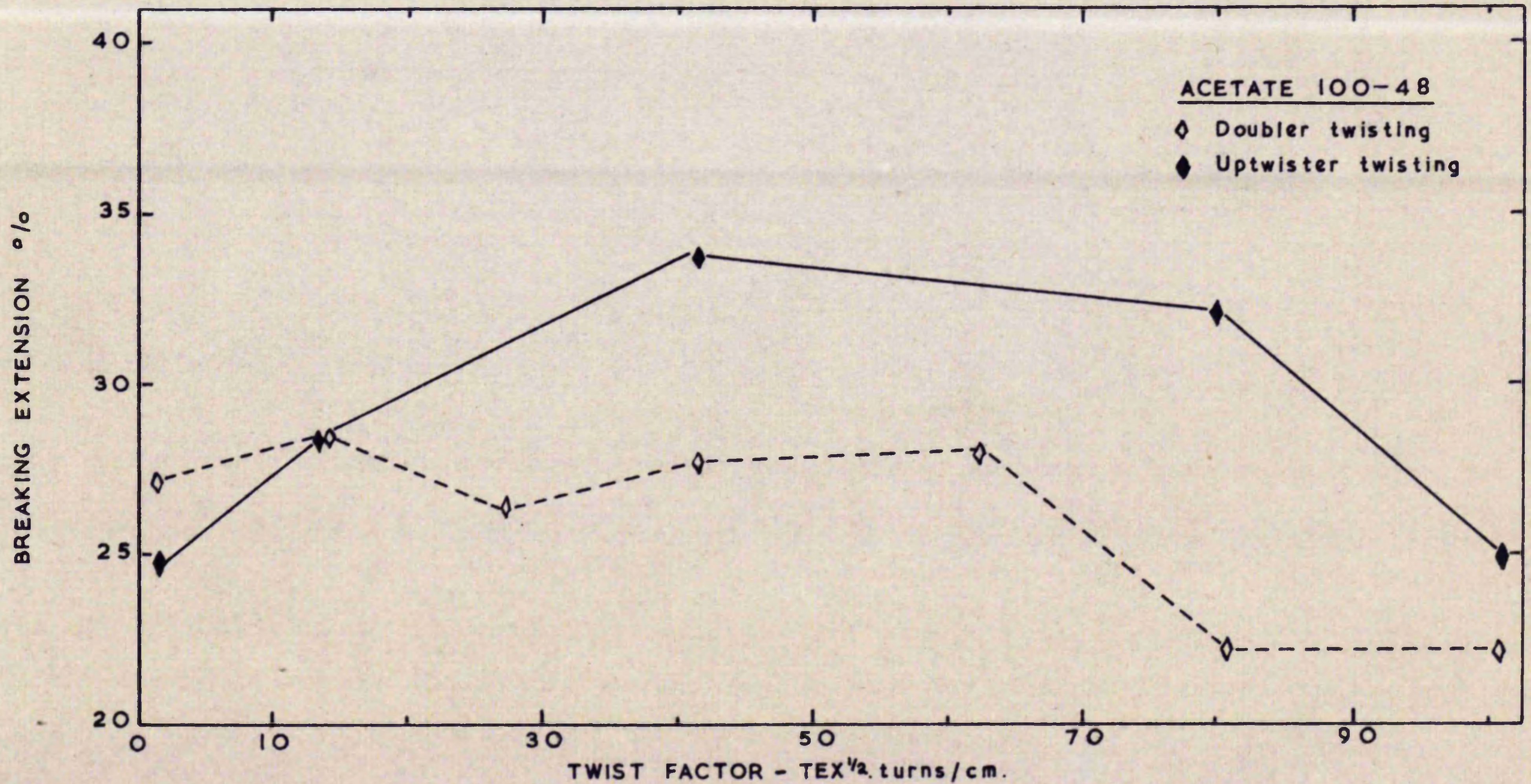


FIG. 3.33A₁ EFFECT OF METHOD OF TWISTING ON BREAKING EXTENSION OF ACETATE YARN.

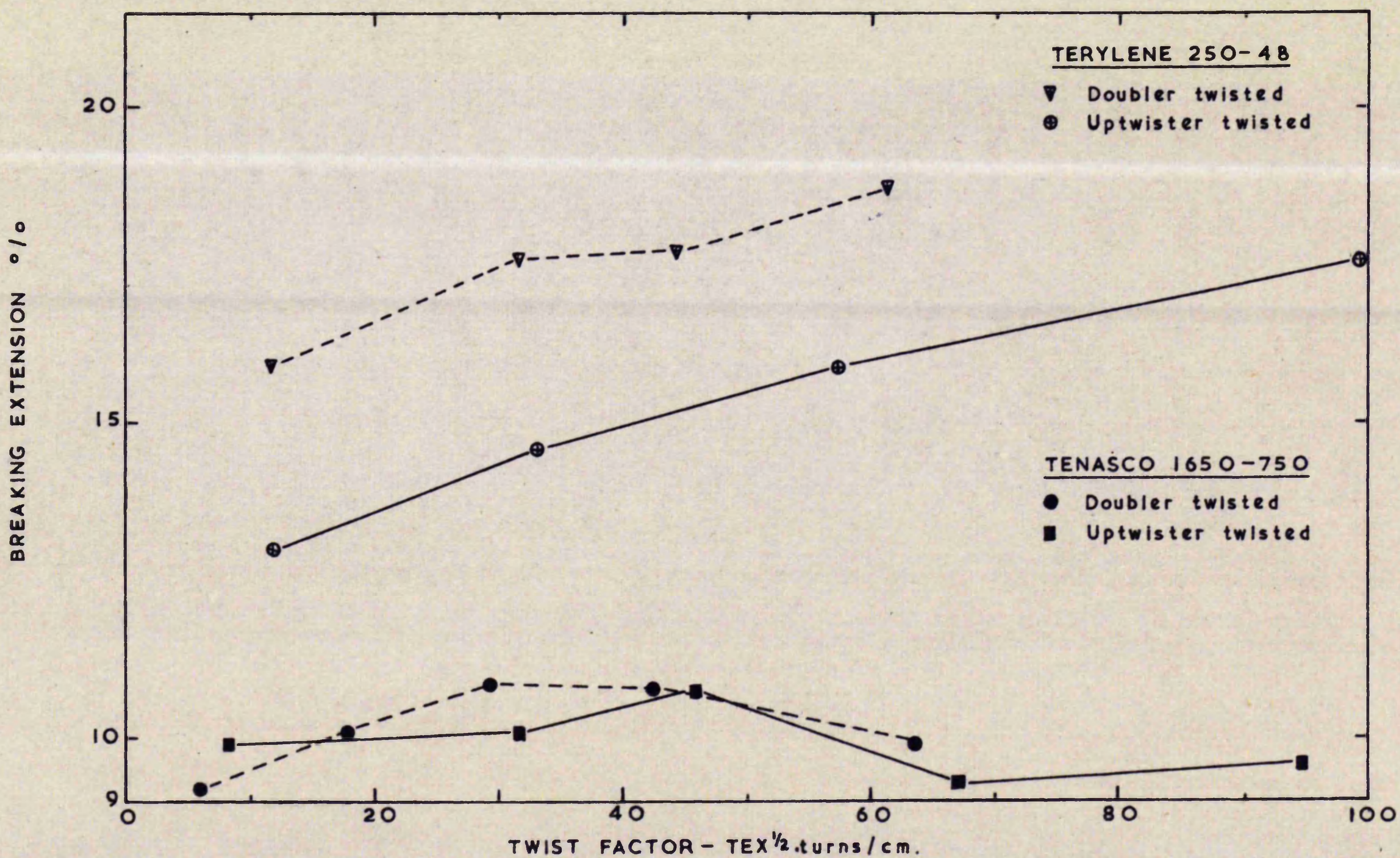


FIG. 3.33A_2 EFFECT OF METHOD OF TWISTING ON BREAKING EXTENSION OF TERYLENE AND TENASCO YARNS.

(3) For acetate 100-48, the general features (initial rise and then fall in breaking extension), which were not so clear in the ring doubler twisted yarn, are observed in the uptwister twisted yarn. This may be due to the higher twisting tensions obtained in the ring doubler method of twisting.

3.4 EFFECT OF TWIST ON TENACITY

3.41 General studies

Values of tenacity (based on the initial count of the yarn - not corrected for the extension) are plotted against initial twist factors in figures ($3.41A_1 - A_4$).

These figures show that

(i) For most of the yarns, the tenacity values show an initial increase to a maximum, followed by a continuous decrease as the twist factor increases. However, the initial increase is not observed in some yarns viz. Tenasco, Nylon and Terylene.

(ii) For all yarn constructions studied, the rate of fall in tenacity increases as the twist factor is increased.

(iii) At a given twist factor (initial) the percentage fall in tenacity is lowest for acetate and highest for Tenasco yarns.

(iv) The results obtained with the different tensile testing instruments are qualitatively similar. However, for a given yarn sample, the tenacity values from the Instron tests are lower than those obtained using the IP2 and Uster automatic tester.

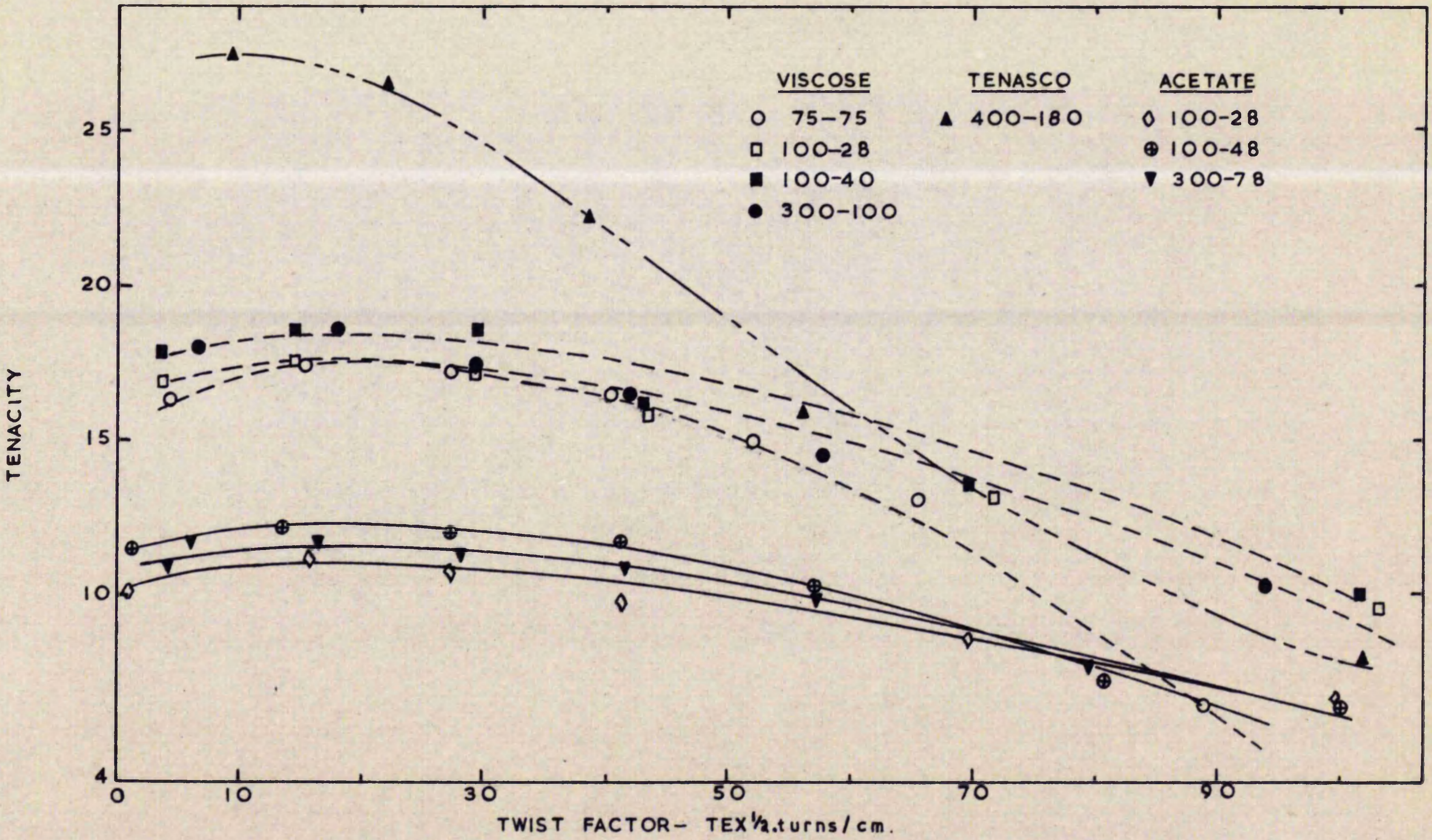


FIG. 3.41A₁ TENACITY OF VISCOSE, TENASCO AND ACETATE YARNS

—USTER TESTS—

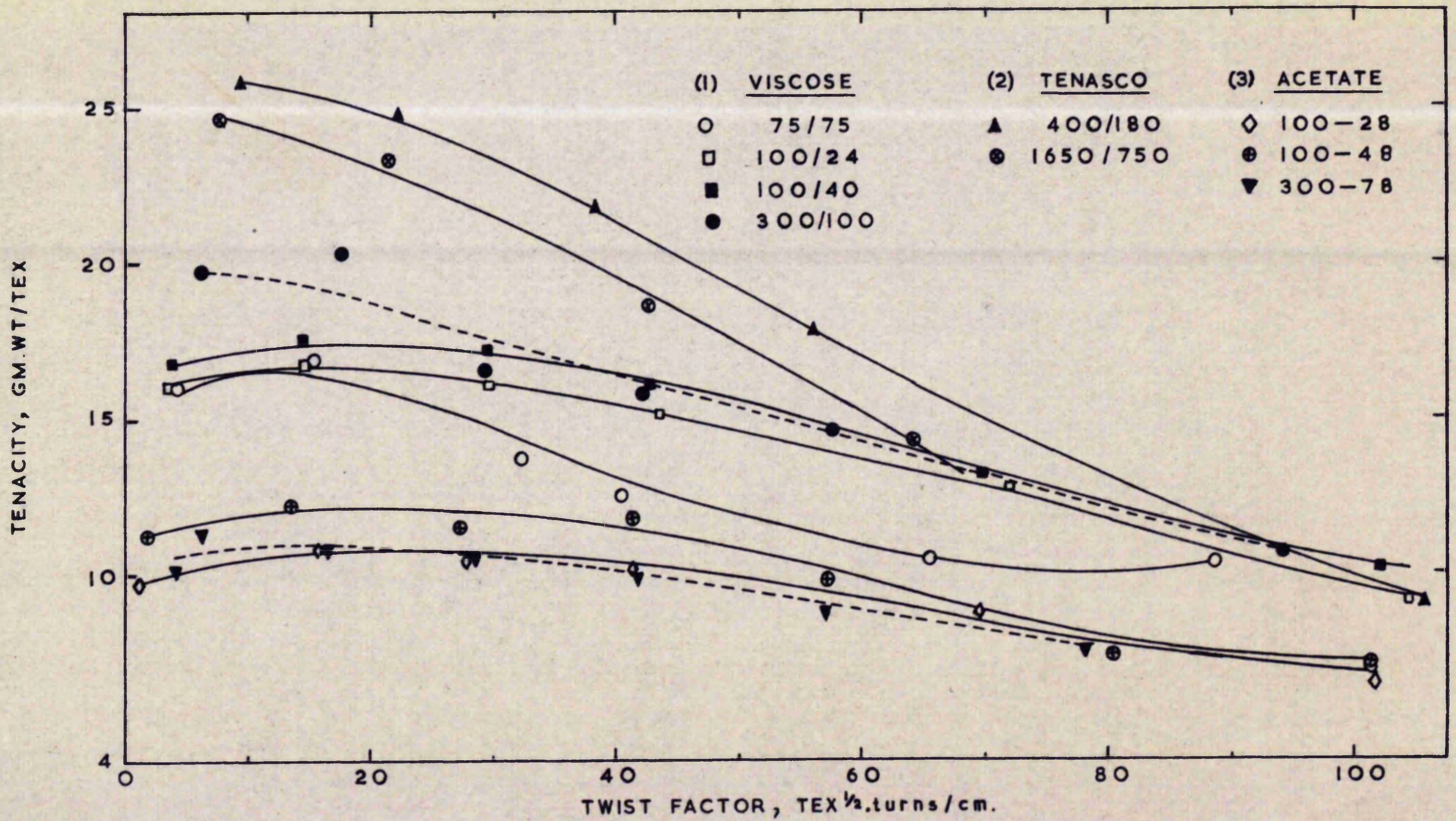


FIG. 3. 41A₂ <u>TENACITY</u> OF VISCOSE TENASCO AND ACETATE YARNS—<u>INSTRON TESTS</u>

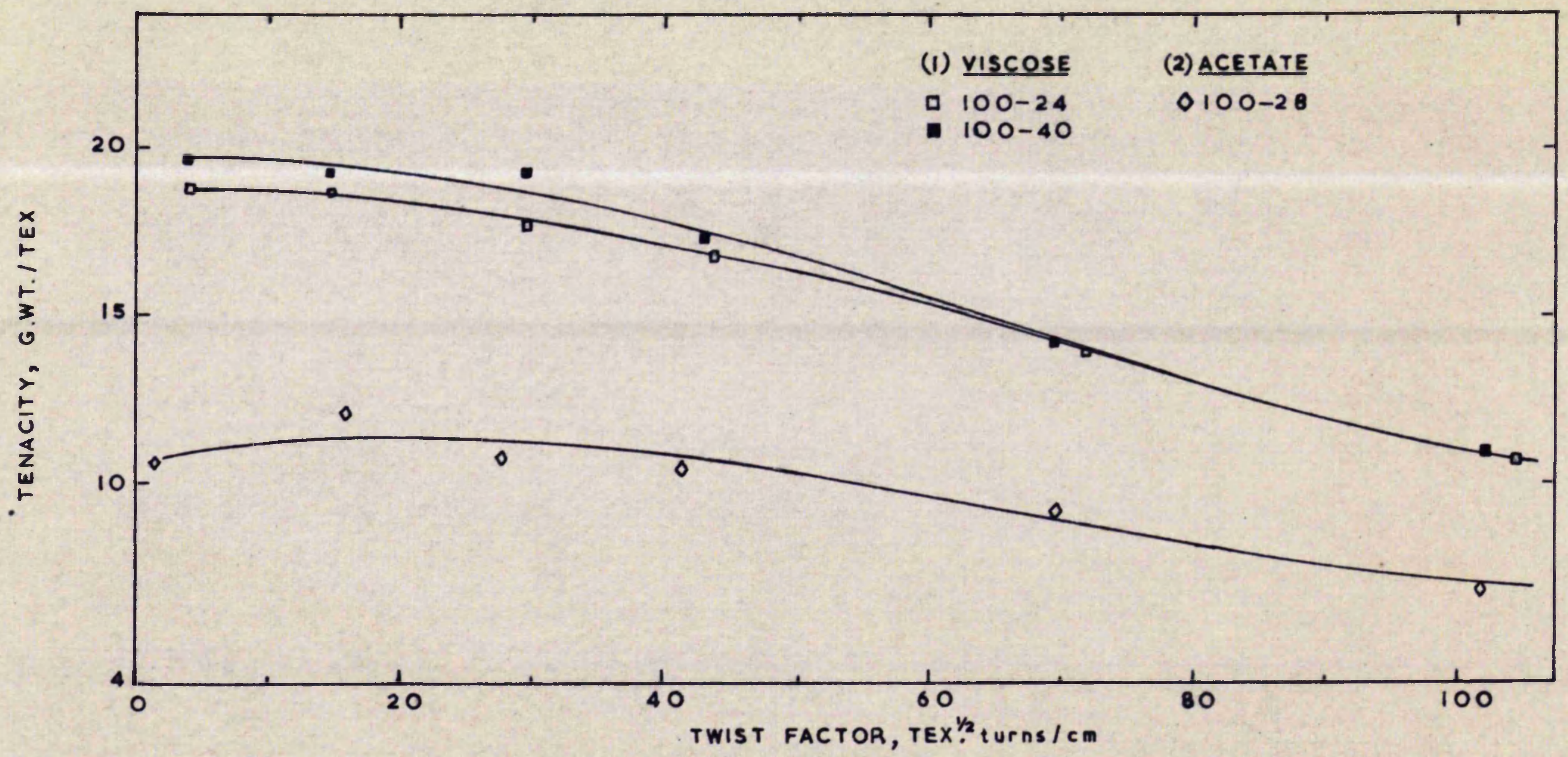


FIG. 3.41 A_3 TENACITY OF VISCOSE AND ACETATE YARNS — $1P_2$ TESTS

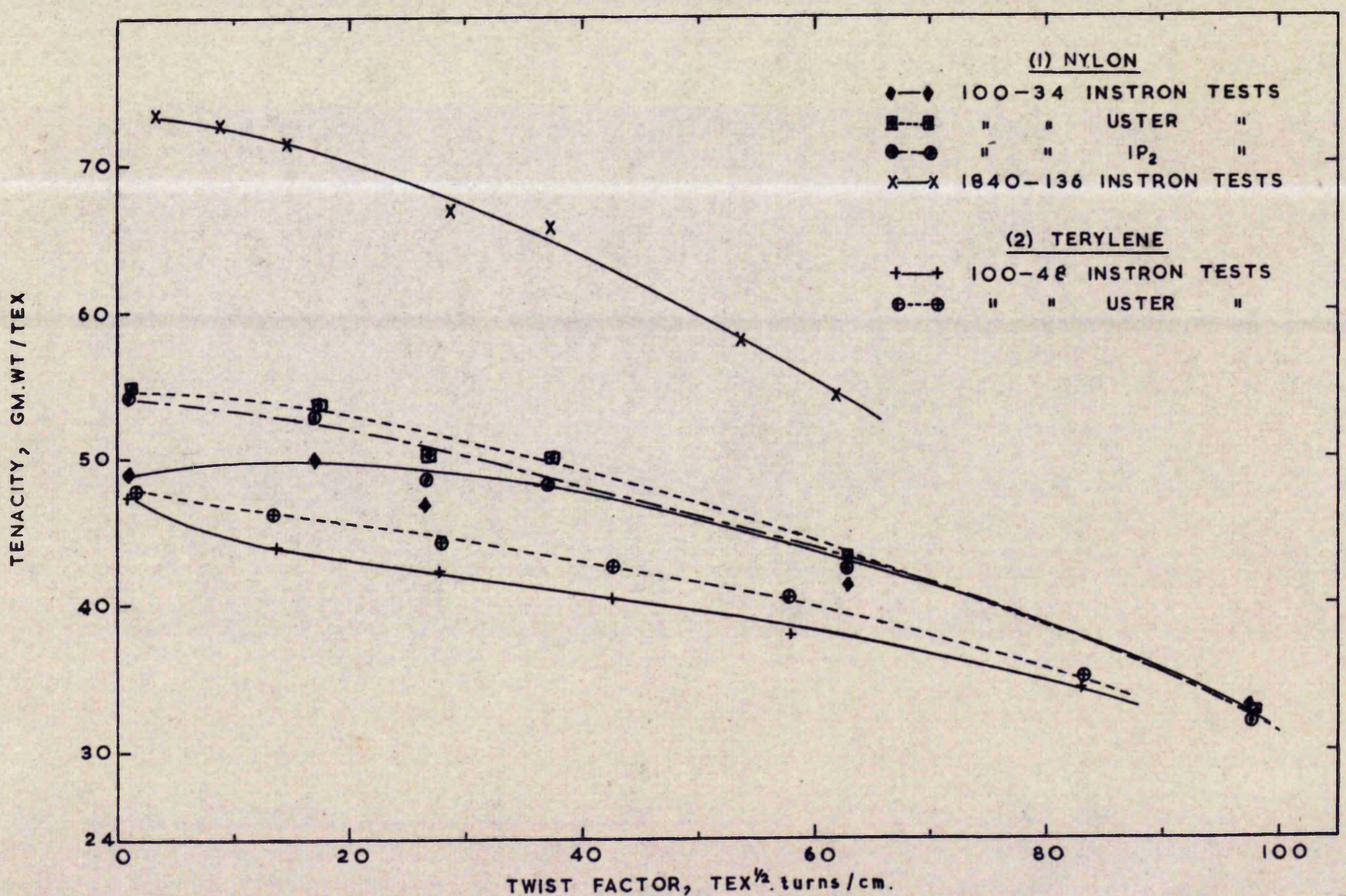


FIG. 3.41A$_4$ TENACITY OF NYLON AND TERYLENE YARNS — INSTRON, USTER AND IP_2 TESTS

3.42 The effect of twisting tension on tenacity

The tenacity results are plotted against the twist factor (initial) as shown in figures ($3.42A_1$-A_4).

These figures show that:

(i) In general, for a given twist factor and material, the tenacity values are low for the yarns twisted under very low tensions. However, above a certain level of tension, there is little further increase in tenacity.

(ii) The initial rise in the tenacity values, which was not present in the yarns twisted under very low tensions, can be observed in those twisted under higher tensions.

(iii) The tenacity of acetate 300-78 yarns remains unchanged over the range of tensions used.

3.43 Effect of method of twisting on tenacity

Results are shown in figures ($3.43A_1$ and A_2). These figures show that:

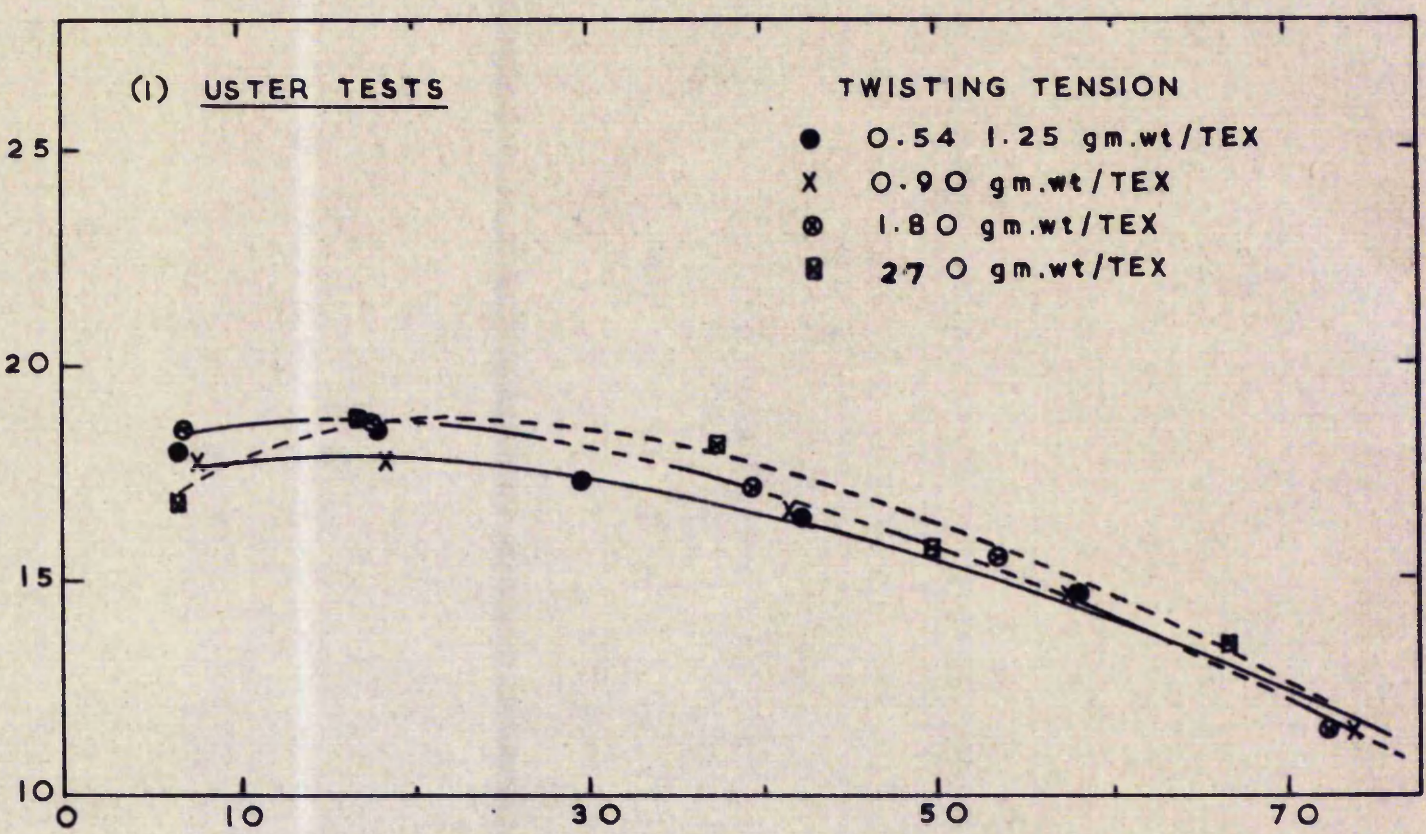
(i) In general, tenacity values are very little affected by the methods of twisting used.

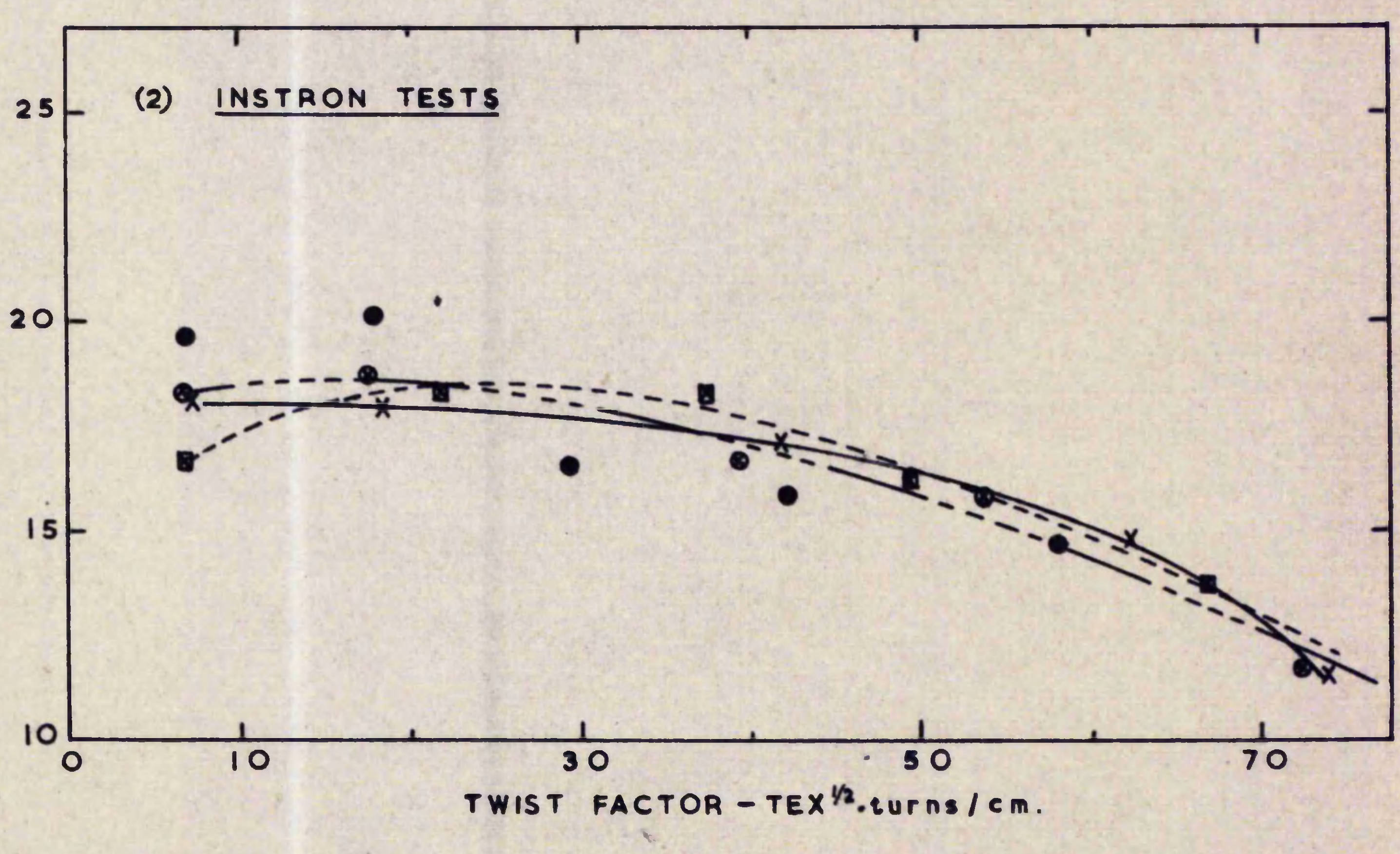
(ii) There is a tendency to obtain somewhat higher values in the doubler twisted yarns as compared to those obtained in uptwister twisted yarns. This is especially so at low twist factors which may be due to the effect of twisting tensions.

3.5 EFFECT OF TWIST ON WORK OF RUPTURE

3.51 General studies

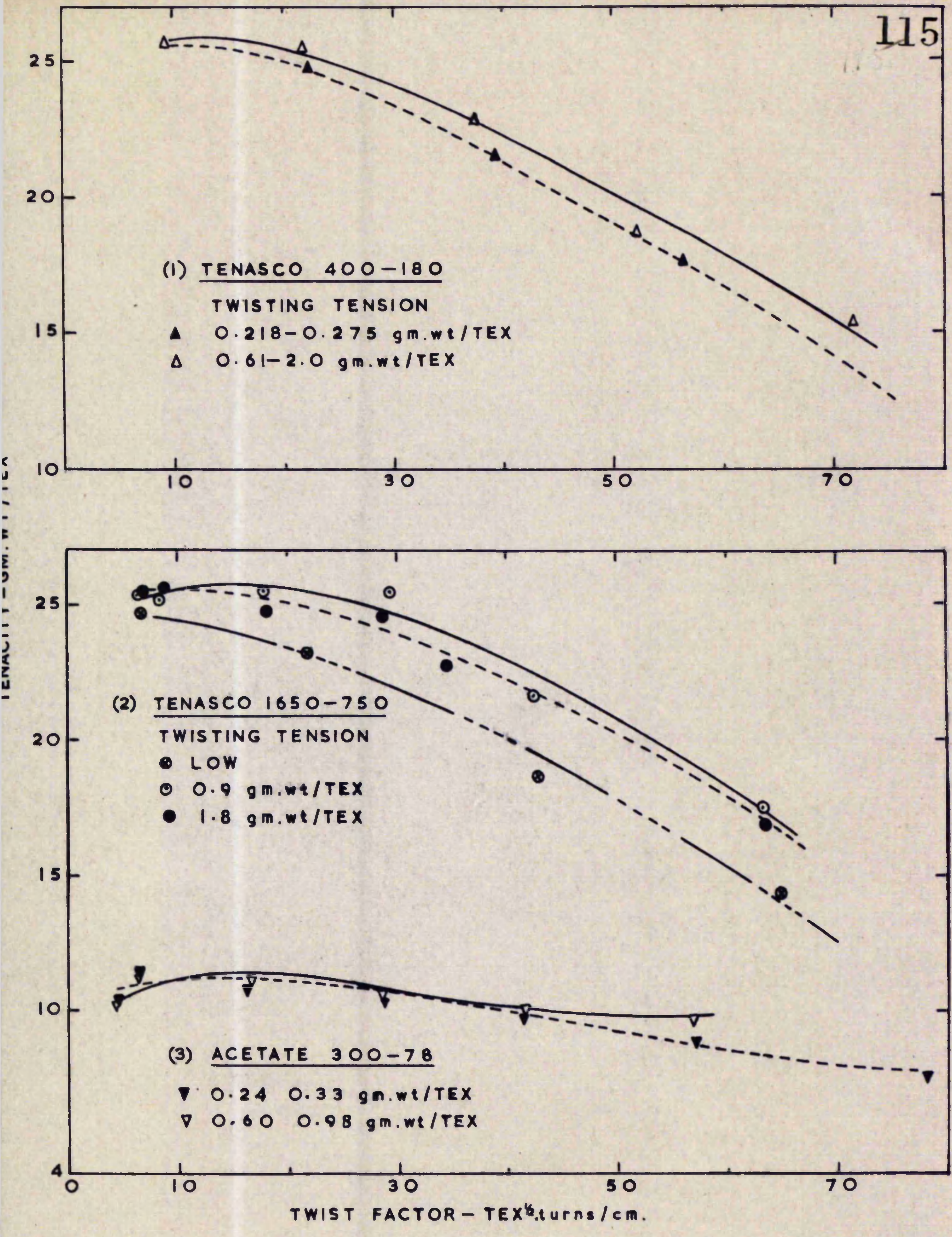
The work of rupture is obtained from the area under the load extension curves (measured with a planimeter) and using the relation:





IG. 3.42A₁ EFFECT OF TWISTING TENSION ON TENACITY OF VISCOSE 300-100

— USTER AND INSTRON TESTS —



42 A_2 EFFECT OF TWISTING TENSION ON TENACITY OF TENASCO AND ACETATE

— INSTRON TESTS —

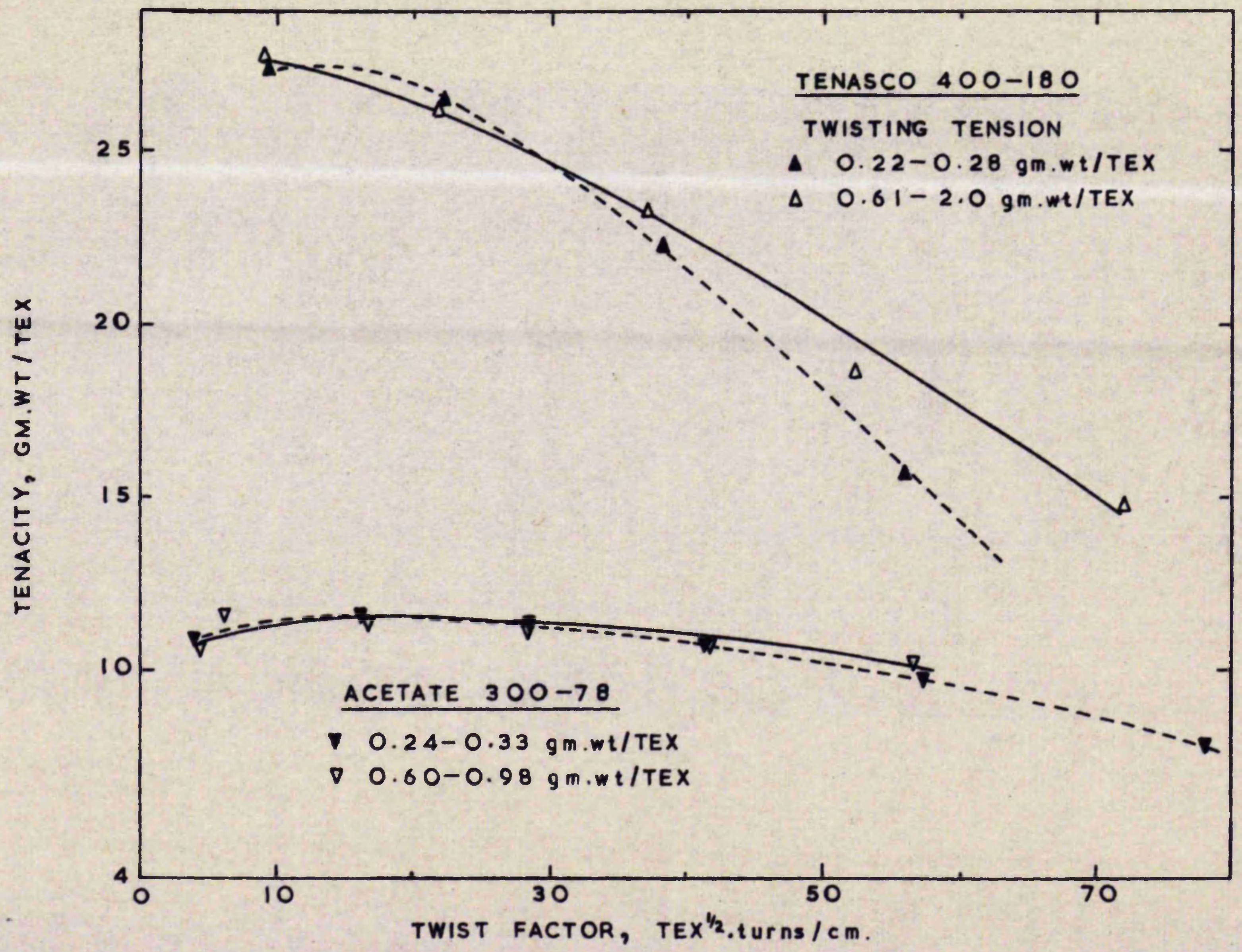


FIG. 3.42A$_3$ THE EFFECT OF TWISTING TENSION ON TENACITY OF TENASCO AND ACETATE YARNS

-USTER TESTS-

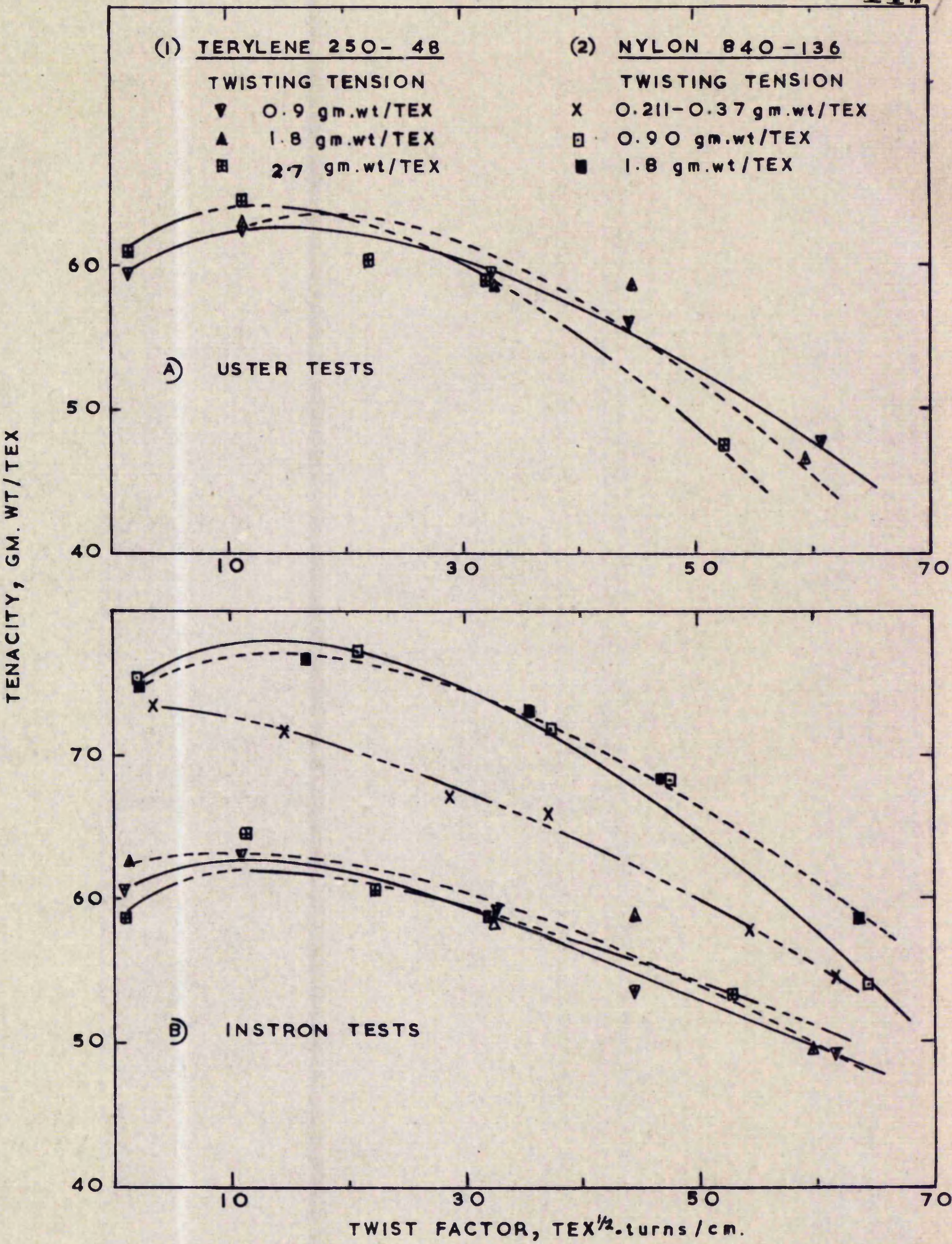


FIG. 3.42A$_4$ EFFECT OF TWISTING TENSION ON TENACITY OF TERYLENE AND NYLON YARNS —USTER AND INSTRON TESTS—

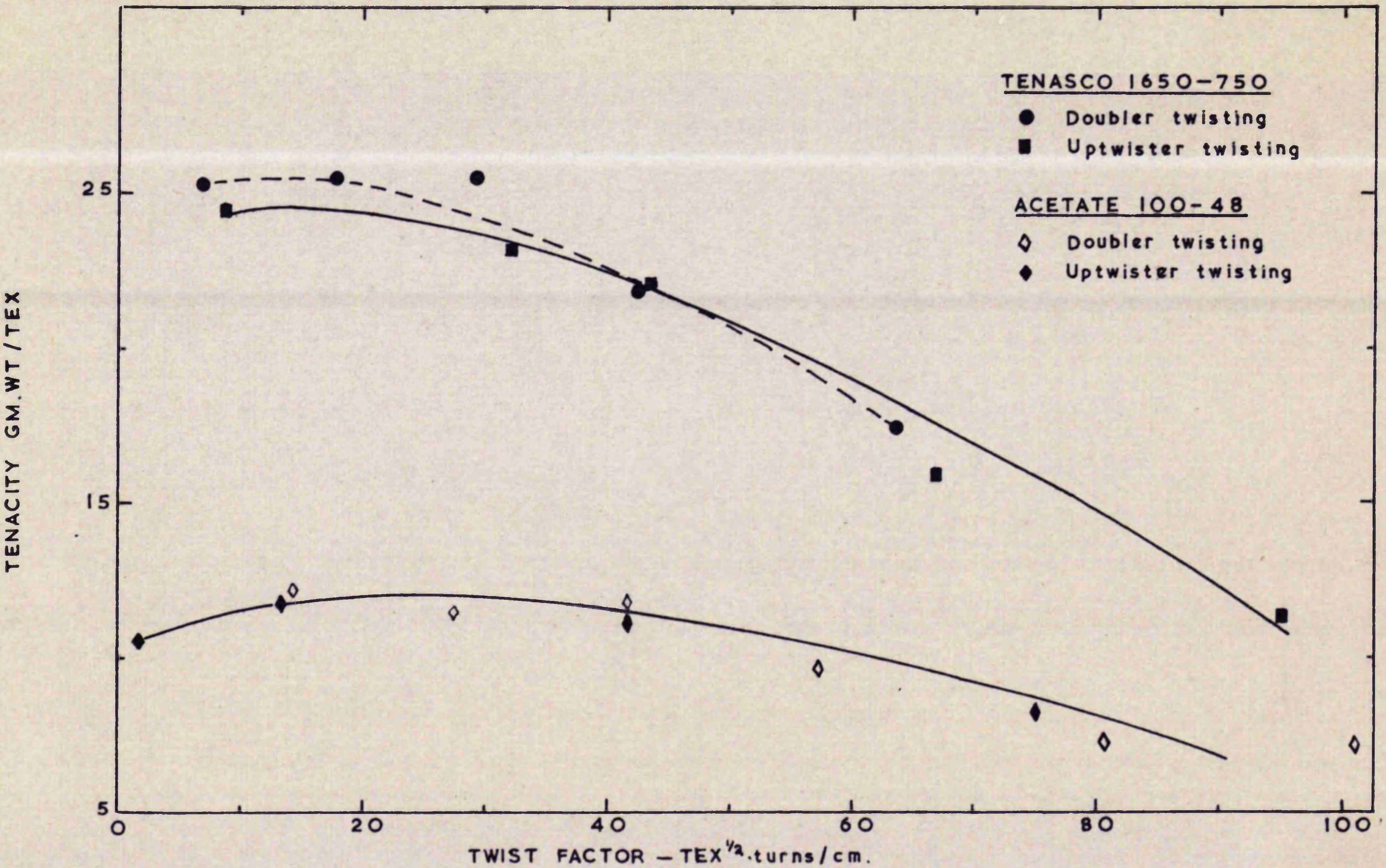


FIG. 3.43A$_1$ EFFECT OF METHOD OF TWISTING ON TENACITY OF TENASCO AND ACETATE YARNS.

—INSTRON TESTS—

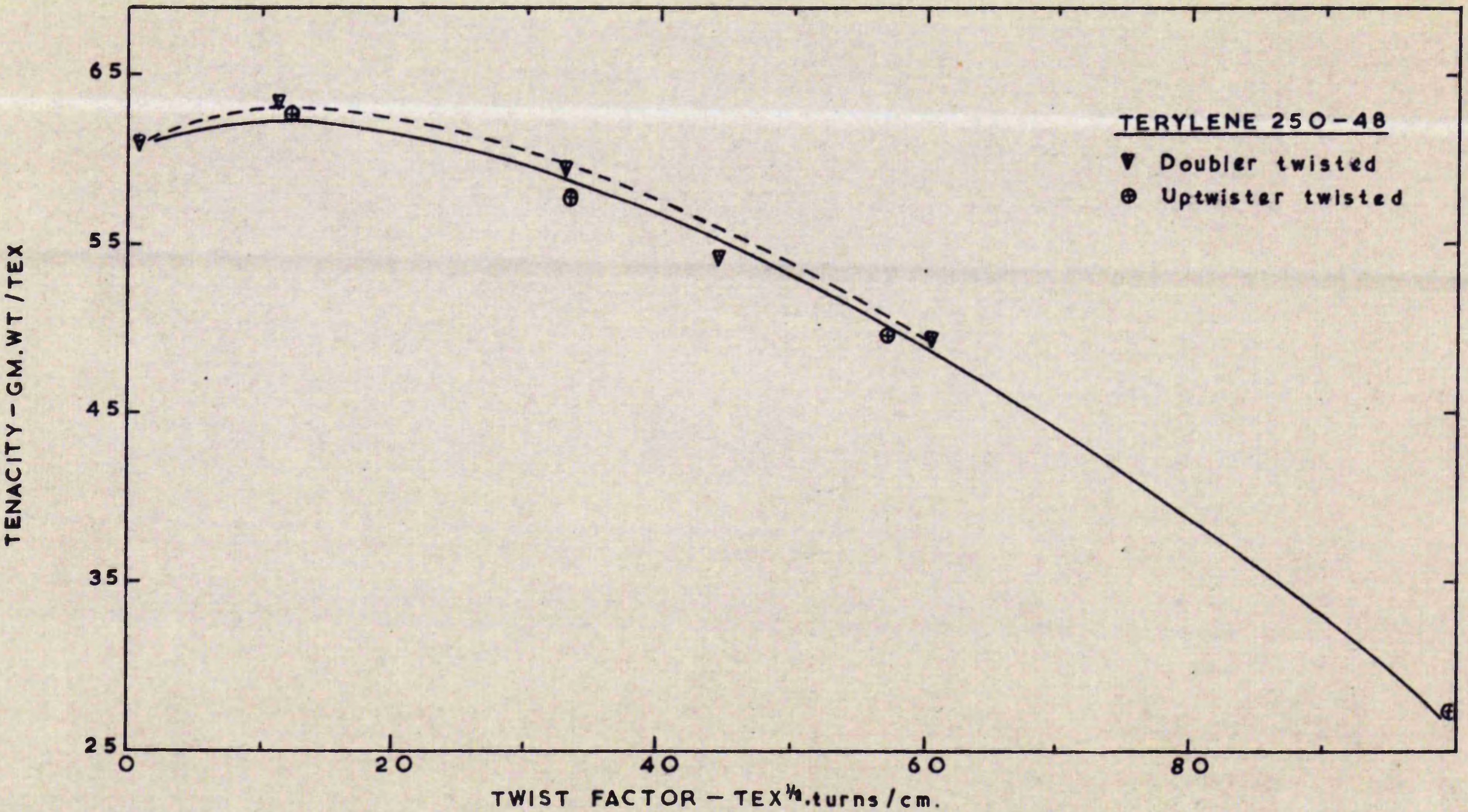


FIG.(3.43 A₂) EFFECT OF METHOD OF TWISTING ON TENACITY OF TERYLENE YARNS.

—INSTRON TESTS—

work of rupture in g.wt./tex.

$$= \frac{\text{area under the load extension diagram in g.wt.-cm.}}{\text{specimen length in cm.} \quad \times \quad \text{yarn count in tex.}}$$

The average of ten curves obtained in the Instron tests was used in each case.

The work factor was calculated as

$$\text{Work factor} = \frac{\text{work of rupture in g.wt./tex for 1 cm.}}{\text{breaking load in g.wt./tex x breaking extension in cm.}}$$

The work of rupture and the work factor values are shown in the figures (3.51A_1 and A_2) and 3.51B_1 and B_2). These figures show that:

(i) For all yarns, the work of rupture initially increases to a maximum and then decreases at higher twist factors, except for nylon 840-136, where it is almost constant.

(ii) The work factor is almost constant for viscose rayon and Tenasco yarns. In nylon and acetate yarns, it initially decreases to a minimum and then remains almost constant; whilst in Terylene, it is nearly constant at low twist factors, but decreases very rapidly at higher twist factors.

3.52 Effect of twisting tension on the work of rupture

Values of the work of rupture and work factor for yarns twisted under varying tension are shown in figures (3.52A_1 ~~and~~ A_3) and figures (3.52B_1 ~~and~~ B_3). It can be seen that

(i) In general, the higher the twisting tension, the lower is the work of rupture and the higher is the work factor.

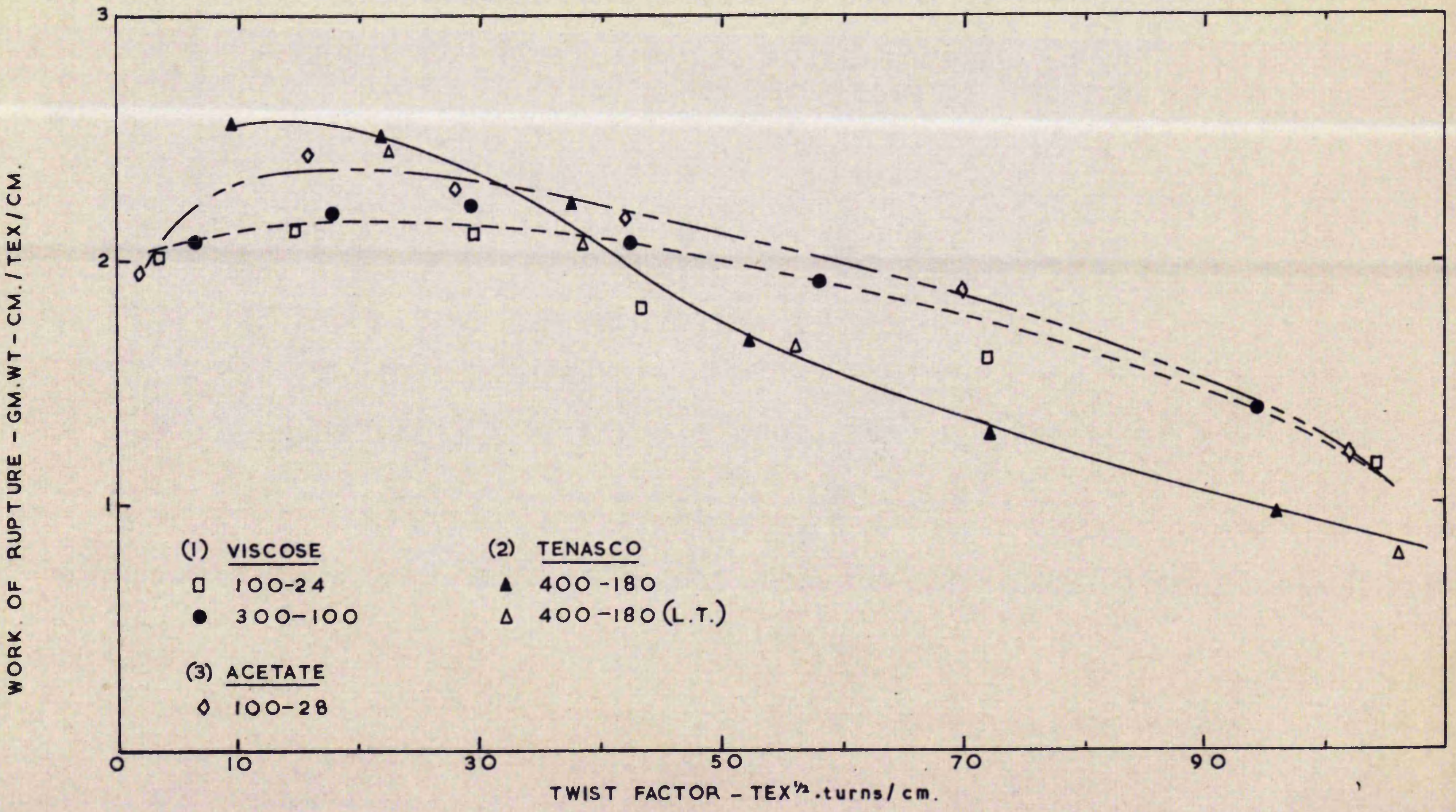


FIG. 3.51A, THE WORK OF RUPTURE FOR VISCOSE, TENASCO & ACETATE YARNS.

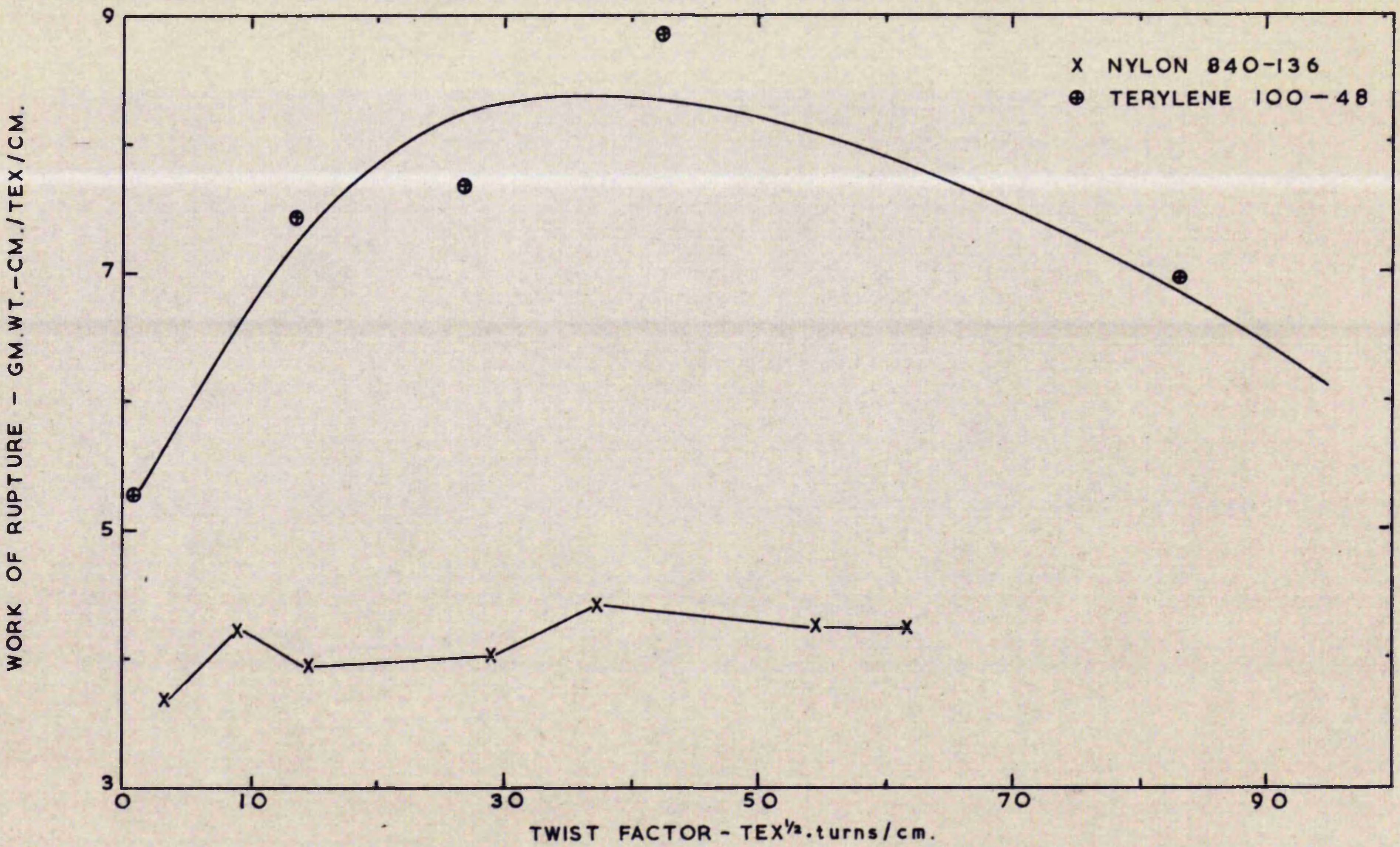


FIG. 3.51A$_2$ THE WORK OF RUPTURE FOR NYLON & TERYLENE YARNS.

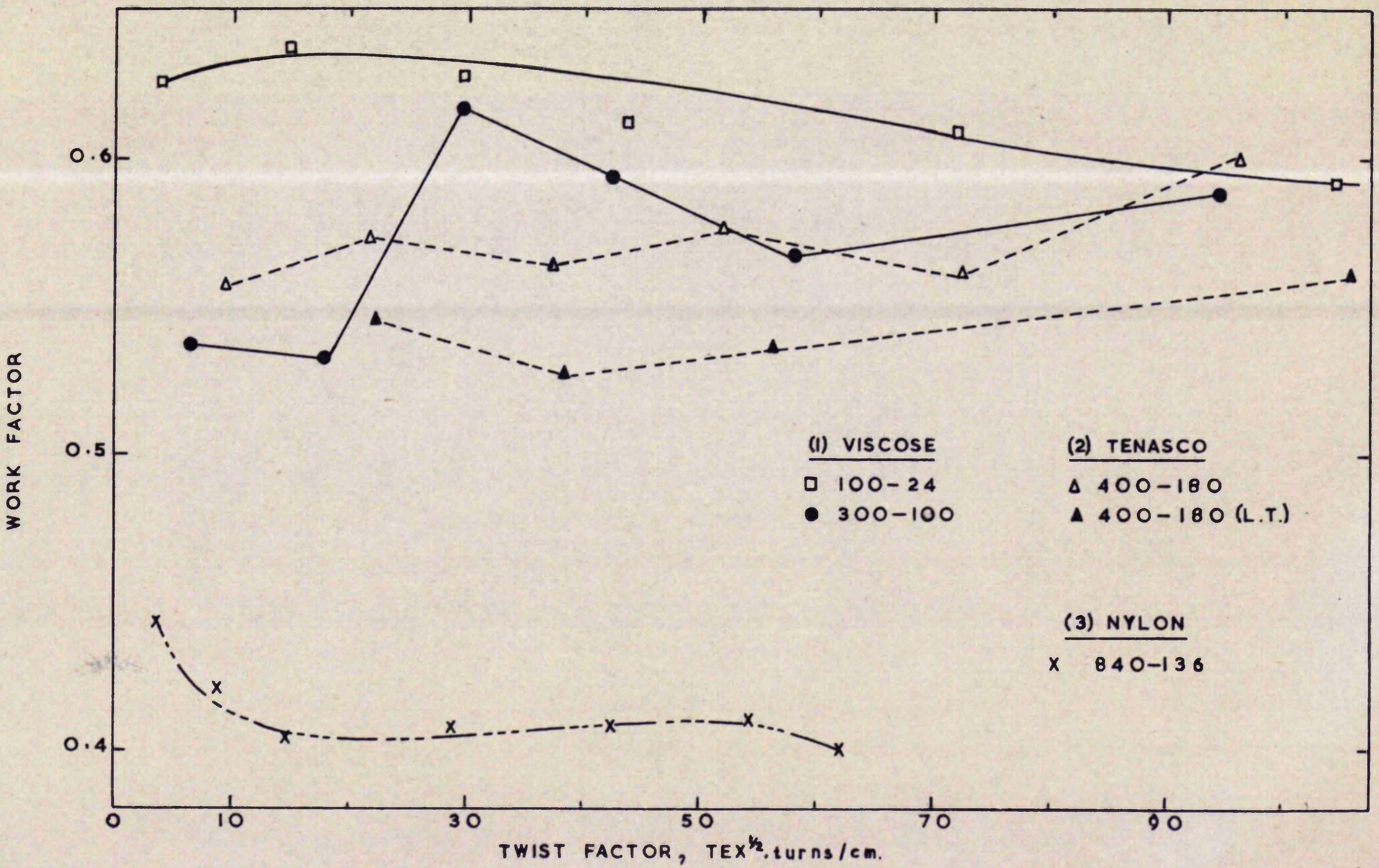


FIG. 3.51B, WORK FACTOR FOR VISCOSE, TENASCO AND NYLON YARNS — INSTRON TESTS

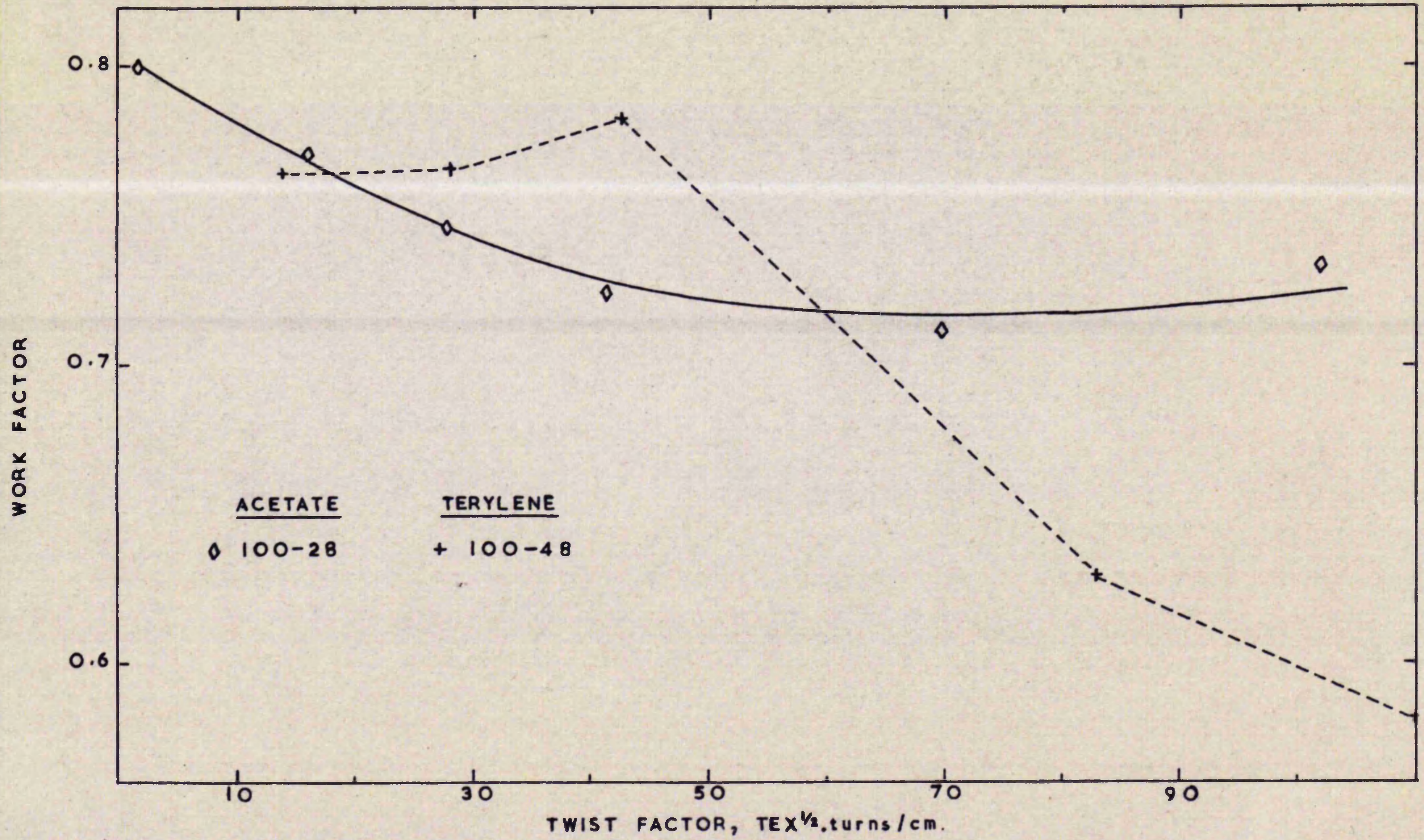


FIG. 3.5? B_2 WORK FACTOR FOR ACETATE AND TERYLENE YARNS INSTRON TESTS

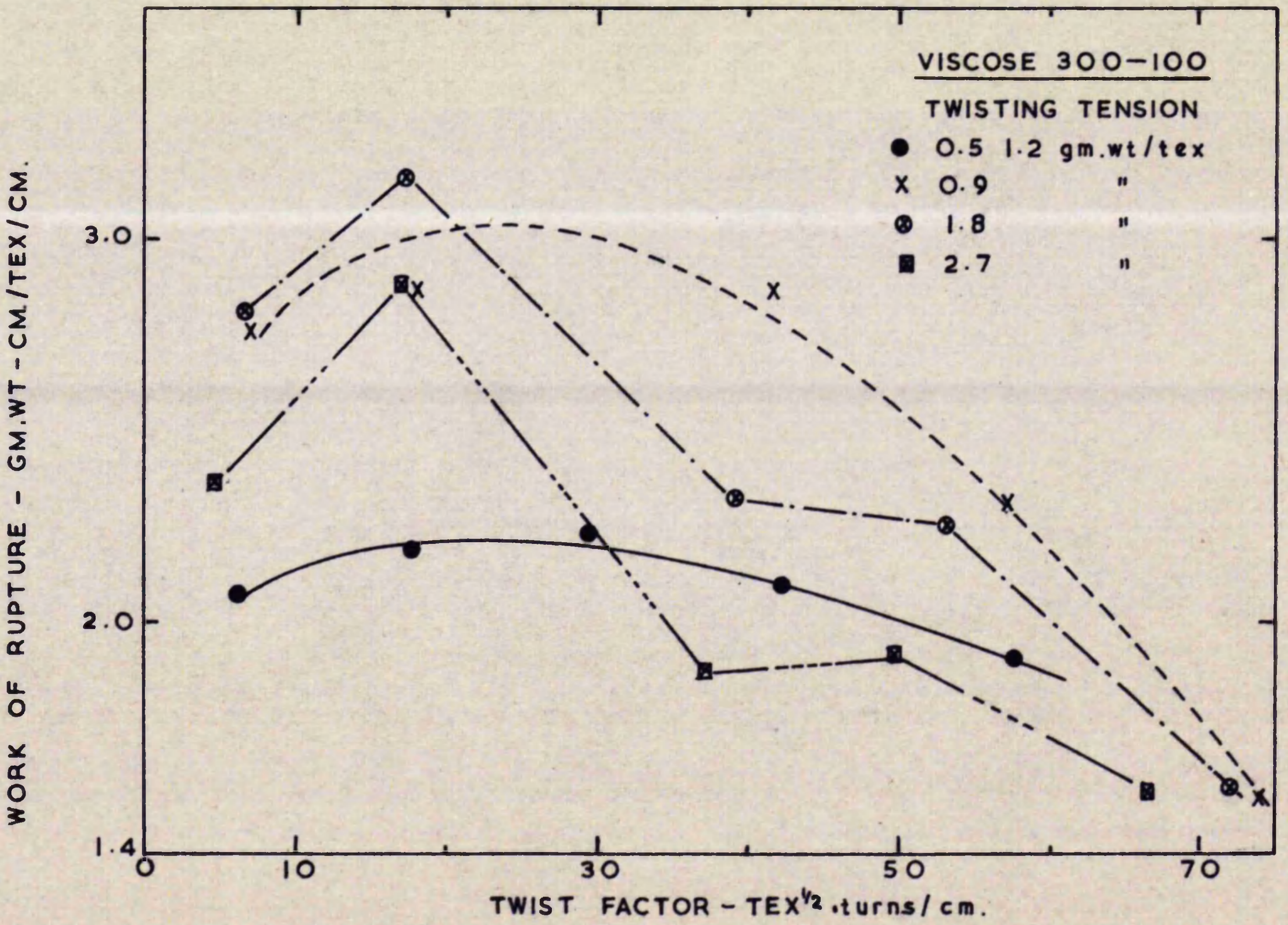


FIG. 3.52 A₁ EFFECT OF TWISTING TENSION ON WORK OF RUPTURE FOR VISCOSE YARNS. —INSTRON TESTS—

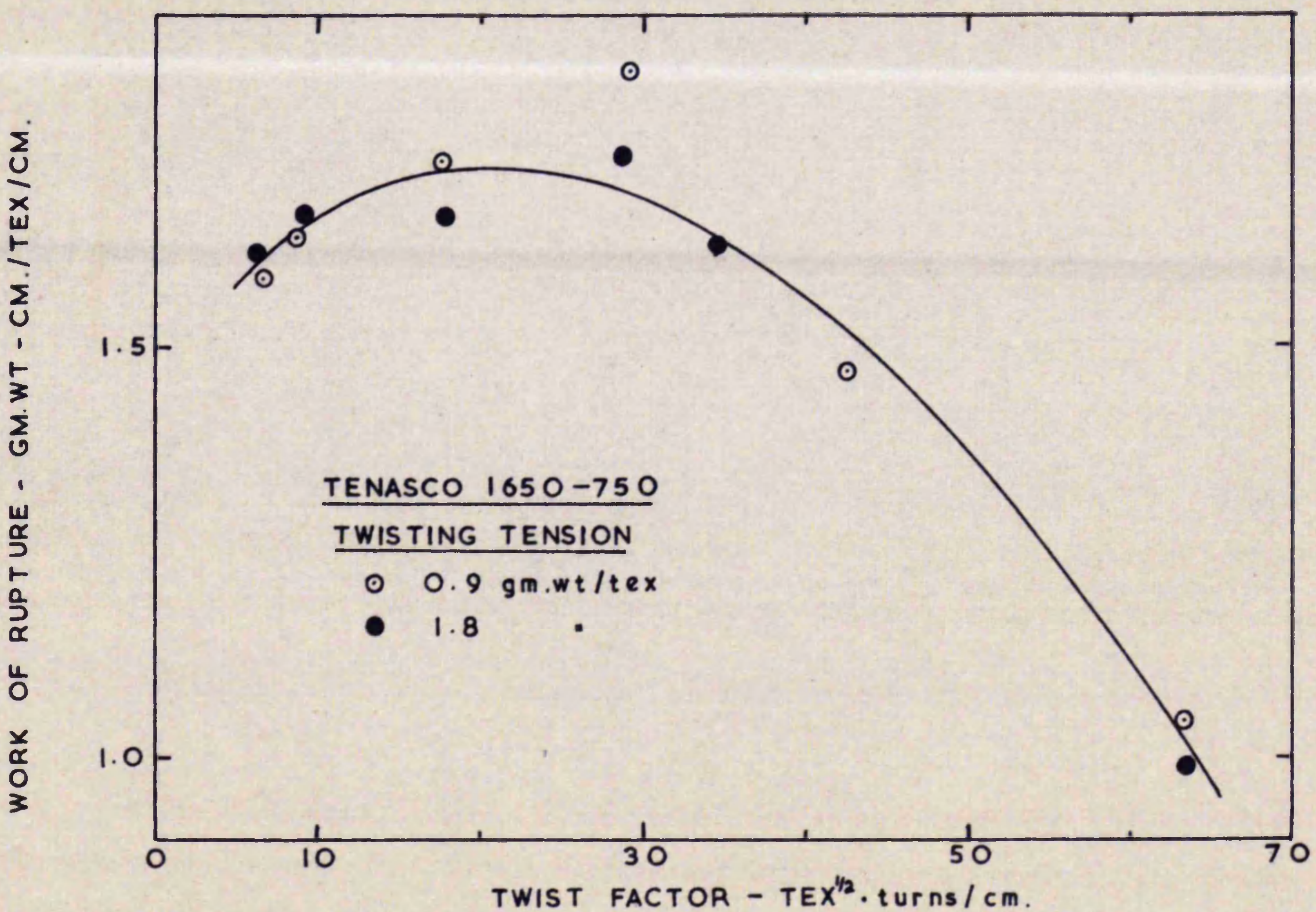


FIG. 3.52 A_2 EFFECT OF TWISTING TENSION ON WORK OF RUPTURE FOR TENASCO YARNS. —INSTRON TESTS—

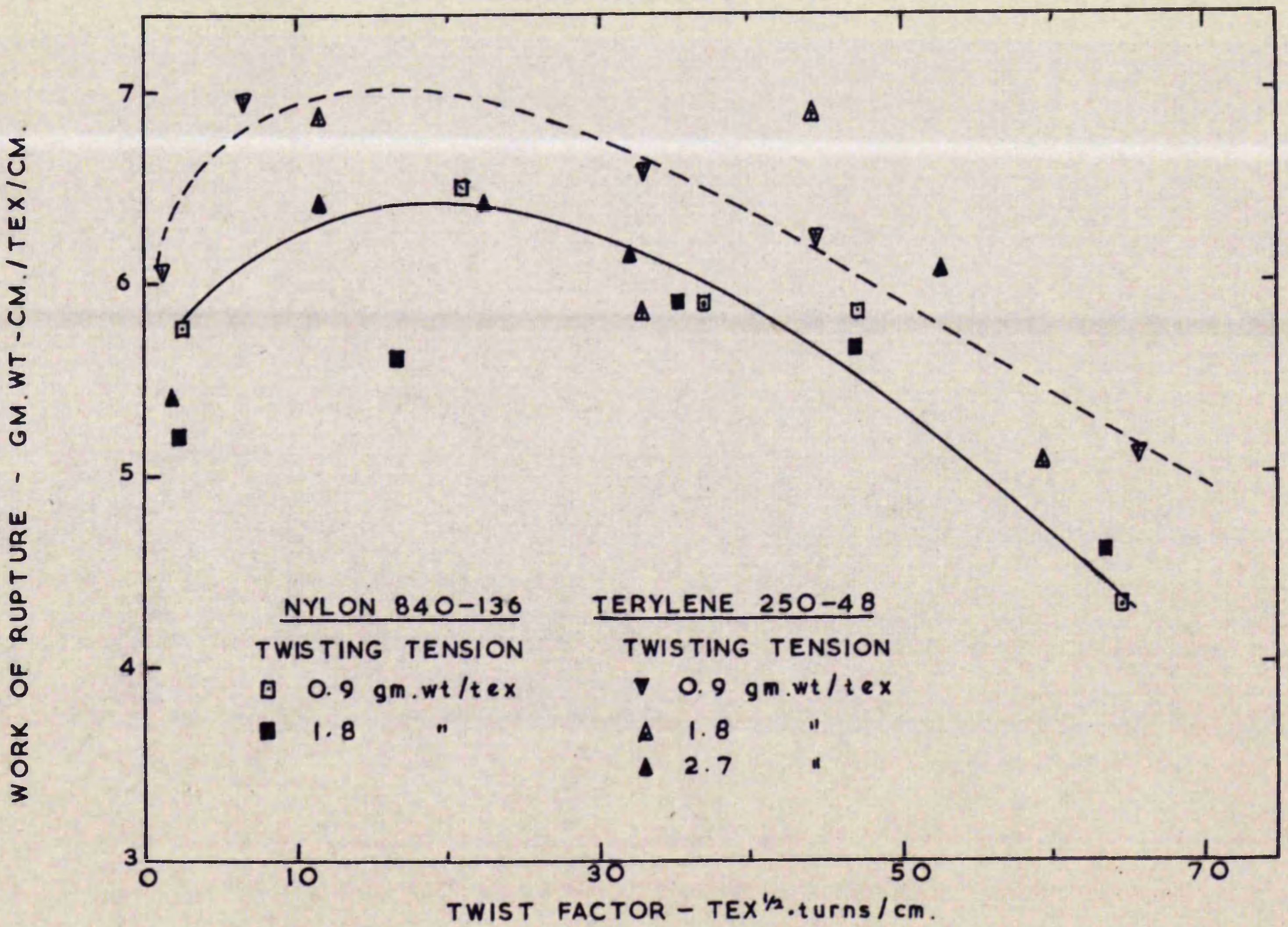


FIG. 3.52A₃ EFFECT OF TWISTING TENSION ON WORK OF RUPTURE OF NYLON & TERYLENE YARNS. — INSTRON TESTS —

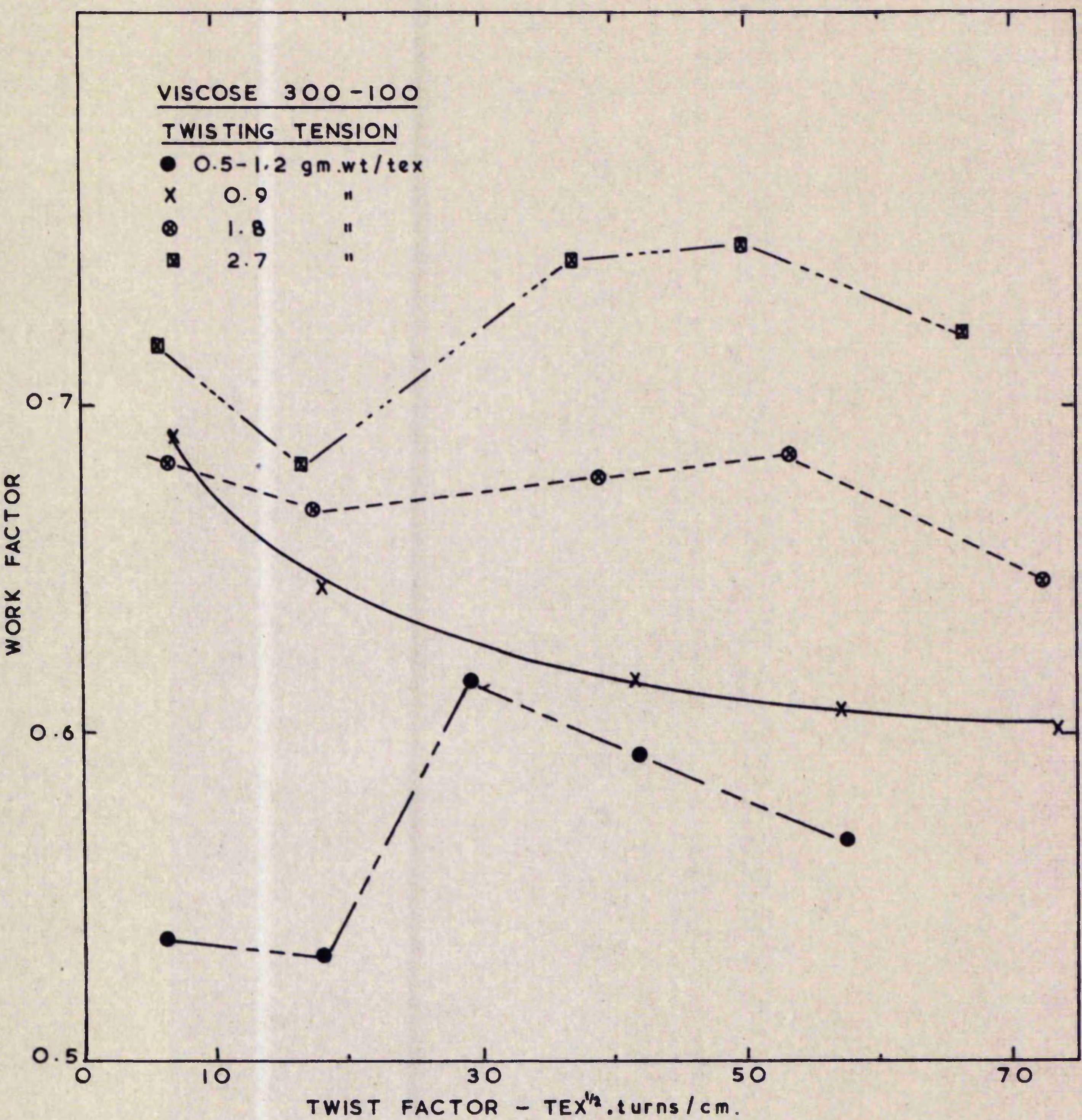


FIG. 3.52B, EFFECT OF TWISTING TENSION ON WORK FACTOR OF VISCOSE YARNS. — INSTRON TESTS —

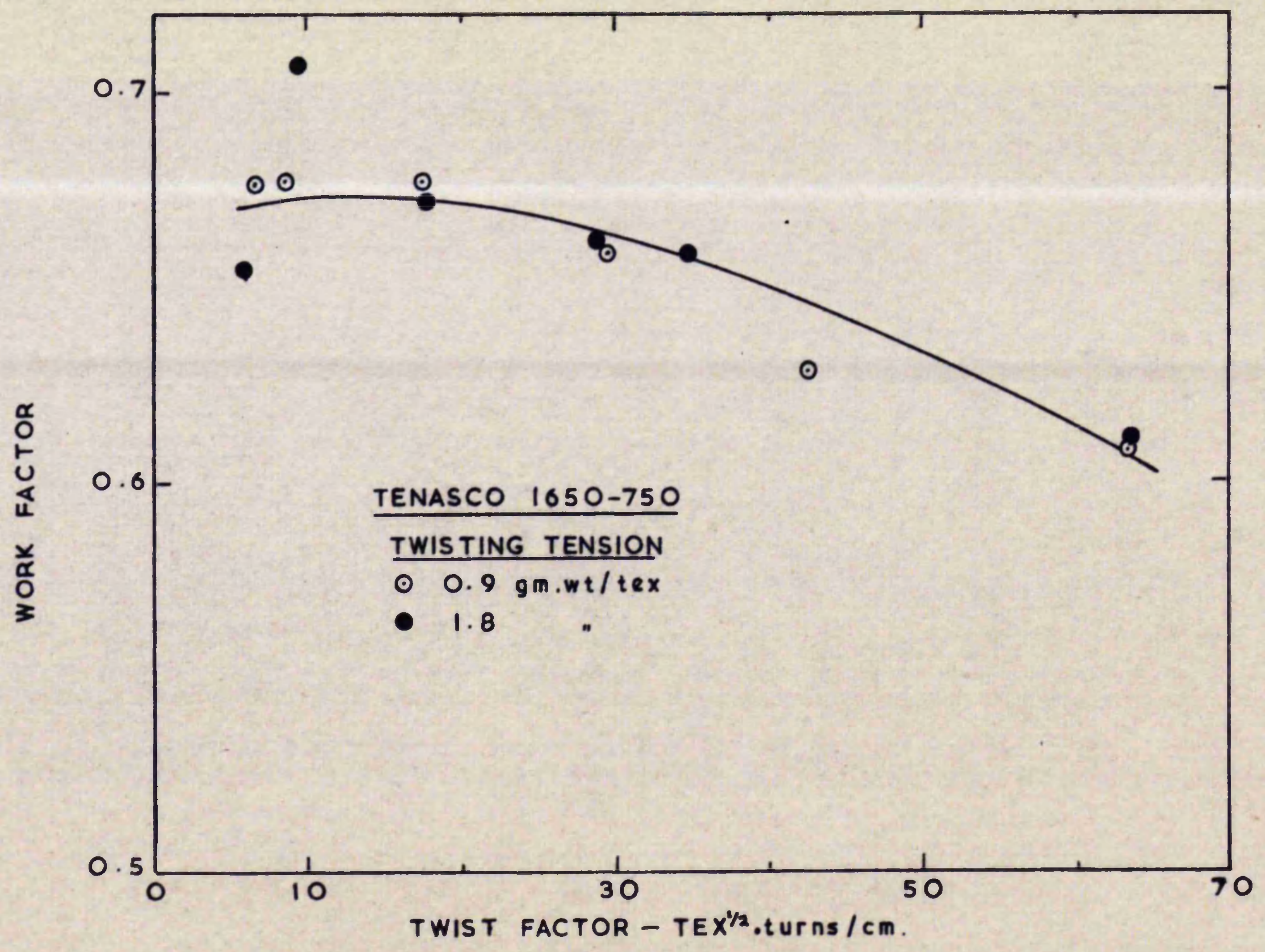


FIG 3.52B$_2$ EFFECT OF TWISTING TENSION ON WORK FACTOR OF TENASCO YARN. — INSTRON TESTS —

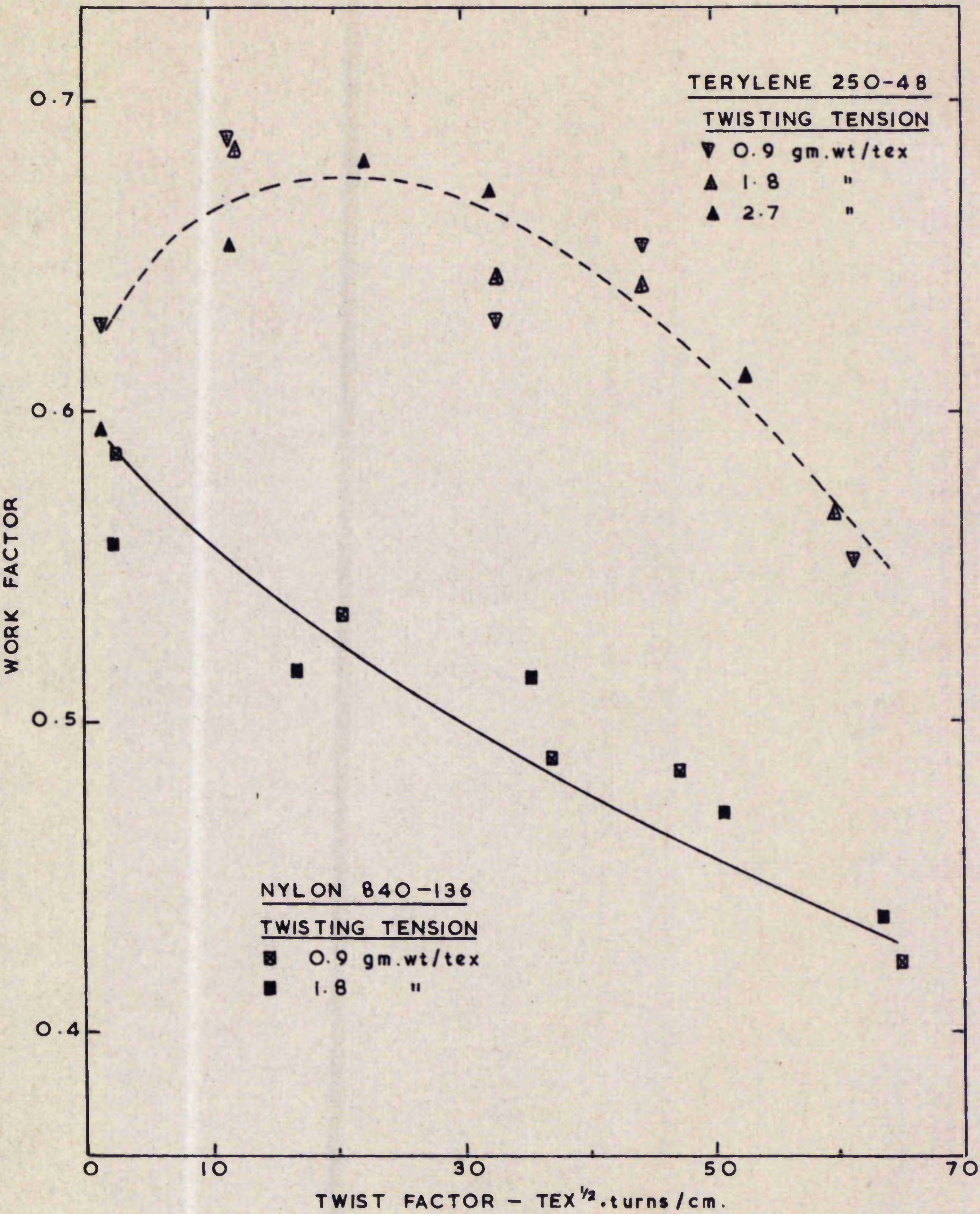


FIG. 3.52 B_3 EFFECT OF TWISTING TENSION WORK FACTOR OF NYLON & TERYLENE YARNS. —INSTRON TESTS—

(ii) The effect of twisting tension in lowering the work of rupture is low at higher twist factors for all yarns tested.

(iii) The work factor of viscose yarns increases as the twisting tension is increased, but it is affected very little by the twist.

(iv) The work factor of nylon and Terylene yarns is affected very little by the twisting tensions used, but it decreases rapidly at all twist factors for high tenacity nylon 840-136, and only at very high twist factors for high tenacity Terylene 250-48 (figures 3.52B_1 and B_3).

3.6 TWIST CONTRACTION FACTOR

The twist contraction factor has been calculated by finding the increase of linear density (tex) in all twisted yarns. The twist contraction factor has been theoretically shown to be independent of the filament properties and to be a function of the surface helix angle (Section 1.35).

For comparison with the theoretical relation, it is very useful to select the proper axis to obtain a linear relation. Theoretical equation ($C_y = \frac{1 + \sec\alpha_o}{2}$) can be shown to give a linear relation when values of Cy (Cy - 1) are plotted against $\tan^2\alpha_o$ or (twist factor-initial)^2. This has been done in figures (3.6A_1 ~~and~~ A_5).

It can be seen that

(i) In general, the contraction factor depends upon the twisting tension used during processing.

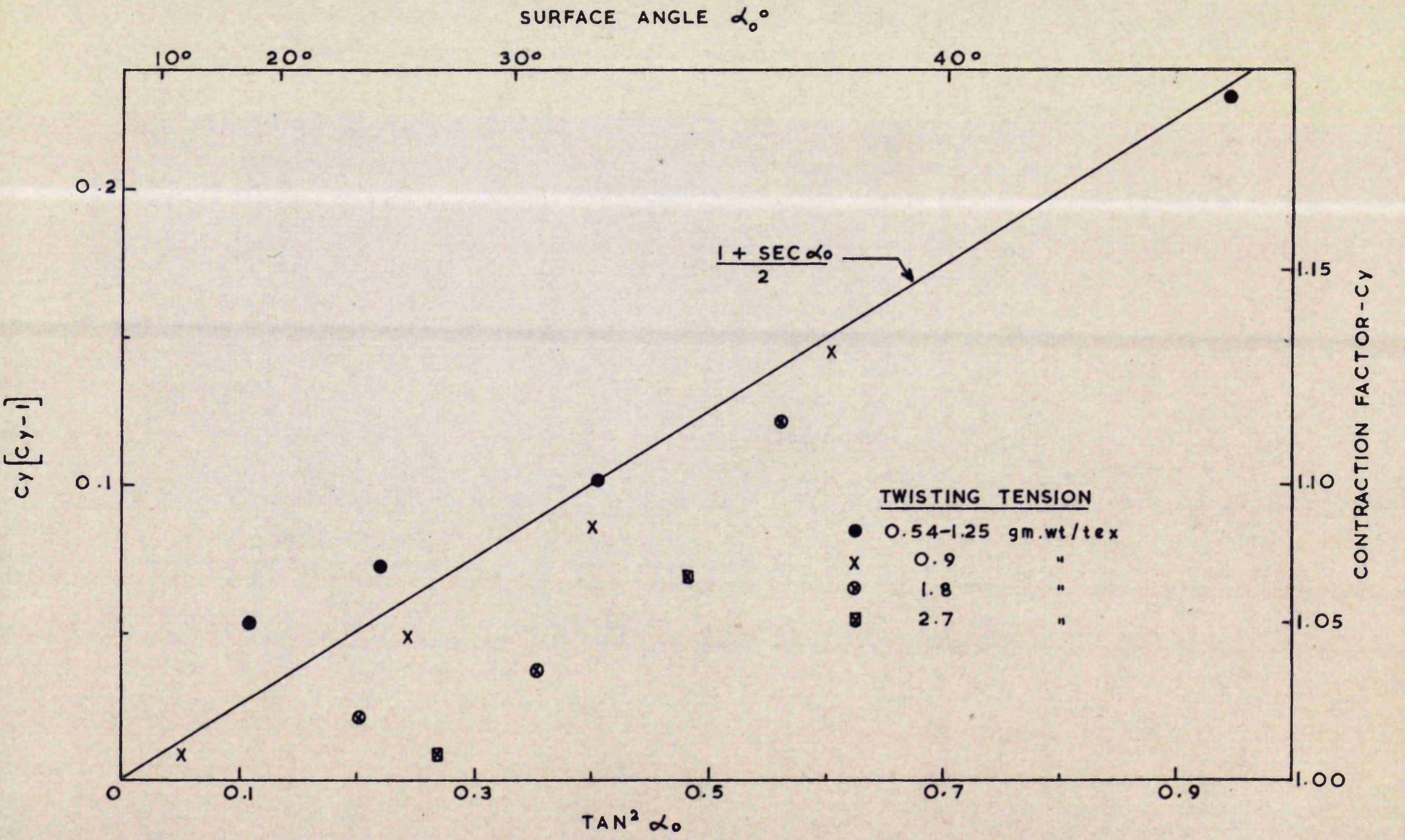


FIG. 3.6 A_1 EFFECT OF TWISTING TENSION ON CONTRACTION FACTOR OF VISCOSE 300-100 YARN.

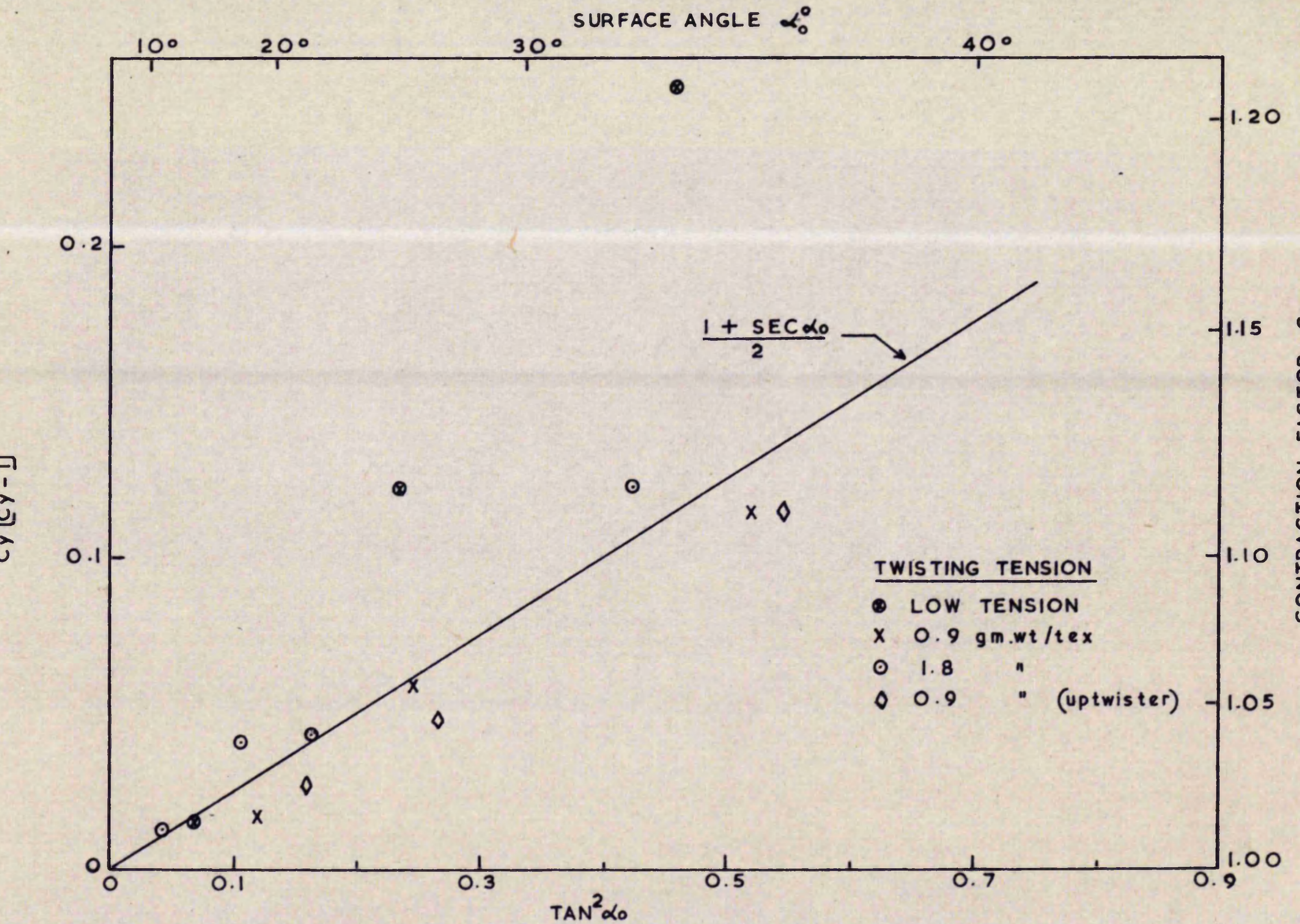


FIG. 3.6 A_2 EFFECT OF TWISTING TENSIONS ON CONTRACTION FACTOR OF TENASCO 1650-750 YARNS.

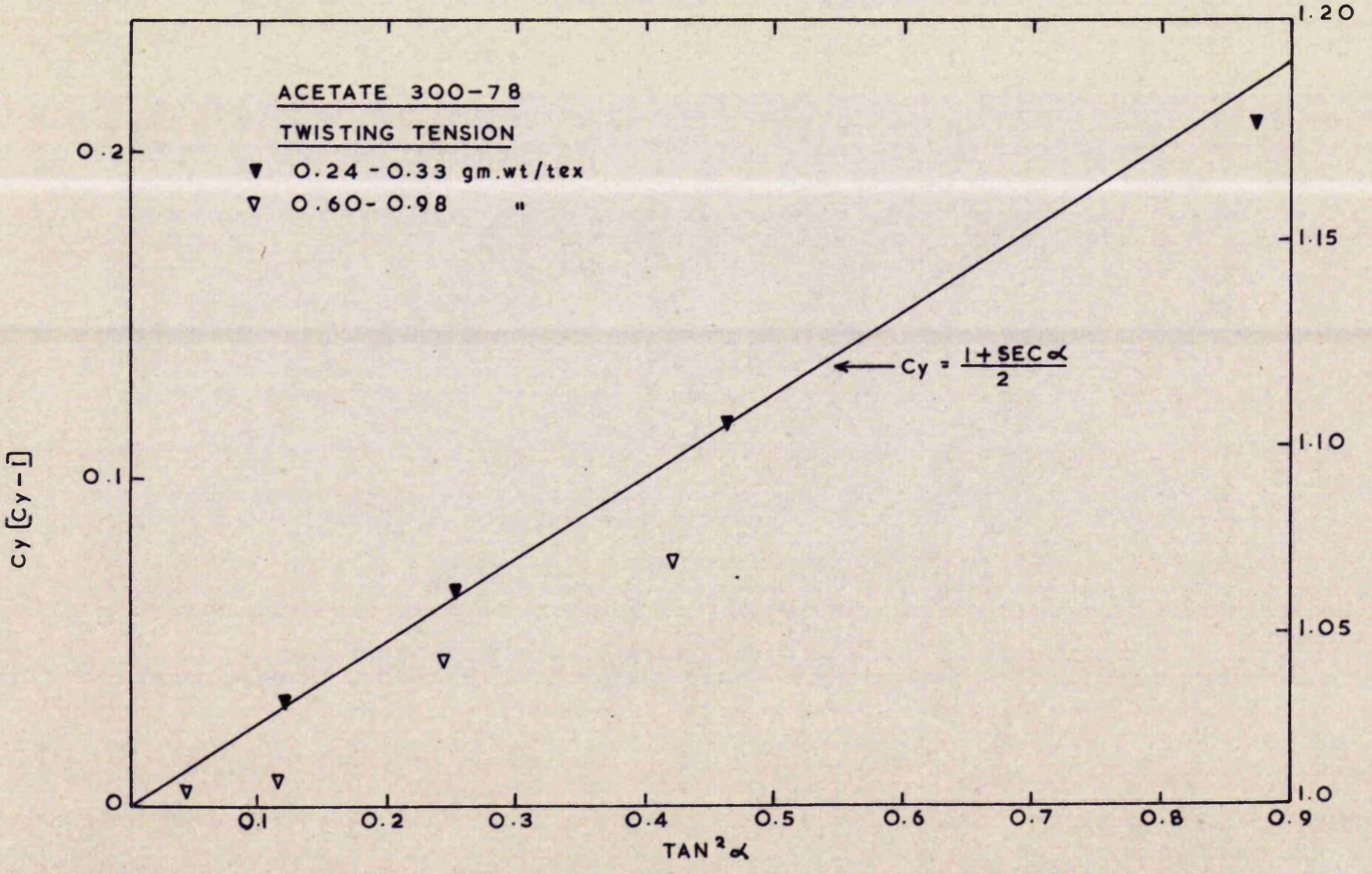


FIG. 3.6A₈ EFFECT OF TWISTING TENSION ON CONTRACTION FACTOR OF ACETATE YARNS.

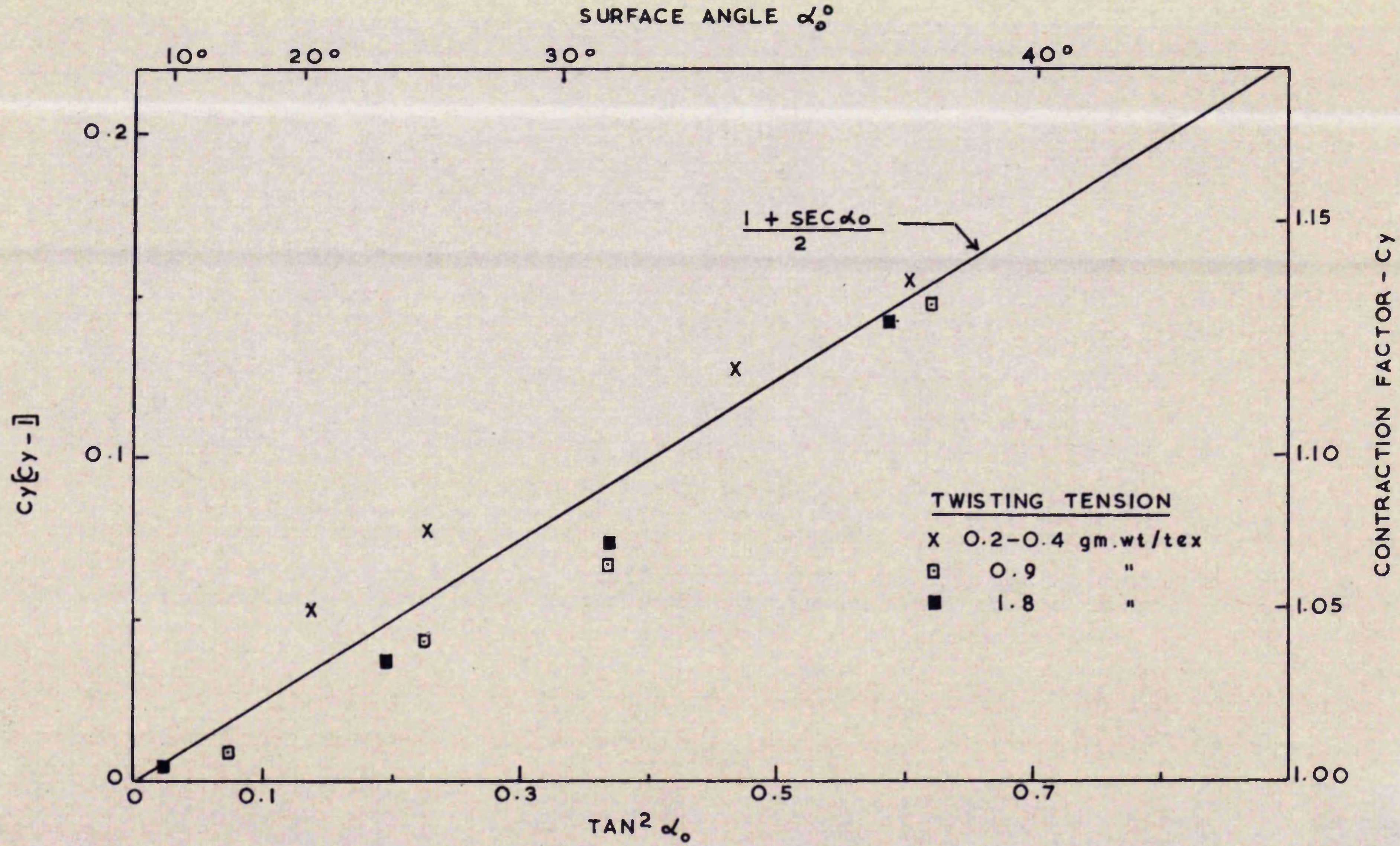


FIG. 3.6 A_4 EFFECT OF TWISTING TENSION ON THE CONTRACTION FACTOR OF NYLON 840-136

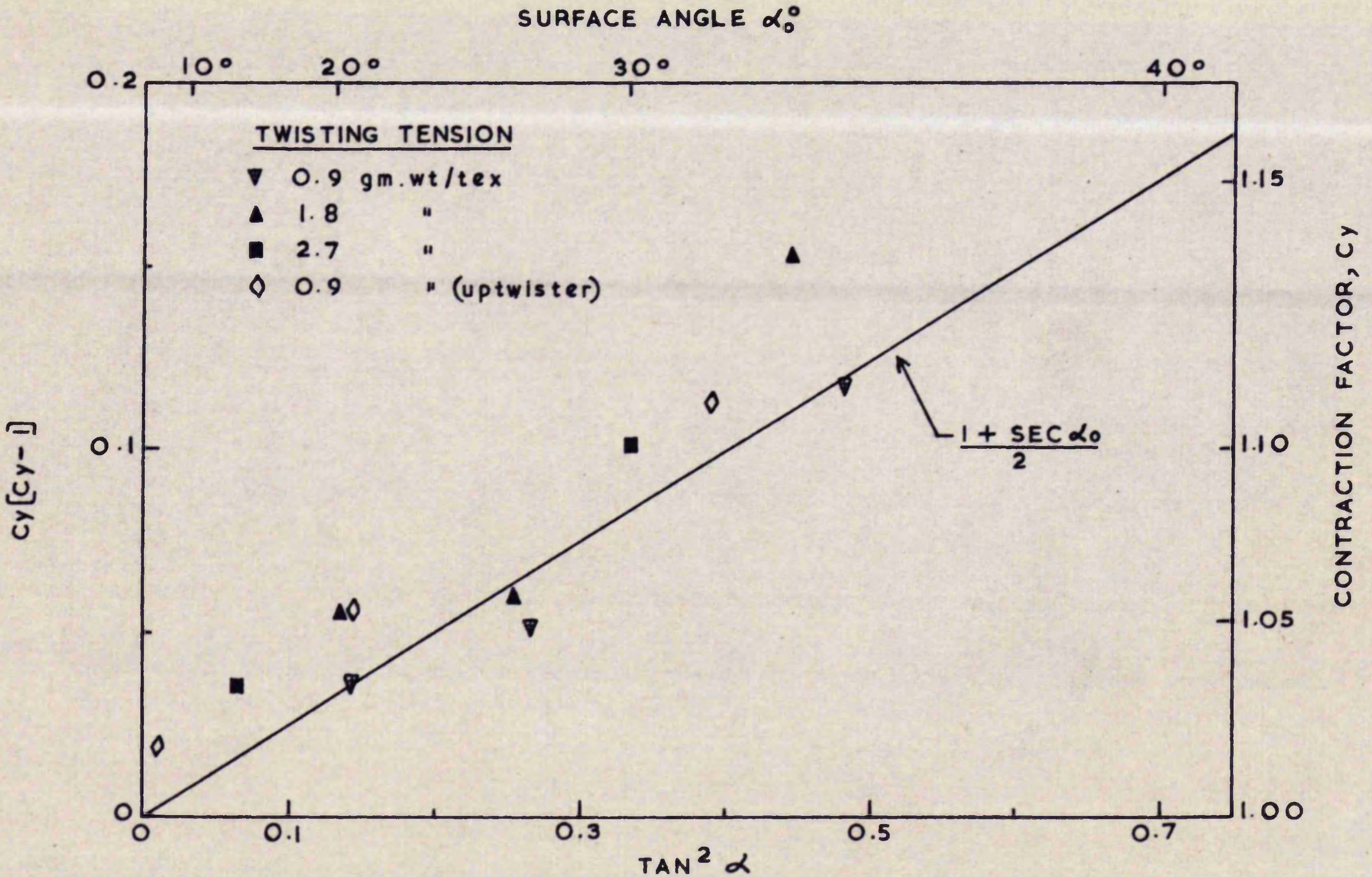


FIG. 3.6 A_5 EFFECT OF TWISTING TENSIONS ON THE CONTRACTION FACTOR OF TERYLENE 250-40

(ii) For a given material, the contraction factor is very high if the twisting tension is lower than a certain level of tension and it is very low when the twisting tension is very much higher than this level. The experimental contraction factor values will lie above or below the theoretical curve, depending upon the twisting tensions.

(iii) When contraction factor values for all materials are plotted against the surface helix angle, no common curve is obtained (figure $3.6B_1$).

(iv) In general, the contraction factor in viscose rayon and acetate yarns is lower than that given by the theoretically predicted relation viz. $\frac{1 + \sec \alpha_o}{2}$. This is especially so at higher twist factors (fig. $3.6A_4$ and A_5).

(v) The effect of heat setting is to increase the contraction factor of all the yarns with higher twists. This increase is very high in nylon yarns as compared with that in viscose rayon and acetate yarns.

3.7 THE VARIABILITY OF THE RESULTS

The coefficient of variation of the yarn properties varied considerably from one yarn to another from values as low as 0.8 to as high as 36%. Table (3.7A) shows the values of the coefficient of variation for the breaking extension and tenacity, below which the majority of the results lie together with few exceptionally high values which occurred. The coefficient of variation (Instron)

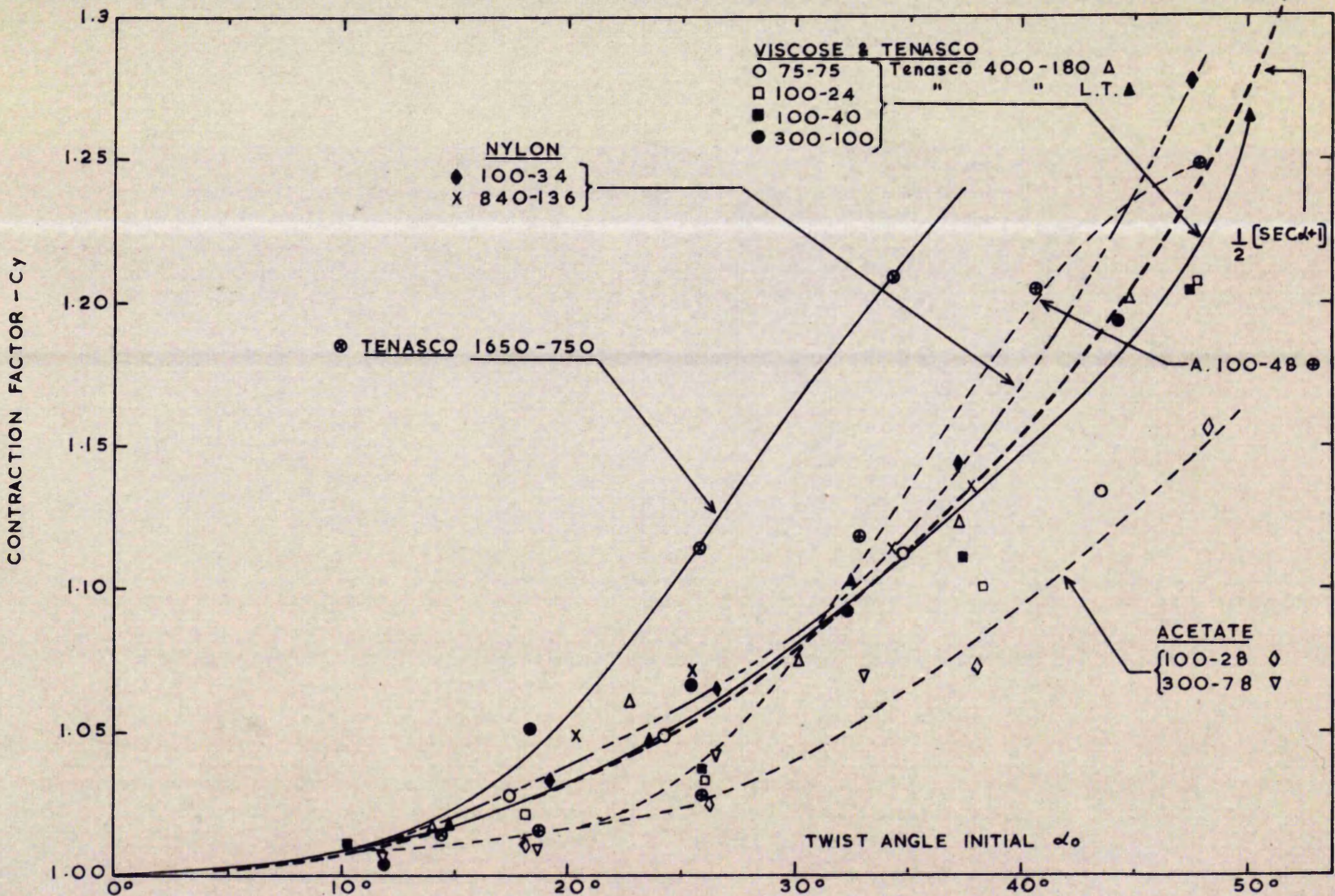


FIG. 3.6 B THE CONTRACTION FACTOR FOR TWISTED CONTINUOUS FILAMENT YARNS.

TABLE 3.7 A

Coefficient of variation % (Instron Tests)

		Breaking Extension	Breaking Load
Value below which most results lie		10	10
Exceptions Yarn	Twist Factor		
(1) Viscose 75-75	88.7	11.2	
Viscose 100-24	104.3	10.3	
Viscose 300-100	39.4	10.6	
(2) Acetate 100-28	69.9	10.8	
	101.9	18.7	
Acetate 100-48	80.2	22.8	
	101.2	20.9	
(3) Nylon 100-34	1.0	12.9	
	16.8	12.9	
(4) Terylene 100-48	0.53	15.1	
	57.90	11.9	
	83.00	11.5	
Terylene 250-48	1.1	11.1	
	11.4	12.0	
	99.7		10.9

has also been included in Appendix I.

It can be seen that

(1) The coefficient of variation increases as the twist is increased.

(2) Low twist nylon and Terylene yarns also showed a high variability.

(3) In general, the coefficients of variation for breaking load values are very low compared with those of the breaking extension.

(4) The coefficients of variation for breaking extension are very high in highly twisted acetate and viscose rayon yarns exhibiting a decrease in breaking extension.

CHAPTER IV

"SUBSIDIARY EXPERIMENTS"

4.1 INTRODUCTION

In chapter I, while discussing the scope for further studies, it has been pointed out that the actual

(1) mechanism of yarn breakage and the yarn deformation behaviour may be different from that generally assumed;

(2) yarn structure may be quite different from the generally accepted idealised one;

(3) properties of filaments constituting the yarn structure may be influenced by the yarn geometry and twisting operation.

Some experimental work was carried out to investigate some of these factors. The experimental techniques, the results and the discussion are reported in the following pages.

4.2 YARN BREAKAGE

Simple theory, on the basis of the idealised structure of twisted yarns, will predict that the breakage is initiated at the yarn centre and moves out to the outside where the filament extension is least. This will continue until the outer filaments reach their breaking extensions. In the constant rate of loading tests, the load previously borne by the broken filaments will be transferred to the others and rupture would occur as an instantaneous cumulative breakdown, but in a constant rate of elongation test, the decrease in yarn tension should be detectable. As discussed in chapter I, Hearle[18] has predicted the theoretical stress-strain relation after breakage is initiated (Figure 1.33D).

However, results reported in the previous chapter show sharp breaks in the twisted yarns even in constant rate of elongation tests. This behaviour may be explained by the picture of the breakage given in chapter 6. Immediately after break, the stress near the break will increase and the stress in distant regions will fall. This will result in a rapid contraction and thus the process of cumulative breakdown at high speed will continue until the breakage is complete.

The speed at which the breakage of filaments progresses along the yarn cross section, will be determined by the amount of twist, the strain recovery behaviour of the material and the specimen length free to contract. High tenacity Tenasco 1650-750 yarn with a breaking extension of 10% twisted to 12 turns per inch on a ring doubler, was used to study the actual mechanism of breakage.

4.21 Experimental

Two sets of experiments were conducted using the Instron Tester:

(A) The effect of rates of extensions and gauge lengths on the load-extension behaviour.

(B) Attempts to arrest the progress of breakage.

In the first set of experiments, the gauge lengths used were 1 cm. 2.5 cm, 5.0 cm, 7.5 cm and 10 cm. The load extension curves were obtained at three ratesof extension (4%, 40% and 400% per min.) at each of the 5 sets of gauge lengths.

In the second set of experiments the gauge length of 1 cm and rate of extension of 40% was used. The Instron is normally supplied with an accessory attachment - the breakage detector. This mechanism detects the decrease in the load signal from the load cell during tensile tests. This breakage detecting mechanism can be employed to stop the cross head or to reverse it automatically as soon as the predetermined amplitude of decreased load signal is detected. This mechanism of automatic cross head reversal at very high speed was found to be a most effective technique to arrest the progress of the breakage.

Attempts were made to use a cine camera technique to photograph the mechanism of breakage, but the speed at which the first group of filaments failed, was found to be undetectable with the equipment used.

4.22 Results

4.22(A) Effect of rates of extension and gauge length on the load extension curves.

Results are given in tables (4.22A, B, C) and figures (4.22A - D). It is seen that:

(i) As the gauge length is decreased from 10 cm to 1 cm, the slope of load extension diagram in the rupture region decreases while the intercept A increases (Fig. 4.22A).

(ii) For longer gauge lengths, the break is instantaneous with load dropping to zero; for gauge lengths of 5 cm or less, the load falls to

TABLE 4.22A

Effect of rate of extension and gauge length on breaking extension and breaking load of Tenasco 1650/750/12(Z) - Ring doubler twisted.

Rates of Extension	Breaking Extension %			Breaking load in gms.wt.			Load in gms.wt. immediately after break		
	4%	40%	400%	4%	40%	400%	4%	40%	400%
Gauge length									
	16.4	19.0	18.4	2050	2425	2750	1000	1000	1250
	16.6	20.6	16.0	2275	2700	2525	950	1000	1200
1 cm.	22.2	19.4	18.0	2525	2425	2800	1000	1700	1000
Avg.	18.4	19.7	17.5	2283	2517	2690	983	1233	1150
	14.8	11.6	12.8	2400	2550	2950	1250	850	150
	12.8	13.2	14.4	2600	2900	3100	850	900	150
2.5 cm.	12.0	10.4	14.4	2350	2400	3050	750	1100	100
	11.6	13.4		2450	2750		750	450	
	14.4	12.8		2400	3100		1000	750	
Avg	13.1	12.3	13.8	2440	2740	3033	920	810	130
	10.2	10.7	12.0	2475	2725	3200	625	200	0
	10.0	10.1	12.4	2625	2600	3260	650	0	0
5.0 cm.	10.4	11.6	10.4	2650	2900	1250	75	75	0
	10.1	11.5	10.8	2590	2850	2750	840	150	0
Avg.	10.2	10.9	11.4	2580	2770	3040	842	142	0
	8.1	8.5	7.6	2700	2900	2750	0	0	0
	7.9	8.5	8.8	2550	3000	2760	0	0	0
7.5 cm.	7.7	8.2	8.8	2300	2800	3100	0	0	0
	7.9	7.5		2450	2750		0	0	0
	7.1	8.6		2225	2800		0	0	0
Avg.	7.9	8.3	8.4	2445	2850	2870	0	0	0
	9.7	10.6	11.6	2600	2600	2950	0	0	0
10.0 cm.	9.2	9.9	11.2	2650	2500	2950	0	0	0
	9.1	8.5	11.6	2675	2575	3100	0	0	0
Avg.	9.3	9.7	11.5	2640	2555	3000	0	0	0

TABLE 4.22B

Effect of Gauge Length on Load Extension Behaviour of Tenasco 1650-750 Uptwister Twisted.

Gauge Length	Twist factor $Tex^{\frac{1}{2}}$ turns/cm.	Average Breaking Load Gm.wt.	Average Breaking Extension %	Load immediately after break				
				Sample No. 1	2	3	4	5
10 cm.	31.6	4650	10.09	sharp drop				
	43.3	4526	10.76					
	66.8	3413	9.26					
	94.9	2603	9.53					
2.5 cm.	31.6	4654	11.89	800	1000	1200	300	900
	43.3	4374	11.61	2000	1500	1000	1100	800
	94.9	2686	10.44	1100	1200	1100	1400	800
1.0 cm.	31.6	4445	16.89	1200	cont.	1400	2900	1100
	43.3	4365	18.60	2000	2300	1600	1700	1000
	66.8	3557	13.96	1600	1400	1600	1800	1600
	94.9	2815	13.95	1200	1200	1000	1400	800

TABLE 4.22 C

Effect of gauge length on load-extension behaviour after break for viscose 300-100 yarns - Ring Twisted.

Twist Factor $Tex^{\frac{1}{2}}$ turns/cm.	Twisting Tension g.wt./ tex.	Maximum Load at Break gm.wt. Gauge Length 1 cm.	Avg.	Load immediately after break - gm.wt. Gauge Length 1 cm.	Avg.	Breaking Extension % Avg. of 5 obs. G.L. 1 cm.	Breaking Extension % Avg. of 10 obs. G.L. 10 cm.	Breaking Load in gm.wt. 10 cm. G.L. Avg. of 10 obs.
7.1	0.9	590 635 600 620 640	617	Continuous drop in load.		35.4	22.7	602
18.1	0.9	600 600 610 615 605	606	210 110 120 160 120	144	27.2	24.7	605
17.5	1.8	580 625 610 585 540	599	100 220 130 290 110	170	35.4	25.3	622
17.0	2.7	590 600 580 580 600	590	80 200 160 100 cont.	135	33.5	22.8	609
41.9	0.9	580 620 595 620 580	599	100 180 210 160 120	154	29.5	27.2	598
39.4	1.8	600 615 580 590 600	597	40 110 170 200 200	144	33.6	20.5	569
37.2	2.7	590 585 615 610 605	601	cont. cont. cont. 240 200	220	22.5	13.7	592
57.3	0.9	540 540 530 545 540	539	190 280 280 225 80	211	26.7	25.6	532
53.2	1.8	540 555 535 555 585	554	180 260 cont. 290 200	232	27.4	20.7	550
49.9	2.7	560 530 540 520 520	534	120 340 280 240 280	252	26.4	15.8	536
73.7	0.9	450 480 450 480 465	465	190 180 170 200 100	168	27.7	21.6	433
72.4	1.8	470 405 420 490 420	441	280 280 280 220 220	256	22.7	20.7	430
66.5	2.7	500 495 470 500 520	497	200 210 300 240 260	242	26.3	15.7	481

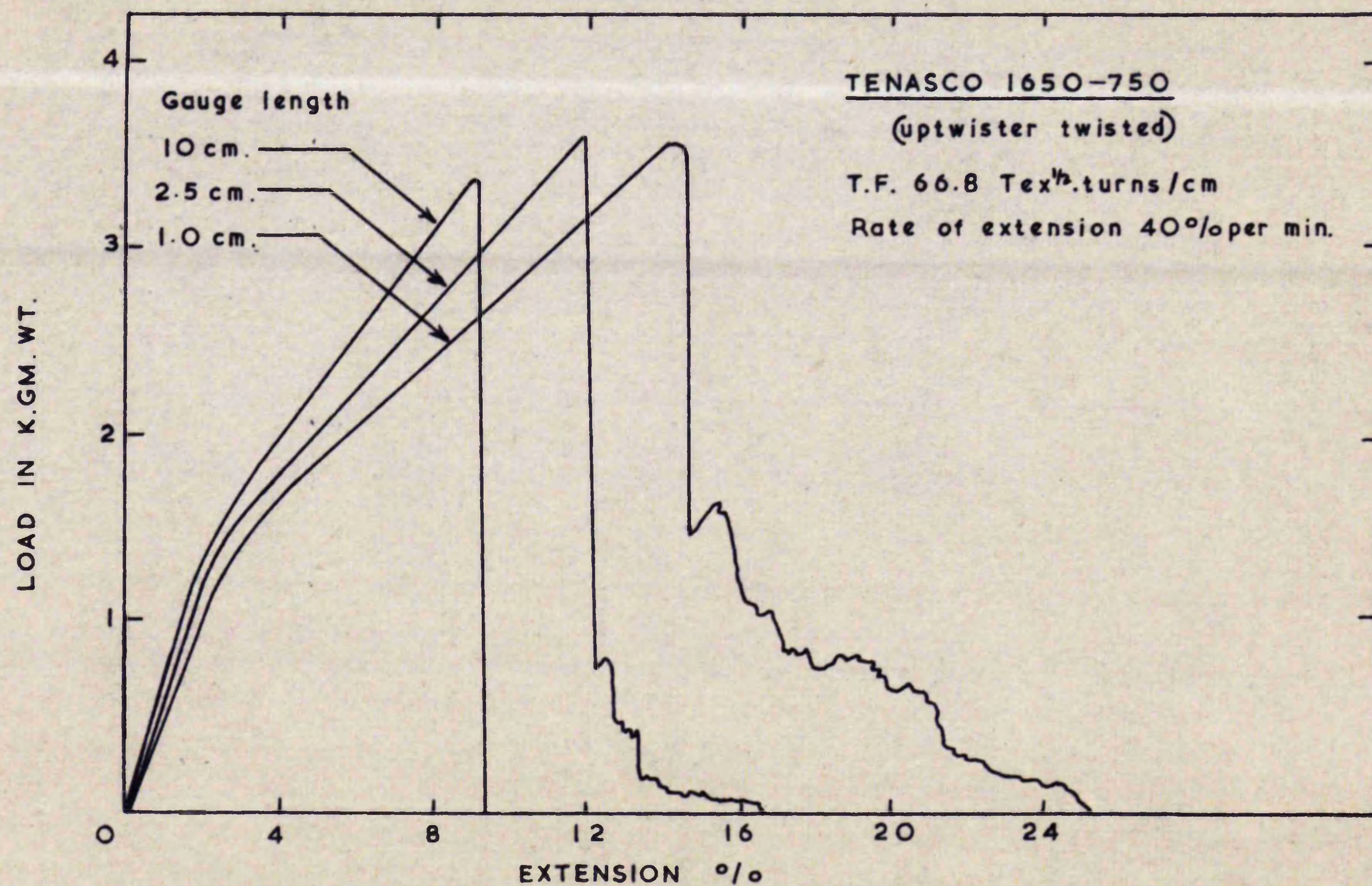


FIG 4.22A EFFECT OF GAUGE LENGTH ON LOAD-EXTENSION DIAGRAM OF TENASCO YARN.

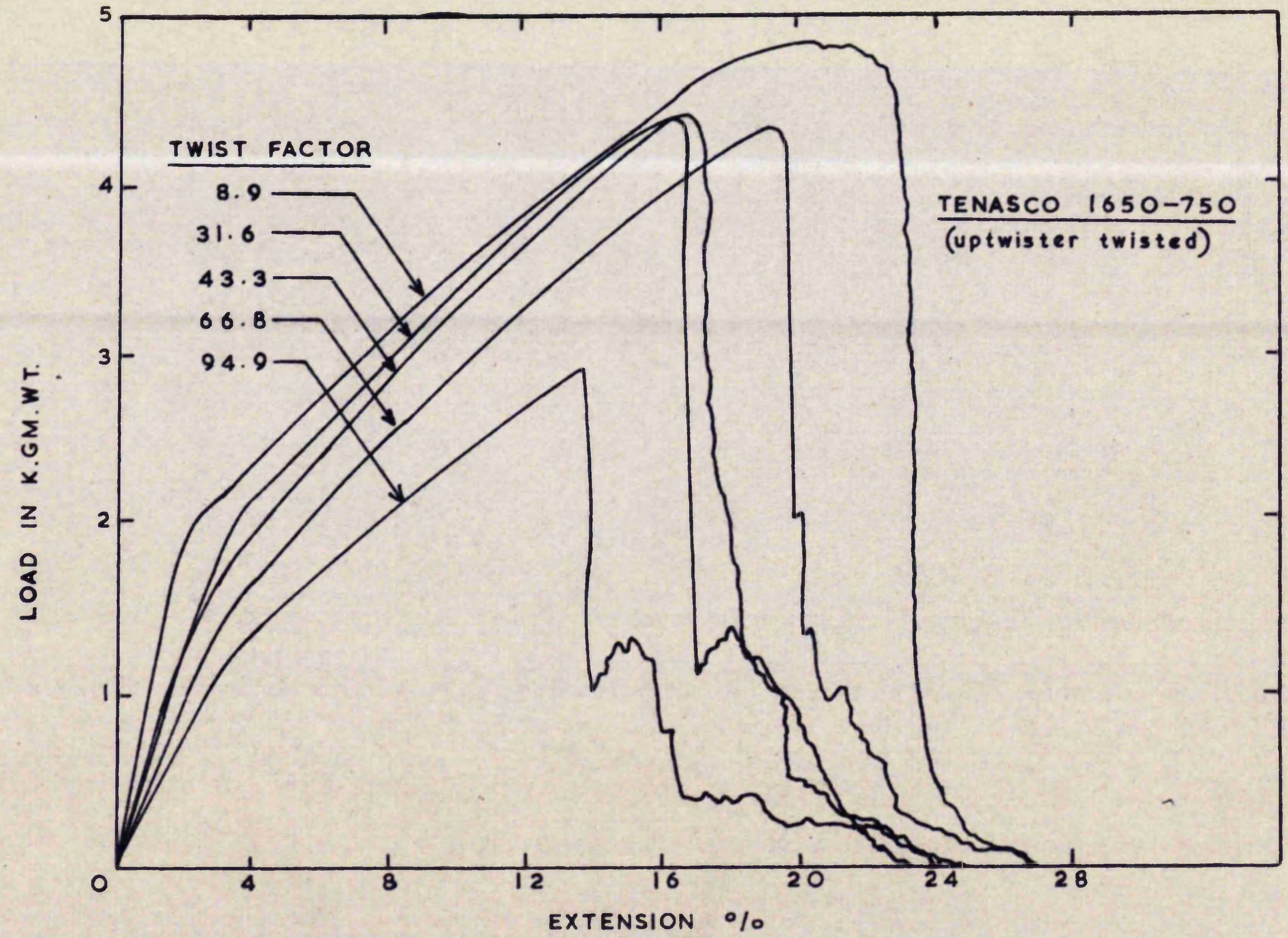


FIG. 4·22B LOAD-EXTENSION DIAGRAM OF TENASCO YARN USING 1 cm. GAUGE LENGTH AND 40% RATE OF EXTENSION / MIN.

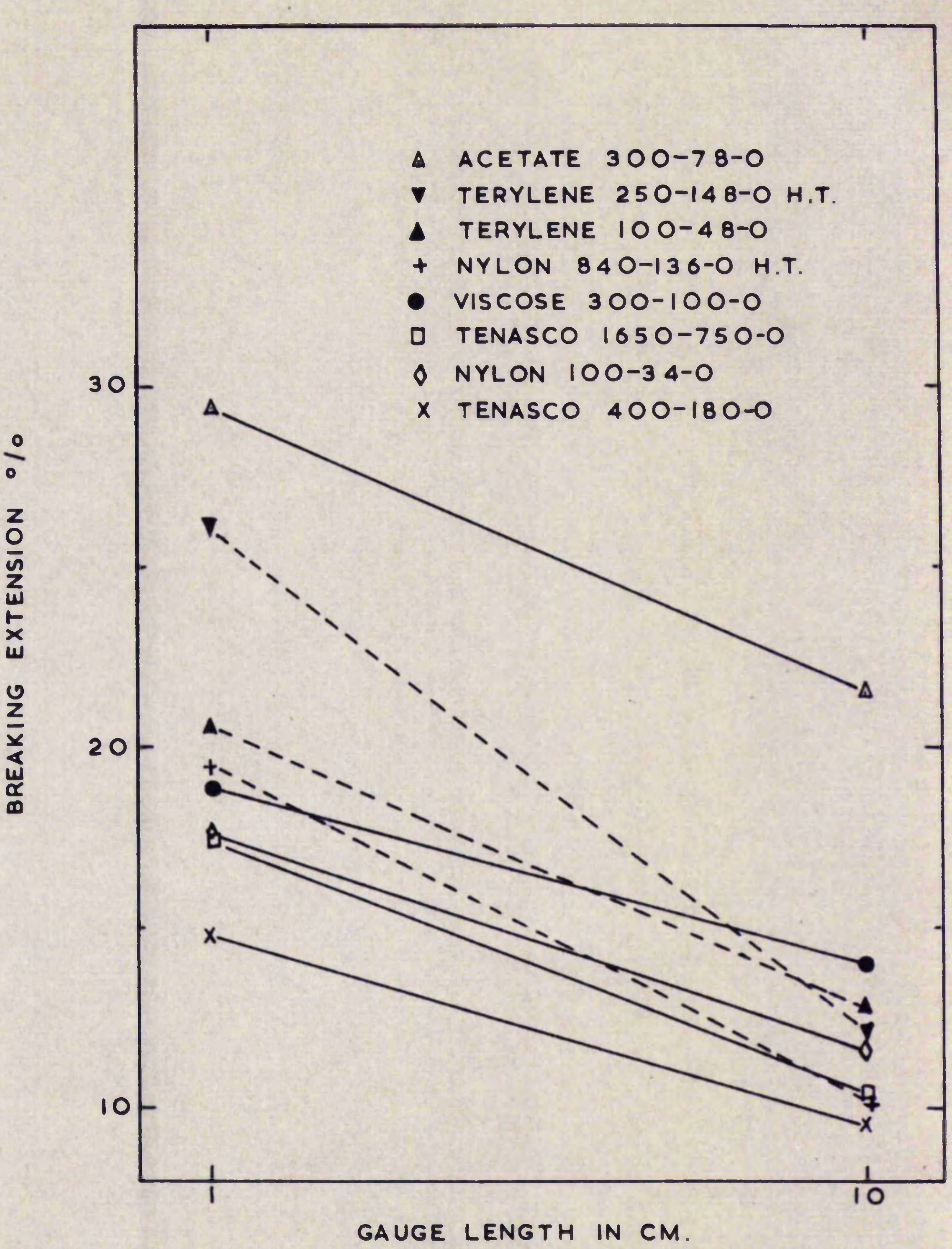


FIG. 4·22C EFFECT OF GAUGE LENGTH ON BREAKING EXTENSION OF ZERO TWIST YARNS.

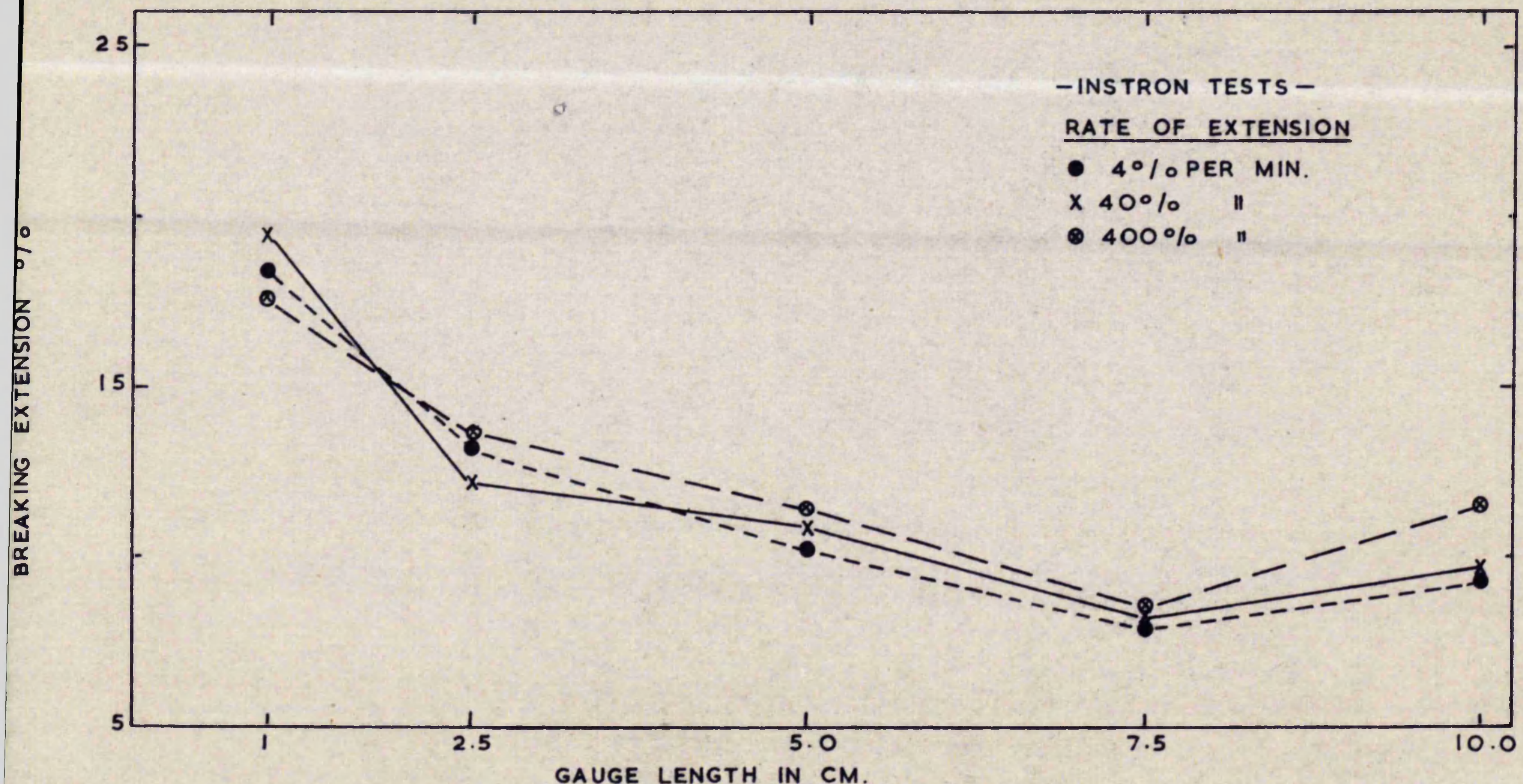


FIG. 4.22D EFFECT OF GAUGE LENGTH ON THE BREAKING EXTENSION OF TENASCO 1650/750/12 YARN.

zero in series of steps as shown in figure 4.22A for Tenasco 1650-750-12 yarn. Similar behaviour is observed in the tests for nylon, viscose rayon and Terylene yarns (Tables 4.22 A,B,C).

(iii) At 1 cm gauge length, the breaking loads are less than the corresponding values at 10 cm gauge length (Tables 4.22 A,B,C).

(iv) As the gauge length is decreased from 10 cm to 1 cm the breaking extension increases (Fig. 4.22 C & D). For Tenasco 1650-750-15 uptwisted yarn this increase is 46.4% (Table 4.22 B).

(v) Both breaking load and breaking extension increase as the rate of extension is increased from 4% per minute to 400% per minute. At 1 cm gauge length the load-extension diagram after break, ends in a series of steps at all rates of extensions. At 10 cm and 7.5 cm gauge length sharp break is observed at all rates of extensions. This may be due to the limited range of extension rates used. At 2.5 cm and 5.0 cm gauge lengths, higher rates of extension show sharp break and lower rates of extension show break in steps (Table 4.22A). However, very slow rate of extension, as in the second observation below will show the similar features as observed at 1 cm gauge length.

4.22(B) Observation of breakage

(1) Fig. (4.22 E) shows a number of twisted Tenasco yarns after the breakage has been intercepted by the sample break detector. The gauge length of 1 cm was used in all these tests.

(2) Fig. (4.22 F) shows the breakage zone as seen from two directions 180^{o} apart. The gauge length of 10 cm was used, but the cross head was

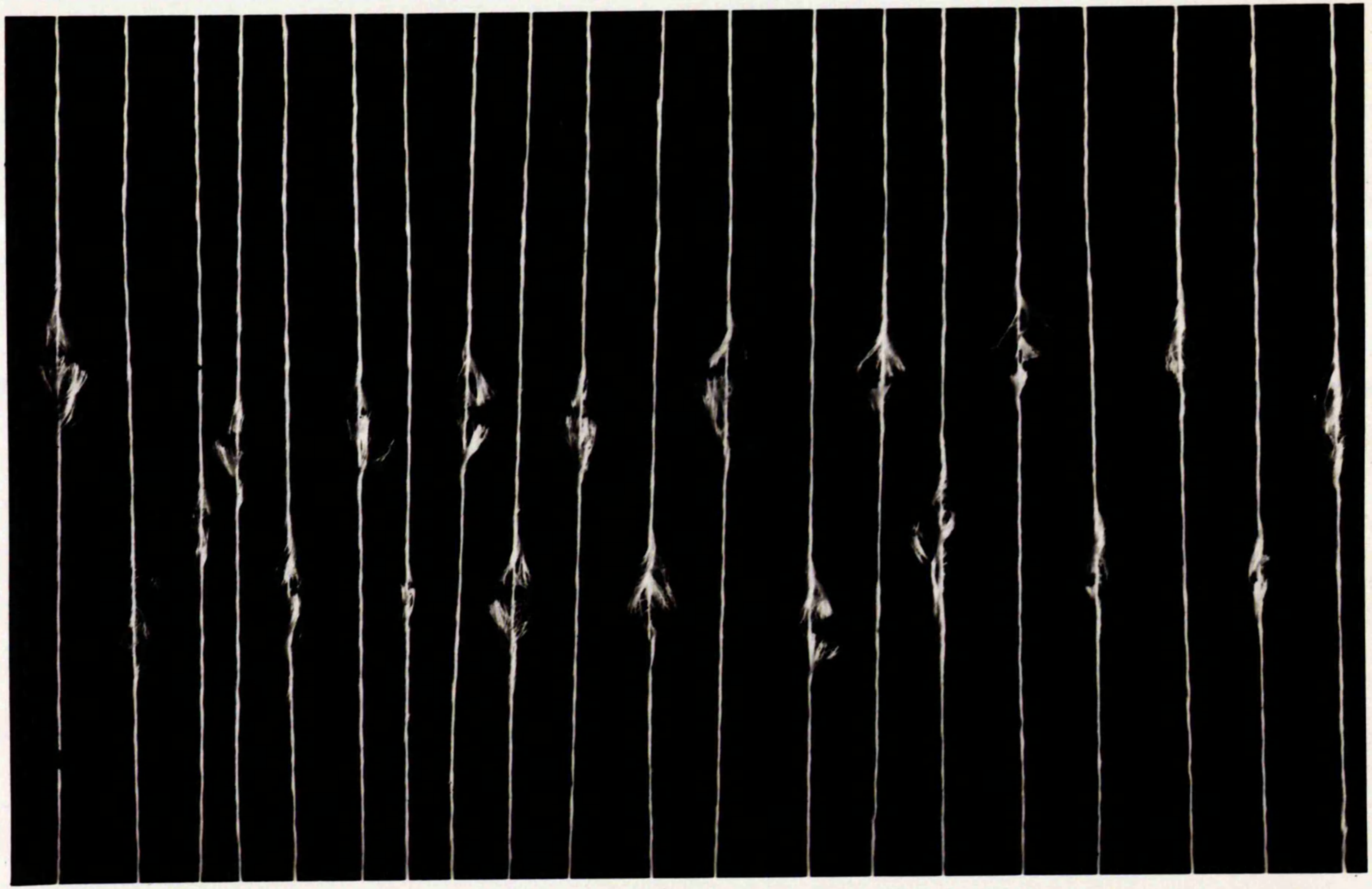


Fig. 4.22 E. Sample of Tenasco yarns in which breakage is interrupted.
Gauge length 1 cm.

A B C

Fig. 4.22 F. Breakage zone of Tenasco 1650-750-12 yarn. Photographs B and C are taken from two directions 180° apart.

manually operated so as to interrupt the breakage.

(3) The lower the twist in the yarns, more is the tendency to obtain a sharp break. However, at very low twists, each filament breaks at random when its own breaking extension is reached (Fig. 4.22 B).

(4) If the surface of the twisted yarn is coated with coloured particles and then the breakage zone is observed:

(a) some coloured and uncoloured filaments will remain intact;

(b) the unbroken group of the filaments will constitute one side of the yarn cross section.

(5) For Tenasco 1650-750-12 ring twisted yarn, the proportion of broken filaments to the unbroken ones was found to be approximately constant in all the test results. This was found by weighing the broken yarn as a whole and the unbroken portion in the tests using 1 cm gauge lengths. (Table 4.22 E).

(6) Similar behaviour is observed in Tenasco 1650-750-15 yarns, uptwisted by British Rayon Research Association.

(7) In acetate and nylon yarns, a few broken filaments were observed to protrude on the yarn surface before the actual break. In acetate yarn tests, the technique to arrest the progress of break was not successful.

(8) Fig. (4.22 G) shows the appearance of the breakage zone in tensile tests using 10 cm gauge lengths and 40% rate of extension. It was not possible to arrest the breakage. However, one can observe a small group of filaments being pulled out from the lower portion of the yarn:

TABLE 4.22 E

Weights of broken and unbroken portions of yarn structure

Breakage Detector Tests at 1 cm. gauge length.

Tenasco 1650-750 Ring doubler twisted.

	Weight of sample (milligram)	Weight of unbroken portion (milligram)	Weight of broken portion %
Sample A			
	10.8	5.2	51.8
	10.6	5.1	51.9
	11.4	4.8	57.9
	10.0	4.0	60.0
	Avg. 10.7	4.8	55.4%
Sample B			
	12.0	4.0	66.6
	11.4	6.4	43.8
	11.6	5.4	53.4
	11.3	4.8	57.5
	Avg. 11.6	5.2	55.2%
Sample C			
	9.2	3.5	61.9
	9.6	3.6	62.5
	10.1	4.6	54.5
	9.6	4.0	58.3
	9.0	2.6	71.1
	Avg. 9.5	3.66	61.4%

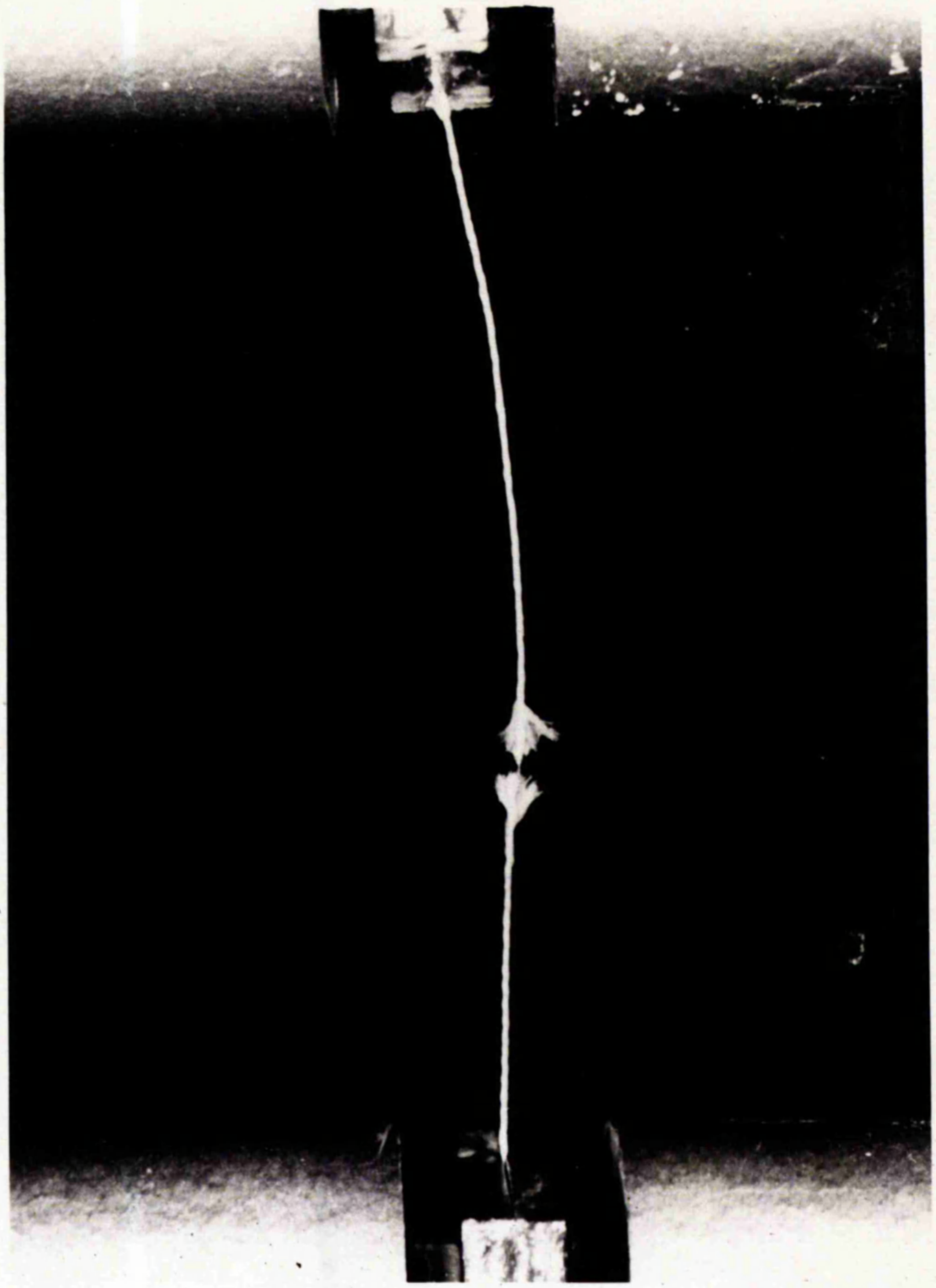


Fig. 4.22 G Breakage zone of Tenasco 1650-750-12 yarn.

Gauge length 10 cm. and Rate of Extension 40% min.

they have evidently broken at some point away from the point at which break started. The load value falls down to zero immediately after break.

4.3 YARN STRUCTURE

Knowledge about the actual yarn structure is of importance in the understanding of yarn properties. Some experiments were carried out to study the differences of the yarn structure from the ideal helical form, as explained in chapter I.

4.31 Experimental

Four sets of experiments were conducted to study the actual structure of highly twisted yarn.

(a) If filaments are extracted from a yarn, they tend to remain set in the configuration which they possessed in the yarn.

(b) If the outside of a twisted yarn is coated with a coloured paste and then the yarn is untwisted at fixed lengths on a twist tester, the surface coated filament layer can be distinguished from the inner uncoated layers, and the form of twisting inferred.

It is necessary to obtain a suitable colouring technique which will avoid colour penetration in the inner layers of yarn structure. After various trials of pastes, a vat colour paste made in methyle alcohol and shellac was found to give satisfactory results. The use of printing pastes and controlled vat dyeing technique were among the other methods tried.

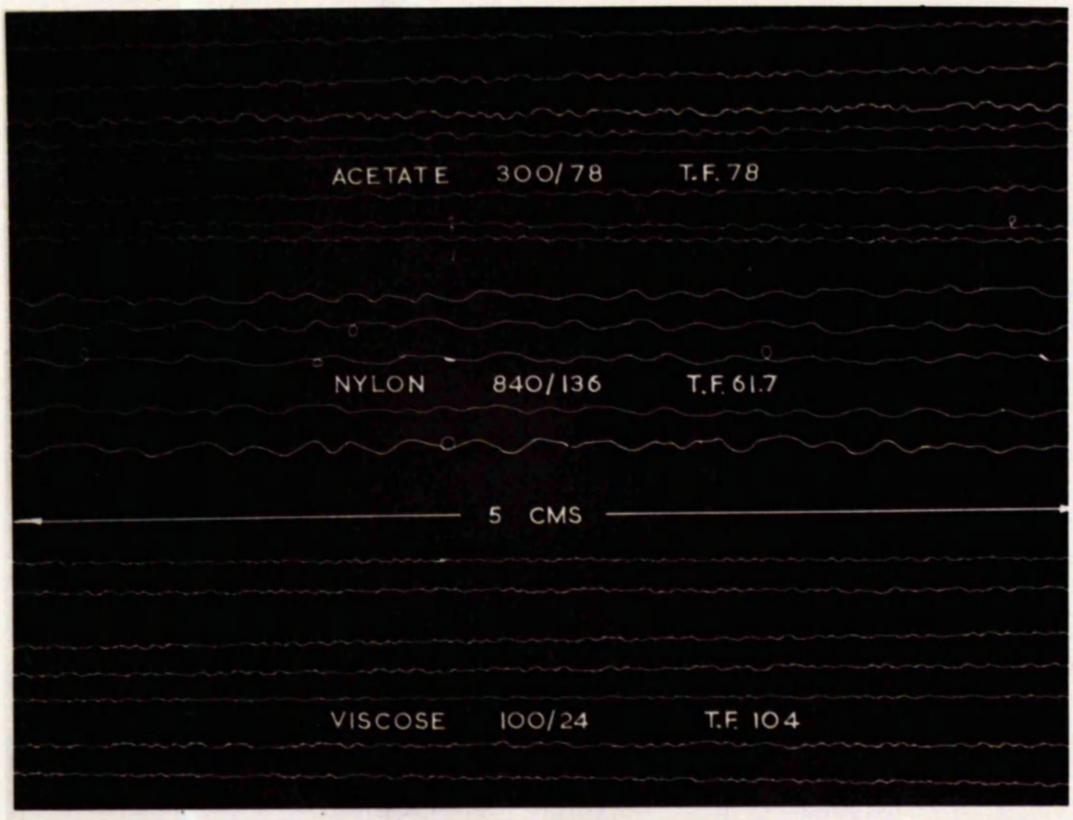
(c) If, in experiment (b), the filaments were extracted, one could identify the portion or the whole of the filaments forming surface or inner layers of yarn structure.

(d) If the length of the filaments between two successive cross-sections of twisted yarns (about 1 mm. apart) is measured by use of the projection microscope, the length distribution of the constituting filaments will qualitatively demonstrate the departure from the idealised geometry of the yarn structure.

4.32 Results

From figure (4.32 A & B) it can be seen that the straight and highly coiled portions of all extracted filaments alternate. This appearance is qualitative evidence that the migration in continuous twisted filament yarns does occur. In the third set of experiments, the coloured and the uncoloured portions of the filaments were found to alternate. This is also evidence of the migration in twisted filament yarns.

All commercially spun continuous filament yarns are found to have some initial twist. The number of filaments constituting these yarns varies from 7 to 1600 filaments. As the number of filaments constituting a given yarn linear density (tex) is increased, the yarns will tend to have a flat ribbon form when wound on winding packages or fed by delivery rollers. Tenasco (1650-750-12) yarn is found to have a ribbon width of 0.8 mm. and thickness of 0.2 mm. If such yarns are twisted on a ring doubler or uptwister, there is a large tendency to twist them in the form of a ribbon. This will



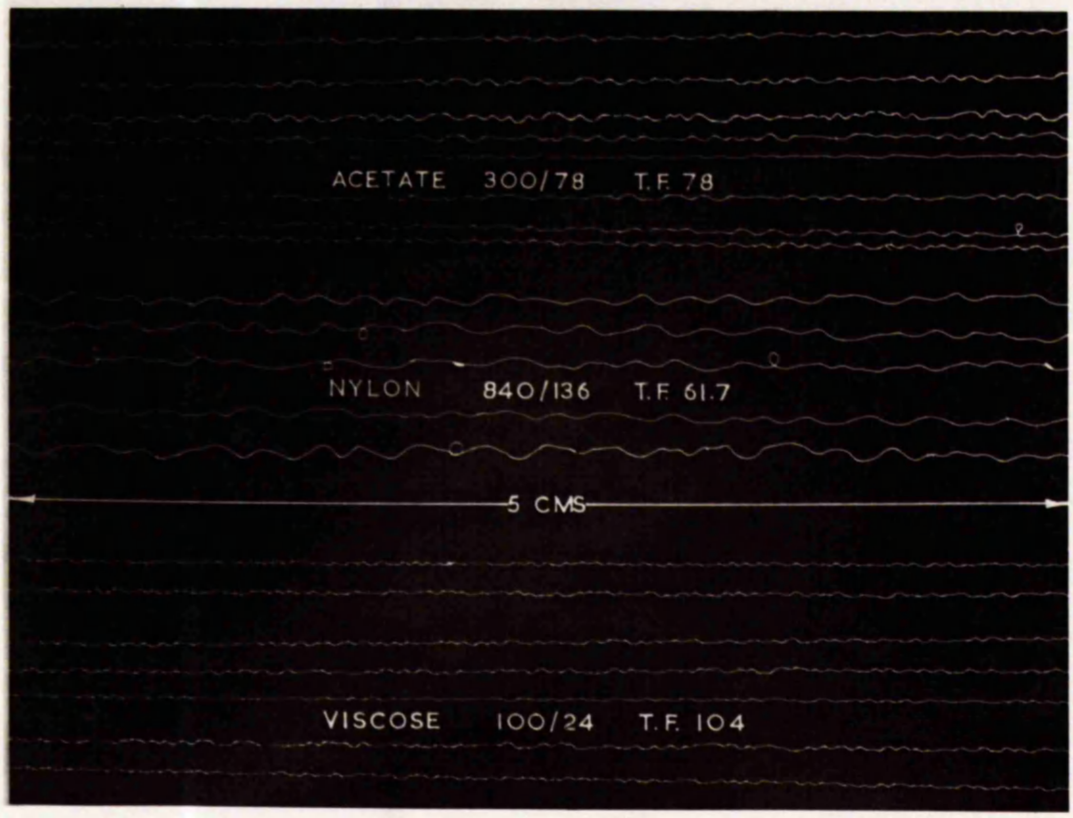


Fig. 4.32 A Form of filaments extracted from twisted yarns.

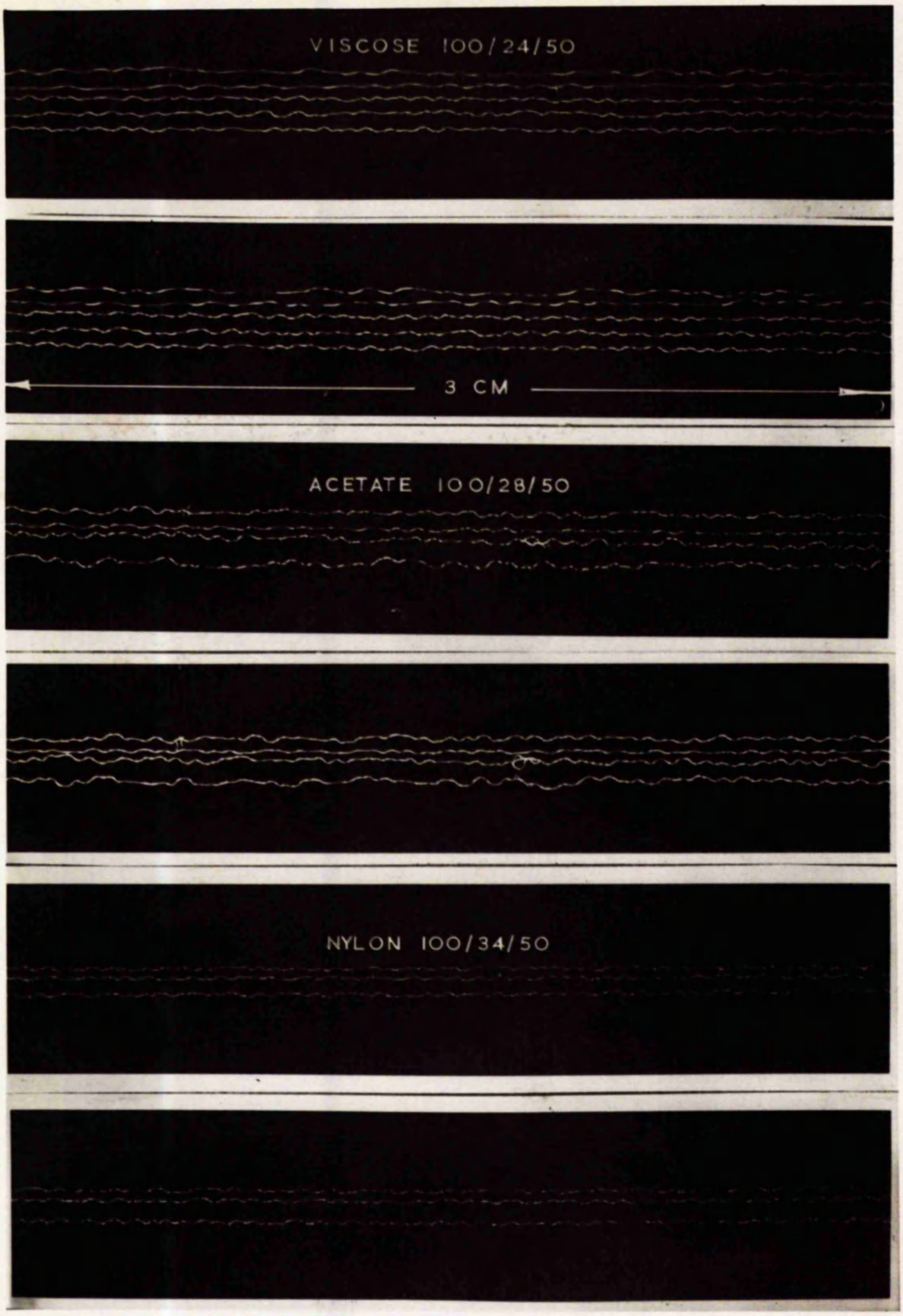


Fig. 4.32 B The upper and lower groups for each yarn are taken from photographs of same filaments viewed from two directions at right angles.

be more so where very low twisting tensions are used. Fig.(4.32 C) shows the ribbon form of Tenasco 1650-750-122 yarn in the process of untwisting at constant length. During initial stages of untwisting the yarn surface layer will be seen to have split by a spiral white line along its axis. The angle of this spiral line depends upon the twist in the yarn.

Fig. (4.32 D) demonstrates the stages in the static twisting of Tenasco 1650-750-1 yarns. As the twist is increased the structure resembles a two ply yarn structure. This may be due to splitting of the ribbon as can be demonstrated in rubber tape twisting, but if twisting tension is sufficiently high, this appearance is less marked.

The length distribution of constituting filaments was found to show a normal distribution, figure (4.32E, E_2), while idealised helical form of yarn structure assumes triangular distribution.

4.33 Discussion

All these experimental techniques suggest that the actual yarn structure is quite different from that generally assumed. Moreover, the number of constituent filaments, method of twisting and twisting tensions will also introduce some differences of yarn structures. If the migration occurs with the period several times greater than the length of one turn of twist, then the ideal helical structure is a good approximation. But if migration pattern repeats over a shorter length or, if buckling occurs then, the ideal model cannot be expected to give the correct results. The ribbon form of twisted structure may

Fig. 4.32C. Tenasco 1650-750-12 yarn in the process of untwisting.

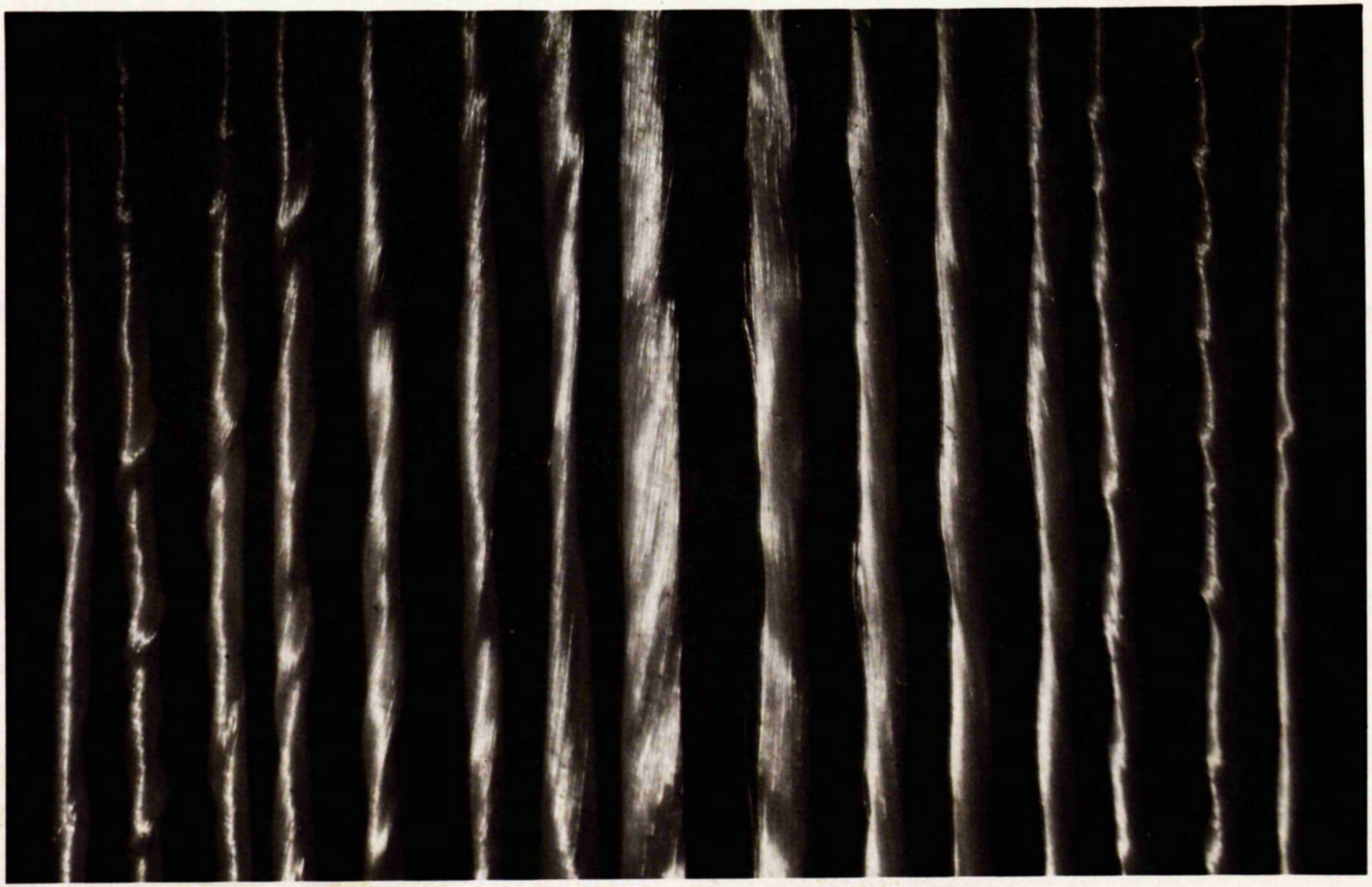


Fig. 4.32 D Appearance of yarns in the static twisting of Tenasco 1650-750-1 yarn.

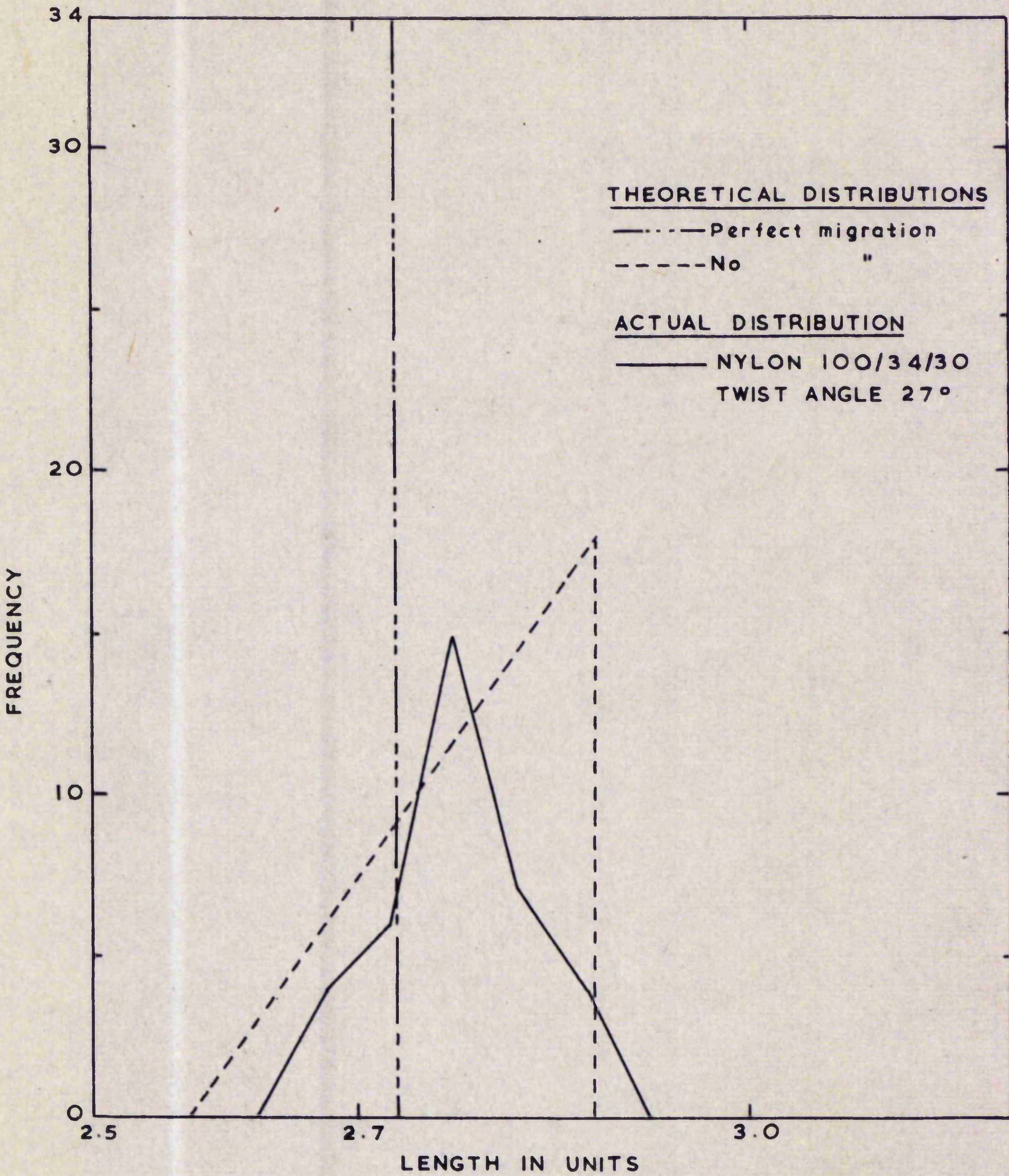


FIG. 4.32 E_1 FREQUENCY DISTRIBUTION OF FILAMENT LENGTHS IN YARN CROSS-SECTION.

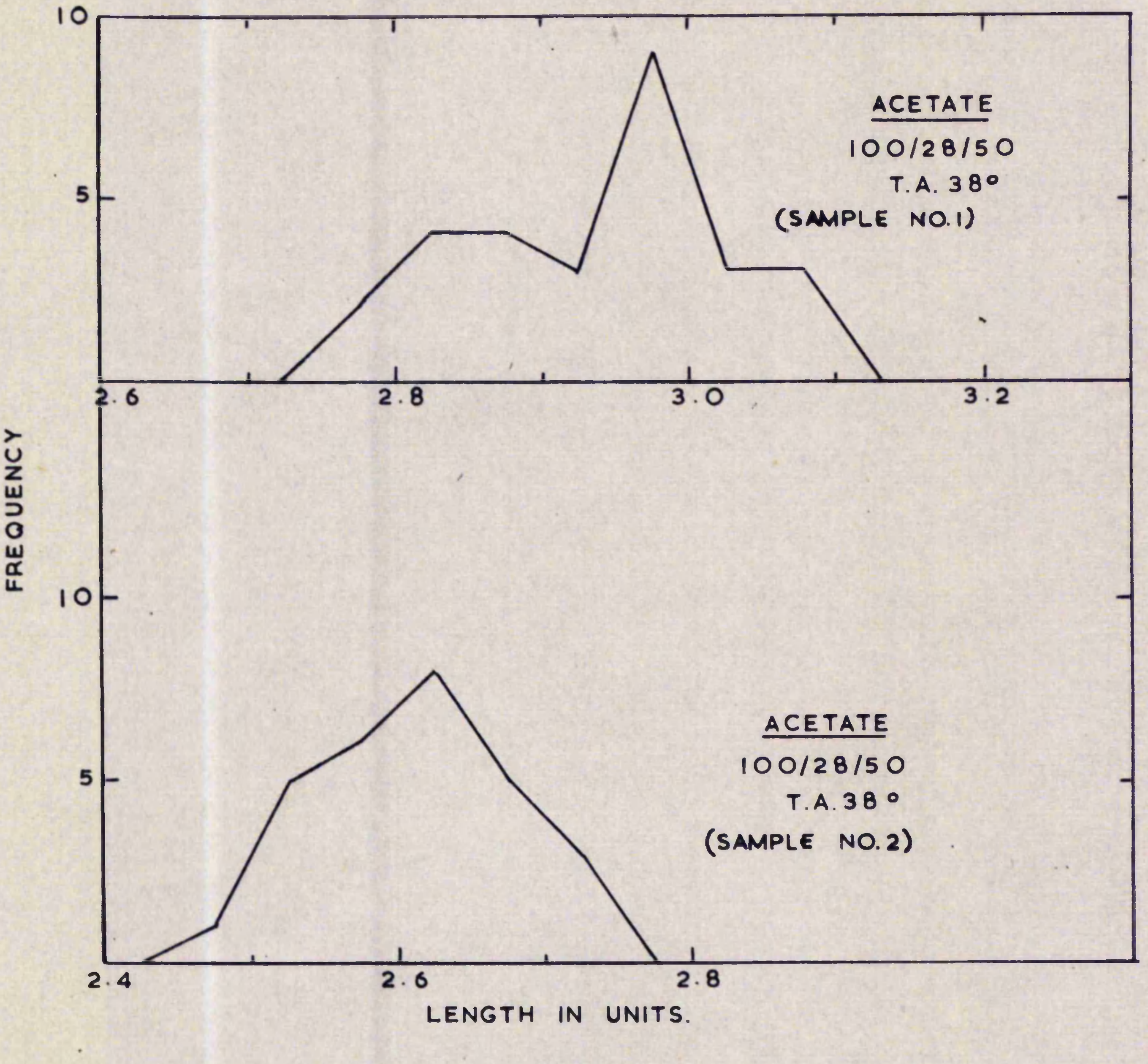


FIG. 4.32E_2 FREQUENCY DISTRIBUTION OF FILAMENT LENGTHS IN YARN CROSS-SECTION.

mean a great departure from the ideal helical structure. If the migration is absent in such a ribbon structure, one could postulate the presence of buckling in the core structure. Tattersall[30] has observed that the yarn produced on the laboratory machine may not have the geometrical properties of a commercially produced yarn.

These techniques will have to be developed to the extent that quantitative results can be obtained.

4.4 RUPTURE PROPERTIES OF FILAMENTS

One of the limitations to the theoretical prediction of rupture properties of twisted continuous filament yarns, is the assumption that the twisting operation and the yarn geometry do not alter the mechanical properties of the filaments constituting the yarn structure.

In testing the validity of such assumptions, the experimental conditions must be controlled to represent those experienced by a filament constituting the yarn structure. The rupture properties of the constituent filaments may be influenced by

(1) the amount of twist and twisting conditions;

(2) the helical path (bending), when the filament forms the outer layer of the yarn structure;

(3) the compressive forces which may influence the extensibility of the central filament;

(4) the combined effect of these factors.

To advance our knowledge about these disturbing factors, preliminary investigations were carried out to study

(a) the rupture properties of filaments extracted from twisted yarn structures;

(b) effect of twist on the rupture properties of single filaments extracted from zero twist yarns.

4.41 Experimental

(A) Tensile tests of extracted filaments.

About 20 cm. lengths of twisted yarns were untwisted on a twist tester. 10 cm. and 1 cm. lengths of these extracted single filaments were then used for tensile tests on the Instron tester. During the process of extraction, precautions were taken to see that filament stretching was avoided.

During mounting these extracted filaments some difficulties were experienced in finding:

(a) the best mounting technique to maintain 1 cm. gauge lengths, initial tension and to obtain the rapidity in tests with the minimum of handling;

(b) a suitable device to avoid slippage or breaks at the clamping jaws.

In mounting 1 cm. gauge lengths, the technique described by Evans and Montgomery[34] was initially tried. This set up consists of a circular jig, resting on an anvil carried in the cross head of the Instron tester. The jig is revolved by a ratchet mechanism to position the successive specimen exactly under the strain-sensing element of the tensile tester (top jaw). The difficulties experienced in the use of this technique were:

1) to obtain a suitable clamping band to keep the bottom tabs in position during the tensile tests;

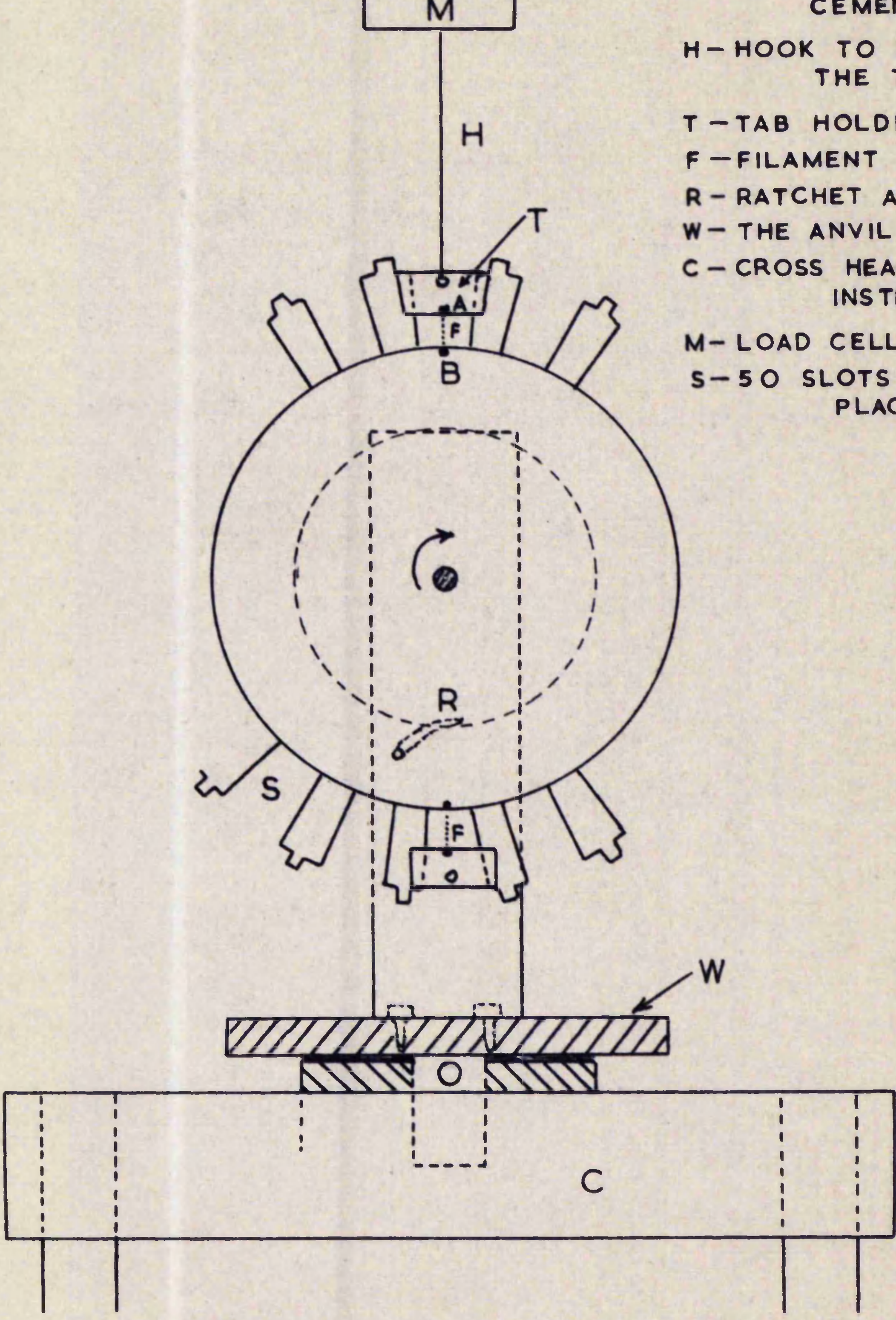
2) to maintain the standard (1 cm.) gaugelength. This difficulty was solved by using a mounting board. The tabs holding the specimen were then transferred to the circular jig.

3) to avoid stretching during the transfer of tabs.

The modified technique used is shown in figure (4.41 A). The circular jig is mounted vertically and rests on an anvil, which is carried in the cross head of the tensile testing machine. The circular disc could be revolved until the bottom fibre fixing point is exactly under the top jaw (Hook Jaw) of the Instron tester. The tabs could be cut to a known weight and used repeatedly. The standard gauge length can be maintained by having a radial slot in the disc between which the filaments could be mounted. One end of the filament can be cemented on to the disc and the other on a special shaped tab resting on the disc. The tab can be punched to be carried by the hook in the top jaw. The arrangement is similar to one used in fibre testing on the Magazine Hair Tester. It is very important to have light weight tabs, as some of the mounted filaments will be hanging freely under the weight of these tabs.

Durofix cement was found to give satisfactory results except for acetate and coarse nylon filaments. During the process of filament extraction, it was found very difficult to take out long lengths of filament from highly twisted yarns without stretching them. This was observed especially in acetate and viscose yarns.

Similar tests for filaments extracted from viscose rayon, acetate and nylon yarns (twisted) were initially carried out on the Cambridge Extensometer - constant rate of extension tests. The technique of mounting 5 ins. lengths of these extracted filaments on a black stiff



A,B - SPECIMEN HOLDING CEMENT

H - HOOK TO HOLD THE TAB

T - TAB HOLDING FILAMENT

F - FILAMENT

R - RATCHET ARRANGEMENT

W - THE ANVIL

C - CROSS HEAD OF INSTRON TESTER

M - LOAD CELL

S - 50 SLOTS EQUALLY PLACED

FIG. 4·41A CIRCULAR JIG FOR TESTING SINGLE FILAMENTS.

paper with cellophane tape and the use of the mechanical jaw was found to give satisfactory results.

(B) Tensile tests of twisted single filaments.

The filaments were extracted from zero twist yarns and twisted under constant tension. Twisting was carried out by using the arrangement as shown in figure (4.41 B). The 4 volt D.C. motor assembly was fixed on the framing of the Instron tester. The specimen of about 15 cm length was then clamped in the jaw fixed on the rotor of the D.C. motor. The other end of the specimen was allowed to hang freely under the tension of the small bull-dog clip. The rotation of the bull-dog clip and hence the other end of the specimen was restricted by means of a horizontal needle attachment which was free to slide vertically to allow for the contraction. The D.C. motor was stopped after a predetermined number on the revolution counter. The actual specimen length was noted. 10 cm lengths of this twisted specimen was then used in the tensile tests. The Mechanical Jaw was used to hold the specimen during the tensile tests.

4.42 Results

The figures (4.42 A & B) show the breaking load and extension behaviour of single filaments extracted from twisted yarns, while figure (4.42 C) show the effect of twist on breaking extension of some acetate and nylon single filaments. The load extension curves in these tests are shown in figures (4.42 D_1 - D_4).

(1) In general, the breaking load and the breaking extension values of twisted single filaments are very little affected at very

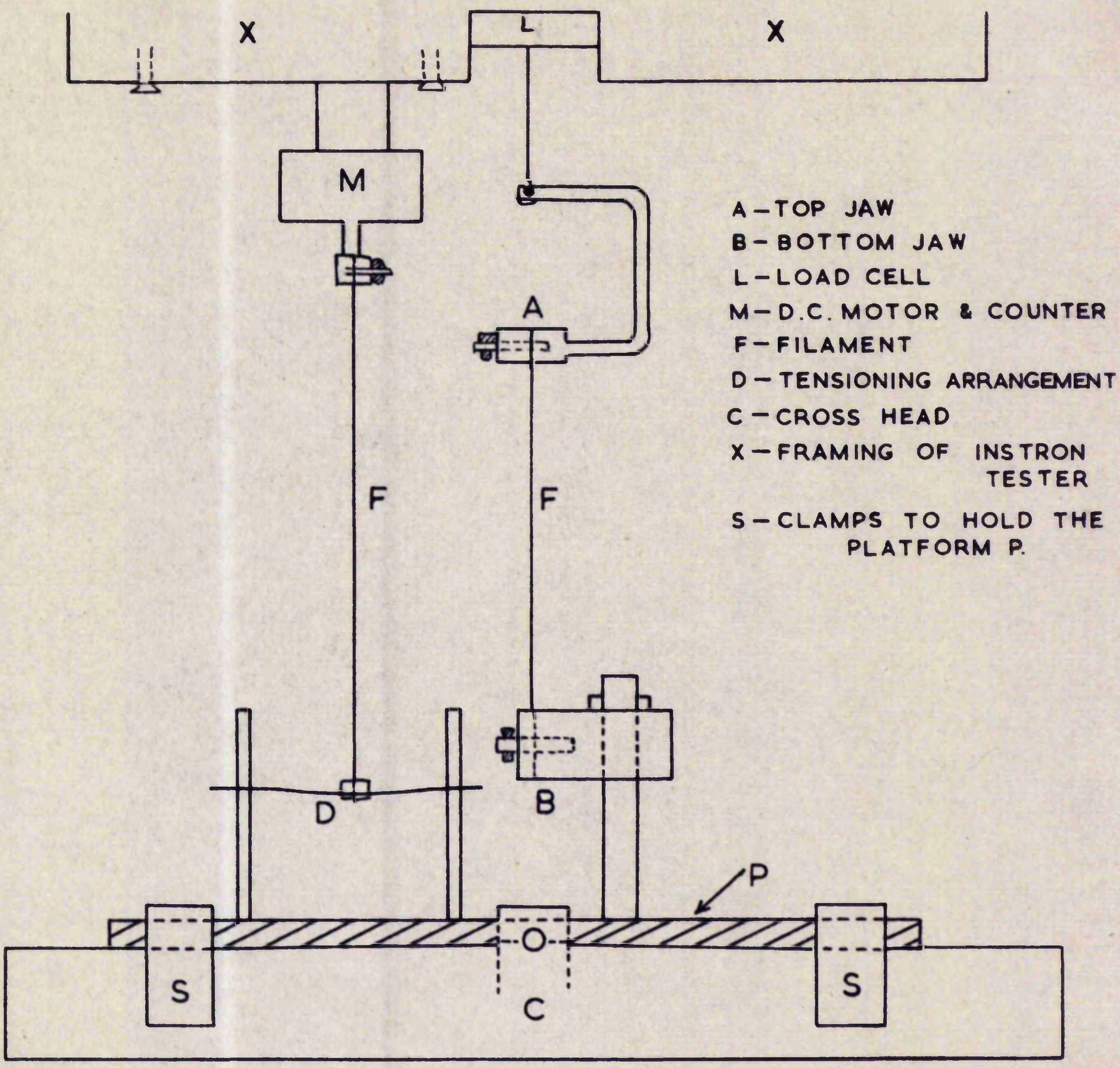


FIG. 4.41B THE EXPERIMENTAL SET-UP FOR TWISTING AND TESTING OF SINGLE FILAMENTS.

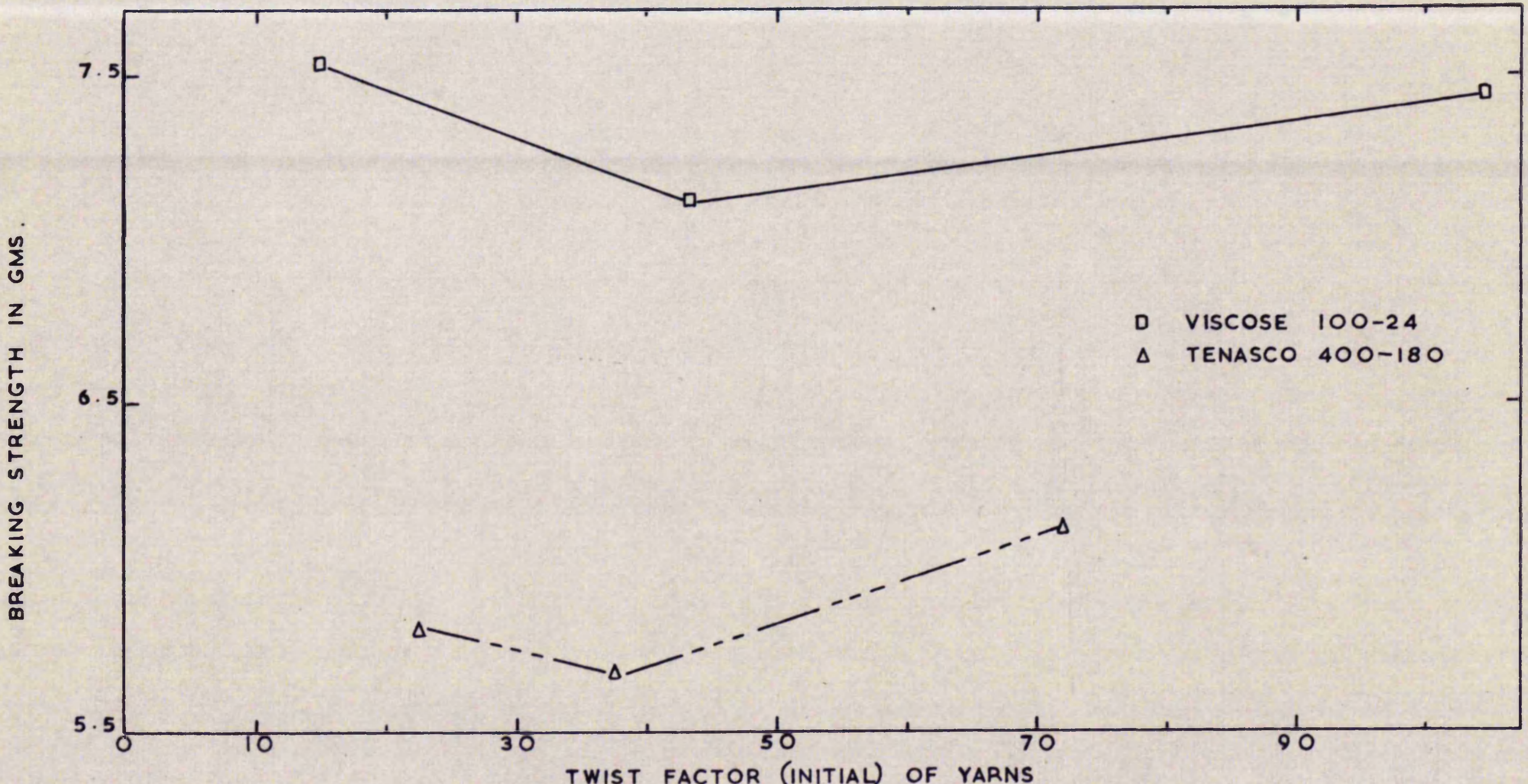


FIG. 4.42A BREAKING LOAD OF THE FILAMENTS EXTRACTED FROM TWISTED YARNS. (INSTRON)

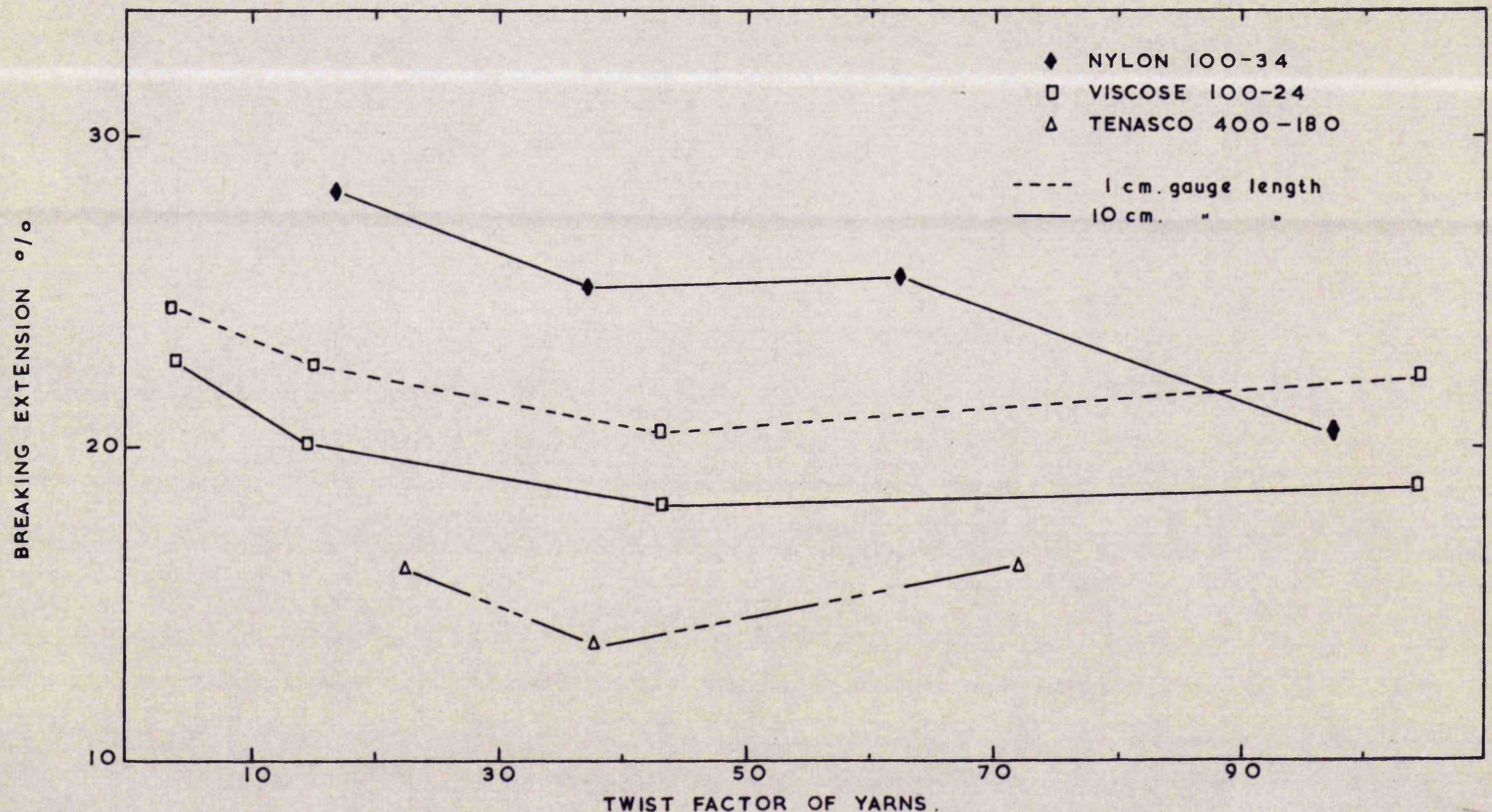


FIG. 4·42B BREAKING EXTENSIONS OF FILAMENTS EXTRACTED FROM TWISTED YARNS. (INSTRON)

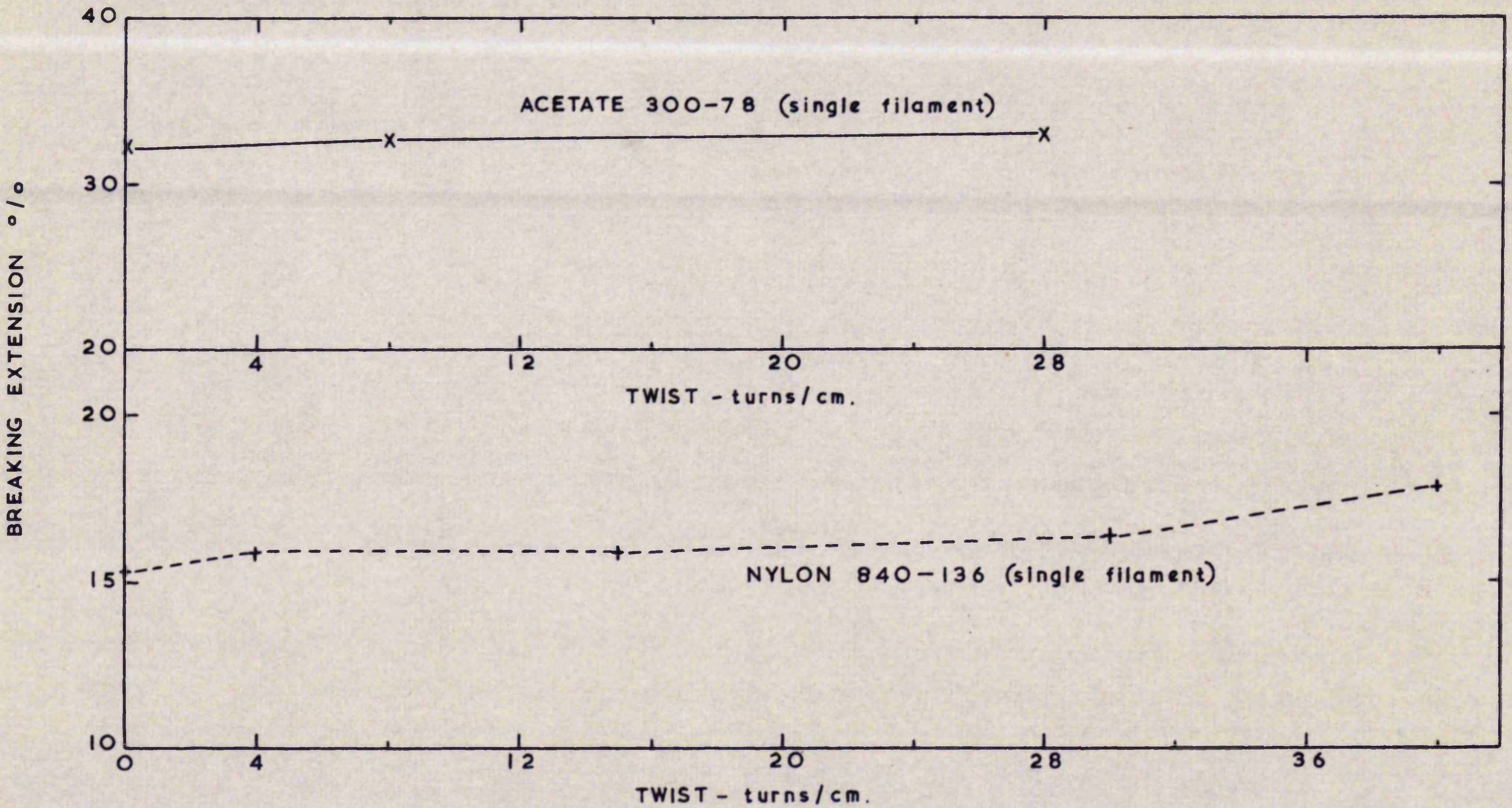


FIG. 4.42 C EFFECT OF TWIST ON BREAKING EXTENSION OF FILAMENTS. (INSTRON)

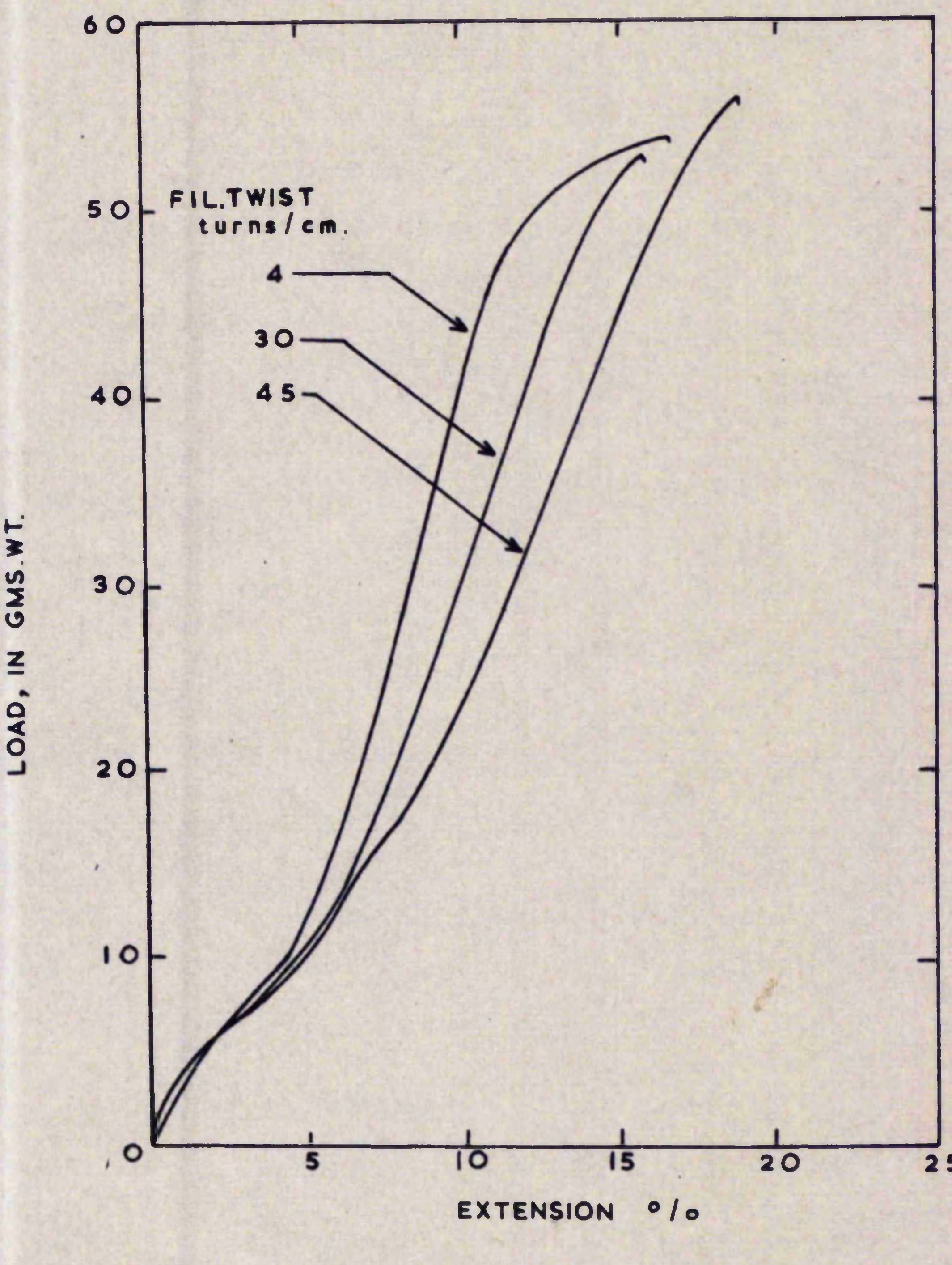


FIG. 4.42D, EFFECT OF TWIST ON LOAD EXTENSION CURVES OF SINGLE FILAMENTS EXTRACTED FROM NYLON 840-136-1

— INSTRON TESTS —

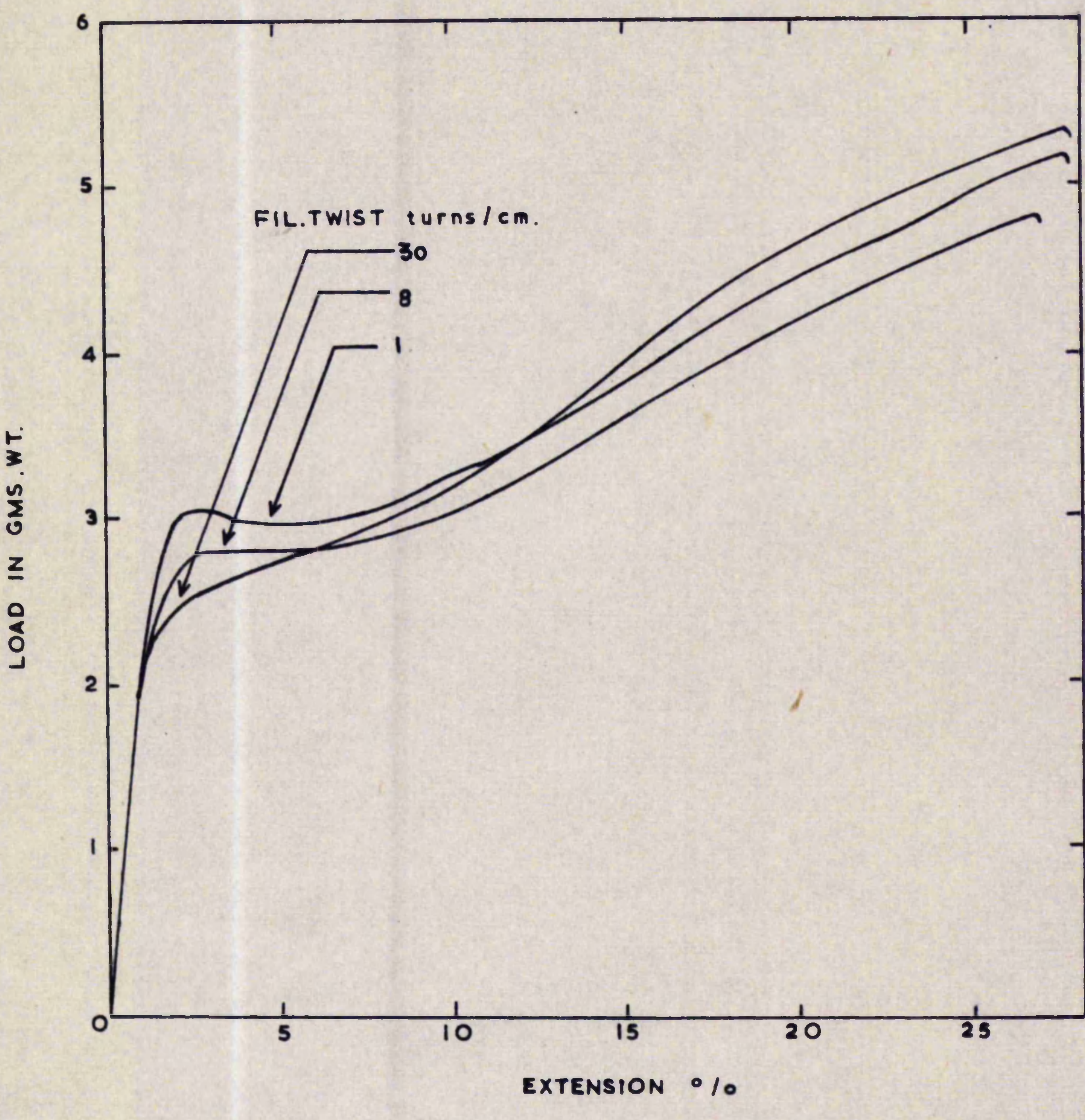


FIG. 4.42 D_2 EFFECT OF TWIST ON LOAD-EXTENSION CURVES OF SINGLE FILAMENTS EXTRACTED FROM ACETATE 300-78-2.5

— INSTRON TESTS —

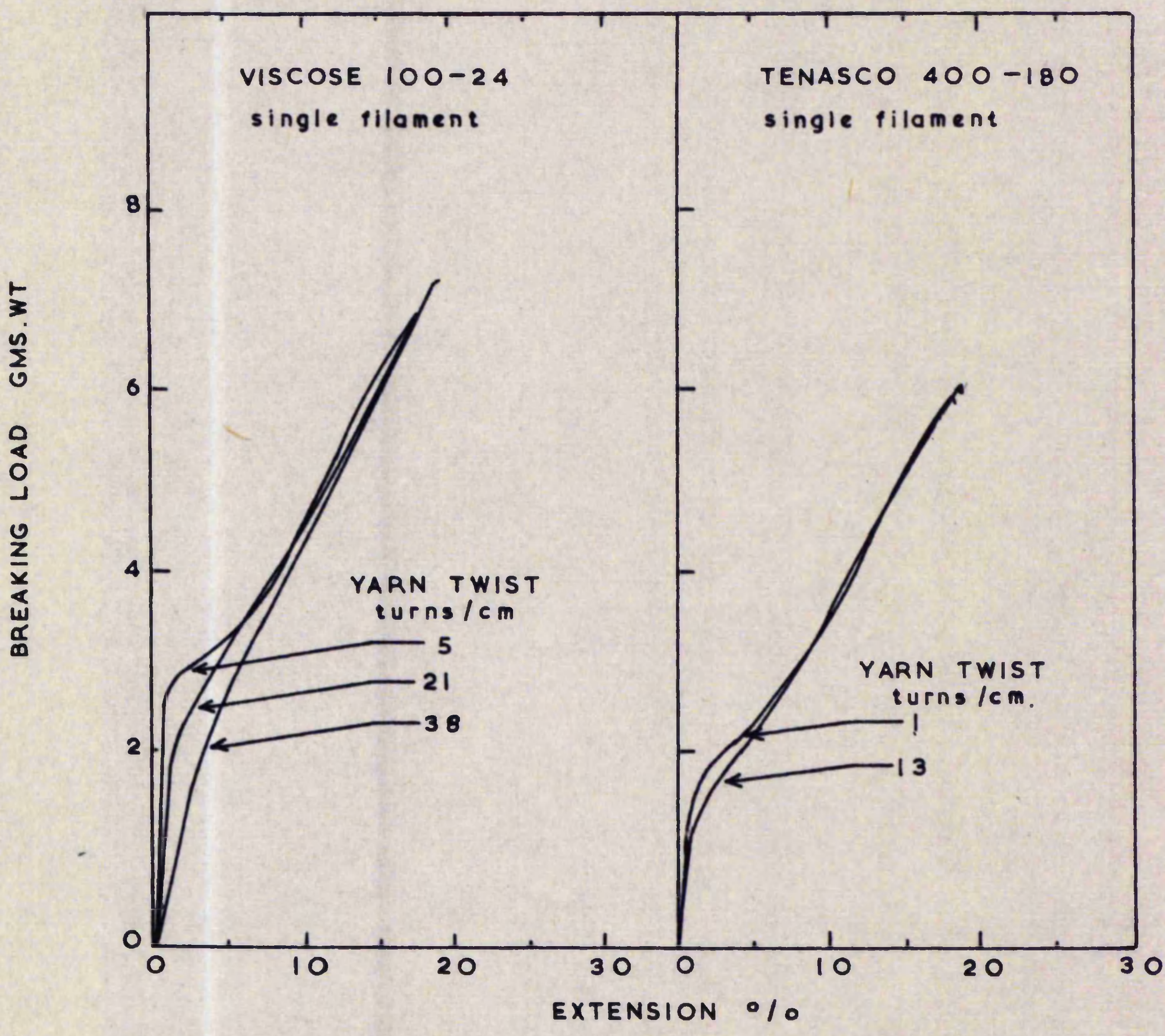


FIG. 4.42D_3 LOAD EXTENSION DIAGRAMS OF EXTRACTED FILAMENTS-VISCOSE AND TENASCO YARNS.

— INSTRON TESTS —

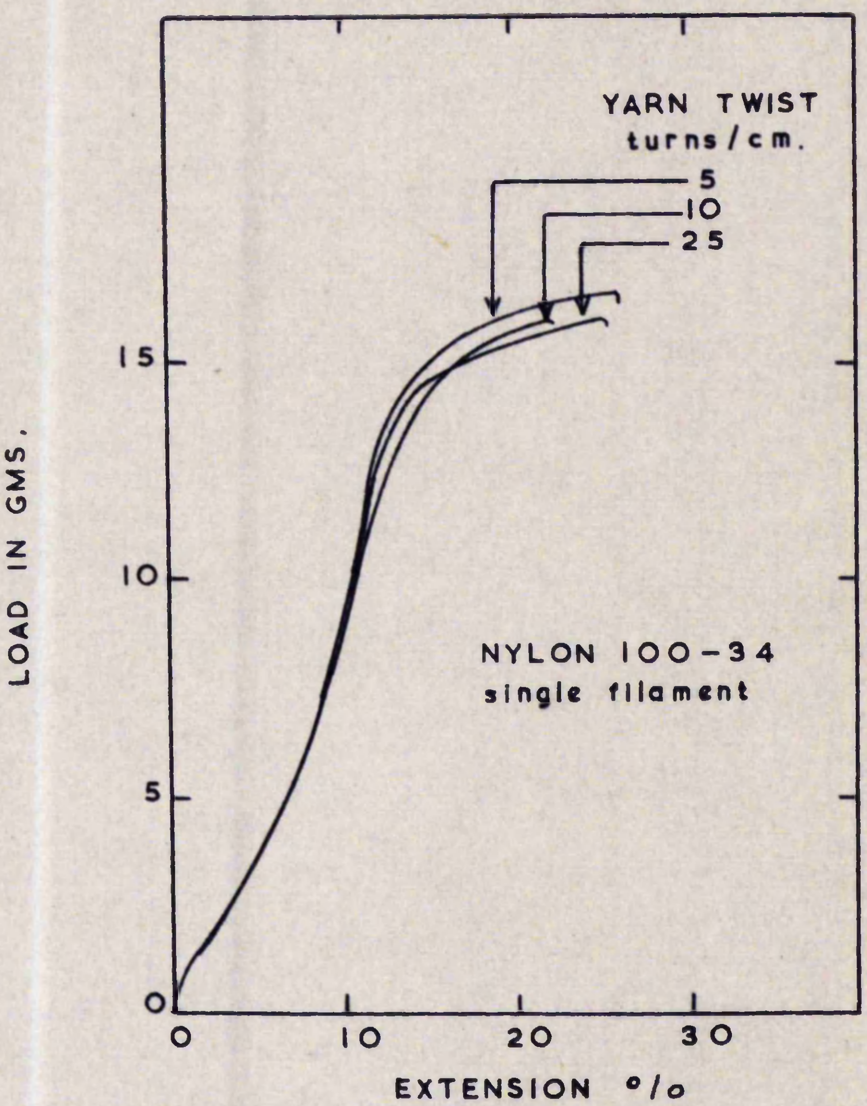


FIG. 4.42D$_4$ LOAD EXTENSION DIAGRAM OF EXTRACTED FILAMENTS - NYLON YARNS

— INSTRON TESTS —

high twists. The breaking extension values show a tendency to increase with twist.

(2) The load-extension diagrams of both twisted and extracted single filaments show the deviations in the bends after the limit of proportionality. This is especially so in acetate filaments (Fig. 4.42 D_1 & D_4).

(3) The breaking extension of single filaments extracted from twisted yarns show decreased extension at high twist factors of yarn (Fig. 4.42 A).

4.43 Discussion

From these results, it can be seen that the twist in single filaments will have very little effect on their rupture properties. Their load extension curves show that viscose and acetate yarns are more affected by the forces applied during twisting. This resulted in decreased initial modulus and less sharp bends in the curves at the limit of proportionality.

In the tests with the extracted filaments, the breaking extension of filaments extracted from highly twisted yarns, especially acetate and viscose rayon, is influenced by the crimp in those filaments.

CHAPTER V

Theoretical Developments

5.1 INTRODUCTION

As discussed in chapter I, the present day theoretical knowledge is limited by certain assumptions made to simplify the theoretical approach. In this chapter the attempt is made to develop the present theories on the prediction of yarn tenacity by considering the effects of

(i) Yarn lateral contraction,

(ii) Large extension values,

and (iii) Both compressive and tensile forces when Hooke's law ceases to hold.

5.2 RELATION OF YARN EXTENSION TO FILAMENT EXTENSION

In practice, the yarn extension at break is found to be of the order of 10 to 30 per cent. The simple relation of yarn extension to that of the filament in the yarn structure is given as

$$\epsilon_f = \epsilon_y \cos^2\theta \qquad \text{......... (5.2a)}$$

where,

ϵ_f = Filament extension

ϵ_y = Yarn extension

θ = The filament helix angle.

Platt et al have analytically shown that this relation is a good approximation when θ and ϵ_y are small.

Moreover, the yarn diameter is found to decrease as the load is increased. For materials having high breaking extension, the yarn diameter at break may be estimated from the measured initial yarn diameter and a factor based on the law of constant volume deformation.

Where both θ and ϵ_y are large, the simple relation (5.2a) will introduce large errors in the determination of filament extension from the yarn extension.

5.21 Geometry of helical yarn structure.

Figure (5.21A) represents the opened out helix from an idealised yarn structure showing

R_o	Helix radius prior to yarn extension
R_b	Helix radius at break
h_o	Yarn length per turn of twist before extension
h_b	" " " " " " at break
θ_o	Helix angle of filament before extension
θ_b	" " " " at break
l_o	Length of filament before extension
l_b	" " " at break.

By definition,

$$\text{Yarn extension } \epsilon_y = \frac{h_b - h_o}{h_o} \qquad \text{.......(5.21a)}$$

and

$$\text{Filament extension } \epsilon_f = \frac{l_b - l_o}{l_o} \qquad \text{.......(5.21b)}$$

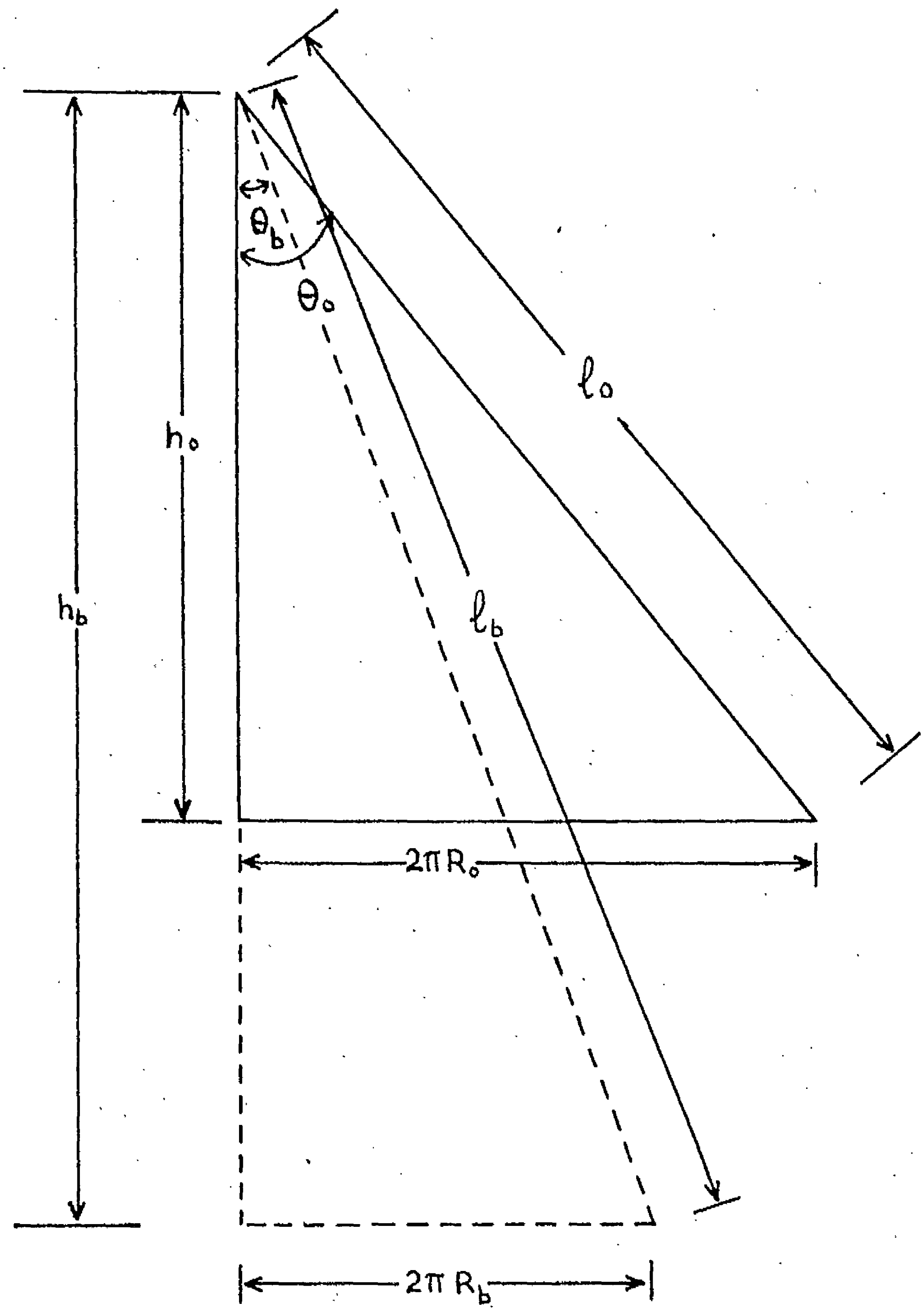


FIG. 5.21A OPENED OUT HELIX FROM AN IDEALISED YARN STRUCTURE.

and, by geometry

$$l_o^2 = h_o^2 + 4\pi^2 R_o^2 \qquad \text{.......... (5.21c)}$$

$$l_b = h_b \sec \theta_b \qquad \text{............ (5.21d)}$$

$$l_o = h_o \sec \theta_o \qquad \text{............ (5.21e)}$$

and

$$\frac{\tan\theta_o}{\tan\theta_b} = \frac{R_o}{R_b} \cdot \frac{h_b}{h_o} \qquad \text{............ (5.21f)}$$

5.22 Yarn contraction ratio.

As the yarn is extended from the length h_o to h_b, the yarn diameter will decrease from R_o to R_b. This yarn deformation behaviour may be mathematically defined by a parameter analogous to a Poisson's ratio:

$$\text{Yarn lateral contraction ratio, } \sigma_y = -\frac{(R_b - R_o)/R_o}{(h_b - h_o)/h_o}$$

$$\text{........ (5.22a)}$$

From equations (5.21a) and (5.22a)

$$\frac{R_b}{R_o} = 1 - \sigma_y \, \epsilon_y \qquad \text{........ (5.22b)}$$

For constant volume deformation,

$$\pi R_b^2 h_b = \pi R_o^2 h_o \qquad \text{........... (5.22c)}$$

or $$\frac{R_b}{R_o} = (1 + \epsilon y)^{-\frac{1}{2}}$$

when y is small this can be expanded and expressed by the approximation:

$$\frac{R_b}{R_o} = 1 - \tfrac{1}{2}\epsilon y \qquad \text{........ (5.22d)}$$

This is a special case of equation (5.22b) with $\sigma y = 0.5$.

By introducing this factor σy in the geometrical relation (5.21f) and by equation (5.21a)

$$\frac{\tan\theta_o}{\tan\theta_b} = \frac{1 + \epsilon y}{1 - \sigma y\, \epsilon y} \qquad \text{........ (5.22e)}$$

5.23 Filament extension

The filament extension ϵf can be related to the yarn breaking extension ϵy, by a function of yarn contraction ratio σy and a filament helix angle θ_o or θ_b. From equations (5.21 a, b, d and e)

$$\epsilon_f = (1 + \epsilon_y)\,\frac{\sec\theta_b}{\sec\theta_o} - 1 \qquad \text{........(5.23a)}$$

Equation (5.23a) can be expressed in terms of θ_o or θ_b as

$$\epsilon_f = (1 + \epsilon_y)\,\sin\theta_o \left[\cot^2\theta_o + \frac{\tan^2\theta_b}{\tan^2\theta_o}\right]^{\frac{1}{2}} - 1$$

or $$\epsilon_f = (1 + \epsilon_y) \left[\frac{\cot^2\theta_b + 1}{\cot^2\theta_b + \dfrac{\tan^2\theta_o}{\tan^2\theta_b}}\right]^{\frac{1}{2}} - 1$$

By simplification and substituting from equation (5.22e)

$$\epsilon_f = (1 + \epsilon_y) \left[1 - \frac{m^2 - 1}{m^2} \sin^2\theta_o\right]^{\frac{1}{2}} - 1$$

.... (5.23b)

or

$$\epsilon_f = (1 + \epsilon_y) \left[1 + (m^2 - 1) \sin^2\theta_b\right]^{-\frac{1}{2}} - 1$$

........(5.23c)

where $m = \dfrac{1 + \epsilon_y}{1 - \sigma_y \epsilon_y}$

As shown in the Appendix, equation (5.23c) can be expanded and approximated by neglecting the terms containing ϵ_y^3 and higher powers of ϵ_y:

$$\epsilon_f = \epsilon_y \left[\cos^2\theta_b - \sigma_y \sin^2\theta_b\right] - \frac{3}{2} \epsilon_y^2 (1 + \sigma_y)^2 \sin^2\theta_b \cos^2\theta_b$$

--------(5.23d)

and when ϵ_y is small

$$\epsilon_f = \epsilon_y \left[\cos^2\theta_b - \quad y \sin^2\theta_b\right]$$

.... (5.23e)

It can similarly be shown that equation (5.23b) can be expanded to obtain equation (5.23e) where $\theta_b = \theta_o$.

If $\sigma_y = \frac{1}{2}$, equation (5.23e) reduces to an expression given by Platt, namely

$$\epsilon_f = \frac{\epsilon_y}{2} \left[3 \cos^2\theta_o - 1\right]$$

.......(5.23f)

when ϵ_y is small and $\sigma_y = o$, equation (5.23d) reduces to the simple relation, $\epsilon_f = \epsilon_y \cos^2\theta$

Thus the simplified relation ($\epsilon_y \cos^2\theta$) may not introduce much error when ϵ_y is very small and the yarn lateral contraction ratio σ_y is zero. However, when both ϵ_y and σ_y are large, the filament extension obtained by equation (5.23d) will be less than that usually calculated by simplified relation. Table (5.23A) and Figure (5.23A) will show the magnitude of errors where such simplifications are assumed.

5.24 Filament extension - alternative method

The relation of filament extension to that of yarn can also be obtained by an alternative method. Yarn contraction ratio can be defined as

$$\sigma_y = - \frac{dr/r}{dh/h} \qquad \ldots\ldots\ (5.24a)$$

Differentiating the geometrical relation (5.21e), we get $2\,\ell\, d\ell = 2\,h\,dh + 8\pi^2\,r\,dr$ multiplying both sides by $\frac{1}{\ell^2}$ and by simplifications

$$\frac{d\ell}{\ell} = \frac{dh}{h}\;\frac{h^2}{\ell^2} + \frac{4\pi^2 r^2}{\ell^2}\;\frac{dr}{r} \qquad \ldots\ldots\ (5.24b).$$

From equation (5.24a) and by definition

$$\epsilon_f = \epsilon_y \left[\cos^2\theta - \sigma_y \sin^2\theta\right] \qquad \ldots\ldots\ (5.24c)$$

TABLE 5.23A

Relation of yarn extension to filament extension

	σ_y=0.00 EQUATION NOS.				σ_y=0.10 EQUATION NOS.				σ_y=0.50 EQUATION NOS			
	5.23c	5.23d	5.23e	5.23b	5.23c	5.23d	5.23e	5.23b	5.23c	5.23d	5.23e	5.23b
	ϵ_y= 0.10											
0°	.100	.100	.100	.100	.100	.100	.100	.100	.100	.100	.100	.100
10°	.0963	.0965	.0969	.0969	.0959	.0962	.0967	.0968	.0944	.0956	.0955	.0959
20°	.0873	.0878	.0883	.0888	.0848	.0852	.0871	.0879	.0784	.0790	.0825	.0835
30°	.0717	.0722	.0750	.0762	.0691	.0691	.0725	.0737	.0558	.0562	.0625	.0647
40°	.0551	.0551	.0587	.0598	.0502	.0502	.0546	.0561	.0298	.0298	.0380	.0407
	ϵ_y = 0.30											
0°	0.300	0.300	.300	.300	.300	.3000	.300	.300	.300	.300	.300	.300
10°	.2876	.2905	.2909	.2918	.2845	.2852	.2900	.2904	.2744	.2775	.2864	.2888
20°	.2502	.2510	.2649	.2684	.2434	.2445	.2614	.2650	.2084	.2159	.2473	.2538
30°	.2004	.1997	.2250	.2316	.1871	.1869	.2175	.2248	.1253	.1306	.1875	.2032
40°	.1467	.1433	.1760	.1856	.1271	.1241	.1637	.1748	.0432	.0405	.1141	.1136

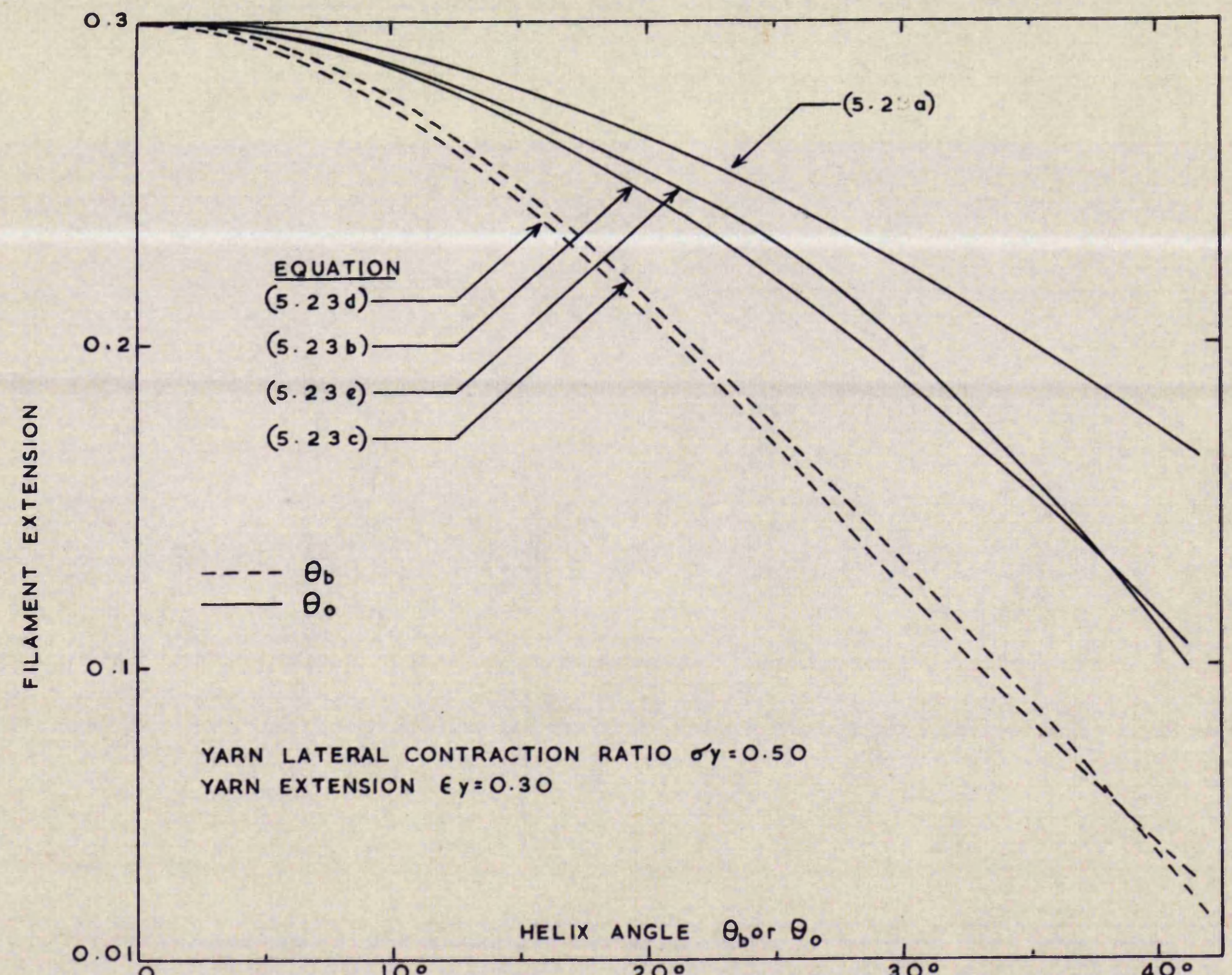


FIG. 5.23A RELATION OF YARN EXTENSION TO FILAMENT EXTENSION.

5.3 YARN TENACITY - INFLUENCE OF TENSILE FORCES ONLY

The fibre ~~[illegible]~~-stress-strain behaviour follows a linear or parabolic relation in the region of failure. When it follows a linear relation the specific stress X can be given as

$$X = A + B\,\epsilon_f \qquad \ldots\ldots\ldots\ldots (5.3a)$$

where A and B are constants.

ϵ_f = the filament extension determined by yarn extension ϵ_y.

From the relation (5.23d) (5.3a)

$$X = A + B\,\epsilon_y \left[\cos^2\theta_b - \sigma_y \sin^2\theta_b\right]$$

$$- B\,\epsilon_y^2 \;\frac{3}{2}\; (1 + \sigma_y)^2 \sin^2\theta_b \;\cos^2\theta_b \qquad \ldots\ldots\ldots (5.3b)$$

Consider an element of yarn cross-section, lying between radii r and $(r + dr)$. the component of the axial specific stress X contributing to the yarn stress can be given as,

$$\text{Yarn tension} = X\,\frac{2\pi r\;dr\;\cos\theta_b}{v}\;\cos\theta_b \qquad \ldots\ldots (5.3c)$$

where v = specific volume of yarn

and yarn tenacity $$Y = \int_0^{\text{yarn radius at break}} \frac{\text{yarn tension}}{\text{cross sectional area of annular ring}}$$

From equations (5.3b and 5.3c)

$$Y = \frac{V}{2\pi R_b^2}\int_0^{R_b} \frac{2\pi}{V}\left\{A + B\epsilon_y\left[\cos^2\theta_b - 6_y \sin^2\theta_b\right] - B\epsilon_y^2 \cdot \frac{3}{2}(1+6_y)^2 \sin^2\theta_b \cdot \cos^2\theta_b\right\}$$

$$\left\{\cos^2\theta_b \cdot r \cdot dr\right\} \quad \ldots\ldots\ldots (5.3d)$$

Integral equations (5.3d) can be expressed in terms of a variable quantity x defined as a ratio

i.e. $x = \dfrac{\text{Length of filament in helix radii } r}{\text{Length of filament at the surface of yarn}}$

and constant C = cosine of the surface helix angle, α

Hence

$$\cos^2\theta_b = \frac{c^2}{x^2}, \quad \sin^2\theta_b = 1 - \frac{c^2}{x^2}, \quad r\,dr = \frac{L^2 x \cdot dx}{4\pi^2}$$

and $\dfrac{L^2}{4\pi^2 r^2} = \dfrac{1}{1-c^2}$

and
$$Y = \frac{2c^2}{1-c^2}\int_c^1 A \cdot \frac{dx}{x} + B\epsilon_y\left[\frac{c^2}{x^2} - 6_y\left(1 - \frac{c^2}{x^2}\right)\right]\frac{dx}{x}$$
$$- B\epsilon_y^2 \cdot \frac{3}{2}(1+6_y)^2\left[\frac{c^2}{x^2}\left(1 - \frac{c^2}{x^2}\right)\right] \cdot \frac{dx}{x}$$

$$= \frac{2c^2}{1-c^2}\left\{A \cdot \log_e x + B \cdot \epsilon_y c^2\left[\frac{1}{-2x^2} + \frac{6_y}{-2x^2}\right] - B\epsilon_y 6_y \log_e x\right.$$
$$\left. - B\epsilon_y^2 \cdot \frac{3}{2}(1+6_y)^2 c^2\left[\frac{1}{-2x^2} - \frac{c^2}{-4x^4}\right]\right\}_c^1$$

$$= \frac{c^2}{1-c^2}\left\{A\log_e\frac{1}{c^2} - B\epsilon_y\sigma_y\log_e\frac{1}{c^2} - B\epsilon_y c^2\left[1+\sigma_y - \frac{1+\sigma_y}{c^2}\right]\right.$$

$$\left. + \frac{3}{2}(1+\sigma_y)^2 \cdot B\epsilon_y \cdot c^2\left[\frac{2-c^2}{2} - \frac{1}{2c^2}\right]\right\}$$

$$= \frac{c^2}{1-c^2}\left\{A\log_e\frac{1}{c^2} - B\epsilon_y\left[\sigma_y\log_e\frac{1}{c^2} - (1+\sigma_y)(1-c^2)\right]\right.$$

$$\left. - \frac{3}{4}(1+\sigma_y)^2 \cdot B\epsilon_y^2(1-c^2)^2\right\}$$

or

$$\text{Yarn tenacity} = A\frac{\log_e \operatorname{Sec}^2\alpha}{\operatorname{Tan}^2\alpha} + B\epsilon_y\left[(1+\sigma_y)\operatorname{Cos}^2\alpha - \sigma_y\frac{\log_e\operatorname{Sec}^2\alpha}{\operatorname{Tan}^2\alpha}\right] - \frac{3}{4}B\epsilon_y^2(1+\sigma_y)^2\sin^4\alpha \quad \text{......... (5.3e)}$$

special cases of equation (5.3e)

(i) when yarn contraction ratio $\sigma_y = o$

$$\text{yarn tenacity} = A\frac{\log_e\operatorname{Sec}^2\alpha}{\operatorname{Tan}^2\alpha} + B\epsilon_y\operatorname{Cos}^2\alpha - \frac{3}{4}B\epsilon_y^2\sin^4\alpha \quad \text{......... (5.3f)}$$

(ii) when yarn extension at break ϵy is small

$$\text{yarn tenacity} = A\,\frac{\log_e \sec^2\alpha}{\tan^2\alpha} + B\,\epsilon_y\left[(1+\sigma_y)\cos^2\alpha - \sigma_y\,\frac{\log_e \sec^2\alpha}{\tan^2\alpha}\right] \qquad \text{......(5.3g)}$$

(iii) when ϵy is small and $\sigma y = o$

$$\text{yarn tenacity} = A\,\frac{\log_e \sec^2\alpha}{\tan^2\alpha} + B\,\epsilon_y \cos^2\alpha \qquad \text{......(5.3h)}$$

Equation (5.3h) is the one reported by Platt.[15]

(iv) when ϵy is small, $\sigma y = o$ and intercept $A = 0$

$$\text{yarn tenacity} = B\epsilon y \cos^2\alpha \qquad \text{......(5.3i)}$$

This relation is the one given by Gegauf,[13] Platt[15] and others.

Figure 5.3A shows graphically the magnitude of error involved by such assumptions. As the change in intercept A is not affected by such alternative assumptions, the change in slope B has been plotted as a relative value.

5.4 YARN TENACITY – (Small extensions)

Influence of tensile and compressive forces

5.41 Compressive stress

Let the fibre stress-strain behaviour, under the influence of tensile stress X and compressive stresses G in two perpendicular directions take the following linear form in the rupture region:

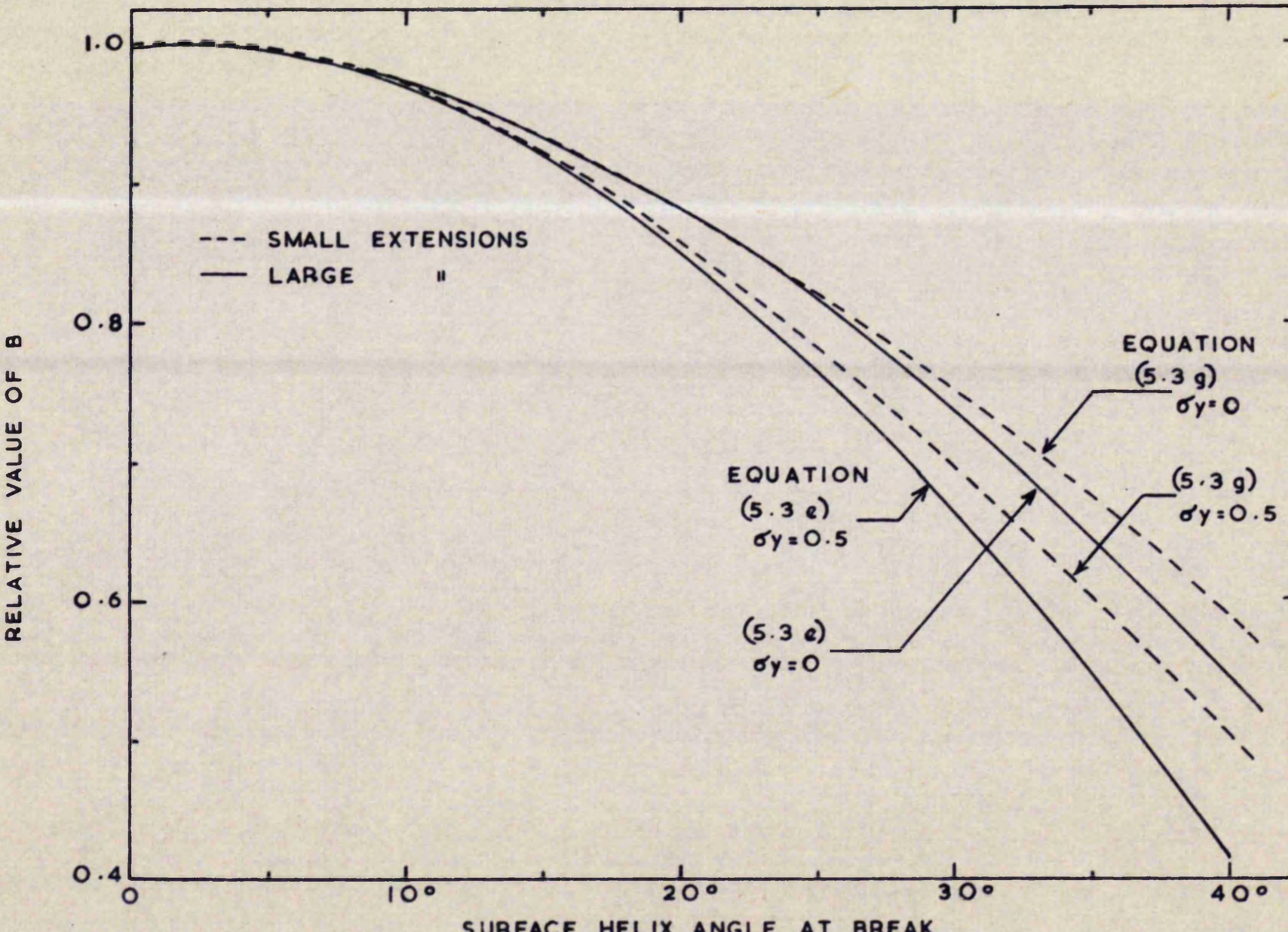


FIG. 5.3A RELATIVE VALUES OF SLOPE B OF A LINEAR STRESS-STRAIN RELATION NEAR BREAK.

$$X = a_1 \epsilon_f - 2 a_2 G + a_3 \qquad \text{.......} (5.41b)$$

where a_1, a_2 and a_3 are constants.

Substituting for value of ϵ_f from equation (5.23e)

$$X = a_1 \epsilon_y \left[\cos^2\theta_b - \sigma_y \sin^2\theta_b\right] - 2a_2 G + a_3$$

or

$$X = a_1 \epsilon_y \left\{ (1 + \sigma_y) \frac{c^2}{x^2} - \sigma_y \right\} - 2a_2 G + a_3 \qquad \text{....}(5.41c)$$

Hearle[18] has given (expression 40) the differential equation governing the radial equilibrium, namely

$$\frac{dG}{dx} = - \frac{X}{x} \qquad \text{.....}(5.41d)$$

substituting for specific stress X from equation (5.41c)

$$\frac{dG}{dx} - 2a_2 \frac{G}{x} = - \frac{a_1 \epsilon_y \left\{ (1 + \sigma_y) \frac{c^2}{x^2} - \sigma_y \right\} + a_3}{x}$$

This is the differential equation governing the radial equilibrium in the yarn. By multiplying both sides of this equation by the appropriate factor $\frac{1}{x^{2a_2}}$

$$\frac{1}{x^{2a_2}} \cdot \frac{dG}{dx} - \frac{2a_2 G}{x^{1+2a_2}} = \frac{a_1 \epsilon_y \left\{ (1 + \sigma_y) \frac{c^2}{x^2} - \sigma_y \right\} + a_3}{x^{1+2a_2}}$$

solving the differential equation

$$\frac{d}{dx}\left[\frac{G}{x^{2a_2}}\right] = - \frac{a_1 \epsilon_y \left\{ (1 + \sigma_y) \frac{c^2}{x^2} - \sigma_y \right\} + a_3}{x^{1+2a_2}}$$

or
$$\frac{G}{x^{2a_2}} = -\int \frac{-a_1 \epsilon_y \left\{(1+\sigma_y)\frac{c^2}{x^2} - \sigma_y\right\} + a_3}{x^{1+2a_2}} \cdot dx$$

$$= \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot \frac{c^2}{x^{2+2a_2}} - \frac{a_1 \epsilon_y \sigma_y - a_3}{2a^2} \cdot \frac{1}{x^{2a_2}} + K$$

where K is the constant of integration

and
$$G = \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot \frac{c^2}{x^2} - \frac{a_1 \sigma_y \epsilon_y - a_3}{2a_2} + K \cdot x^{2a_2}$$

At the surface of the yarn $x = 1$ and $G = 0$

$$0 = \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot c^2 - \frac{a_1 \sigma_y \epsilon_y - a_3}{2a_2} + K$$

or

$$K = \frac{a_1 \sigma_y \epsilon_y - a_3}{2a_2} - \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot c^2$$

consequently

$$G = \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot \frac{c^2}{x^2} \left\{1 - x^{2+2a_2}\right\}$$

$$- \frac{a_1 \sigma_y \epsilon_y - a_3}{2a_2} \left\{1 - x^{2a_2}\right\} \quad \text{.......... (5.41e)}$$

5.42 Yarn tenacity

Consider a yarn element lying between radii r and $(r + dr)$ in the yarn cross-section. The forces contributing to the yarn tension are:

(i) component of axial stress X parallel to yarn axis

(ii) component of the tangential stress G parallel to the yarn axis.

Thus,

$$\text{Yarn tension} = X\frac{2\pi r dr}{v}\cdot\cos^2\theta - G\frac{2\pi r dr}{v}\cdot\sin\theta\cdot\sin\theta$$

or

$$= \frac{L^2}{2\pi v}\left[X\cdot\frac{c^2}{x^2} - G\left(1-\frac{c^2}{x^2}\right)\right]x\,dx \qquad \text{....(5.42a)}$$

where

$x = \frac{l}{L}$, $c = \cos\alpha$ $\frac{c}{x} = \cos\theta$, $r dr = \frac{L^2 x\,dx}{4\pi^2}$

l = length of filament in helical path for 1 turn of twist

L = value of l at the surface of yarn

α = surface helix angle at break

But

$$\text{Yarn tenacity} = \frac{\text{yarn tension}}{\text{yarn cross sectional area}}$$

$$Y = \frac{Y \cdot L^2}{\pi R^2 \cdot 2\pi Y} \int_c^1 \left[X \cdot \frac{c^2}{x^2} - G\left(1 - \frac{c^2}{x^2}\right) \right] x \, dx$$

Since, $\frac{L^2}{4\pi R^2} = \frac{1}{1-c^2}$

$$Y = \frac{2}{1-c^2} \int_c^1 \left[X \cdot \frac{c^2}{x^2} - G\left(1 - \frac{c^2}{x^2}\right) \right] x \cdot dx$$

or

$$Y = \frac{2}{1-c^2} \int_c^1 \left\{ \left[a_1 \epsilon_y \left[(1+\sigma_y) \frac{c^2}{x^2} - \sigma_y \right] - 2a_2 G + a_3 \right] \frac{c^2}{x^2} - G\left(1 - \frac{c^2}{x^2}\right) \right\} x \, dx$$

Re-arranging

$$Y = \frac{2}{1-c^2} \int_c^1 \left\{ a_1 \epsilon_y (1+\sigma_y) \frac{c^4}{x^4} - a_1 \epsilon_y \sigma_y \frac{c^2}{x^2} + a_3 \frac{c^2}{x^2} - G\left[2a_2 \cdot \frac{c^2}{x^2} + 1 - \frac{c^2}{x^2} \right] \right\} x \, dx$$

Substituting for G from equation (5.41c)

$$Y = \frac{2c^2}{1-c^2} \int_c^1 a_1 \epsilon_y (1+\sigma_y) \frac{c^2}{x^3} - \frac{a_1 \epsilon_y \sigma_y - a_3}{x}$$

$$- \left[\frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \cdot \frac{c^2}{x^2} \left(1 - x^{2+2a_2}\right) - \frac{a_1 \epsilon_y \sigma_y - a_3}{2a_2} \left(1 - x^{2a_2}\right) \right]$$

$$\left[\frac{2a_2 - 1}{x} + \frac{x}{c^2} \right] dx$$

Rearranging,

$$Y = \frac{2c^2}{1-c^2}\int_c^1 \left\{ \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \left[2(1+a_2)\frac{c^2}{x^3} + \frac{c^2}{x^3}(2a_2-1) \right.\right.$$

$$\left. + c^2(2a_2-1)x^{2a_2-1} + x^{2a_2+1} - \frac{1}{x} \right] - \frac{a_1 \epsilon_y \sigma_y - a_3}{2a_2}\left[2a_2 \cdot \frac{1}{x} \right.$$

$$\left.\left. - \frac{2a_2-1}{x} + (2a_2-1)x^{2a_2-1} + \frac{x^{2a_2+1}}{c^2} \right] \right\} dx$$

Simplifying,

$$= \frac{2c^2}{1-c^2}\int_c^1 \left\{ \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \left[3\frac{c^2}{x^3} + (2a_2-1)c^2 x^{2a_2-1} \right.\right.$$

$$\left. + x^{2a_2+1} - \frac{1}{x} \right] - \frac{a_1 \epsilon_y \sigma_y - a_3}{2a_2}\left[\frac{1}{x} - \frac{x}{c^2} + (2a_2-1)x^{2a_2-1} \right.$$

$$\left.\left. + \frac{x^{2a_2+1}}{c^2} \right] \right\} dx$$

Integrating,

$$Y = \frac{2c^2}{1-c^2}\left\{ \frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)} \left[-\frac{3}{2}\frac{c^2}{x^2} + \frac{2a_2-1}{2a_2}\cdot x^{2a_2} c^2 \right.\right.$$

$$\left. + \frac{x^{2a_2+2}}{2a_2+2} - \log_e x \right]_c^1 - \frac{a_1 \epsilon_y \sigma_y - a_3}{2a_2}\left[\log_e x - \frac{x^2}{2c^2} \right.$$

$$\left.\left. + \frac{2a_2-1}{2a_2}\cdot x^{2a_2} + \frac{x^{2a_2+2}}{c^2(2+2a_2)} \right]_c^1 \right\}$$

$$= \frac{c^2}{1-c^2}\left\{\frac{a_1 \epsilon_y (1+\sigma_y)}{2(1+a_2)}\left[\frac{1}{1+a_2} - \frac{c^{2a_2+2}}{1+a_2} + \frac{2a_2-1}{a_2}c^2 - \frac{2a_2-1}{a_2}c^{2a_2+2}\right.\right.$$

$$\left. -3c^2+3+\text{Log}_e c^2\right] - \frac{a_1\epsilon_y\sigma_y - a_3}{2a_2}\left[\frac{1}{c^2(1+a_2)} - \frac{c^{2a_2+2}}{c^2(1+a_2)}\right.$$

$$\left.\left. + \frac{2a_2-1}{a_2} - \frac{2a_2-1}{a_2}c^{2a_2} - \frac{1}{c^2} + 1 + \text{Log}_e\frac{1}{c^2}\right]\right\}$$

Simplifying

$$= \frac{c^2}{1-c^2}\left\{\frac{a_1\epsilon_y(1+\sigma_y)}{2(1+a_2)}\left[\frac{4+3a_2}{1+a_2} - \frac{2a_2^2+2a_2-1}{a_2(1+a_2)}c^{2a_2+2}\right.\right.$$

$$\left. - \frac{1+a_2}{a^2}c^2 + \text{Log}_e c^2\right] - \frac{a_1\epsilon_y\sigma_y - a_3}{2a_2}\left[\frac{3a_2-1}{a_2} - \frac{a_2}{1+a_2}\cdot\frac{1}{c^2}\right.$$

$$\left.\left. - \frac{2a_2^2+2a_2-1}{a_2(1+a_2)}c^{2a_2} + \text{Log}_e\frac{1}{c^2}\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.42b)$$

Equation (5.42b) can be expressed as

$$Y = f_1(c, a_1, a_2, a_3, \sigma_y) + f_2(c, a_1, a_2, \sigma_y)\epsilon_y$$

$$\text{i.e. } Y = \frac{a_3}{2a_2}\left[\frac{3a_2-1}{a_2} - \frac{a_2}{1+a_2}\cdot\frac{1}{c^2} - \frac{2a_2^2+2a_2-1}{a_2(1+a_2)}c^{2a_2} + \text{Log}_e c^2\right]$$

Equation (5.42a) can be expressed as

$$+ \epsilon_y \left\{ \frac{a_1(1+\sigma_y)}{2(1+a_2)} \left[\frac{4+3a_2}{1+a_2} - \frac{2a_2^2+2a_2-1}{a_2(1+a_2)} c^{2a_2+2} + \log_e c^2 - \frac{1+a_2 c^2}{a_2} \right] \right.$$

$$\left. - \frac{a_1 \sigma_y}{2a_2} \left[\frac{3a_2-1}{a_2} - \frac{a_2}{1+a_2} \cdot \frac{1}{c^2} - \frac{2a_2^2+2a_2-1}{a_2(1+a_2)} c^{2a} + \log_e c^2 \right] \right\}$$

or

$$Y = a_3 f_1(c, a_2) + a_1 \epsilon_y \left[(1+\sigma_y) f_2(c, a_2) - \sigma_y f_1(c, a_2) \right] \qquad \ldots\ldots\ldots\ldots (5.42c)$$

Special cases of equation (5.42c)

(i) when yarn contraction ratio $\sigma_y = 0$

$$Y_{\sigma_y=0} = a_3 f_1(c, a_2) + a_1 \epsilon_y f_2(c, a_2) \ldots\ldots\ldots\ldots (5.42d)$$

(ii) when stress-strain curve is not much different from one following Hooke's law relation to break.

Then $a_3 = 0$ and $a_1 = B$ and $a_2 = \sigma_1$ Poisson's ratio for the fibre.

$$Y = B \epsilon_y \left\{ (1+\sigma_y) f_3(c, \sigma_1) - \sigma_y f_4(c, \sigma_1) \right\} \ldots\ldots\ldots\ldots (5.42e)$$

Further if $\sigma_y = 0$ equation (5.42e) will reduce to one given by Hearle[18] (equation 57).

(iii) When yarn contraction ratio $\sigma_y = 0.5$

$$Y = a_3 f_1(c, a_2) + \frac{a_1 \epsilon_y}{2}\left[3 f_2(c, a_2) - f_1(c, a_2)\right] \qquad \text{.........} (5.42f)$$

In the practical application of this expression, constants a_1, a_2 and a_3 must be experimentally determined.

CHAPTER VI

6.1 INTRODUCTION

In this chapter the experimental results shown in chapter III are discussed in the light of the theoretical predictions given in chapter I and the further developments put forward in chapter V. The limitations of such theoretical predictions are also discussed in the light of the results obtained in the subsidiary experiments (chapter IV), the processing methods and conditions, the instrument characteristics, and structural parameters such as the contraction factor.

6.2 LOAD-EXTENSION DIAGRAM

The curves shown in figure 3.21B show qualitatively most of the features in the manner predicted in figure 1.33e. As the twist is increased, (i) the initial linear portions of the curves decrease in slope, (ii) the bends in the curves at the limit of proportionality become less sharp, (iii) above the limit of proportionality, the linear portions of the curves decrease in slope and (iv) at break the load suddenly drops to zero.

The bends in the curves at the limit of proportionality, become less sharp as twist increases. This is due to the fact that in a twisted yarn, the different filaments in a yarn cross-section reach the limit of proportionality over a range of yarn extensions. In other words, the outerfilaments may be following the initial linear portion of the stress-strain curve, while the centre filament has already passed the limit of proportionality.

The linear portion in the curves, after the bends and near break, decreases in slope as the twist is increased. This is due to the fact that, at a given yarn extension, ϵ_y, the load contribution to the twisted yarn, by the filaments forming the pseudocore structure will be lower than that contributed by the filaments forming the core structure. In a twisted yarn structure, the surface filaments will be contributing a proportionately lower load due to the maximum obliquity factor and the lower extension suffered by them. On the other hand, at a given yarn extension ϵ_y, all the filaments in the low twist yarns will be contributing their full share as the obliquity effect is low and all suffer the same extension as the yarn.

At very low twists (twist factors below 10 tex$^{\frac{1}{2}}$ turns/cm.) each filament is more or less independent of the others and breaks when its breaking extension is reached. In such a case, the load extension curve ends in a series of steps corresponding to the breakage of each filament as shown in figure (3.21A). This behaviour is seen prominently in nylon and Terylene yarns, but somewhat less so in viscose rayon and Tenasco yarns. However, for higher twists, break occurs sharply with the load suddenly dropping to zero. Transformation from separate breakage of individual filaments to the sharp break of the coherent twisted yarn, is an indication that the transverse forces are large enough to produce the friction needed to prevent the filaments slipping over one another.

6.21 Effect of twisting tension

The load-extension curve is also influenced by the twisting tension used during processing. The use of higher tensions during twisting appears to cause the occurrence of a higher modulus, higher stress value for the limit of proportionality and a lower breaking extension. The tenacity and the strain values of the limit of proportionality will be little affected. This behaviour is diagrammatically illustrated in figure (6.21A). If permanent deformation (OA) is assumed to occur, then the yarn processed under higher tension would follow a stress-strain curve ABC which on transferring back to the origin will show similar behaviour to that obtained in Instron test results (figure 3.22A).

The tension during twisting is obviously an important practical factor, which can be expected to have different effects on different materials, depending on their elastic behaviour. Immediately after twisting, the yarn is usually wound under tension onto some package such as cheese or a ring bobbin. Stress relaxation will occur on these packages, but the degree of the stress relaxation will depend upon the twisting and the winding tensions used and on the recovery

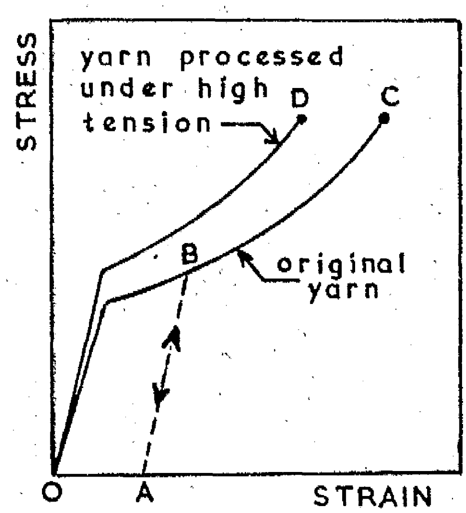


FIG. 6·21A EFFECT OF PROCESSING TENSION ON STRESS-STRAIN CURVE properties of the filaments. Thus it should be noted that the twisting tension used is a disturbing factor which must be borne in mind in considering how the properties vary with twist.

6.22 Linear relation near break

As shown in figures (3.23 B and C), the constants A and B of the linear stress-strain relation near break vary with twist. In most of the results both intercept 'A' and slope 'B' initially rise as the twist is increased and then falls. This behaviour is similar to that of the tenacity values and can be explained in the similar way as discussed in section (6.5).

When twisting tension is higher, the change in the intercept 'A' is very erratic. This may be expected as permanent deformation occurs. In figure 6.22A the relative values of slope 'B' for viscose rayon and Tenasco twisted yarns, are plotted against surface helix angle at break. As discussed in section (6.5), the absolute values are related to those at 10^{o} surface angle at break. This was useful in eliminating the variability factor. From figure 6.22A, it can be seen that, experimental points for Tenasco, lie below the theoretical curves, while those for viscose above it. The simple relation ($\cos^2\alpha$) gives less satisfactory agreement. The expression assuming influence of compressive forces show quite good agreement with experimental values. (Equation 5.42e)

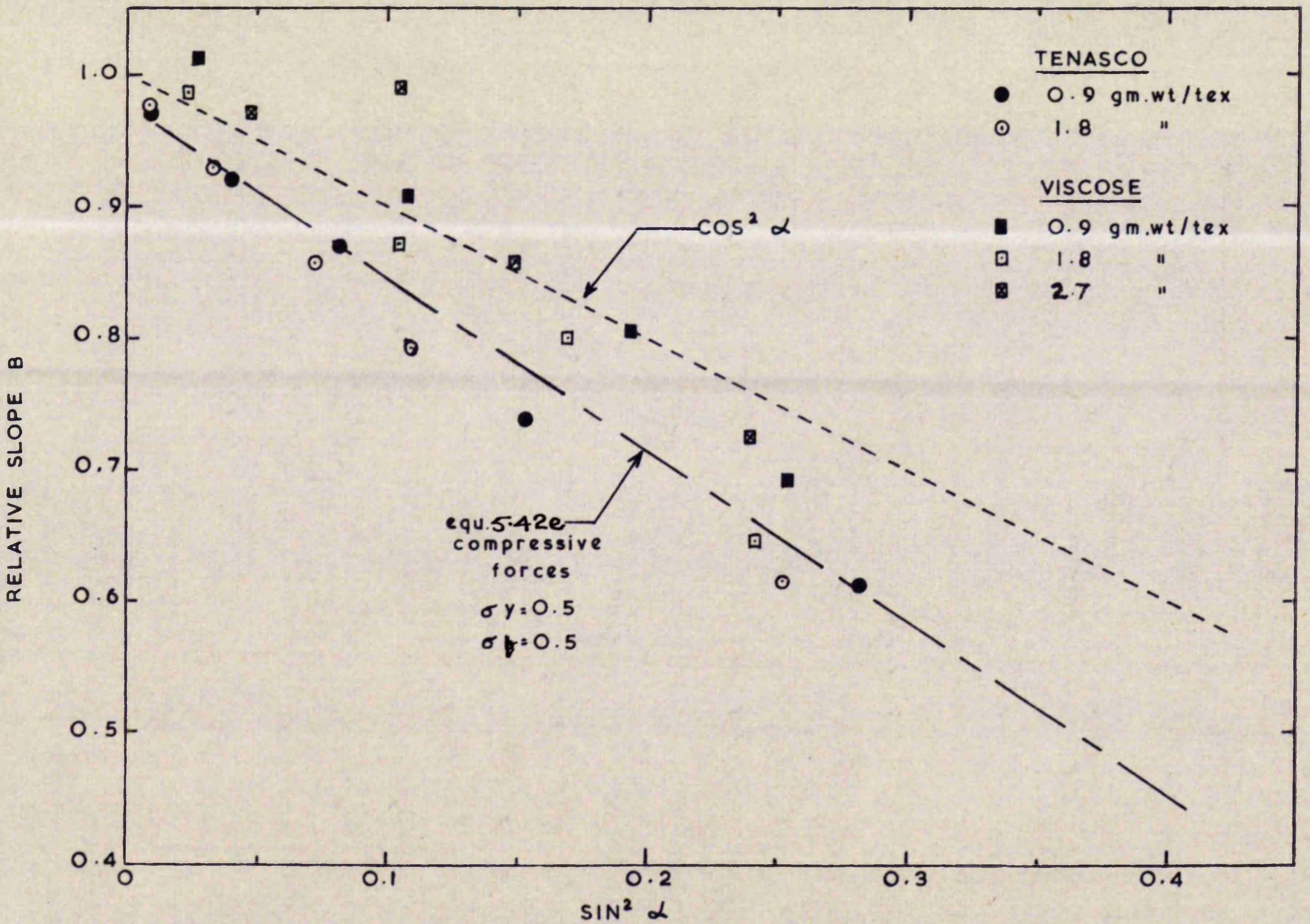


FIG. 6.22A EFFECT OF TWIST ON RELATIVE SLOPE B OF TENASCO 1650/750 & VISCOSE 300-100 YARNS.

6.3 YARN BREAKAGE

6.31 Breaks in long specimen lengths of yarns

The behaviour at break differs from what has been theoretically postulated (section 4.2). The theoretical view represented by figure (1.33D), assumes that once one filament has broken, it ceases to contribute to the yarn tension, but this assumption will not hold good in the actual yarn structure, where the filament migration, frictional forces and transverse forces are disturbing factors.

In the twisted yarn, break occurs almost instantaneously, even in the constant rate of extension tests. Oncethe break has started it becomes complete more rapidly than the instrument can detect. However, the behaviour of the load extension curve after the initiation of break, depends upon the sensitivity of the load measuring unit and the recorder, the specimen length, rate of extension etc.

The actual mechanism of break can be explained by the following picture: when the break is initiated, the broken filament ceases to support the load at the point of breakage, but provided there is some friction, it will still remain an effective part of the yarn at positions remote from the point of break. The actual mechanism of break can be demonstrated by figure (6.3A). The initiation of the break introduces an uneven distribution of stress. Near the

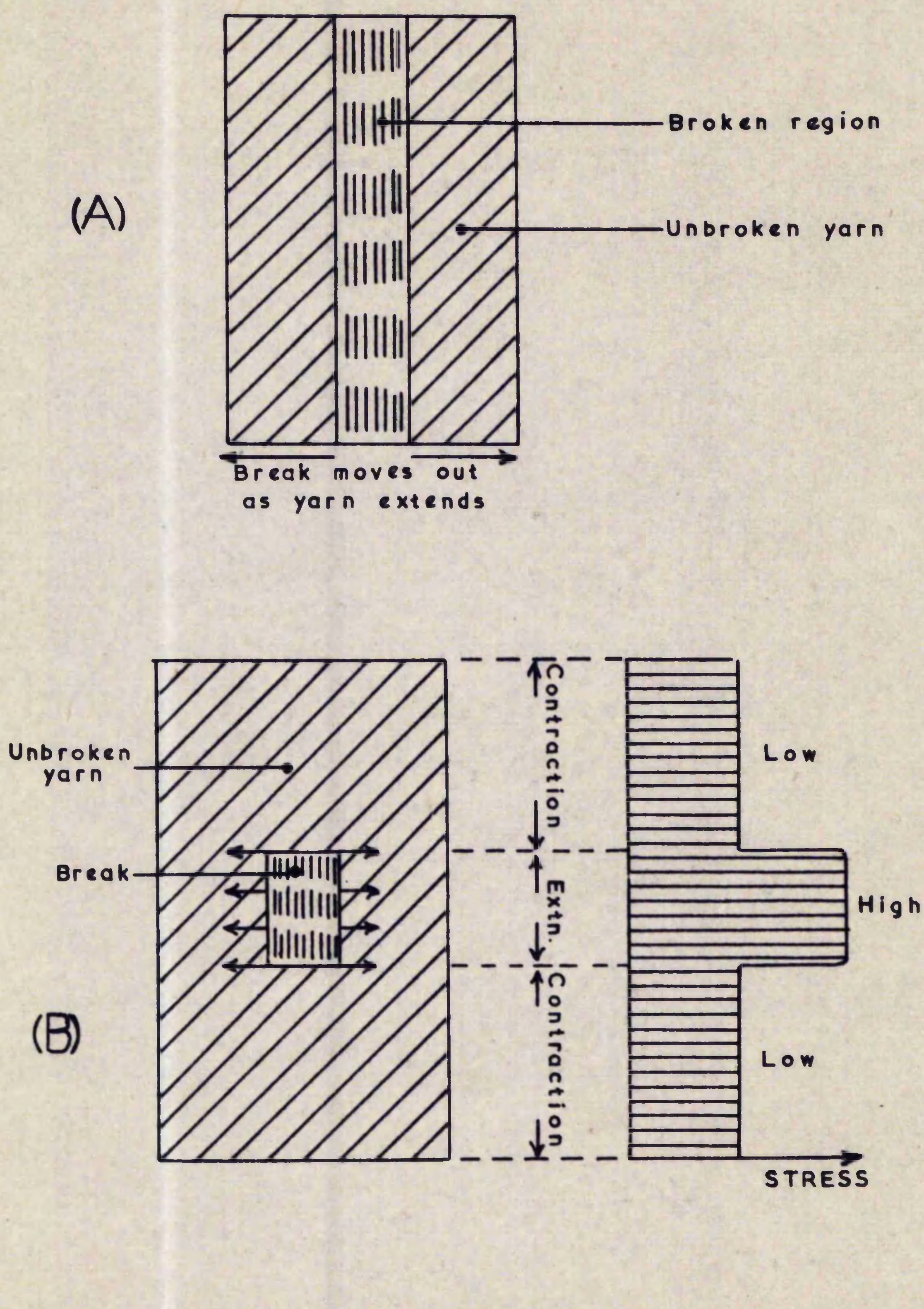


FIG. 6·3A MECHANISM OF BREAK

break, there will be less material to carry the load and the stress will be greater than in regions distant from the break. The total load which the yarn can bear must be reduced, and so the stress in the distant regions must fall and they will contract, further extending the breaking region and causing the break to continue until breakage is complete. Thus even in constant rate of elongation tests the breakage will be sharp.

6.32 Breaks in short specimen lengths of yarn

As reported in section 4.2, the picture of the breakage zone depends upon the gauge length used in the tensile tests at constant rate of extensions. When 1 cm. gauge lengths were used, the load extension diagram of Tenasco 1650-750-12 yarn, after break, ends in a series of steps at all rates of extensions (4%, 40% and 400% per min.) but when the gauge length was increased to 2.5 and 5.0 cm. the higher rates of extension show sharp break, while the lower rates of extension show a series of steps. This behaviour could be expected only if the above picture is the correct picture of the yarn break. As the gauge length is decreased, the instantanous contraction in the regions distant from the break will be proportionately less and so also the speed at which the mechanism of break continues to occur. Further, the fact that at 1 cm. gauge length the rate of extension has no effect on the percentage drop in load immediately after the initiation of break, indicates that the yarn breakage is not initiated by the failure of a single filament (central) but by a group of filaments forming core or surface structure.

From the photograph of a breakage region (fig. 4.22F), three thoughts regarding the breakage of this first group of filaments occur:

(a) Break starts in outside filaments and moves inwards - fig. 6.31 A (1);

(b) Break starts at core and then progresses in one preferred direction - fig. 6.31 A (2);

(c) The structure acts as one composed of separate units which break separately - fig. 6.31 A (3).

In any case, it seems, without doubt, that the mechanism of the breakage must be quite different than so far postulated by the simple idealised structure of the twisted yarns.

The load-extension diagram after the onset of break, shows the manner in which the yarn structure continues to fail (figures 4.22 A & B). This behaviour does not support the theoretical predictions[18]. It shows that the filaments fail as a group. The breakage is initiated by the failure of the largest group of filaments in the yarn cross-section. It would be very interesting to study the failure of this first group by the use of superspeed camera technique.

One curious effect observed is the fact that the breaking load decreases and the breaking extension increases as the gauge length is increased. This may be due to the uneven stress distribution during

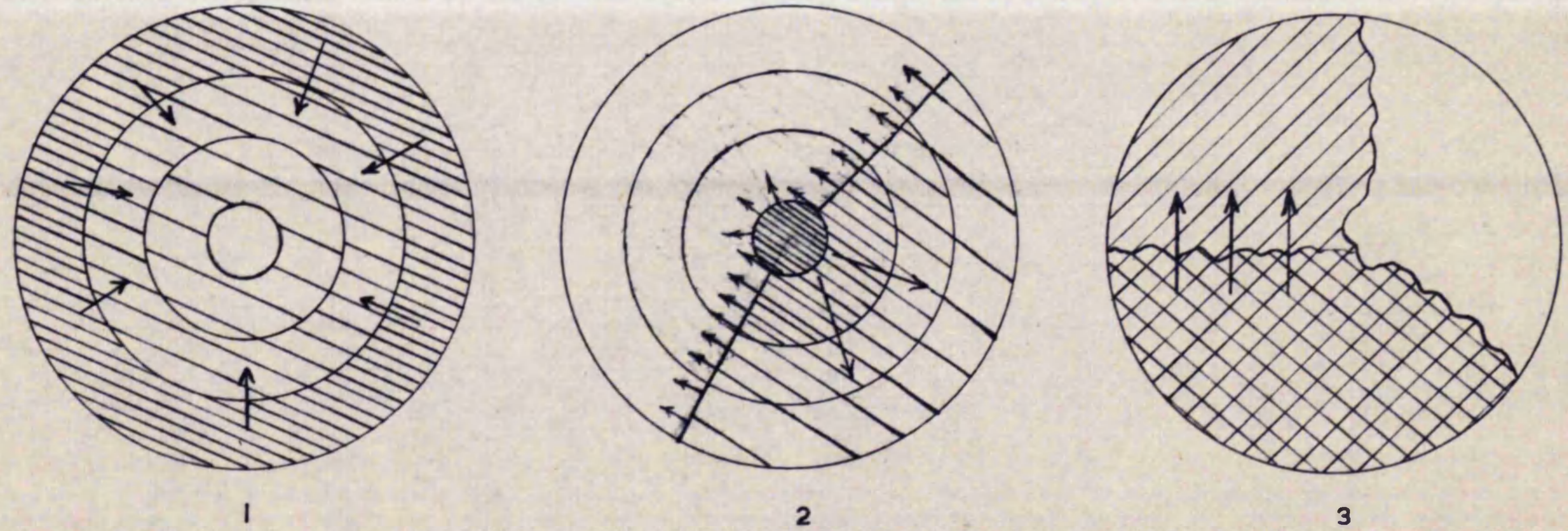


FIG. 6.31A THE PROGRESS OF BREAK ALONG THE YARN CROSS-SECTION.

BREAK IS INITIATED IN THE DARK SHADED REGION AND MOVES TOWARDS THE LESS DARK SHADED REGION.

extension or to the jaw effect. If the actual yarn structure is not far different from the idealised helical structure, the uneven stress distribution should not occur. The jaw effect will result in adding a fixed length to the actual yarn extension which will be independent of the gauge lengths used. This will increase the breaking extension of yarns as the gauge length is decreased but it will not affect the breaking load of yarns unless it results in the break at the jaw.

6.4 BREAKING EXTENSION

The results shown in chapter III confirm the observation of everyone[20,21,22,23] except Platt[15], that the breaking extension of most materials varies with twist. The values for viscose rayon and Tenasco show least variation with twist except for the sharp decrease in the viscose 75/75 yarn with 43^{o} twist angle. For acetate, the breaking extension is low for the low twist yarns, but increases to a maximum and then decreases as the twist is increased. This decrease is observed in acetate yarns with twist angle higher than 35^{o}. For nylon and Terylene yarns, the breaking extension increases continuously as the twist is increased. However, the results obtained from the Uster automatic strength tester shows initial decrease and then continuous increase in breaking extension of Terylene 100-48 yarns. This behaviour is not observed in the

Instron test results.

There are several possible explanations of these effects described in the following sections.

6.41 Influence of mutual support

The filaments constituting the yarn structure do not have identical properties. The extensions at break range over a distribution of values. In a zero twist yarn, each filament will break when its own breaking extension is reached. The maximum load will occur either as soon as the first, and least extensible filament, breaks or after a few filaments have broken. Hence the yarn breaking extension, taken as the extension corresponding to the maximum load value in the load extension diagram, will be low as compared to that of the last filament to break. This difference in yarn extension and the extension of the last filament to break depends upon the variability of the filaments. The within-yarn variability of filament extension at break is low for viscose rayon and Tenasco yarns while it is high for acetate, nylon and Terylene yarns.

As the twist is increased the breaking regions of the filaments will no more be distributed throughout the test length of the yarn, but will get localised at a single point in the yarn. Twist introduces mutual cohesion between the filaments. This will result in preventing the weak place in one filament extending more than the strong places in its near neighbours. This mutual supporting effect delays the initiation of break and in effect increases the breaking extension as the twist is increased. The influence of this effect will reach its full contribution as soon as the transverse forces are large enough to produce the friction needed to prevent the fibre slippage over one another.

This factor explains the initial rise in breaking extension of acetate, and some viscose rayon and Tenasco yarns. However, it fails to explain the continuous increase of extension in nylon and Terylene yarns and the decrease in acetate yarns with high twists. It also explains the behaviour of viscose yarns showing least variations at higher twists.

6.42 Influence of twist and compressive forces

If filaments are allowed to contract during twisting, this may increase their breaking extension (fig. 4.42c). But where the filament diameter is very much smaller than that of the yarn, the actual twist in the single filament is very low and insufficient to increase its breaking extension.

The filaments in the yarn are also under the influence of large transverse stresses. In the centre of the highly twisted yarn they may be about 1/3 of the value of the axial forces in the fibres[18]. These compressive forces may influence the filament breaking extension. As the magnitude of these compressive forces is minimum for a yarn element following outer structure, its breaking extension will be least affected. Thus in an extreme case the breakage may be initiated by surface filaments, but the complete failure of yarn structure will occur almost instantaneously. In practice, this is not so; the yarn breakage can be interrupted. No information is available on which to estimate the effect of such transverse stresses.

6.43 Influence of twisting conditions

The fibre properties may alter as a result of the forces imposed on the fibres during twisting. It is necessary to consider both the total twisting tension, and the distribution of tension between the filaments during twisting. The sum of the axial components of the tensions developed in different filaments is equal to the twisting tension. The tension experienced by the filaments will depend on their position in the yarn structure and on the twisting tension. It will be at a maximum in the filaments following the surface helix and at a minimum in the central filament. The filaments may suffer a permanent deformation if the twisting tension is high enough to extend the filaments beyond their elastic limits.

Two extreme structures that may result on twisting a parallel strand of filaments are as follows:

(A) a structure with perfect migration,

(B) a structure with no migration.

In either of these cases, the filaments may or may not be permanently deformed on twisting into yarn.

A) Perfect Migration

Perfect migration will only occur if the reaction to the changing conditions of individual filament tensions is instantaneous. If such a structure is obtained by twisting a parallel strand at higher tensions, two extreme structures can be considered: permanently deformed or buckled structure. Twisting tension may extend the filaments during yarn formation. This deformation may be of elastic or permanent form.

(i) On removing the twisting tension, if the deformation is completely non-recoverable, a permanently deformed structure will result. As all the filaments will suffer the same degree of deformation, the yarn breaking extension will be independent of twist and will be determined by the breaking extension of the central filament. The breaking extension of the central filament will be less than that of the filaments in zero twist yarn.

(ii) On removing the twisting tension, if the deformation is completely recovered, the buckled yarn structure will occur. Since all filaments in the yarn structure have suffered equal deformation during twisting, the central filament will have to get buckled to allow the recovery of deformation in the surface filament. The degree of buckling can be theoretically predicted from equation (5.23a). It will depend on both the deformation suffered during twisting and the recovery behaviour. This assumes the absence of inter fibre slippage resulting in redistribution of strain.

Such a buckled structure, with perfect migration pattern, will show higher breaking extensions as the twist is increased.

B) <u>No Migration</u>

If such a structure is obtained by twisting parallel strands of filament, buckling will always occur except in the cases where the recovery of deformation produced during twisting is insufficient. The behaviour of the structure with no migration can be studied by considering three structures: two extremes and one intermediary case.

(i) The structure where recovery of twisting deformation does not occur - no buckling.

(ii) The structure where perfect recovery of the twisting deformation and buckling occurs.

(iii) The structure where partial recovery of the twisting deformation occurs - no buckling.

The first case will result in the contraction factor equal to one which is not practically observed. In the second case, the yarn breaking extension will increase as the twist is increased. This increase will be due to the failure of surface filaments before that of the buckled core structure. While in the third case, the yarn breaking extension will be determined by the failure of the surface filaments.

Actual contraction factor may be taken as the measure of the deformations during twisting. Theoretical contraction factor $(\frac{\sec\alpha_o+1}{2})$ assumes perfect migration and complete strain recovery. However, if the migration is imperfect or absent, the fact that the actual contraction factor is equal to the theoretically predicted one, cannot be taken as the criteria of the complete strain recovery. A lower contraction factor will mean some permanent deformation.

The effect of twisting conditions on the yarn breaking extension can be theoretically explained by assuming buckled theoretical model as follows:

Consider a parallel strand of filaments with ϵ_b' as their breaking extensions. If such a strand is twisted to obtain a twisted yarn having surface helix angle α_o, the following three typical structures can be studied for its breaking extension behaviour:

(1) The central filament has recovered the twisting strain completely while the surface filaments have been permanently deformed. Here contraction factor $C_y = 1$

(2) The filaments following the average helix angle have recovered completely, the central filament has buckled while the surface filament has some permanent deformation. Here the contraction factor is one given by theoretical expression $C_y = \frac{\sec\alpha_o + 1}{2}$

(3) The filaments following the surface helix have recovered the twisting strain completely, while the whole of the inner structure buckled. The maximum buckling occurs in the central core structure.

Here the contraction factor $C_y = \operatorname{Sec}\alpha_o$

If appropriate values of C_y are substituted, the same general expression can be given for calculating the yarn breaking extension in each of these cases.

From figure (6.43A);

If the surface filament is to initiate the break.

$$\epsilon_y = \frac{(1+\epsilon_b) L_o \cos\alpha_b - L_c}{L_c} \qquad \text{..........(6.43a)}$$

where

$L_o =$ The length of parallel strand fed.

$L_c =$ The yarn length on twisting and relaxation.

$\alpha_b =$ The surface helix angle at break.

Substituting $\dfrac{L_o}{L_c} = C_y$

$$\therefore\ \epsilon_y = (1+\epsilon_b)\cos\alpha_b \cdot C_y - 1 \qquad \text{..........(6.43b)}$$

If the central filament is to initiate the yarn failure then $\alpha_b = 0$ and

$$\epsilon_y = (1+\epsilon_b) C_y - 1 \qquad \text{..........(6.43c)}$$

It can be seen that the surface filament will be the first to reach its breaking extension whenever such a yarn structure is being extended.

Values of ϵ_y obtained in substituting the appropriate values of C_y are shown in figures ($6.43B_1 - B_3$), and compared with experimental results, it can be seen that Nylon and Terylene yarn structures

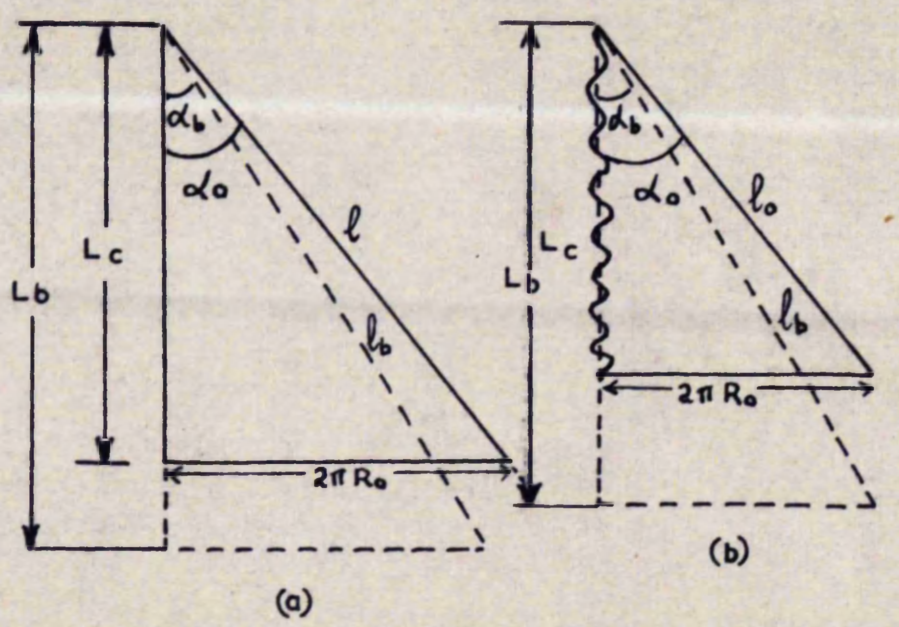


FIG. 6.43A EFFECT OF TWISTING TENSION ON HYPOTHETICAL YARN STRUCTURES.

(a) CENTRAL FILAMENT HAVING NO STRAIN
(b) " " BUCKLED

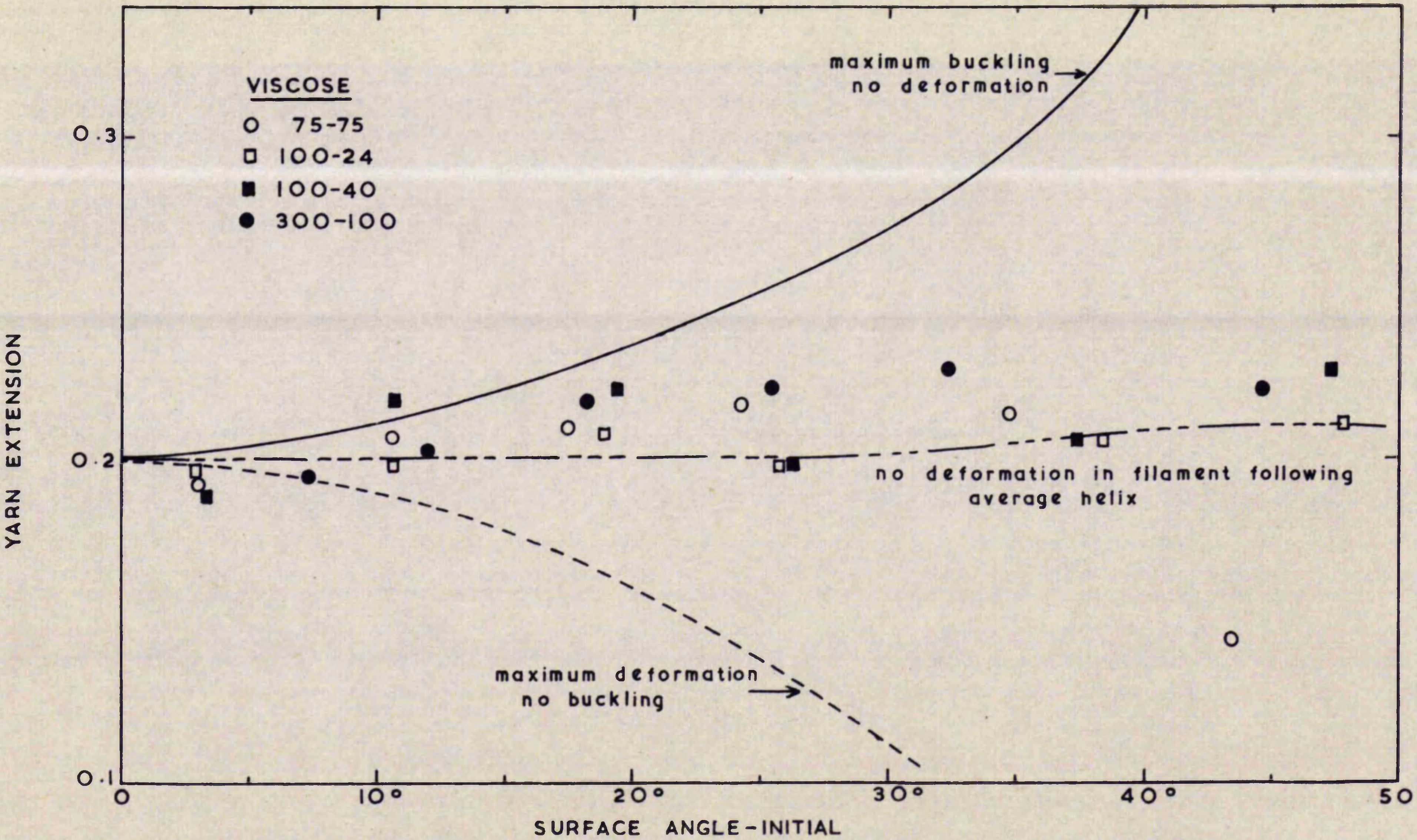


FIG. 6.43 B₁ BREAKING EXTENSION OF VISCOSE YARNS.

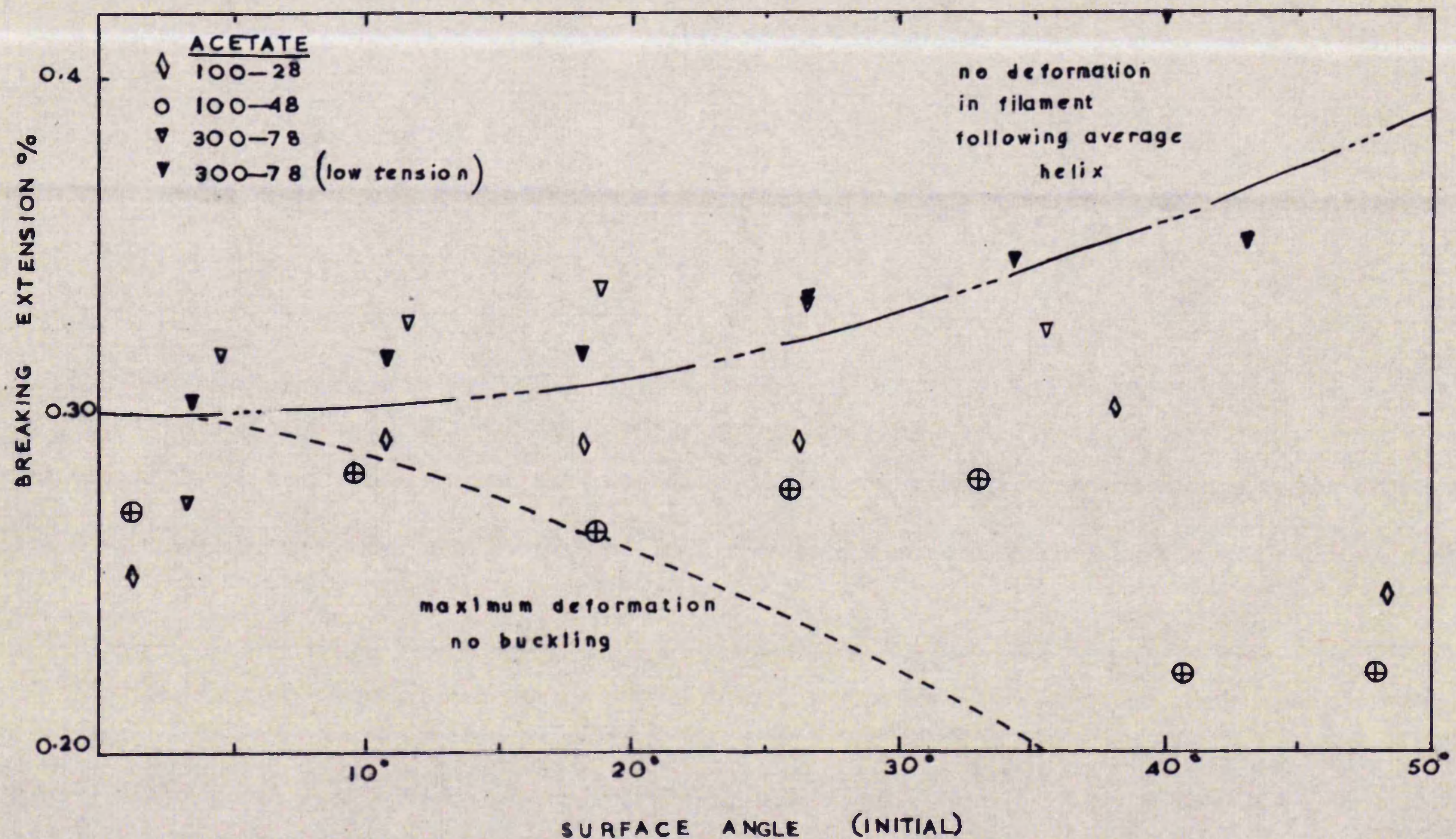


FIG. 6.43 B_2 BREAKING EXTENSION OF ACETATE YARNS

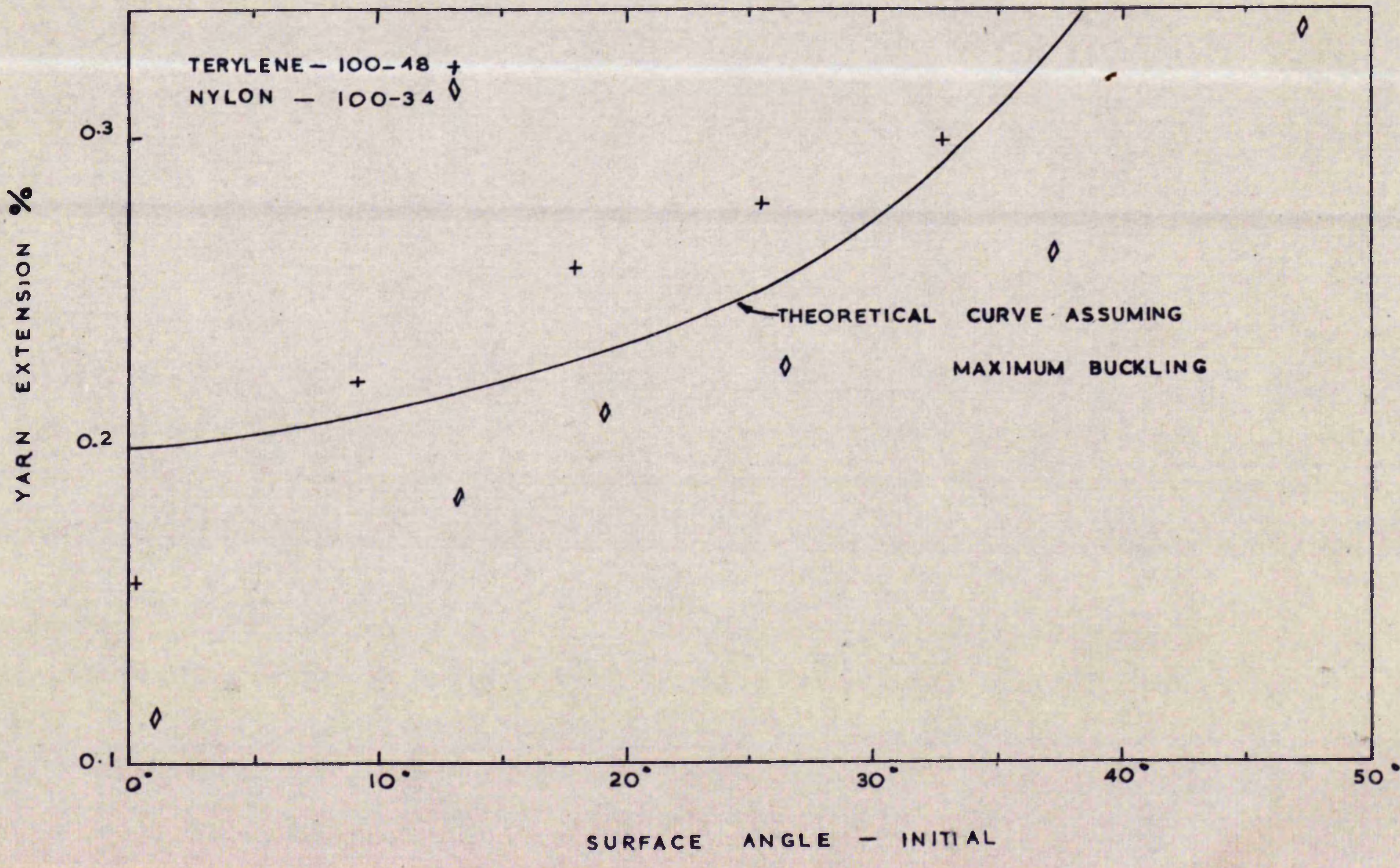


FIG. 6.43 B_3 BREAKING EXTENSION OF NYLON AND TERYLENE YARNS

may be close to the third case while viscose and acetate to the second case. In the case of acetate yarn, the structure at higher twist factors has a tendency to go near the second structure. Similarly the highly twisted nylon structure has a tendency to reach the second stage of structure.

The limitationsto the prediction equation (6.43b) are:

(a) The extent of migration: this will increase the value of the surface helix angle at which the decreases will be expected and will increase the rate and the extent at which the initial rise may occur.

(b) The elastic properties of material: this factor will determine the actual contraction factor and hence the degree of buckling and permanent deformation.

Thus the hypothetical model of buckled idealised yarn structure and the three special cases discussed will help to explain the contradictory results obtained by some workers. Grover and Hamby[20] observed higher initial increase in the breaking extension of viscose rayon, Orlon and acetate twisted yarns, followed by a rapid decrease in yarns with twist factors higher than 50 tex$^{\frac{1}{2}}$ t.p.c. (25° - 30°). While Taylor[21] and Platt[15] observed that the yarn breaking extension remains fairly constant. These results can be predicted by assuming the structural differences due to the twisting conditions. Grover and Hamby[20] used static method of twisting where the extent of filament migration may be lower than that expected in the yarns in the continuous process of twisting. Moreover, the contraction factors, observed by Grover and Hamby[20] are very much lower than those observed by Taylor et al[21] (fig. 1.52 $B_1 - B_5$), especially so at higher twist factors. More evidence should be made available to determine the extent to which the buckling and the permanent deformation in fact happens.

6.44 The influence of testing conditions

The breaking extensions of very low twist nylon and Terylene yarns are found to be very low in the Instron tests as compared with those obtained in the Uster and IP2 tests. Some initial fall in the breaking extension is observed in Uster results for Terylene 100/48 and some nylon and Dacron yarns reported by others (fig. 1.34 $B_1 - B_5$).

This is mainly due to the fact that the breakage of a very low twist yarn is not sharply indicated in tests carried out on the constant rate of loading machines (IP2, Uster, or Goodbrand pendulum type). The coefficient of variation in breaking extension results will also be high. In the Uster tester, the break is detected by the movement of the feeler (spring lever) pressing against the yarn. Where there is a sharp break, as in the twisted yarns, this will act satisfactorily, but in a very low twist yarn, it may not act until the most extensible filament has broken. If the variability in the breaking extension of individual filaments is high, as it is in the extensions of Terylene and nylon yarns, apparently higher extension values will be registered in tests on these very low twist yarns. Viscose rayon and acetate yarns, as supplied by spinners, do have some initial twist which is sufficient to reduce this variability effect.

6.45 The influence of the combined factors

While the exact part played by all the above factors is uncertain, it is possible to give a general explanation of most of the observed effects.

At low twist factors, the supporting effect will be most prominent in increasing the breaking extension as the twist is increased. As soon as the contribution due to this factor reaches its maximum, the influence of the two compressive forces and the twisting conditions play an important part. The combined effect of these two factors in the highly twisted yarns, will result in the failure of the surface filaments to initiate the yarn breakage. If the material easily suffers a permanent deformation, then this results in a fall in the breaking extension, but if the elastic recovery is good, then there will be buckling in the core filaments and the breaking extension will increase even at higher twist factors. Cellulose acetate comes into the first category; nylon, Terylene, Dacron and Fortisan in the second; and viscose rayon and Tenasco appear to be fairly evenly balanced between the two.

6.5 YARN TENACITY

6.51 General discussion

In most of the results in figures ($3.41A_1 - A_4$) tenacity first rises as the twist is increased and then falls. The initial rise is due to the effects of twist on a variable material, which results in the support of weak places in one filament by the neighbouring filaments as explained in section 6.41. Where the initial rise is absent, this is presumably because the variability of the filaments is so small that its effect is less than that of the decrease in tenacity which becomes predominant in all cases at higher twists. Another factor that may result in the absence of initial rise is the imperfect yarn structure resulting in uneven stress distribution.

6.52 Relative tenacity

At higher twist factors the decrease in tenacity is expected theoretically. For the quantitative comparison with theory, it was found very convenient to relate the actual tenacity values to a standard value. Since the tenacity of zero twist yarn is very much affected by the influence of the variability factor, the tenacity value at surface helix angle of 10° at break is taken as standard tenacity. For comparison with theory, it will be convenient to plot these relative values against $\sin^2 \alpha_b$, since such a plot will be of the linear form on the simplest theoretical expression.

6.52A Hooke's law relation

The filament stress-strain curve near break may be of a linear form. For some materials, the Hooke's law assumption can be taken as a good approximation. As shown in figure $6.52A_1$, the relative tenacity values, when plotted against twist angle at break, show a common curve for all materials. This is especially so if the relative tenacity values are not corrected for the breaking extension. (figures $6.52A_2$ and A_3). Nylon yarn shows higher values of relative tenacity as its breaking extension increases with twist; while the relative tenacity values of viscose yarns will be very little affected. In comparing with the theory, it is more logical to correct the relative tenacity values for breaking extension, but the theoretical approach assumes that the yarn breaking extension is not influenced by twist in the yarn.

From these figures, it can be seen that Hooke's law relation can be taken as a good approximation. The simple relation ($\cos^2\alpha$) shows less agreement with the experimental values. The experimental values show quite good agreement with theoretical curve obtained by assuming Hooke's law relation, influence of both compressive and tensile forces and filament. Poisson's ratio $\sigma_f = 0.5$ (equ.5.42e). This is especially so at higher twist angles. The relative values for Tenasco yarns (Tenasco 400-180) lie below the theoretical curves as does the relative slope B (fig. $6.22A_1$).

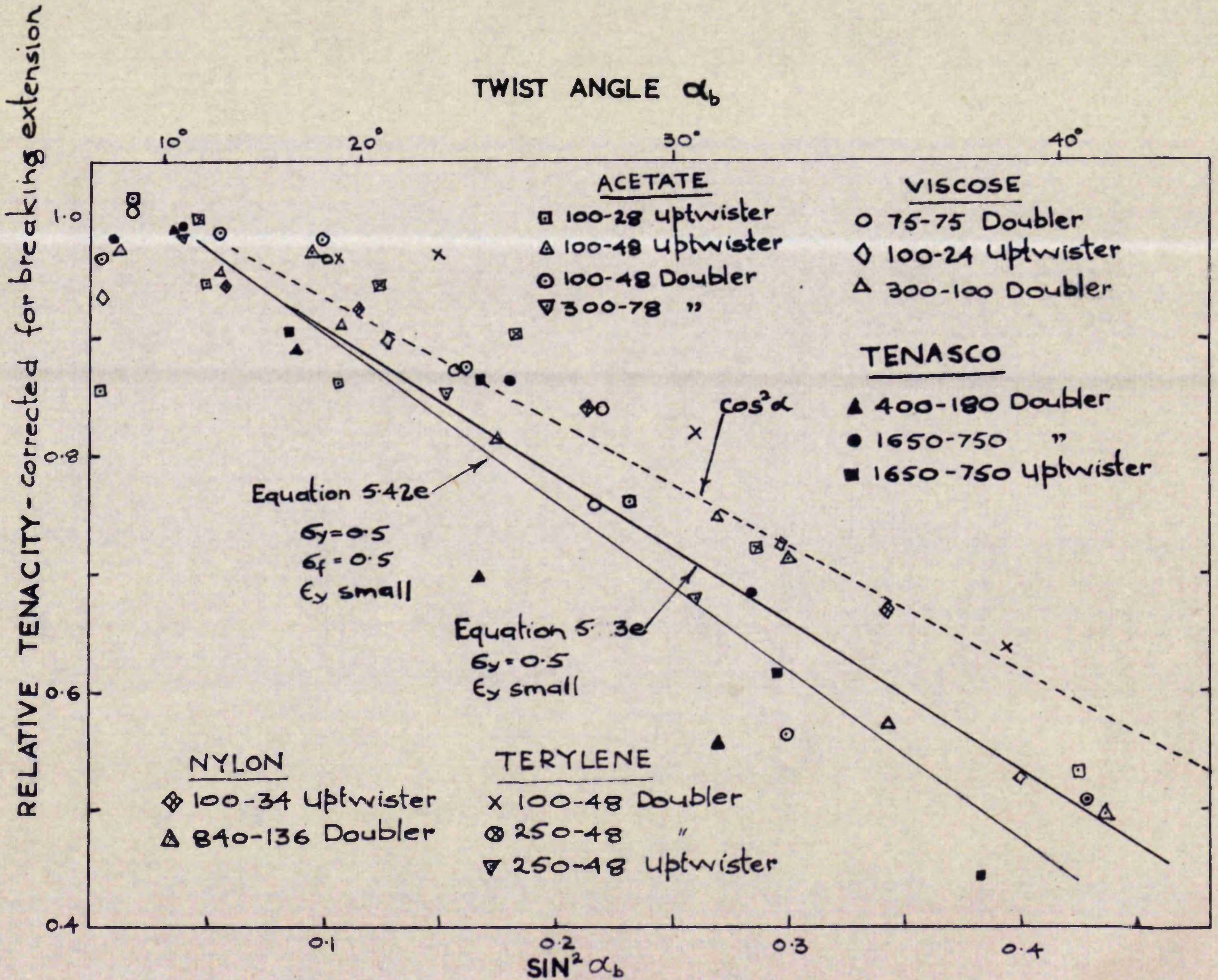


FIG.652A. RELATIVE VALUE OF TENACITY PLOTTED AGAINST α_b

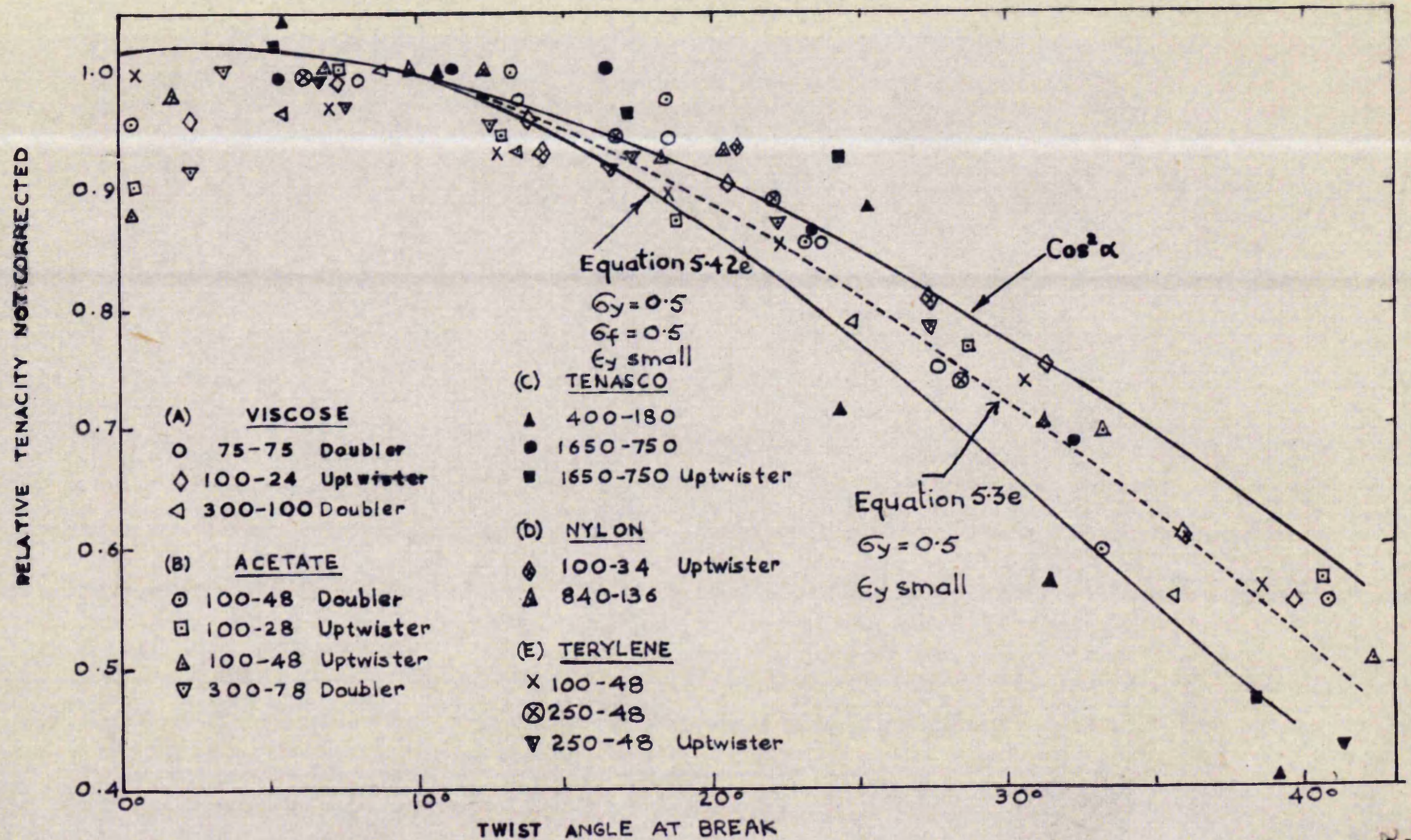


FIG. 6·52A₂ RELATIVE VALUE OF TENACITY PLOTTED AGAINST TWIST ANGLE AT BREAK

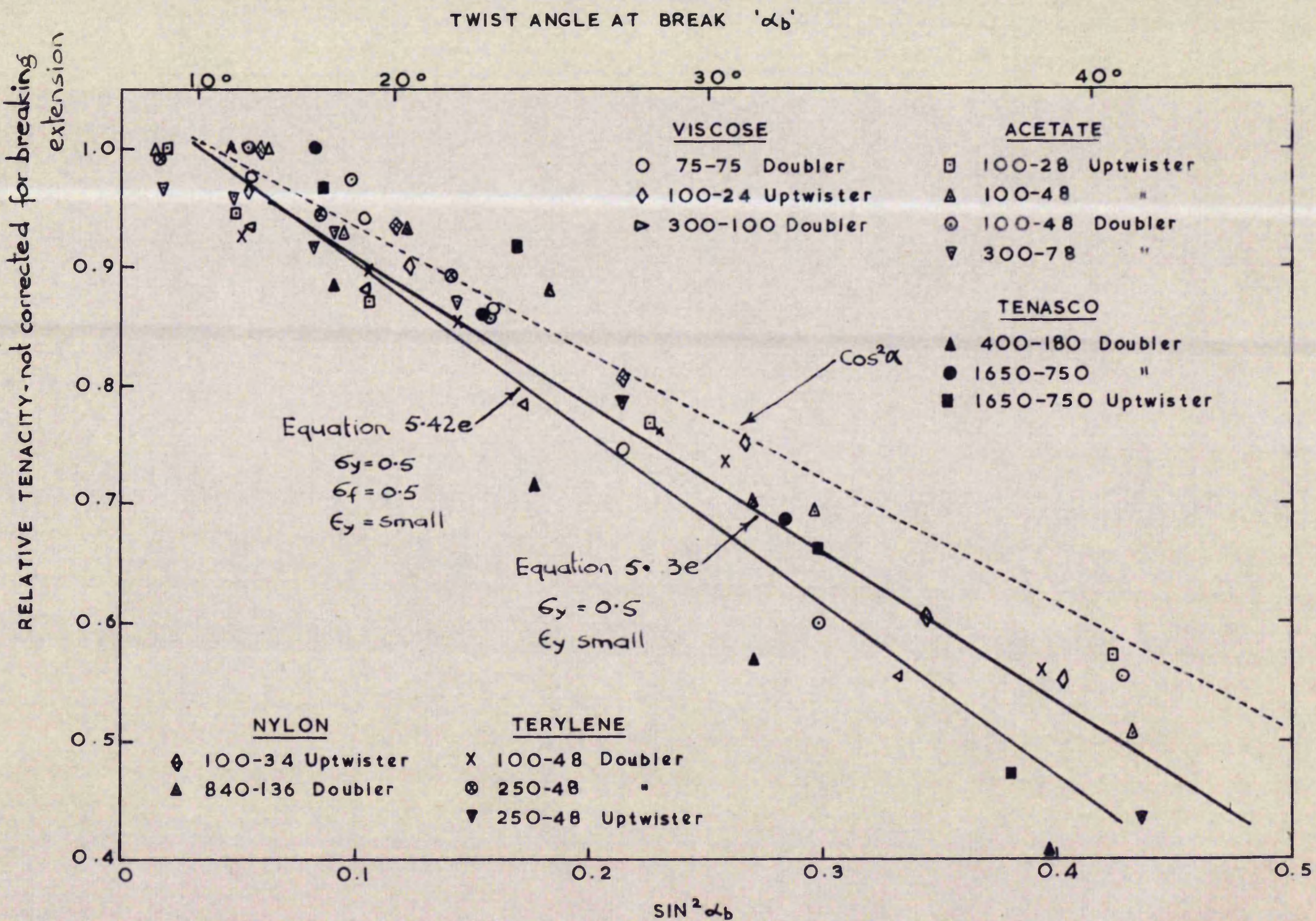


FIG. 6·52A3 RELATIVE VALUE OF TENACITY PLOTTED AGAINST TWIST ANGLE AT BREAK.

6.52B Deviation from Hooke's law relation

Where the stress-strain relation near break is of the linear form, the simple expression for tenacity is given as

$$Y_{\alpha_b} = A \frac{\log_e \sec^2\alpha_b}{\tan^2\alpha_b} + B\epsilon_y \cos^2\alpha_b \qquad \ldots\ldots\ldots(6.52a)$$

where,

A = Intercept in g.wt./tex.

B = Slope

α_b = Twist angle at break.

The relative tenacity.

$$\frac{Y_{\alpha_b}}{Y_o} = \frac{A}{Y_o} \frac{\log_e \sec^2\alpha_b}{\tan^2\alpha_b} + \frac{B\epsilon_y}{Y_o} \cdot \cos^2\alpha_b \qquad \ldots\ldots\ldots(6.52b)$$

where Y_o is the standard value of yarn tenacity given by the relation

$$Y_o = A + B\epsilon_y \qquad \ldots\ldots\ldots(6.52c)$$

If we define

a = Intercept in g.wt./tex expressed as a ratio of the breaking stress Y_o

b = slope expressed as a ratio of the breaking stress Y_o

Then $$a + b\epsilon_y = 1$$

and relative tenacity = $a \dfrac{\log_e \sec^2\alpha_b}{\tan^2\alpha_b} + b\epsilon_y \cdot \cos^2\alpha_b$

In special cases

(1) where α_b is very small and is positive or negative.

It can be shown that

$$\frac{\log_e \sec^2\alpha_b}{\tan^2\alpha_b} \simeq \cos^2\alpha_b$$

And therefore,

$$\text{Relative Tenacity} \simeq (a + b\epsilon_y)\cos^2\alpha_b$$
$$\simeq \cos^2\alpha_b$$

Thus a simple relation assuming Hooke's law will be a good approximation, even though the stress-strain behaviour near break is of the linear form. This will be especially so as value of a is very low as compared to that of $b\epsilon_y$

(2) where α_b is large and a is positive

$$\frac{\log_e \sec^2\alpha_b}{\tan^2\alpha} > \cos^2\alpha_b \quad (\alpha_b > 20^\circ)$$

and this will result in the under estimation of the relative tenacity values when predicted by the simple relation assuming Hooke's law.

(3) where α_b is large and a is negative

$$\frac{\log_e \sec^2\alpha_b}{\tan^2\alpha_b} > \cos^2\alpha_b$$

and $$b\cos^2\alpha_b > \cos^2\alpha_b$$

Here, the simple relation assuming Hooke's law will over estimate the relative tenacity values.

The value of constant 'a' will be negative for fibres like nylon, Tenasco, high tenacity Terylene, Fortisan etc. Figures ($6.52B_1$ and B_2) illustrate this behaviour.

In calculating the slope and the intercept factors, it is very important to obtain the stress function X which will represent the stress-strain relations of fibre over a wide range of extensions. In a yarn structure twisted to 40° twist angle, the surface filaments will be experiencing extensions as low as 13% of that experienced by a central filament (equation 5·23d). In other words, for acetate, where breaking extension is 30%, the stress function X should represent the stress-strain behaviour of the fibre over the extension values from 4% to 30%. In actual practice, approximations have to be made.

Platt and Neal Truslow[17] have observed that the intercept factor 'a' is negative for acetate yarns (Table 6.52I). In the present work it is found to be a positive one. This is due to the method of obtaining these constants. The phenomenon that acetate yarns do show

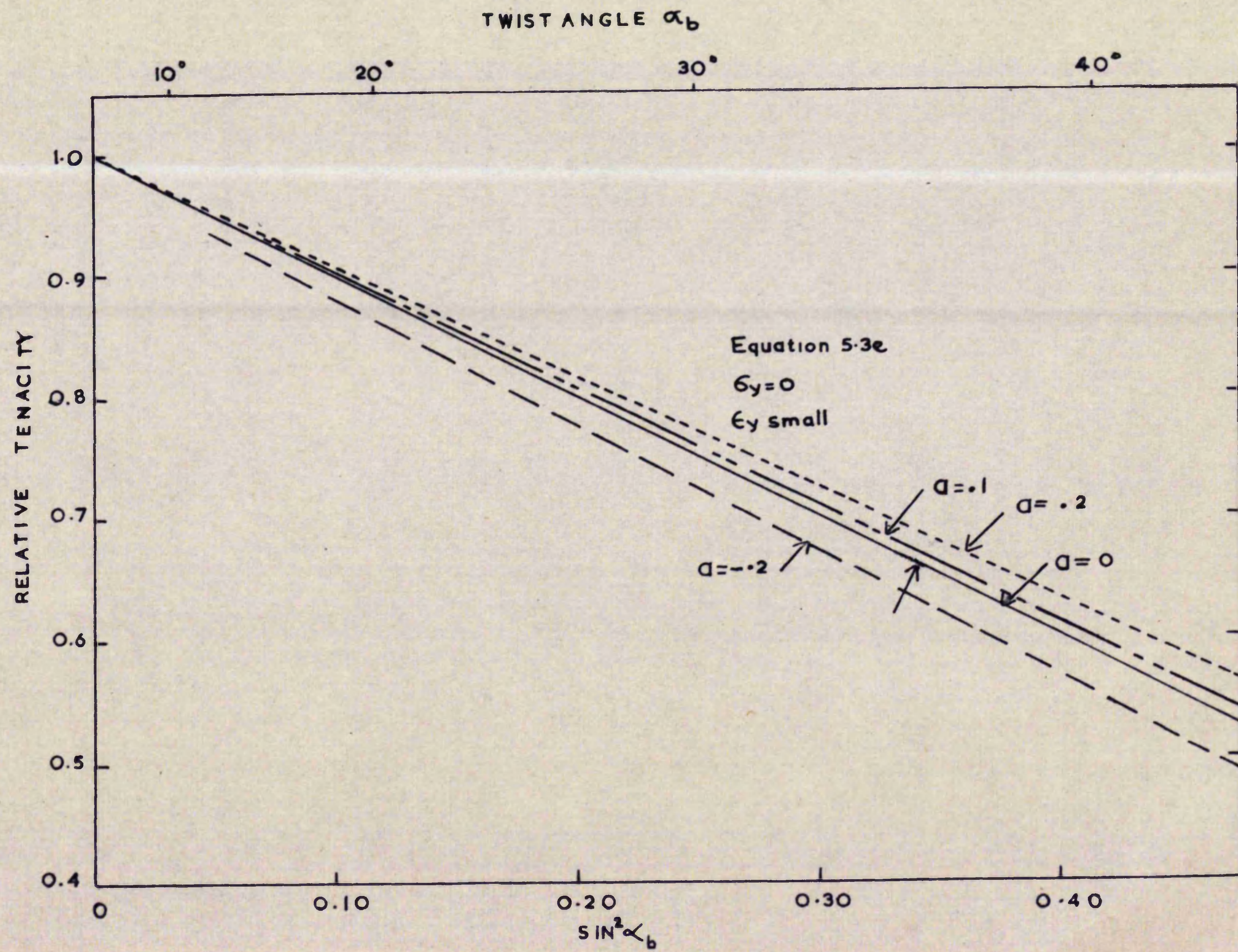


FIG. 6·52 B₁ DEVIATION FROM HOOK'S LAW RELATION

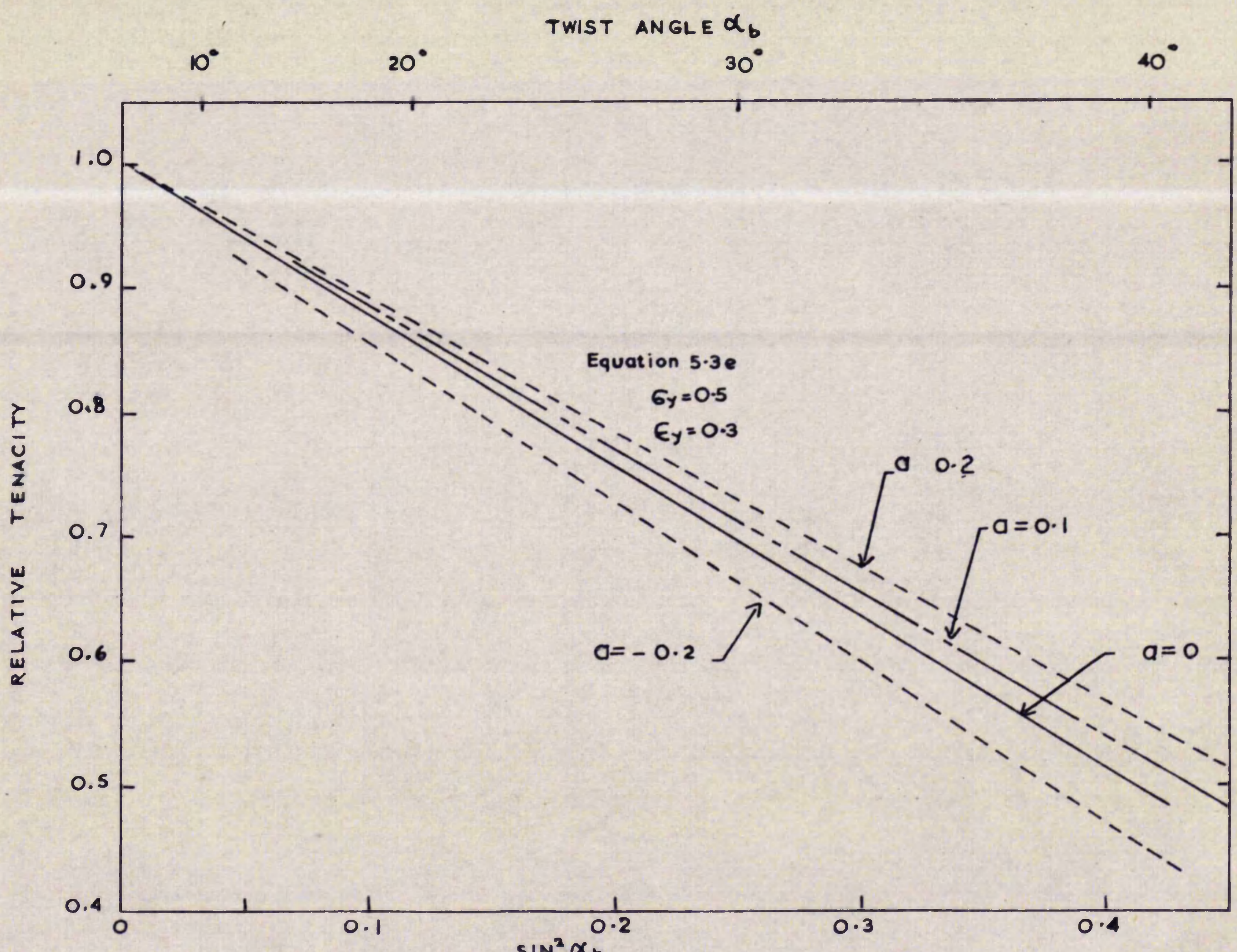


FIG. 6.52 B_2 DEVIATION FROM HOOK'S LAW RELATION.

TABLE 6.52 I

Values of 'a' intercept constant for various fibres

	Intercept factor 'a'	Filament Breaking Ext. %
A) Neal Truslow		
Viscose Regular	0.25	29
Viscose H.T.	0.024	18
Acetate	-.187	30
Nylon	0.455	28
Glass	0.000	5
Orlon	0.111	16
Dacron Regular	0.820	23
Dacron H.T.	0.735	13
Vicara	0.228	30
B) Present Work		
Viscose	0.169	20
Tenasco	0.070	18
Tenasco	0.264	10
Acetate	0.356	30
Seraceta	0.298	35
Nylon	-0.348	15

fairly rapid loss in strength as the twist is increase, has been attributed to the negative value of intercept factor. However, large extensions in acetate yarns, lateral contraction ratio, compressive forces may be influencing the tenacity behaviour.

Thus where Hooke's law ceases to hold, these disturbing factors will limit such an approach. Also actual slope and intercept factors of viscose and acetate yarns are affected by twisting tensions used (section 3.23).

Relative tenacity (corrected) values are plotted in figures ($6.52C_1 - C_4$). It can be seen that, the theoretical equation (5.3e) assuming yarn lateral contraction ratio $\sigma_y = 0.5$ and large extension values (ϵ_y) shows better agreement than that shown by the simplified relation assuming $\sigma_y = 0$ and ϵ_y small. However, both nylon and Terylene yarns show higher relative values than those predicted by these equations (fig. $6.52C_4$). This may be attributed to their breaking extension behaviour with twist and also the fact that their stress-strain behaviour near break is less linear in form (parabolic).

In testing the validity of equation 5.42e, assuming influence of compressive forces, the elastic constants a_1, a_2 and a_3 will have to be experimentally determined. The computation of these constants from the observed tenacity values for yarns with varying twist angles, will evolve some approximations and difficult calculations.

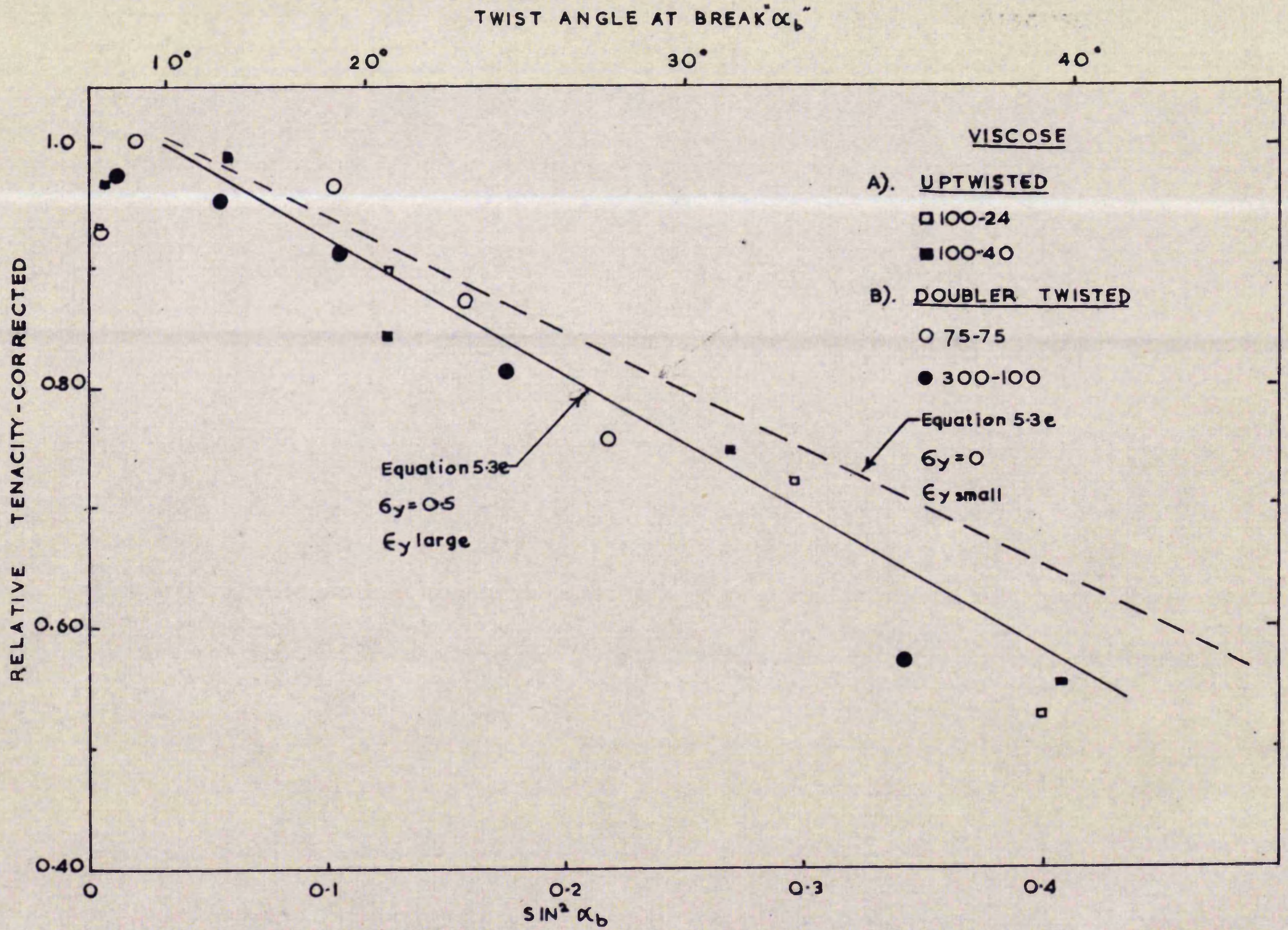


FIG. 6·52 C₁ RELATIVE VALUES OF TENACITY PLOTTED AGAINST $\sin^2 \alpha_b$

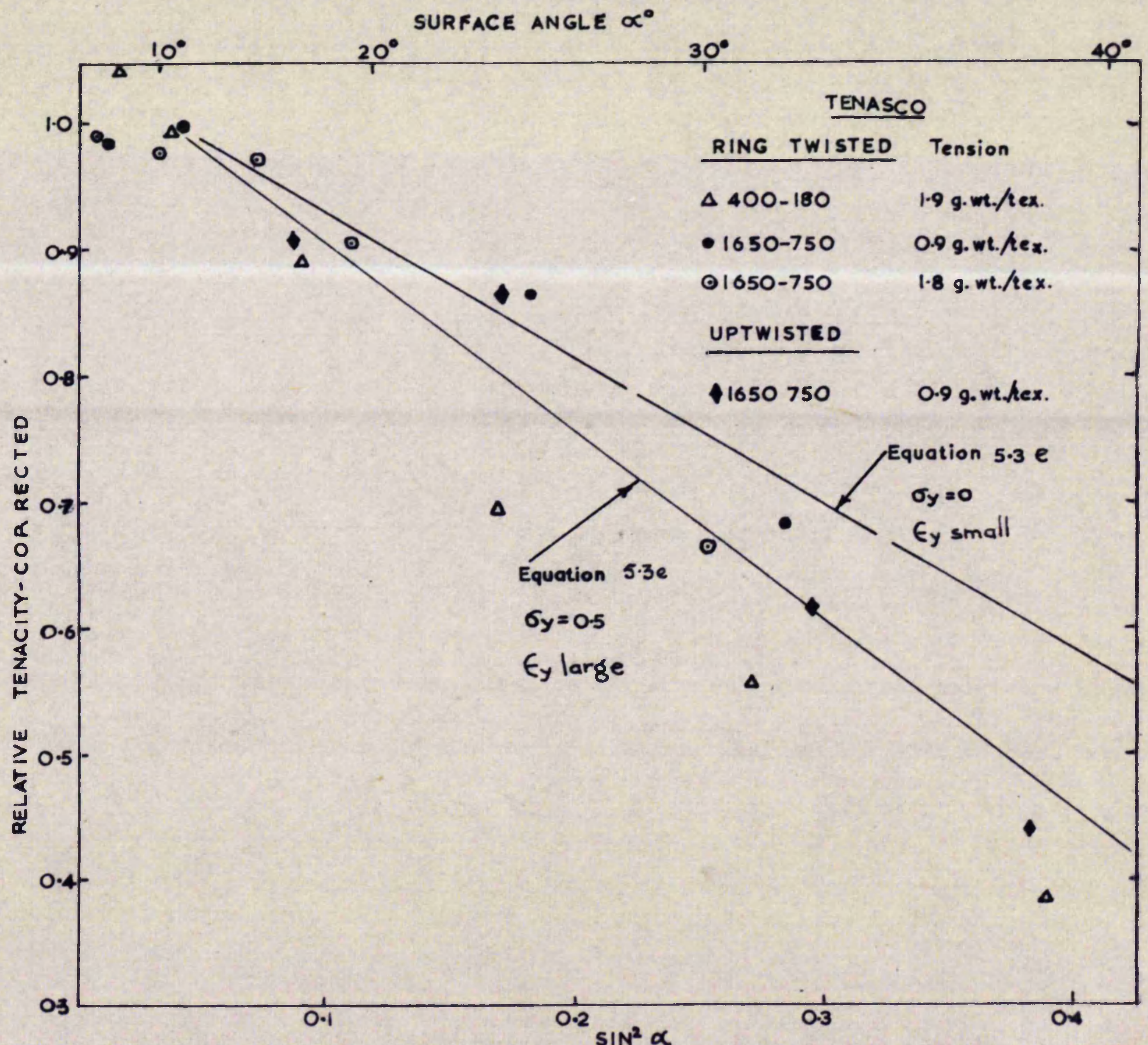


FIG. 6.52·C_2 RELATIVE VALUES OF TENACITY PLOTTED AGAINST $SIN^2 \alpha$

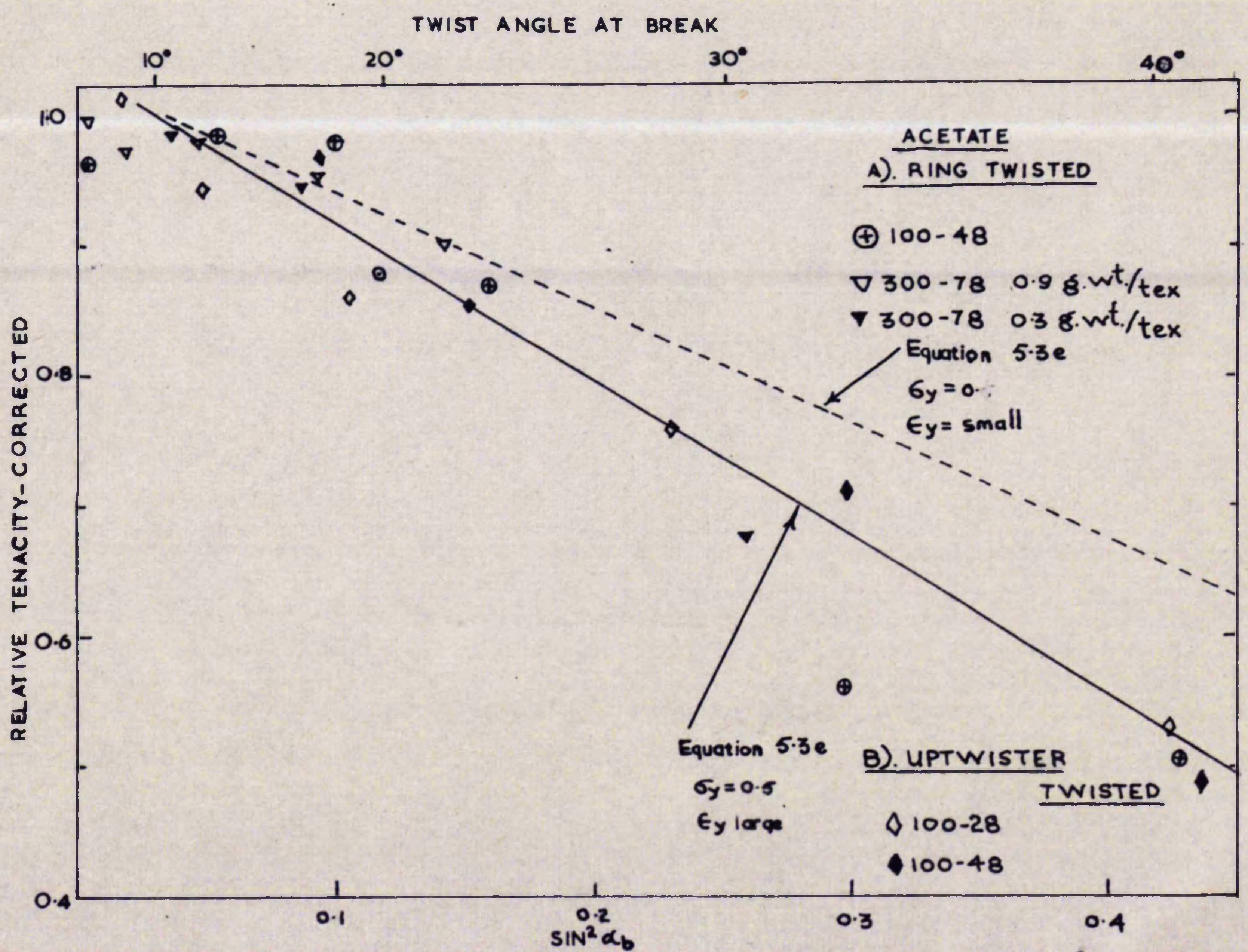


FIG. 6.52 C_3 RELATIVE VALUE OF TENACITY PLOTTED AGAINST $SIN^2\alpha_b$

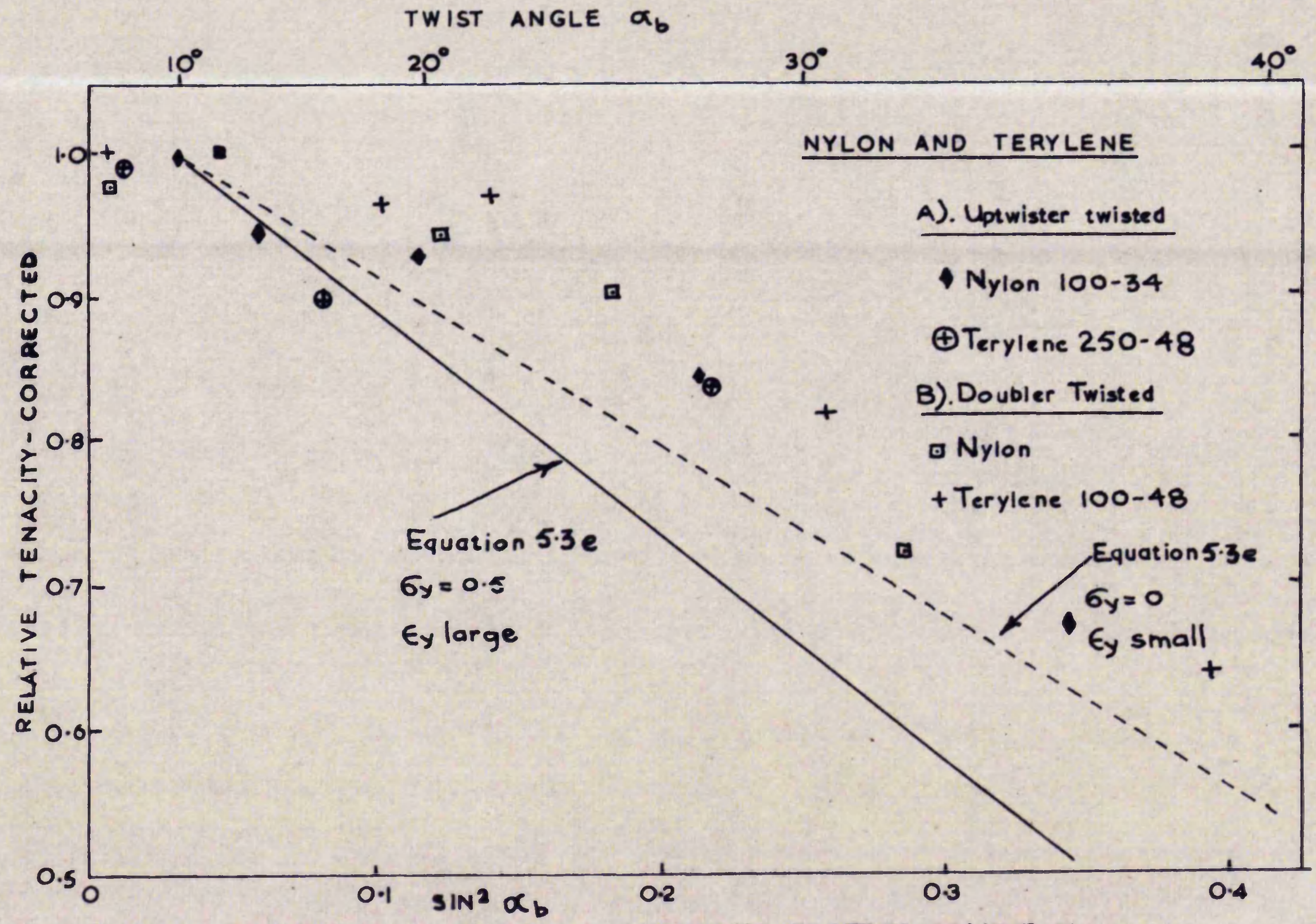


FIG. 6·52 C_4 RELATIVE VALUES OF TENACITY PLOTTED AGAINST α_b

6.54 Relative tenacity - influence of twisting conditions

As discussed in section (6.44), the actual yarn structure may deviate from the theoretical idealised model. This may result in the buckling of the central filament and the permanent deformation of the surface filaments. The extent of buckling will depend upon the material properties. The state of equilibrium may reach when the outward compressive force of the buckled inner structure will balance the inward radial compressive forces due to the residual stresses in the surface filaments. The nylon and Terylene yarns will show higher values of yarn extension so also the relative tenacity values. This factor affecting the relative values can be easily calculated from equation (6.43b). The factor $\frac{(1+\epsilon_{y_0})\cos\alpha_b C_y - 1}{\epsilon_{y_0}}$ will be maximum when $C_y = \sec\alpha_0$ and minimum when $C_y = 1$. The value of this factor will be 1, when contraction factor C_y can be determined by the complete recovery of strains in the filaments following helix angle α_b.

6.6 WORK OF RUPTURE

Work of rupture has been observed to follow the similar behaviour as does the tenacity values. (figures 3.51A). It initially increases to a maximum and then decreases continuously. This increase is not observed in Tenasco yarns twisted under low tensions. The work of rupture of nylon and Terylene (high tenacity) yarns is least affected by twist.

These results are expected theoretically.

Work of rupture = work factor x tenacity x breaking extension.

If work factor is assumed to be constant, the work of rupture will depend upon the behaviour of the breaking extension and tenacity values with twist. If yarn breaking extension is not influenced by the twist, the relative work of rupture should exhibit, both qualitatively and quantitatively, the features observed in yarn tenacity values.

From figure (3.51B), it can be seen that the work factor is very little affected with twist for all yarns except nylon and Terylene, where it decreases at very high twist factors.

The relative work of rupture values (figures 6.6A_1) and A_2) show all the features of tenacity behaviour for all yarns except high tenacity nylon and Terylene yarns. The decrease in the relative work of rupture for Tenasco yarns is very high and also the tenacity value. The work of rupture for nylon and Terylene (H.T.) yarns remain fairly constant at all twist factors. This may be due to the fact that actual breaking extension values are higher than the predicted ones.

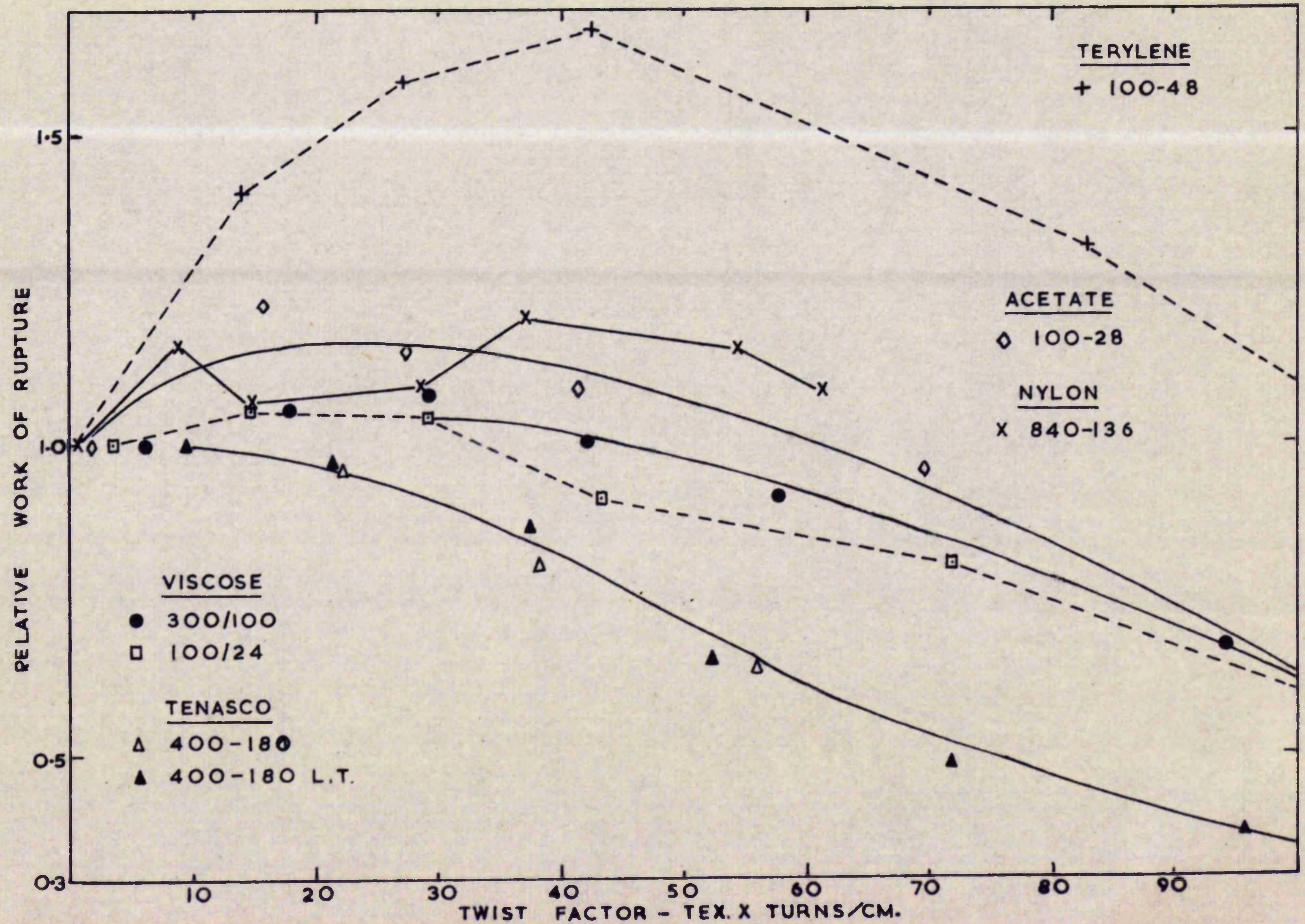


FIG. 6.6A RELATIVE WORK OF RUPTURE FOR TWISTED YARNS

REFERENCES

1. F.T. Pierce. J.Text. Inst., 1937, 28, T45.

2. E. R. Schwarz. J.Text. Inst., 1933, 24, T105.

3. H. J. Woods. J.Text. Inst., 1933, 24, T317.

4. E. R. Schwarz and Killian. J.Text. Inst., 1936, 27, T237.

5. F. T. Pierce. Text. Res. J., 1947, 17, 123.

6. L. R. G. Treloar. J.Text. Inst., 1956, 47, T348.

7. M. M. Platt. Text. Res. J., 1950, 20, 665.

8. E. R. Schwarz. Text. Res. J., 1951, 21, 125.

9. W. S. Hamburger et al. Text. Res. J., 1952, 22, 695.

10. W. E. Morton and Yen. J.Text. Inst., 1952, 43, T60.

11. W. E. Morton. Text. Res. J., 1956, 24, 325.

12. S. Kakiage J.Text.Mach.Soc.Japan, 1958, 16, 15.

13. G. Gegauff. Bull. Soc. Ind. Mulhouse, 1907, 77, 153.

14. M. Platt et al. Text. Res. J., 1954, 24, 132.

15. M. Platt et al. Text. Res. J., 1950, 20, 1.

16. S. A. Shorter. J. Text. Inst., 1957, 48, T99.

17. N. Truslow. Text. Bull., 1954, 80, 106.

18. J. W. S. Hearle. J. Text. Inst., 1958, 49, T389.

19. W. S. Hamburger. "Edger Marbury Lecture 1955", A.S.T.M. U.S.A.

20. E. B. Grover and D. S. Hamby. "Textile Processing of synthetics – continuous filament". School of Textiles, North Carolina State College, 1956, Appendix.

REFERENCES - cont'd

21. J. T. Taylor et al. W.A.D.C. Technical Report Nos. 52-55, 1952, March.

22. F. J. Alexander and C. H. Sturley. J. Text. Inst., 1952, 43, P1.

23. Shrinagbhushan. Text. Dig., 1956, 17, 4.

24. Symes. B.N.S. Outlook 1958, 1.

25. Celanese Corporation of America. Technical Bulletin Fortisan-36 TO-20-A.

26. E. Braschler. "Die Festigkeit von Baum woll gespinsten" Leeman and Company, Zurich and Leipzig, 1935,10.

27. J. Gregory. J. Text. Inst., 1950, 41, T14.

28. W. E. Morton and J. W. S. Hearle. J. Text. Inst., 1957, 48, T159.

29. E. J. Sparke. J. Text. Inst., 1950, 41, P69.

30. G. H. Tattersall. J. Text. Inst., 1958, 49, T295.

31. D. S. Hamby. Text. World, 1954, 104, 111.

32. A. B. Marathe. M.Sc.Tech. Thesis, C.O.S.T. Manchester, 1958.

33. R. Meredith. J. Text. Inst., 1945, 36, T110.

34. T. F. Evans and D. J. Montgomery. Text. Res. J., 1953, 23, 690.

APPENDIX I

APPENDIX - Yarns investigated

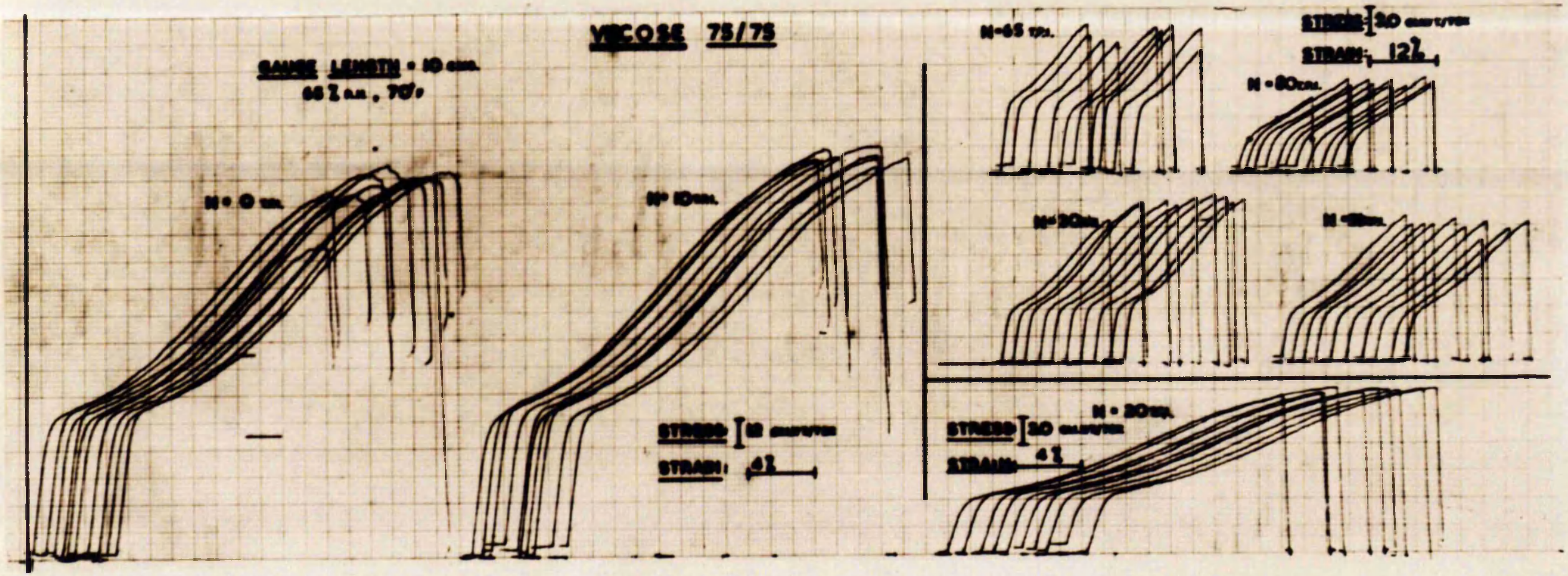
MATERIAL	NOMINAL den./fil. no./t.p.i.	ACTUAL VALUES T.P.C.	TEX	TWIST FACTOR $tex^{\frac{1}{2}}$ turns/cm.	DIAMETER in 10^{-2} cm.	BREAKING LOAD in gms.wt. Uster	Instron	C.V.% (Instron tests)	BREAKING EXTENSION in % Uster	Instron	C.V.% (Instron tests)	a) Twisting Method b) Traveller Number c) Twisting Tension
(25)	250/48/0.75	0.20	28.72	1.1	2.82	1712	1746	1.3	10.3	9.7	9.1	
	(High Tenacity)	2.11	28.36	11.2	2.06	1775	1794	1.3	11.7	15.9	8.8	a) Ring doubler
		6.07	29.36	32.9	1.98	1748	1737	2.2	12.3	17.6	8.0	b) 21's and 19½'s
		8.16	29.74	44.5	2.01	1670	1604	1.2	13.0	17.7	5.6	c) 0.9 g.wt/tex
		10.88	31.36	60.9	2.03	1503	1549	4.2	14.3	18.8	6.9	
(26)	250/48/0.75	0.20	27.52	1.1	2.82		1721	2.0		9.8	11.1	
	(High Tenacity)	2.14	28.38	11.4	2.04	1790	1791	1.1	11.7	15.9	12.0	a) Ring doubler
		6.06	29.00	32.6	1.92	1705	1660	1.3	12.2	15.8	6.3	b) 19's, 17½'s & 18's
		8.25	29.08	44.5	1.93	1695	1709	1.6	12.4	18.3	7.9	c) 1.8 g.wt/tex
		10.70	31.24	59.8	1.98	1450	1532	2.6	13.2	18.0	7.1	
(27)	250/48/0.75	0.20	28.50	1.1	2.58	1745	1665	1.8	9.8	8.2	5.6	
	(High Tenacity)	2.13	27.60	11.2	1.92	1780	1771	1.4	11.8	15.2	8.1	a) Ring doubler
		4.20	28.56	22.4	1.88	1720	1729	1.3	10.7	14.9	4.9	b) 15's and 14½'s
		5.97	29.28	32.3	1.88	1723	1709	1.2	11.7	16.3	7.4	c) 2.7 g.wt/tex
		9.58	30.14	52.6	1.91	1433	1612	2.8	11.2	18.6	7.9	

VISCOSE	75/75/3.5	1.49	8.54	4.3	1.30	139	136	5.7	18.5	19.2	8.77	
(1)		5.24	8.58	15.4	1.12	149	145	1.9	20.2	20.7	3.25	a) Ring doubler
(Courtaulds Ltd.)		9.16	8.78	27.2	1.10	150	148	1.5	20.8	21.0	4.14	
		13.39	8.96	40.1	1.07	148	147	4.1	21.9	21.9	6.71	b) 26's
		17.44	8.88	52.0	1.04	133	-	-	20.8	-	-	
		21.18	9.50	65.3	1.04	124	126	4.7	20.9	21.5	4.43	c) 0.585 - 1.14 g.wt/
		28.48	9.68	88.7	1.06	63	63	5.8	10.5	14.4	11.2	tex
(2)	100/24/2.5	1.04	11.64	3.6	1.51	196	186	3.2	21.0	19.5	4.82	
		4.34	11.64	14.8	1.31	203	194	3.1	20.0	19.9	5.21	a) Uptwister
		8.47	11.88	29.2	1.24	205	192	2.9	21.4	20.8	5.94	b) -
		12.51	12.04	43.5	1.23	190	182	4.5	19.4	19.7	7.36	c) -
		20.08	12.82	71.9	1.25	169	164	4.5	19.7	20.6	7.67	
		27.84	14.08	104.3	1.25	137	131	9.5	20.5	21.0	10.27	
(3)	100/40/2.5	1.16	11.32	3.9	1.55	202	189	2.6	19.8	18.8	5.33	
		4.29	11.44	14.5	1.33	215	201	1.9	21.0	21.9	4.78	a) Uptwister
		8.64	11.62	29.5	1.28	216	200	4.7	22.2	22.1	7.09	b) -
		12.48	11.74	42.7	1.23	190	189	4.9	18.5	19.8	8.28	c) -
		19.76	12.58	69.9	1.23	172	166	4.2	20.2	20.7	8.54	
		27.64	13.68	102.1	1.25	137	139	7.9	21.3	22.6	6.55	
(4)	300/100/2.5	1.10	33.40	6.3	3.85	602	660	3.2	20.8	19.5	1.59	
		3.07	33.54	17.8	2.18	624	673	5.6	22.2	20.4	3.77	a) Ring doubler
		4.99	35.08	29.5	2.09	608	582	3.4	23.4	21.9	6.09	b) 19's
		7.06	35.67	42.2	2.13	586	568	2.9	24.0	22.1	1.68	c) 0.538 - 1.25 g.wt/
		9.56	36.47	57.7	2.11	534	536	2.1	24.1	22.8	3.42	tex
		14.93	39.89	94.3	2.07	414	427	4.8	22.5	22.1	5.44	
(5)	300/100/2.5	1.23	33.38	7.1	3.75	592	602	1.99	23.1	22.7	1.21	
		3.13	33.66	18.1	2.34	599	605	1.6	25.4	24.7	3.14	a) Ring doubler
		7.08	34.92	41.9	2.21	580	598	2.58	22.2	27.2	3.98	b) 17½'s, 19½'s and 20's
		9.55	36.00	57.3	2.13	526	532	4.01	21.3	25.6	4.21	c) 0.9 g.wt/tex
		12.00	37.68	73.7	2.07	433	433	6.61	18.6	21.6	8.31	
(6)	300/100/2.5	1.16	33.42	6.7	3.71	618	608	3.17	20.1	23.2	3.70	a) Ring doubler
		3.03	33.28	17.5	2.34	624	622	1.60	20.6	25.3	3.28	b) 14½'s, 16½'s, 17's
		6.76	34.00	39.4	2.13	580	569	1.93	18.9	20.5	10.65	
		9.07	34.44	53.2	2.09	536	550	3.04	16.8	20.7	7.24	c) 1.8 g.wt/tex
		11.91	36.96	72.4	2.00	428	430	7.06	16.7	20.7	10.8	

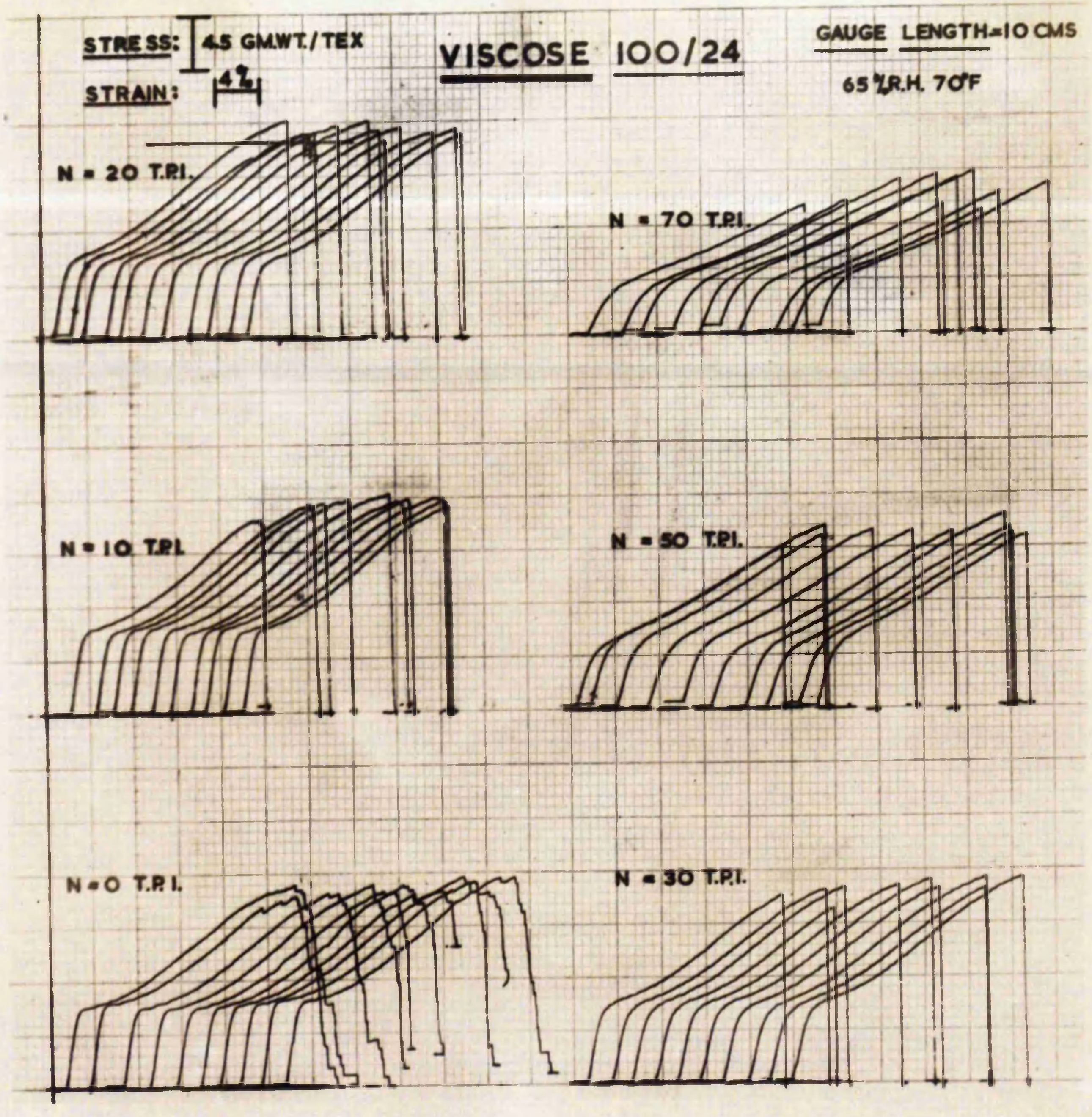
(7)	300/100/2.5	1.05	33.50	6.1	3.60	570	560	3.92	17.2	20.3	4.20	
		2.96	32.92	16.9	2.20	620	609	3.71	18.8	22.8	4.54	a) Ring doubler
		6.54	32.28	37.2	2.02	583	592	2.18	14.2	13.7	7.00	b) 9's, 12's, 14½'s
		8.67	33.12	49.9	1.90	519	536	4.69	13.7	15.8	8.30	c) 2.7 g.wt/tex
		11.23	35.04	66.5	1.97	474	481	6.02	13.5	15.7	7.66	
TENASCO (8)	400/180/3	1.39	45.92	9.4	2.86	1262	1186	3.0	19.4	17.8	4.41	
(Courtaulds Ltd.)		3.26	46.78	22.3	2.57	1240	1166	4.1	20.2	18.2	4.75	a) Ring doubler
		5.53	48.20	38.4	2.52	1069	1056	2.1	18.7	17.8	2.89	b) 26's
		7.88	50.54	56.0	2.57	806	904	3.7	17.3	17.3	2.08	c) 0.218 - 0.275 g.wt/
		13.89	58.12	105.9	2.73	465	536	9.2	14.9	15.8	4.37	tex
(9)	400/180/3	1.39	45.92	9.4	2.86	1262	1186	3.0	19.4	17.8	4.41	
		3.19	46.76	21.8	2.50	1224	1199	2.2	18.9	17.0	2.43	a) Ring doubler
		5.35	48.75	37.4	2.49	1135	1119	4.8	20.0	17.1	4.59	b) 18's
		7.42	49.49	52.2	2.49	922	933	3.3	16.4	15.4	3.74	c) 0.609 - 1.99 g.wt/
		10.03	51.60	72.0	2.41	753	801	7.2	15.9	14.7	3.64	tex
		12.91	55.18	95.9	2.45	587	630	7.1	14.4	14.1	6.07	
(10)	1650/750/1(Z)	0.57	193.9	7.9	6.23		4774	1.6		9.9	3.46	
		1.54	196.9	21.7	5.31		4606	2.5		9.8	4.66	a) Ring doubler
		2.91	215.5	42.7	5.32		4050	3.4		10.1	4.92	b) -
		4.21	234.5	64.5	5.12		3374	4.7		9.7	5.24	c) -
(11)	1650/750/1(Z)	0.49	193.9	6.9	6.97		4910	2.2		9.2	0.8	
		0.64	194.3	8.9	5.20		4915	6.7		9.5	6.1	a) Ring doubler
		1.26	194.9	17.7	5.76		4953	3.9		10.1	3.2	
		2.07	197.2	29.1	5.35		5025	4.9		10.9	1.9	b) 8's, 7's & 6's
		2.97	204.7	42.6	5.34		4482	4.0		10.7	4.1	c) 0.9 g.wt/tex
		4.33	213.9	63.3	5.31		3734	5.6		9.9	3.5	
(12)	1650/750/1(Z)	0.49	194.2	6.8	6.39		4929	2.2		9.3	0.89	
		0.65	192.8	9.0	5.19		4945	4.0		9.6	7.1	a) Ring doubler
		1.28	195.2	17.9	5.11		4876	.12		9.9	4.2	b) 6's and 4's
		2.02	200.7	28.6	5.07		4944	4.6		10.6	4.2	c) 1.8 g.wt/tex
		2.44	200.9	34.6	5.28		4605	4.8		10.7	1.8	
		4.31	214.4	63.1	4.92		3632	4.9		9.6	2.9	

(13)	1650/750/1	0.64	194.32	8.9	5.20		4775	2.9		9.9	5.2	
	(Z)	2.24	199.88	31.6	5.56		4650	1.9		10.1	3.9	a) Uptwister
		3.03	203.80	43.3	5.40		4526	1.7		10.8	4.5	b) -
		4.56	215.20	66.8	5.18		3413	4.4		9.3	4.2	c) 0.7 g.wt/tex
		6.26	230.16	94.9	5.05		2603	8.3		9.5	8.1	
ACETATE												
(14)	100/28/0.75	0.34	11.76	1.2	1.77	119	116	4.83	21.9	25.2	5.17	
	(Celanese)	4.64	11.38	15.6	1.32	129	125	3.73	30.0	29.2	8.69	a) Uptwister
		8.14	11.52	27.6	1.28	124	121	1.80	27.9	29.2	6.39	b) -
		12.09	11.68	41.3	1.29	115	120	2.84	26.9	29.3	7.48	c) -
		20.00	12.22	69.9	1.25	106	108	4.18	27.7	30.5	10.81	
		28.05	13.20	101.9	1.27	85	88	9.09	19.4	24.9	18.69	
(15)	100/48/1.25	0.50	11.16	1.7	1.74	129	125	3.14	28.1	27.2	3.55	
	(Seraceta	4.20	10.70	13.8	1.29	130	131	2.21	25.5	28.2	4.85	a) Ring doubler
	Courtaulds)	8.34	10.86	27.5	1.29	130	125	4.05	26.9	26.4	8.84	b) 26's
		12.41	11.00	41.2	1.25	129	129	4.84	28.9	27.7	5.37	c) 0.538 - 0.823 g.wt/
		16.57	11.96	57.3	1.24	123	118	4.84	30.5	28.0	7.51	tex
		22.10	13.18	80.2	1.23	95	98	7.87	19.9	22.2	22.8	
		27.69	13.36	101.2	1.27	89	94	6.39	17.5	22.3	20.9	
(16)	A100/48/1.25	0.5	11.16	1.2	1.74		117.7	3.6		24.9	5.4	
		4.10	10.24	13.1	1.30		122.3	2.6		28.9	7.2	a) Uptwister
		12.41	11.20	41.6	1.29		124.5	0.8		33.9	3.4	b) -
		21.03	12.72	75.0	1.35		105.6	2.2		32.2	4.8	c) 1.6 g.wt/tex
		31.10	13.68	115.0	1.36		83.5	4.1		24.5	9.1	
(17)	300/78/2.5(Z)	0.83	33.52	4.8	2.48	359	344	1.79	28.1	27.2	4.31	
	(Seraceta	1.10	33.68	6.4	2.43	392	379	2.37	32.9	31.9	4.48	a) Ring doubler
	Courtaulds)	2.93	33.65	16.9	2.26	382	379	2.29	32.7	32.9	4.46	b) 19's
		4.89	33.80	28.4	2.21	378	364	2.68	32.0	33.8	5.36	c) 0.594 - 0.978 g.wt/
		7.11	34.96	42.0	2.23	378	354	1.67	35.7	33.4	4.40	tex
		9.46	35.87	56.6	2.18	365	354	3.74	36.6	35.8	6.62	
(18)	300/78/2.5(Z)	0.81	33.26	4.7	2.66	365	337	3.39	29.2	30.3	2.56	
	(Seraceta	1.10	33.68	6.4	2.43	392	379	2.37	32.9	31.9	4.48	a) Ring doubler
	Courtaulds)	2.79	33.56	16.2	2.42	391	364	1.48	33.8	31.7	3.35	b) 26's
		4.81	34.30	28.2	2.15	388	362	2.57	35.4	31.9	7.55	c) 0.238 - 0.332 g.wt/
		7.02	35.28	41.7	2.27	388	349	2.90	38.0	33.4	4.62	tex
		9.47	36.76	57.4	2.29	366	328	5.60	37.3	32.6	9.2	
		12.50	39.18	78.0	2.38	308	300	3.90	34.0	35.1	5.5	

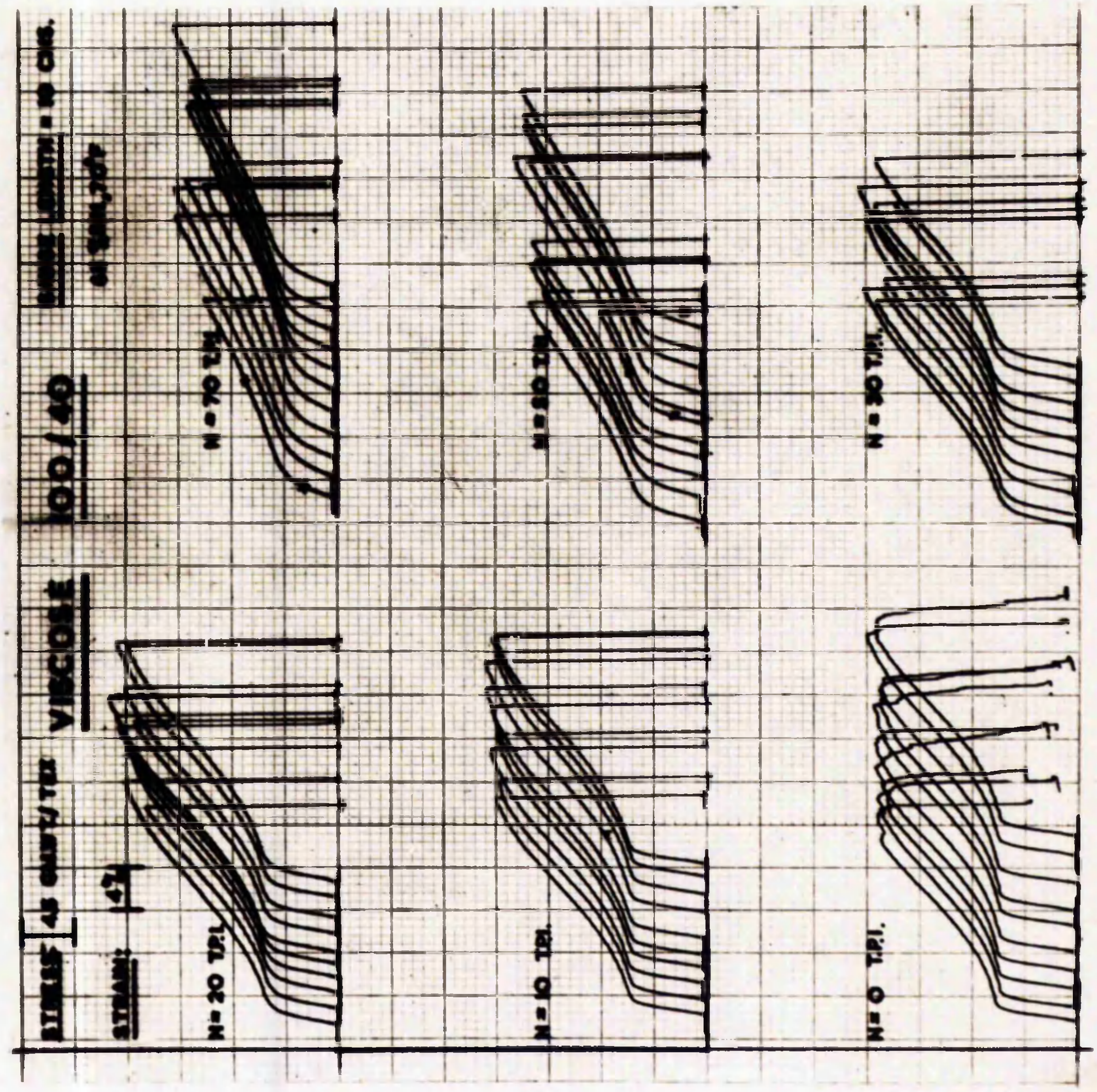
NYLON 100/34/0.75	0.3	11.32	1.0	2.12	-	558	3.1	-	11.5	12.9	
(19)(Medium Tenacity)	4.95	11.54	16.8	1.41	622	578	1.8	23.0	18.5	12.9	a) Uptwister
(British	7.62	11.76	26.1	1.34	591	555	1.0	24.2	21.1	3.6	b) -
Nylon Spinners	10.66	12.04	37.0	1.37	605	582	2.3	22.5	22.6	9.3	c) -
Ltd.)	17.48	12.96	62.7	1.38	565	540	4.5	28.1	26.3	9.6	
	25.63	14.46	97.5	1.35	474	486	4.5	30.9	33.5	7.8	
(20) 840/136/1	0.33	94.7	3.2	4.95		6955	3.0		11.2	5.5	
(High Tenacity)	0.89	96.4	8.7	4.42		6995	2.7		13.9	3.3	
	1.51	96.8	14.9	4.15		6925	2.8		13.6	3.0	a) Ring doubler
	2.90	99.4	28.9	4.08		6640	2.9		14.8	3.5	b) 19's
	3.69	101.5	37.3	4.13		6685	4.0		16.5	6.0	c) 0.211 - 0.372 g.wt/ tex
	5.26	105.4	54.2	4.12		6100	5.5		17.8	7.1	
	5.94	107.6	61.7	4.11		5815	2.2		19.5	3.7	
(21) "	0.24	96.02	2.3	5.30		7247	1.6		13.5	2.7	
	2.10	96.90	20.7	4.14		7471	.7		15.8	3.3	a) Ring doubler
	3.74	99.92	37.3	4.04		7185	1.3		16.8	1.2	b) 14½'s and 15's
	4.72	101.96	47.5	4.08		6959	1.3		17.7	3.8	c) 0.9 g.wt/tex
	6.18	108.56	64.4	4.07		5866	3.4		18.9	3.5	
(22) "	0.24	95.24	2.3	5.43		7145	2.4		12.9	6.45	
	1.69	95.72	16.5	3.91		7320	1.4		14.1	4.34	a) Ring doubler
	3.55	98.68	35.3	3.96		7211	2.1		15.7	3.76	b) 10's, 11's and 11½'s
	4.66	101.80	47.1	4.16		6942	1.9		16.7	6.4	c) 1.8 g.wt/tex
	6.11	107.12	63.2	3.98		6257	2.9		18.0	7.3	
TERYLENE (23) 100/48/0.75	0.16	11.14	0.5	1.94	529	-	-	24.2	15.9	15.1	
(I.C.I. Ltd.)	4.08	11.14	13.6	1.28	508	489	7.5	21.3	22.1	7.9	a) Ring doubler
(Medium	8.13	11.50	27.6	1.24	506	488	2.6	23.8	25.9	4.2	b) 26's
Tenacity)	12.40	11.66	42.5	1.24	497	501	2.1	27.5	27.9	10.6	c) 0.89 - 1.39 g.wt/ tex
	16.38	12.50	57.9	1.25	507	473	3.2	35.1	28.9	11.9	
	23.13	12.87	83.0	1.26	451	445	5.1	34.5	32.1	11.5	
	30.10	14.34	113.9	1.33	382	370	3.1	34.6	32.6	7.2	
(24) 250/48/0.75	0.2	27.52	1.05			1746	1.3		9.7	9.1	
(High Tenacity)	2.26	27.96	11.97	2.07		1751	1.2		12.9	5.8	
	6.13	29.08	33.08	1.99		1883	2.7		14.5	6.9	a) Uptwister
	10.23	31.48	57.37	1.97		1566	3.4		15.9	6.1	b) -
	16.78	35.32	99.71	2.12		967	10.9		17.6	9.2	c) 0.83 g.wt/tex

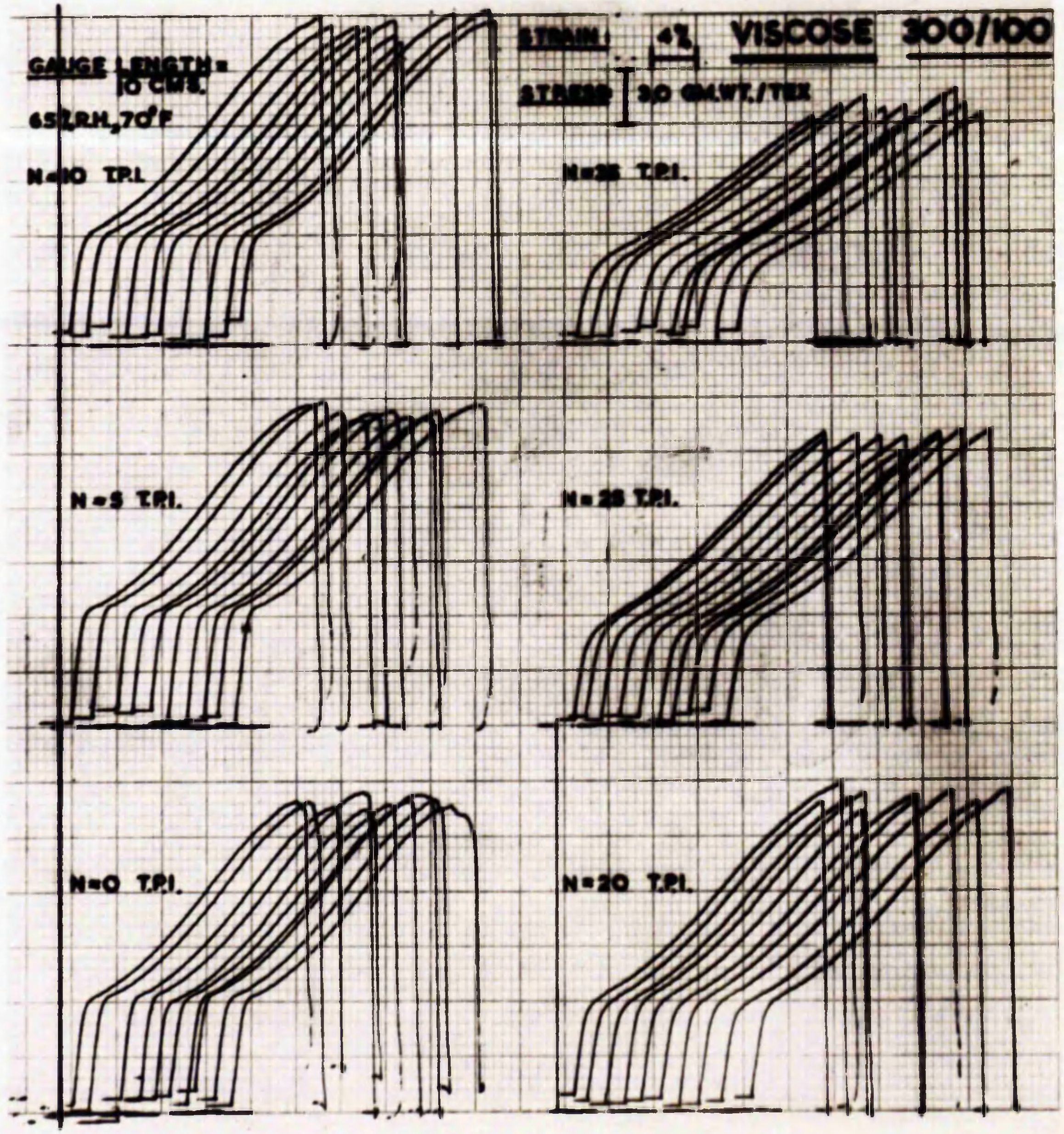


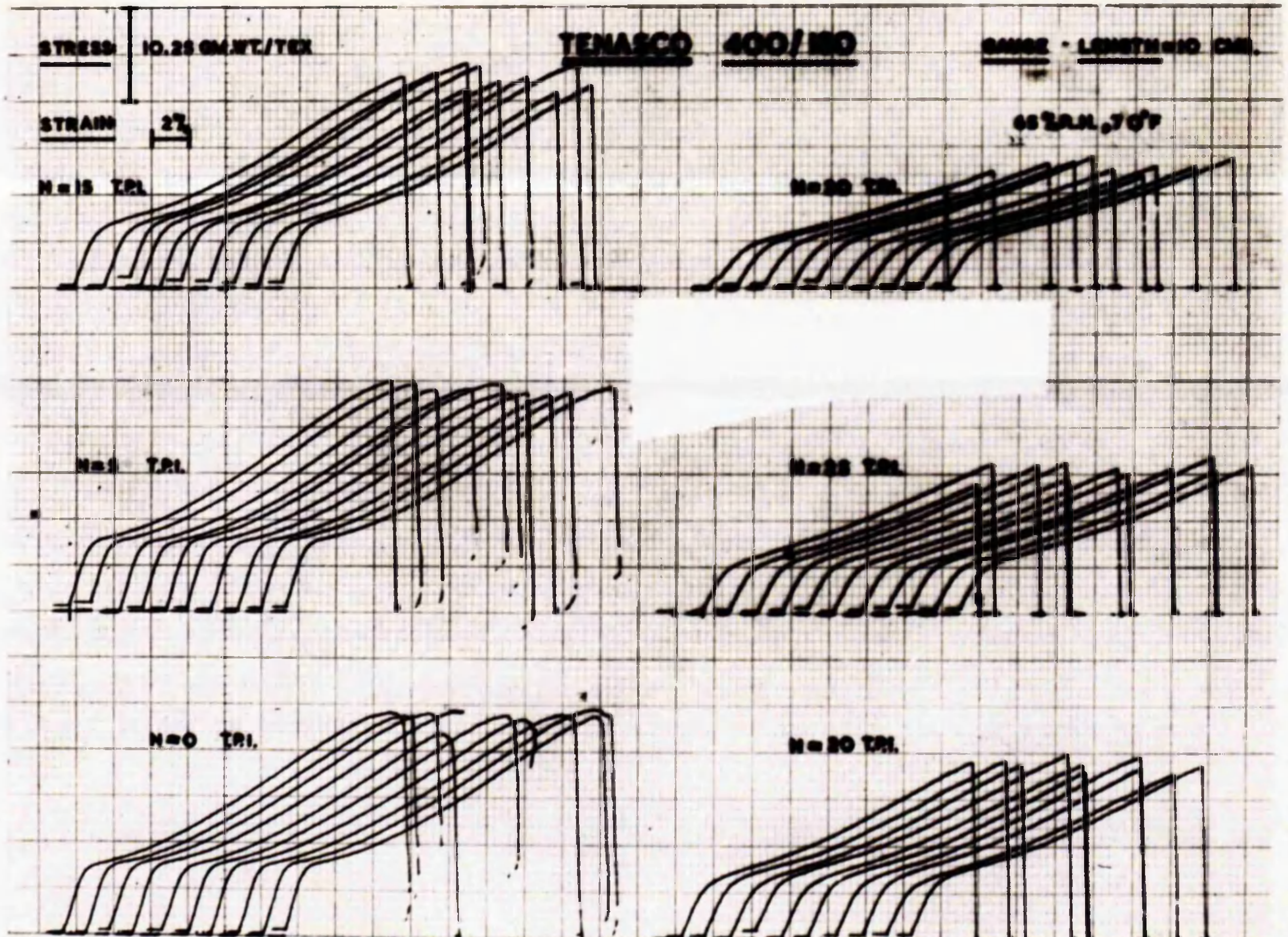
VISCOSE 75/75
GAUGE LENGTH = 10 cms.
65% R.H., 70°F
N = 0 t.p.i.
N = 65 t.p.i.
N = 80 t.p.i.
N = 20 t.p.i.
STRESS
STRAIN: 4%
STRAIN: 12%

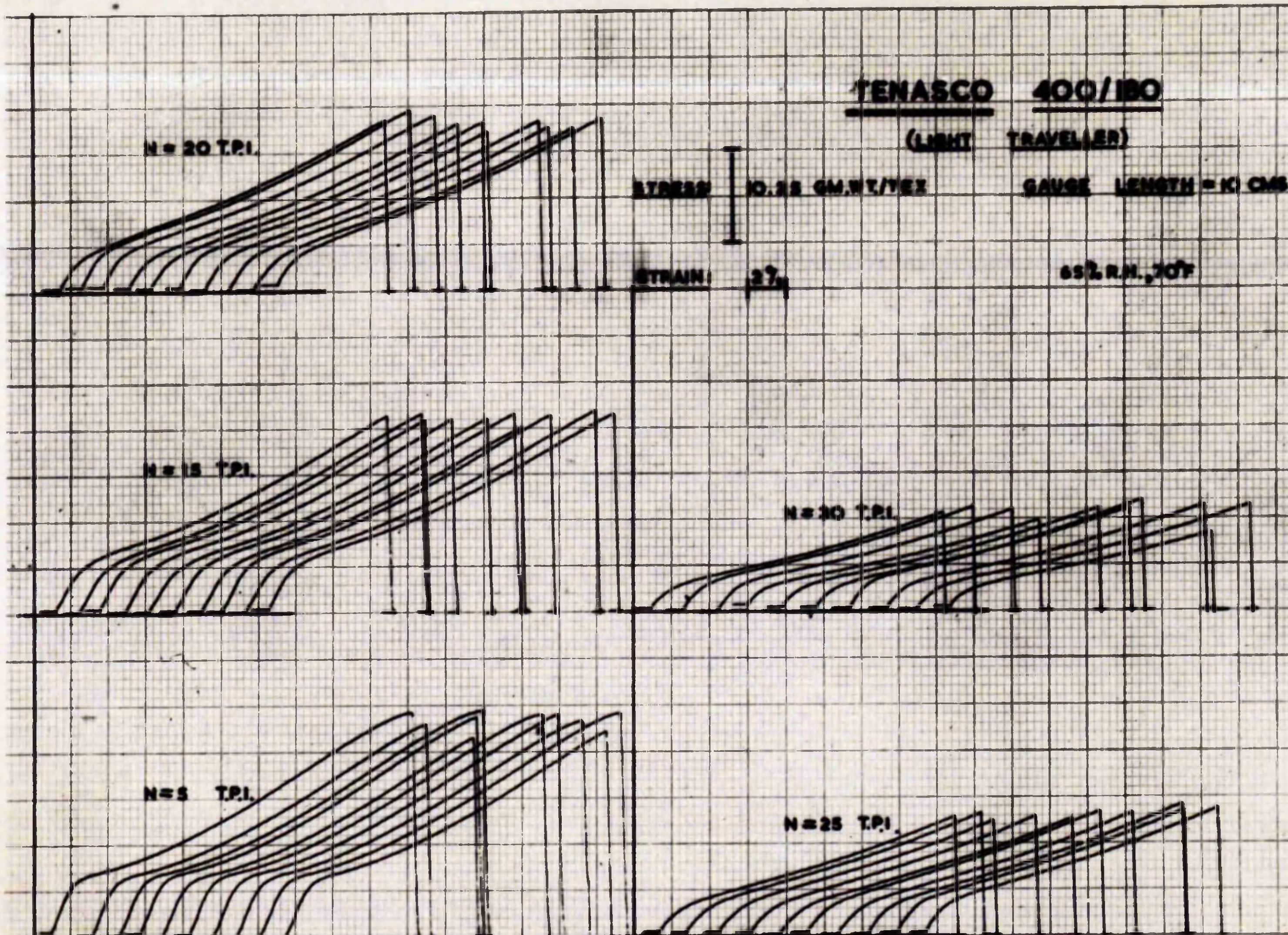


STRESS: 4.5 GM.WT./TEX
STRAIN: 4%
VISCOSE 100/24
GAUGE LENGTH = 10 CMS
65% R.H. 70°F
N = 20 T.P.I.
N = 70 T.P.I.
N = 10 T.P.I.
N = 50 T.P.I.
N = 0 T.P.I.
N = 30 T.P.I.


STRESS: 45 GMS/TEX
STRAIN: 4%
VISCOSE
100/40
N = 70 T.P.I.
N = 30 T.P.I.
N = 20 T.P.I.
N = 10 T.P.I.
N = 0 T.P.I.

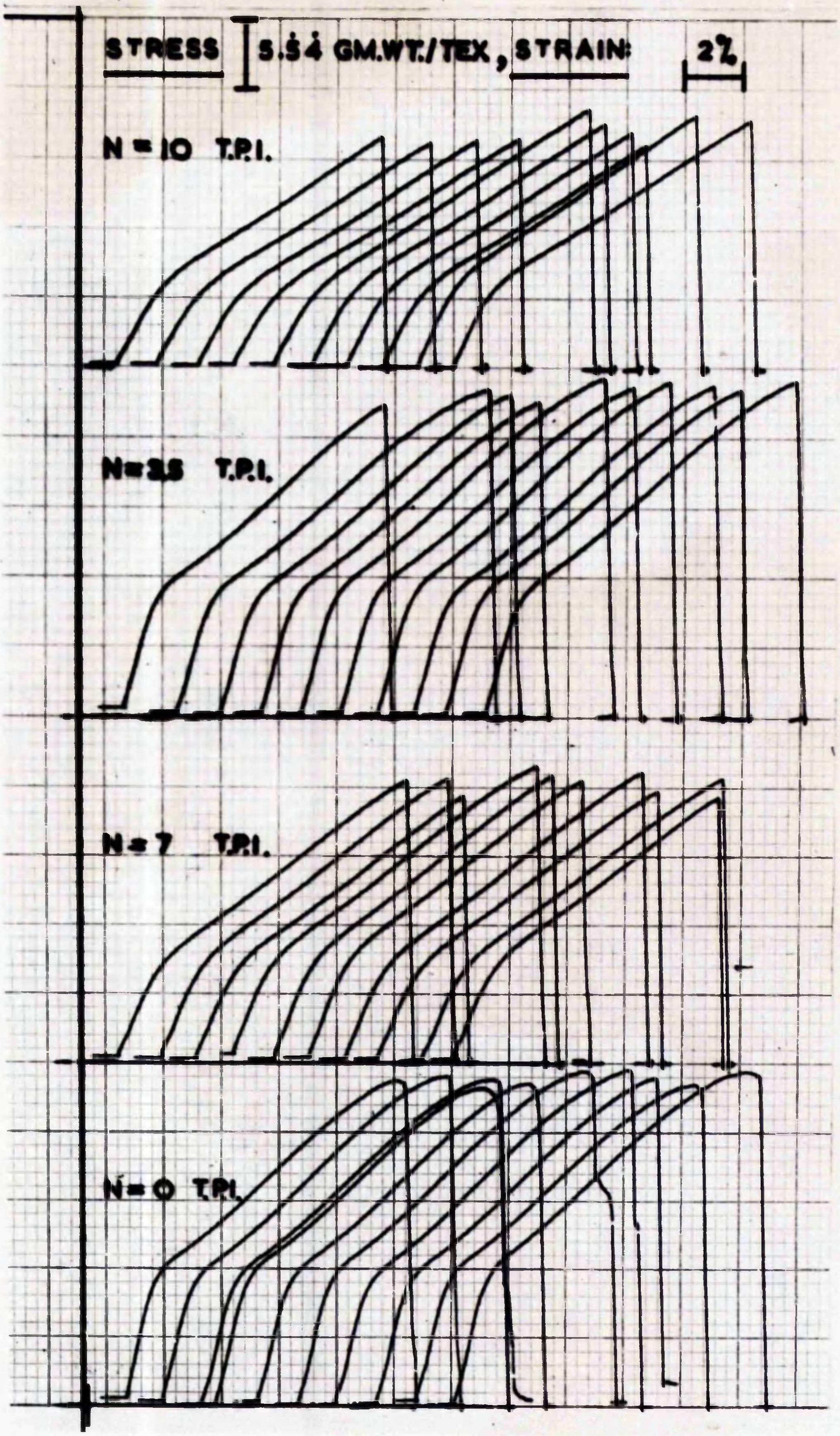

STRAIN: 4%
VISCOSE 300/100
STRESS 3.0 GM.WT./TEX
GAUGE LENGTH = 10 CMS.
65% R.H., 70°F
N=10 T.P.I.
N=35 T.P.I.
N=5 T.P.I.
N=25 T.P.I.
N=0 T.P.I.
N=20 T.P.I.

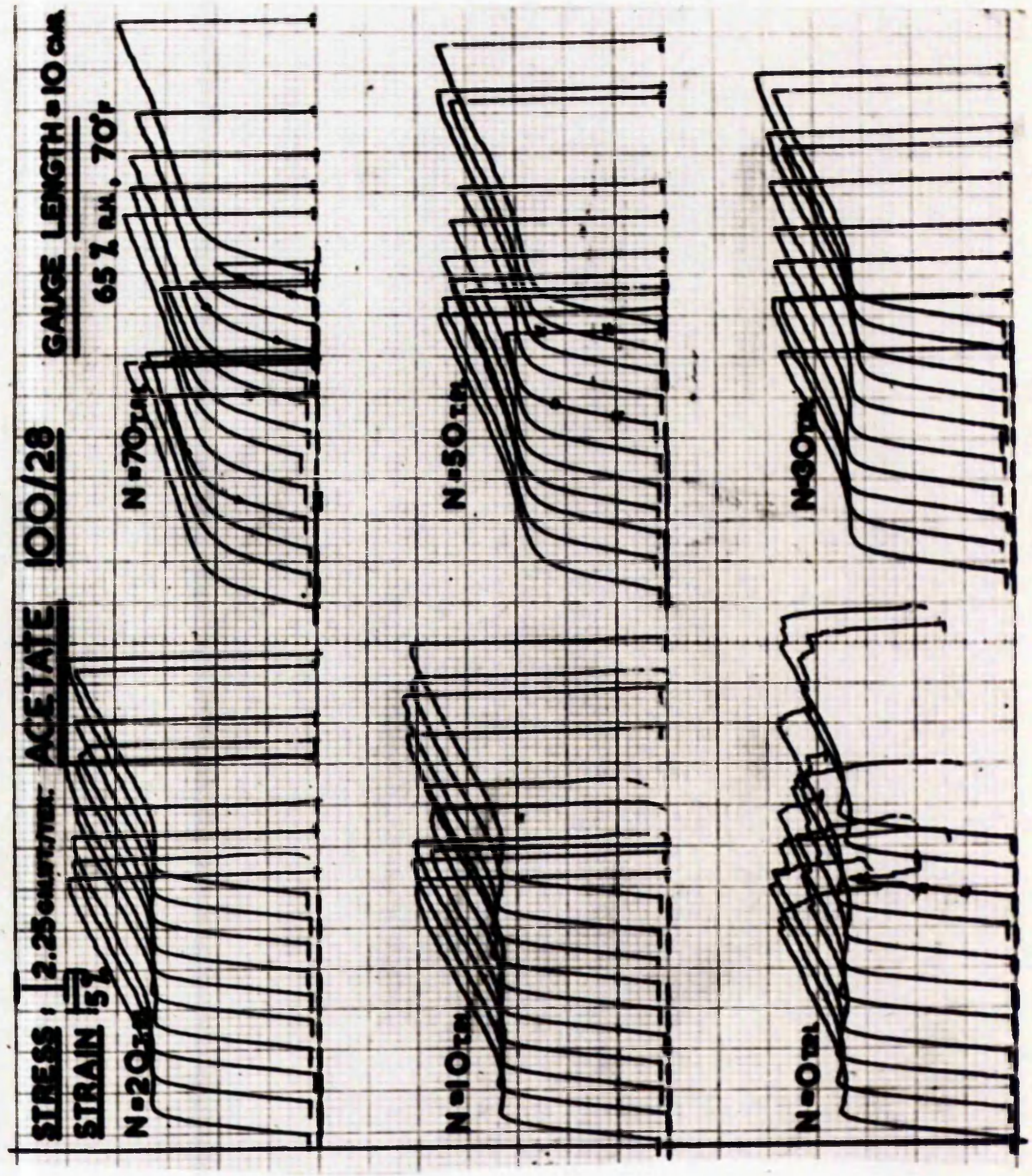

STRESS: 10.25 GM.WT./TEX
STRAIN 2%
TENASCO
65% R.H., 70°F
N=15 T.P.I.
N=30 T.P.I.
N=5 T.P.I.
N=25 T.P.I.
N=0 T.P.I.
N=20 T.P.I.


TENASCO 400/180
(LIGHT TRAVELLER)
STRESS:
10.25 GM.WT./TEX
STRAIN:
2%
GAUGE LENGTH = 10 CMS.
65% R.H., 70°F
N = 20 T.P.I.
N = 15 T.P.I.
N = 5 T.P.I.
N = 30 T.P.I.
N = 25 T.P.I.

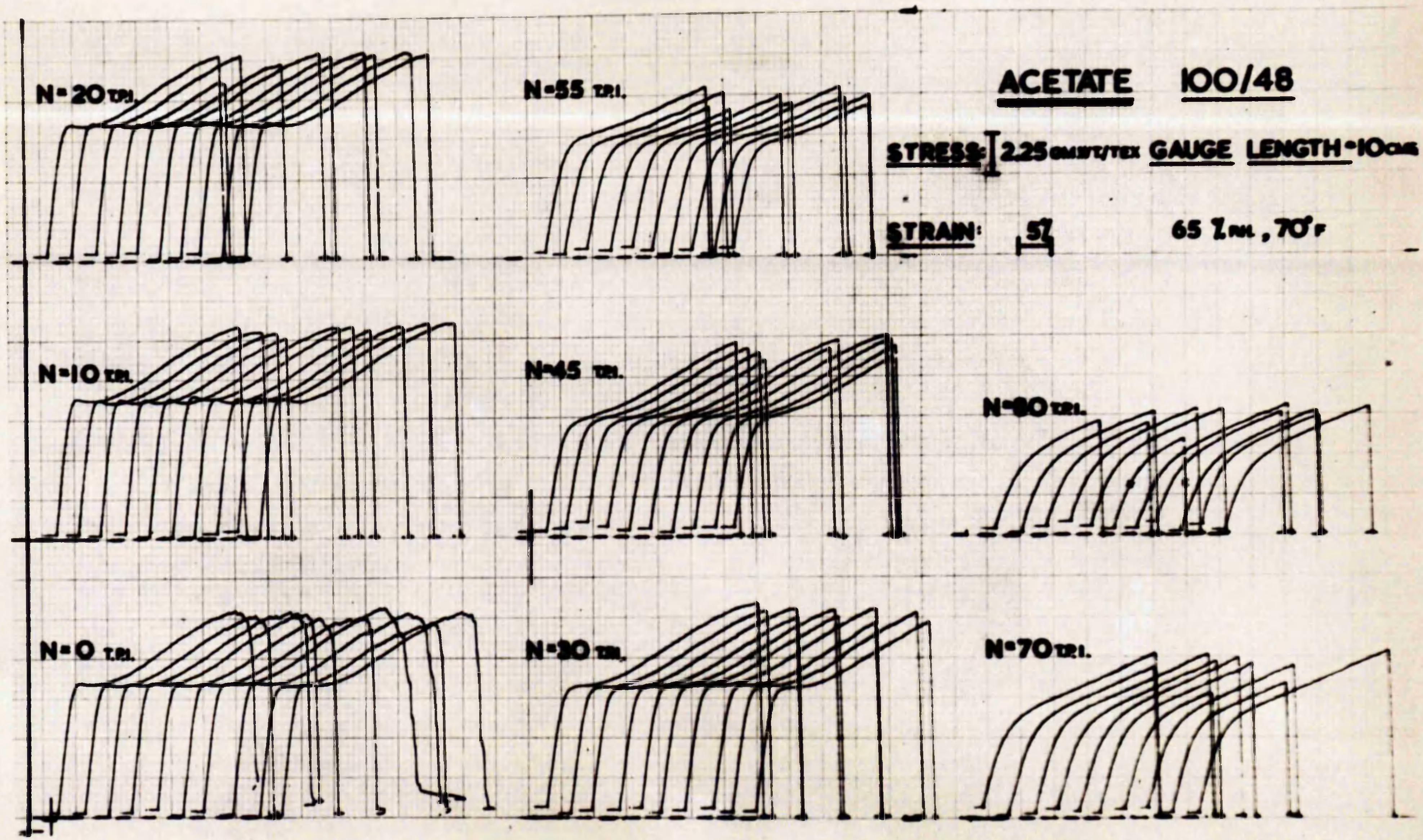
TENASCO 1650/750

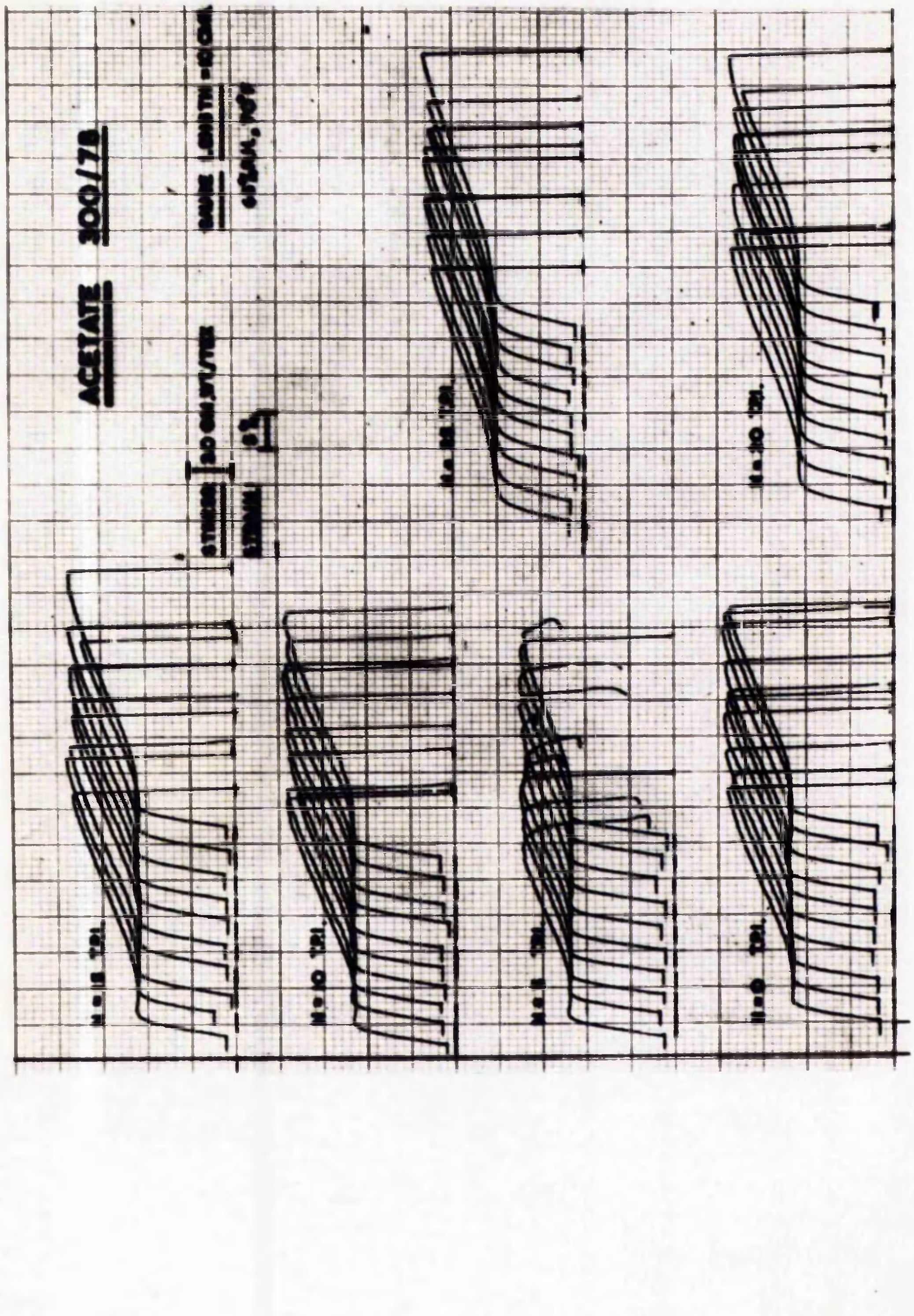
GAUGE LENGTH = 10 CMS. 65% R.H., 70° F

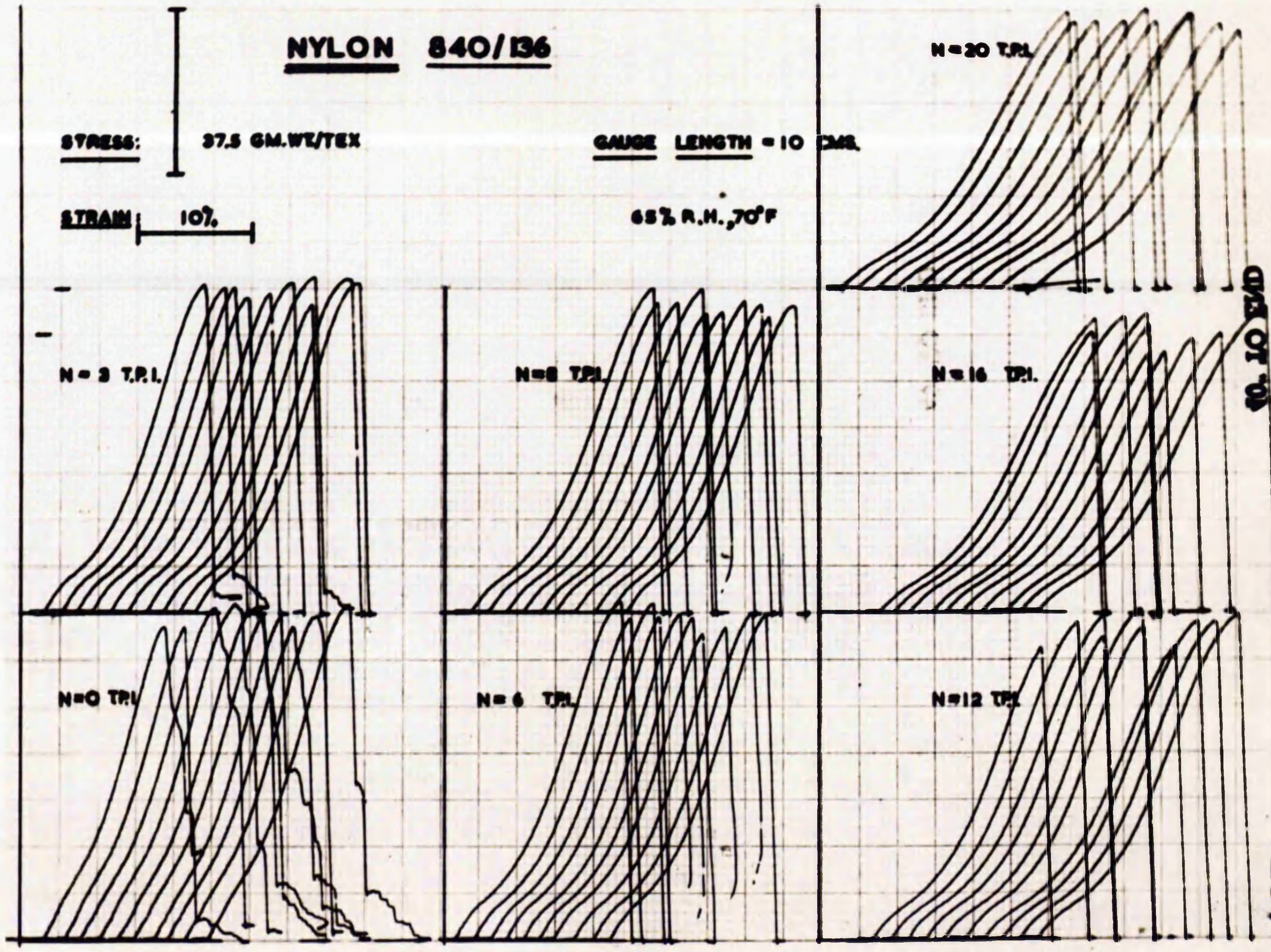


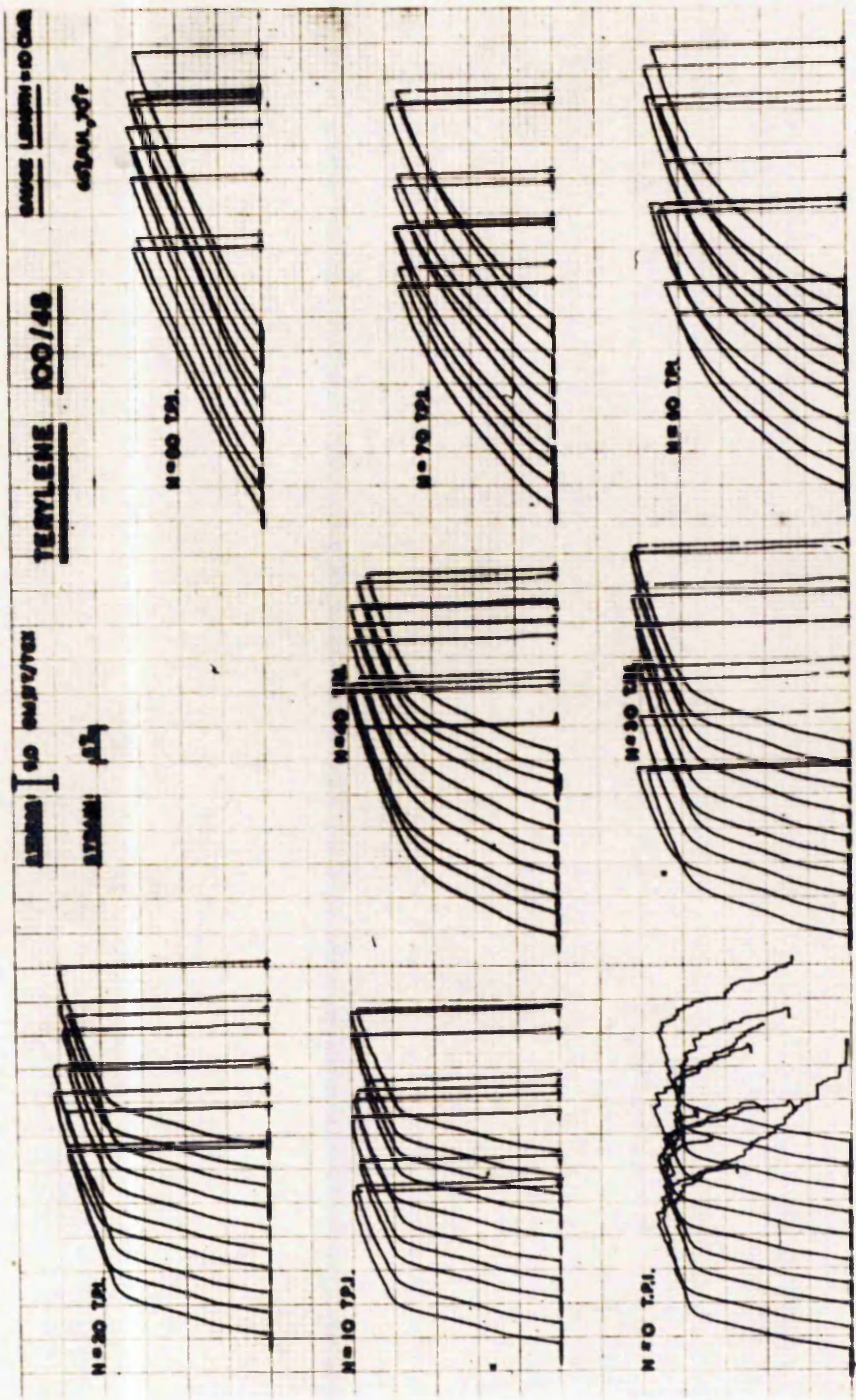


STRESS
STRAIN
ACETATE 100/28
GAUGE LENGTH = 10 CMS.
65% R.H., 70°F
N=20
N=70
N=10
N=50
N=0
N=90


ACETATE 100/48
STRESS: 2.25 GMS WT/TEX GAUGE LENGTH = 10 CMS
STRAIN: 5%
65 % RH, 70°F
N = 20 T.P.I.
N = 55 T.P.I.
N = 10 T.P.I.
N = 45 T.P.I.
N = 80 T.P.I.
N = 0 T.P.I.
N = 30 T.P.I.
N = 70 T.P.I.


ACETATE 300/78


NYLON 840/136
STRESS: 37.5 GM.WT./TEX
STRAIN: 10%
GAUGE LENGTH = 10 CMS.
65% R.H., 70°F
N = 20 T.P.I.
40. TO END
N = 3 T.P.I.
N = 8 T.P.I.
N = 16 T.P.I.
N = 0 T.P.I.
N = 6 T.P.I.
N = 12 T.P.I.


TERYLENE 100/48
N=20 T.P.I.
N=10 T.P.I.
N=0 T.P.I.
N=40 T.P.I.
N=30 T.P.I.
N=70 T.P.I.

APPENDIX III

Approximation for equation No. 5.23c

$$\epsilon_f = (\epsilon_y + 1)\left[1 + (m^2-1)\sin^2\theta_b\right]^{-1/2} - 1 \qquad \text{.......... (1)}$$

By binomial expansion and neglecting $\sin^6\theta_b$ and higher powers of $\sin\theta$

$$\epsilon_f = \epsilon_y - (1+\epsilon_y)\cdot\frac{m^2-1}{2}\sin^2\theta_b$$

$$+ \frac{3}{2}\left(\frac{m^2-1}{2}\right)^2(1+\epsilon_y)\sin^4\theta_b \text{........(2)}$$

$$\because m = \frac{1+\epsilon_y}{1-\sigma_y\epsilon_y} = 1 + \epsilon_y(1+\sigma_y) + \epsilon_y^2\sigma_y(1+\sigma_y) + \cdots\cdots$$

$$\therefore m^2 - 1 = 2\epsilon_y(1+\sigma_y) + \epsilon_y^2(1+\sigma_y)(1+3\sigma_y) + 2\epsilon_y^3\sigma_y(1+\sigma_y)^2 + \cdots$$

And

$$\left(\frac{m^2-1}{2}\right)^2 = \epsilon_y^2(1+\sigma_y)^2 + \epsilon_y^3(1+\sigma_y)^2(1+3\sigma_y) + \cdots$$

substituting for (m^2-1) in equation (2)

$$\epsilon_f = \epsilon_y - (1+\epsilon_y)\Big[\epsilon_y(1+\sigma_y) + \epsilon_y^2\frac{1+\sigma_y}{2}(1+3\sigma_y)$$

$$+ \epsilon_y^3\sigma_y(1+\sigma_y)\Big]\sin^2\theta_b + \frac{3}{2}(1+\epsilon_y)\left[\epsilon_y^2(1+\sigma_y)^2 + \epsilon_y^3(1+\sigma_y)^2(1+3\sigma_y)\right]\sin^4\theta_b.$$

By simplification and neglecting ϵ_y^3 and higher powers of ϵ_y

$$\epsilon_f = \epsilon_y - \epsilon_y(1+\sigma_y)\sin^2\theta_b - \epsilon_y^2\frac{(1+\sigma_y)}{2}(1+3\sigma_y)\sin^2\theta_b$$

$$- \epsilon_y^2(1+\sigma_y)\sin^2\theta_b + \frac{3}{2}\epsilon_y^2(1+\sigma_y)^2\sin^4\theta_b$$

or

$$\epsilon_f = \epsilon_y\left[1 - (1+\epsilon_y)\sin^2\theta_b\right] - \epsilon_y^2(1+\epsilon_y)\sin^2\theta_b\left[\frac{1+3\epsilon_y}{2} + 1 - \frac{3}{2}(1+\epsilon_y)\sin^2\theta\right]$$

$$= \epsilon_y\left[\cos^2\theta_b - \epsilon_y\sin^2\theta_b\right] - \epsilon_y^2(1+\epsilon_y)^2\sin^2\theta_b\frac{3}{2}(1-\sin^2\theta)$$

$$= \epsilon_y\left[\cos^2\theta_b - \epsilon_y\sin^2\theta_b\right] - \epsilon_y^2(1+\epsilon_y)^2\frac{3}{2}\cdot\sin^2\theta_b\cdot\cos^2\theta_b \quad \text{..........} \quad (3)$$

Similarly equation No.523bcan be shown to approximate to

$$\epsilon_f = \epsilon_y\left[\cos^2\theta_o - \epsilon_y\sin^2\theta_o\right] \quad \text{.........} \quad (4)$$

But when ϵ_y is small θ_o is approximately equal to θ_b and equation (3) and (4) will be identical.